Industrial Policies and Economic Development

Ernest Liu[*] [†]

January 28, 2017

Job Market Paper

This draft reflects work in progress. Please visit
http://economics.mit.edu/grad/liuer
for the most recent version.

Abstract

Many currently and previously developing countries have adopted industrial policies that push resources towards certain "strategic" sectors, and the economic reasoning behind such polices is not well understood. In this paper, I construct a model of a production network where firms purchase intermediate goods from each other in the presence of credit constraints. These credit constraints distort input choices, thereby reducing equilibrium demand for upstream goods and creating a wedge between the potential sales ("influence") and actual sales by upstream sectors. I analyze policy interventions and show that, under weak functional form restrictions, the ratio between a sector's influence and sales is a sufficient statistic that guides the choice of production and credit subsidies. Using firm-level production data from China, I estimate my sufficient statistic for each sector and show that it correlates with proxy measures of government interventions into the sector. Using a panel of cross-country input-output tables and sectoral production tax rates, I show that the tax rates for developing countries in Asia also correlate with the model-implied intervention measure.

[*]Department of Economics, MIT. Email: liuer@mit.edu.

[†]I am indebted to Daron Acemoglu and Abhijit Banerjee for their guidance and support throughout this project. I also thank Marios Angeletos, David Atkin, Vivek Bhattacharya, May Bunsupha, Esther Duflo, Sebastian Fanelli, John Firth, Greg Howard, Yan Ji, Pooya Molavi, Scott Nelson, Harry Di Pei, Jeff Picel, Frank Schilbach, Ludwig Straub, Tavneet Suri, Linh Tô, Ivan Werning, and especially Arnaud Costinot, Daniel Green, Alp Simsek, Jesse Shapiro, and Rob Townsend for conversations and helpful comments. All errors are of course my own.

1 Introduction

Industrial policies are broadly defined as the selective interventions that attempt to alter the structure of production towards certain sectors. Such policies are not only widely adopted in developing countries today, but also played a prominent role in the developmental stage for many now-advanced economies. Prime historical examples of industrial policies include Japan in the 1950s and 1960s and South Korea and Taiwan in the 1960s and 1970s. In all of these cases, the government heavily promoted "strategic" upstream sectors that supply to many others sectors. A wealth of policy instruments was adopted during these periods, including various forms of tax incentives and subsidized credit, and in the case of Taiwan, direct state involvement in production. In Korea, the explicit industrial movement was termed the "Heavy-Chemical Industry" drive, and for almost a decade firms in selected industries received policy loans with significantly reduced interest rates (Amsden 1989, Woo-Cumings 2001). Total policy loans directed towards the targeted sectors accounted for 45% of the total domestic credit of the banking system in 1977 (Hernandez 2004). Many of the largest manufacturing conglomerates in Korea today originated during this era.

By their nature, these policies seek to affect the development of the aggregate economy through selective intervention in a few sectors. Understanding the effects of such intervention therefore requires modeling the linkages among sectors in the economy. Moreover, the frequent use of subsidized or targeted loans suggests financing constraints play an important role in the design of these policies. Motivated by these facts, this paper develops a framework for studying optimal industrial policy in a general equilibrium setting with financial frictions and network linkages among sectors.

In my model, production requires factor inputs as well as intermediate goods produced by other sectors, and firms face credit constraints when purchasing some of these inputs for production. These credit constraints distort input choices and endogenously affect sectoral input-output linkages, thereby reducing demand for upstream goods that are subject to constraints. In equilibrium, the constraints generate a wedge between the total sales of the affected upstream sectors and the elasticity of aggregate output with respect to sectoral Hicks-neutral productivity shocks. This elasticity, known as the sectoral "influence" in the production networks literature, can be interpreted as the potential sectoral sales absent market imperfections (Hulten 1978). I analyze policy interventions and show that, under weak functional form restrictions, the ratio between a sector's influence and sales—which I refer to as the sectoral "sales gap"—is a sufficient statistic that summarizes the inefficiencies in the input-output network and could guide policy interventions that expand sectoral production. Specifically, I show that starting from a decentralized equilibrium without distortionary taxes, a sector's sales gap captures the ratio between social and private marginal return to spending resources in the sector on production inputs and on credit. Moreover, if production functions are iso-elastic, the same sufficient statistic captures the optimal sectoral subsidies to labor, which is the value-added input in the model. These results are potentially surprising because sectors with the highest sales gaps are not necessarily the sectors in which firms are most constrained; instead, they are upstream sectors that directly or indirectly supply to many constrained downstream sectors. In fact, my results imply that even if the private returns to credit are equalized across all firms in the economy, a benevolent planner might still want to direct credit to upstream sectors in order to improve production efficiency.

The sales gap is a sufficient statistic for network inefficiencies because while sales capture the relative sectoral size under equilibrium production in the presence of frictions, influence captures the relative sectoral size under optimal production. The distance between the two vectors thus reveals a direction in which production efficiency can be improved. This finding can be viewed as an "anti-network" result similar to Hulten's: as long as we know the sales gap—the difference between a sector's potential and actual sales—knowledge of the underlying frictions in the input-output system becomes irrelevant for welfare analysis.

I conduct two distinct empirical exercises to estimate sales gap and examine its correlations with proxy measures of government interventions. The first exercise focuses on China, whose socialist roots and strong legacy of state intervention makes it a particularly interesting setting to apply my results. Relying on firm-level manufacturing census, I estimate firm production elasticities and the distribution of credit distortions for the manufacturing sectors, and I use these estimates to compute sectoral influence and sales gap based on the observed Chinese input-output table. I find that private firms in sectors with higher sales gaps tend to receive more external loans and pay lower interest rates, and that the sectoral presence of Chinese State-Owned Enterprises (SOEs) is heavily directed towards sectors with higher sales gaps. My theory suggests that these selective interventions can enhance welfare by effectively subsidizing upstream production and potentially ameliorating the network inefficiencies. My findings therefore allow for a positive reappraisal of the selective state interventions in China and provide a counterpoint to the prevailing view (e.g. Song et al. 2011) that SOEs are a sign of sectoral inefficiency.

My second empirical exercise compares across countries. Using a panel of cross-country input-output tables, I construct the sales gap measure for a set of *developing* countries based on the input-output tables from a set of *developed* countries, adopting a strategy that is similar in spirit to Rajan and Zingales (1998) and Hsieh and Klenow (2009). I show that, as a group, developing countries tend to have higher sales gaps in tertiary and heavy manufacturing sectors and lower sales gaps in primary and light industrial sectors. Moreover, I show that the sectoral sales gaps of a set of developing countries in Asia strongly and positively correlate with a measure of sectoral subsidies adopted by these countries. The pattern is largely absent or even reversed in developing countries from the other continents, which on average have had worse economic performances in recent years than their Asian counterparts. These results are consistent with the hypothesis that governments in countries with strong economic performances are better at understanding the network distortions and are adopting policies to address them.

Literature Review My paper is related to a large body of development macro literature on the misallocation of resources, including Banerjee and Duflo (2005), Jeong and Townsend (2005), Restuccia and Rogerson (2008), Hsieh and Klenow (2009), Banerjee and Moll (2010), Song et al. (2011), Buera et al. (2011), and Rotemberg (2014), among many others. The broad purpose of this literature is to study the implications of micro-level financial frictions on aggregate productivity. My paper draws on this literature but provides a different focus. Rather than studying how financial constraints distort the efficient use of resources *within* a sector, I study how constraints endogenously affect input-output linkages and distort the relative size of sectors, generating the misallocation of resources *across* sectors in a production network.

The importance of sectoral linkages for economic development was first pointed out by Hirschman (1958), who argues that industrial policies should target and promote sectors with the strongest linkages. His work has inspired an early and substantial development economics literature that aims to measure the Hirschmanian linkages and study their relationships with economic performance and industrial policies, including Chenery and Watanabe (1958), Rasmussen (1965), Yotopoulos and Nugent (1973), Chenery et al. (1986), Jones (1976), and Shultz (1982), among others. My paper revisits this topic using a model with neoclassical microfoundations to formalize the implications of linkages for industrial policies.

My modeling approach embeds cross-sector input-output linkages into a static version of the competitive entry model with the convex-concave technologies of Hopenhayn (1992) and Hopenhayn and Rogerson (1993). The model sits squarely within the class of generalized Leontief models as defined in Arrow and Hahn (1971, pp. 40, *Leontief economy*). This class of generalized Leontief models has been extensively studied in the early general equilibrium literature, including Hulten (1978), who shows that without market imperfections and under aggregate constant returns to scale, sectoral influence is equal to sales, an equivalence that is broken in my model due to financial frictions.

A modern revival of this Leontief input-output approach, often referred to as the "production networks" literature, imposes functional forms on generalized Leontief models and more explicitly studies how productivity shocks transmit through input-output linkages. Key contributions to this literature include Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999), Shea (2002), and Acemoglu et al. (2012). Several papers in this literature embed financial frictions into production networks: Jones (2013) and Bartelme and Gorodnichenko (2015) model financial frictions through implicit wedges or distortions in factor prices à la Hsieh and Klenow (2009), and Altinoglu (2015) and Bigio and La'O (2016) model frictions through working capital constraints. These papers aim to characterize how linkages amplify sectoral financial frictions and study their implication on aggregate output. Relatedly, Baqaee (2016) studies a production network model with monopolistic markups, which also create wedges between marginal product and marginal cost of production inputs. Market imperfections in the models of these papers also break Hulten's equivalence theorem.

My theoretical analysis differs substantively from those offered by the current production networks literature. First, because the decoupling of influence and sales is at the heart of my analysis, I conduct a detailed characterization of how sectoral constraints and network structure affect the sales gap. Second, I show that under aggregate constant returns to scale, sectoral size is proportional to *influence* under optimal production, even though it is proportional to *sales* under equilibrium production in the presence of constraints. I further show that their ratio, the sales gap, exactly captures the ratio between social and private marginal returns to spending productive resources in a sector, thus providing a direction in which production can be improved through policy intervention. These results do not rely on the Cobb-Douglas or the constant elasticity-of-substitution assumptions imposed by the production networks literature.

Baqaee (2015) and Acemoglu et al. (2016) observe that in a production network under Cobb-Douglas technology assumption, productivity shocks travel downstream through input-output linkages from suppliers to buyers, while demand shocks travel upstream. My paper shows that these results can be generalized without the specific functional form assumptions, and I apply these intuitions to show how credit constraints affect allocations and distort the relative size of sectors. First, as an application of the non-substitution theorem by Samuelson (1951), demand shocks have no effect on equilibrium prices in a generalized Leontief model and affect equilibrium quantities only through *backward linkages* or, in other words, by traveling upstream. Second, even without the Cobb-Douglas assumption, productivity shocks in a sector travel through *forward linkages*, affecting the unit cost of production hence equilibrium prices of downstream buyers. Equilibrium prices of upstream sectors are unaffected, and output quantities in upstream sectors change in response to downstream productivity shocks only through the changes in demand induced by these shocks. Lastly, financial frictions in a generalized Leontief model serve both as a productivity shock and a shock to intermediate demand that emanates from the constrained sectors. The productivity shock aspect of financial frictions propagates downstream by lowering aggregate output, while the demand shock aspect propagates upstream and suppresses the relative size of upstream sectors.

My model prominently features pecuniary externalities and thus relates to the literature on the inefficiency of general equilibrium with prices in additional constraints, including seminal work by Greenwald and Stiglitz (1986) and Geanakoplos and Polemarchakis (1986). More broadly, my policy analysis leverages the fact that in the presence of input-output linkages, policy instruments need not directly target the source of distortions in order to improve welfare. My analysis therefore relates to the second-best literature initiated by Lipsey and Lancaster (1956) and more recently contributed by Farhi and Werning (2013), Kilenthong and Townsend (2016), and Korinek and Simsek (2016), among others.

While I provide closed-form solutions for second-best policies under Cobb-Douglas assumptions, my main results study welfare changes in response to a marginal change in production subsidies. This approach is related to a series of papers in public finance literature, including Ahmad and Stern (1984), Deaton (1987), and Ahmad and Stern (1991), that study the welfare effect of marginal tax reforms.

The empirical setting of my analysis builds on a broad empirical and policy literature on state intervention and industrial policies, including Pack and Westphal (1986), Chenery et al. (1986), Amsden (1989), Wade (1989), Wade (1990), Westphal (1990), Page (1994), Pack (2000), Noland (2004), and more recently, Rodrik (2004, 2008) among others. Compared to the Computable General Equilibrium approach adopted by some papers in the policy literature, such as Dervis et al. (1982) and Robinson (1989), my work is microfounded as I explicitly model firm-level incentives and their behavior under credit constraints.

My first empirical exercise focuses on China and relates to a large literature that studies the growth experience of the country, including Brandt et al. (2008), Song et al. (2011), Zhu (2012), Bai et al. (2014), Storesletten and Zilibotti (2014), Aghion et al. (2015), and Hsieh and Song (2015). Most notably, I borrow from Song et al. (2011) in modeling SOEs as unconstrained profit maximizers, an assumption that plays a central role in this empirical analysis. The observation that Chinese SOEs are more present in upstream sectors has also been made by Li et al. (2015), who adopt the "upstreamness" measure by Antras et al.

(2012).

My second empirical exercise uses cross-country input-output tables to test the relationship between the sales gap measure and a measure of sectoral tax rates in developing countries. I use observed input-output tables for *developed* countries to predict undistorted production technologies for *developing* countries, an exercise that is similar in spirit to Rajan and Zingales (1998) and Hsieh and Klenow (2009). Bartelme and Gorodnichenko (2015) conduct a similar exercise to infer unconstrained input-output production technologies, though they examine a different empirical relationship, one that is between aggregate TFP and a measure of total linkages across industries.

The rest of the paper is organized as follows: Section 2 provides the theoretical results, Section 3 conducts the empirical exercise based on firm-level Chinese manufacturing data, Section 4 conducts the cross-country analysis, and section 5 concludes.

2 Theory

2.1 Model Setup

Economic Environment There is a representative consumer who consumes a unique final good (with price normalized to 1) with non-satiated preferences and supplies labor L inelastically. There are S intermediate production sectors in the economy, each producing a differentiated good that is used both for intermediate production and also the production of the unique final good. I will refer to the output of sector i as good i and refer to the S goods altogether as "intermediate goods".

The final good is produced competitively by combining intermediate goods under production function

$$F\left(Y_1, \cdots, Y_S\right)$$

where Y_i is the units of good i used for final production. I assume $F(\cdot)$ is differentiable, has constant-returns-to-scale, and is strictly increasing and jointly concave in its arguments.

Production of intermediate goods is modeled as a two-stage entry game. In the first stage, a large measure of identical, risk-neutral, and atomistic potential entrants freely choose whether to set up a firm in any sector, taking the expected profit and cost of entry as given. In the second stage, firms that have entered in sector i produce an identical and perfectly substitutable good i. To build a firm in sector i, an entrant ν pays a fixed cost κ_i units of the final good and acquires a production technology

$$q_i(\nu) = h_i \cdot z_i(\nu) f_i\left(\ell_i(\nu), m_{i1}(\nu), \cdots, m_{iS}(\nu)\right),$$

where $\ell_i(\nu)$ is the amount of labor employed by firm ν in sector i, $m_{ij}(\nu)$ is the amount of good j used as intermediate inputs for production of good i by firm ν, and $z_i(\nu)$ captures firm-specific Hicks-neutral productivity. Lastly, h_i is a sector-wide Hicks-neutral productivity shock common to all firms in sector i, which is introduced for notational purposes and is normalized to $h_i \equiv 1$ unless explicitly noted.

The model formulation implicitly assumes no joint production—each industry produces only one good. We make the following assumptions on f_i and F:

Assumption 1. *Production functions f_i and F are continuously differentiable and strictly concave. Furthermore,*

1. *$F(\cdot)$ satisfies the Inada conditions:*

$$\lim_{Y_i \to 0} \frac{\partial F(Y_1, \cdots, Y_i, \cdots, Y_S)}{\partial Y_i} = \infty, \quad \lim_{Y_i \to \infty} \frac{\partial F(Y_1, \cdots, Y_i, \cdots, Y_S)}{\partial Y_i} = 0.$$

2. *$f_i(0, m_{i1}, \cdots m_{iS}) = 0$ and $\frac{\partial f_i(\ell_i, m_{i1}, \cdots m_{iS})}{\partial \ell_i} > 0$ at all input levels. That is, every firm needs labor to produce and output is always strictly increasing in labor.*

Financial Constraints Financial frictions in this network economy are modeled as pledgeability constraints faced by firms in the intermediate goods sectors. I assume the cost of a subset of production inputs has to be paid before production takes place, and each entrepreneur ν has an exogenous amount of expendable funds $W_i(\nu)$. Formally, for each firm ν in industry i, there is a subset of inputs $K_i \subset \{1, \cdots, S\}$ that is subject to constraints of the following form:

$$\sum_{j \in K_i} p_j m_{ij} \leq W_i(\nu) \tag{1}$$

where p_j is the price of good j, and m_{ij} is the amount of good j used for production. I use K_i to denote the set of constrained intermediate inputs and X_i to denote the set of unconstrained inputs, with $K_i \cup X_i = \{1, \cdots, S\}$. In this static production model, the left-hand side of the financial constraint (1) can be interpreted as an upfront payment requirement on certain inputs, before the firms make sales and are able to recover the input expenditures. The right-hand side of the constraint captures the total available funds to cover such upfront costs, which can be interpreted as the sum of entrepreneurial wealth and the total bank credit available to the entrepreneur to purchase the constrained inputs.

Inputs in K_i that are subject to the constraint and can be thought of as capital goods (e.g. machinery, equipments and computers) or services that can be subject to hold-up problems (such as outsourced R&D services), for which trade credit is difficult to obtain and costs must be incurred upfront. The unconstrained inputs in X_i can be thought of as material or commodity inputs—such as intermediate materials for the production of consumer goods (e.g. textiles), commoditized services, and energy inputs—for which trade credit is more available (Fisman 2001) such that the input cost can be paid after production is carried out.

The fact that labor input is unconstrained is not important for my theoretical results: the same sufficient statistic will capture the ratio between social and private marginal return to spending additional resources on any production input, including labor, whether or not the input is constrained. On the other hand, when I apply my model to data in sections 3 and 4, I take the empirical stance that labor is unconstrained. This assumption is motivated by the empirical evidence that firms in developing countries do not seem to be constrained in labor choices (Cohen 2016, De Mel et al. 2016). Note also that that the fixed cost of entry κ_i does not appear in constraint (1), but this is without loss of generality: I can always relabel the amount of exogenous expendable funds as $\tilde{W}_i$ and define $W_i \equiv \tilde{W}_i - \kappa_i$. Similarly, we can also reinterpret the constraint (1) as requiring only a fraction of the cost of capital inputs to be paid upfront by relabeling W_i with a multiplicative constant.

In Appendix C I consider several different formulations of financial frictions. Appendix C.1 reformulates the model with financial frictions in the form of a monitoring cost that is linear in the amount of credit delivered. The linear monitoring cost creates an exogenous wedge between marginal product and marginal cost of inputs, similar to the implicit wedges in Hsieh and Klenow (2009), Jones (2013), and Bartelme and Gorodnichenko (2015), and our results survive in that environment. I relax the constraint formulation (1) in Appendices C.2 and C.3 by successively introducing input-specific requirement for upfront payment (with the left-hand-side of constraints taking the form of $\sum_{j\in S} \eta_{ij}(\nu) p_j m_{ij}$ for $\eta_{ij}(\nu) \in [0,1]$) and partial pledgeability of revenue (by introducing $\delta_i(\nu) p_i q_i(\nu)$ to the right-hand-side of constraints). I show that all of my theoretical results survive when production inputs have varying degrees of upfront-payment requirement $\eta_{ij}(\nu)$. When revenue pledgeability is also introduced, the constraint formulation nests the pledgeability constraints in Bigio and La'O (2016) and my results still hold if within-sector firm heterogeneity is removed, an assumption maintained by other papers in this literature.

My theory focuses on intermediate production, and I assume the final good producer operates without any credit constraints.

Firm's Profit Maximization Problem Firms choose inputs in order to maximize profit subject to the credit constraint in (1):

$$(\mathbf{P_{firm}}) \max_{\{m_{ij}\}_{j=1}^{S},\ell} p_i q_i\left(\nu, \ell, \{m_{ij}\}\right) - \sum_{j=1}^{S} p_j m_{ij} - w\ell \quad \text{subject to (1).}$$

Free Entry Before production takes place, there is a large (unbounded) pool of prospective entrants into any industry, and all potential entrants are identical ex-ante. After incurring the fixed cost of entry κ_i units of the final good, firms independently draw Hicks-neutral productivities $z_i(\nu)$ and expendable funds $W_i(\nu)$ from a sector-specific distribution with a compact, non-negative support and cumulative distribution function $\Phi_i(\cdot)$. Because all same-sector firms with identical productivity and wedges make the same allocation choice, I abuse the notation and use ν as the index for both the random draws of $(z_i(\nu), W_i(\nu))$ and also for the firm with these draws.

To make entry decisions, prospective entrepreneurs form rational expectations on the variable profits $\pi_i(\nu) \geq 0$, the maximand of $(\mathbf{P_{firm}})$. The expected profit net of fixed cost in sector i is $(\mathbb{E}_\nu[\pi_i(\nu)] - \kappa_i)$. If this value were negative, no firm would want to enter. In any equilibrium where entry is unrestricted, an assumption I maintain, this value cannot be strictly positive, hence

$$\kappa_i = \mathbb{E}_\nu[\pi_i(\nu)]. \tag{2}$$

Equilibrium

Definition 1. A *decentralized equilibrium* is a collection of prices $\{p_i\}_{i=1}^S$, wage rate w, measure of firms $\{N_i\}_{i=1}^S$, firm-level allocations $\{\ell_i(\nu), m_{i1}(\nu), \cdots, m_{iS}, q_i(\nu)\}_{i=1,\cdots S}$, production inputs for the final good $\{Y_i\}_{i=1}^S$, aggregate consumption C, net aggregate output Y, and aggregate labor supply L such that

1. The representative consumer maximizes utility subject to his budget constraint, such that:

$$wL = C.$$

2. A firm ν in each sector i solves the constrained profit maximization problem ($\mathbf{P_{firm}}$), taking wage rate w, prices $\{p_i\}_{i=1}^S$, its own productivity $z_i(\nu)$, and expendable funds $W_i(\nu)$ as given.

3. Free-entry drives ex-ante profits to zero in all sectors such that equation (2) holds.

4. Production inputs for the final good solve the profit maximization problem of the final producer

$$(Y_1, \cdots, Y_S) = \arg\max_{\{\tilde{Y}_i\}} F\left(\tilde{Y}_1, \cdots, \tilde{Y}_S\right) - \sum p_i \tilde{Y}_i. \tag{3}$$

5. All markets clear:

$$\text{(labor)} \qquad L = \sum_i L_i \tag{4}$$

$$\text{(interm. good } j \text{ for all } j) \qquad Q_j = Y_j + \sum_i M_{ij} \tag{5}$$

$$\text{(final good)} \qquad Y = F(Y_1, \cdots, Y_S) - \sum_i \kappa_i N_i, \tag{6}$$

where capital case letters L_i, M_{ij}, Q_j denote total sectoral quantities:

$$L_i \equiv N_i \int_\nu \ell_i(\nu)\, d\Phi_i(\nu)$$

$$M_{ij} \equiv N_i \int_\nu m_{ij}(\nu)\, d\Phi_i(\nu)$$

$$Q_i = N_i \int_\nu q_i(\nu)\, d\Phi_i(\nu).$$

6. The net aggregate output equals consumption:

$$Y = C.$$

Before I introduce government expenditure in section 2.4, net aggregate output Y always equals aggregate consumption C, and I use the two terms interchangeably.

Example $\mathcal{E}$ There is no closed-form solution for equilibrium allocations without additional functional form assumptions. To make the discussion concrete, I sometimes refer to a specific three-sector example that is nested under my model. I refer to the example as $\mathcal{E}$ and the setup is as follows. There are $S = 3$ intermediate production sectors in the economy, and these sectors form a vertically connected production network: good 1 is produced upstream using labor only, good 2 is produced by combining good 1 and labor, and good 3 is produced downstream by combining good 2 and labor. I assume the final good is produced linearly from the downstream good 3:

$$F = Y_3.$$

I remove heterogeneity across firms within a sector, and I drop the firm index ν to simplify notation. The firm-level production functions take the iso-elastic form:

$$q_1 = \ell_1{}^{\alpha_1}, \quad q_2 = \ell_2{}^{\alpha_2} m_{21}{}^{\sigma_2}, \quad q_3 = \ell_3{}^{\alpha_3} m_{32}{}^{\sigma_3},$$

where q_i is the firm-level output, ℓ_i is the labor input for production, and $m_{i,i-1}$ is the amount of good $i-1$ used by a firm in the production of good i. I normalize $\alpha_1 \equiv \alpha_i + \sigma_i$ for sectors $i = 2, 3$ so that the concavity of production is constant across all three sectors (to avoid carrying additional constants and obfuscating notation for this example). I normalize firm-level productivity to $z_i \equiv 1$.

All intermediate goods are subject to credit constraints. I also assume W_i is constant for all firms in sector i:

$$p_1 m_{21} \leq W_2, \quad p_2 m_{32} \leq W_3. \tag{7}$$

The flow of inputs and outputs in the network is represented in figure 1.

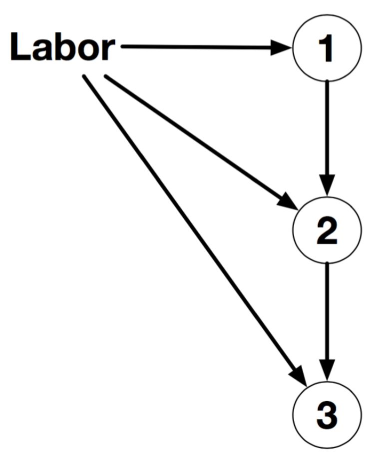


Figure 1: Illustration of the Vertical Production Economy

2.2 Equilibrium Characterization

Firm-level Allocations Under Assumption 1, the solution to firm ν's profit maximization problem post-entry $(\mathbf{P_{firm}})$ is characterized by the first-order conditions with respect to inputs, which can be re-arranged into the following set of expenditure share equations:

$$\frac{w\ell_i(\nu)}{p_i q_i(\nu)} = \frac{\partial \ln f_i(\nu)}{\partial \ln \ell} \tag{8}$$

$$\frac{p_j m_{ij}(\nu)}{p_i q_i(\nu)} = \frac{\partial \ln f_i(\nu)}{\partial \ln m_{ij}} \quad \text{for all } j \in X_i \tag{9}$$

$$\frac{p_j m_{ij}(\nu)}{p_i q_i(\nu)} = \varphi_i^K(\nu) \frac{\partial \ln f_i(\nu)}{\partial \ln m_{ij}} \quad \text{for all } j \in K_i \tag{10}$$

The first two equations are standard: because labor and intermediate goods $j \in X_i$ are unconstrained, their expenditure shares $\frac{w\ell_i(\nu)}{p_i q_i(\nu)}$ and $\frac{p_j m_{ij}(\nu)}{p_i q_i(\nu)}$ are equal to the respective output elasticity of the firm production functions evaluated at equilibrium quantities of inputs. On the other hand, this equivalence breaks down for intermediate goods $j \in K_i$ that are subject to credit constraints, reflected by firm-specific wedge $\varphi_i^K(\nu) \leq 1$, which shows up in the expenditure share equation for the constrained inputs. When the credit constraint binds, $\varphi_i^K(\nu) < 1$, and it distorts input expenditure downwards relative to the efficient level.

For each firm, the wedge $\varphi_i^K(\nu)$ is pinned down by equilibrium prices and firm-specific random draws. The inverse wedge $\left(\varphi_i^K(\nu)\right)^{-1}$ can be interpreted as the firm's private return to spending on constrained inputs, while $\left(\varphi_i^K(\nu)\right)^{-1} - 1$ captures the marginal gains of having additional working capital and is the interest rate the firm is willing to pay to obtain credit.

Definition 2. The *private return to spending on input j* for firm ν in sector i is the ratio between the marginal product and marginal cost of input j:

$$PR_i^j(\nu) \equiv p_i \frac{\partial q_i(\nu)}{\partial m_{ij}} \Big/ p_j.$$

Similarly, the *private return to spending on labor* is

$$PR_i^\ell(\nu) \equiv p_i \frac{\partial q_i(\nu)}{\partial \ell_i} \Big/ w.$$

Lemma 1. *Consider the inverse of the wedge on intermediate input in sector i,* $\left(\varphi_i^K(\nu)\right)^{-1}$.

1. *Let* $\eta_i(\nu) \geq 0$ *be the Lagrange multiplier on the financial constraint (1) for the firm ν's profit maximization problem* $(\mathbf{P_{firm}})$. *We have*

$$\left(\varphi_i^K(\nu)\right)^{-1} = 1 + \eta_i(\nu).$$

2. $\left(\varphi_i^K(\nu)\right)^{-1}$ *captures the private return to spending on constrained inputs:*

$$\left(\varphi_i^K(\nu)\right)^{-1} = PR_i^j(\nu) \quad \textit{for all } j \in K_i.$$

3. $\left(\varphi_i^K(\nu)\right)^{-1} - 1$ *captures the firm's marginal gains from having access to additional working capital W_i:*

$$\left(\varphi_i^K(\nu)\right)^{-1} - 1 = \frac{\mathrm{d}\pi_i(\nu)}{\mathrm{d}W_i(\nu)}.$$

The variable profit earned by firm ν can be found by subtracting variable costs from revenue:

$$\pi_i(\nu) \quad = \quad p_i q_i(\nu) - w\ell_i(\nu) - \sum_{j=1}^{S} p_j m_{ij}(\nu).$$

For labor and unconstrained intermediate inputs, the lack of a wedge (that differs from one) in (8) and (9) implies that the private return to spending on these inputs is 1 and the marginal product of these inputs is equal to their marginal costs:

$$PR_i^{\ell}(\nu) = PR_i^j(\nu) = 1 \text{ for } j \notin K_i. \tag{11}$$

To simplify notations, in what follows I let $\alpha_i(\nu)$ denote firm ν's equilibrium output elasticity with respect to labor, which is also equal to the labor expenditure share because labor is not subject to the credit constraint. Let $\sigma_{ij}(\nu)$ and $\omega_{ij}(\nu)$ respectively denote the output elasticity and (potentially distorted) expenditure share of intermediate input j:

$$\alpha_i(\nu) \equiv \frac{\partial \ln f_i(\nu)}{\partial \ln \ell} = \frac{w\ell_i(\nu)}{p_i q_i(\nu)}, \quad \sigma_{ij}(\nu) \equiv \frac{\partial \ln f_i(\nu)}{\partial \ln m_{ij}}, \quad \omega_{ij}(\nu) \equiv \frac{p_j m_{ij}(\nu)}{p_i q_i(\nu)}.$$

Sectoral Allocations Let N_i be the number of firms that enter sector i in equilibrium. Recall sectoral total output and inputs are defined as

$$Q_i = N_i \mathbb{E}_\nu[q_i(\nu)], \quad M_{ij} = N_i \mathbb{E}_\nu[m_{ij}(\nu)], \quad L_i = N_i \mathbb{E}_\nu[\ell_i(\nu)],$$

where I use $\mathbb{E}_\nu[\cdot]$ to replace $\int_\nu \cdot \, d\Phi_i(\nu)$ in Definition 1. The sectoral total expenditure on labor as a share of sectoral revenue, which I denote as $\alpha_i \equiv \frac{wL_i}{p_i Q_i}$ (without the index for firms, ν), can be expressed as a weighted average of firm-level labor share, with weights being each firm's output:

$$\begin{aligned}
\alpha_i \equiv \frac{wL_i}{p_i Q_i} \quad &= \quad \frac{\mathbb{E}_\nu[w\ell_i(\nu)]}{\mathbb{E}_\nu[p_i q_i(\nu)]} \\
&= \quad \mathbb{E}_\nu\left[\frac{w\ell_i(\nu)}{p_i q_i(\nu)} \frac{q_i(\nu)}{\mathbb{E}_\nu[q_i(\nu)]}\right] \\
&= \quad \mathbb{E}_\nu\left[\alpha_i(\nu) \frac{q_i(\nu)}{\mathbb{E}_\nu[q_i(\nu)]}\right]
\end{aligned}$$

The sectoral expenditure share of intermediate inputs, which I denote as $\omega_{ij} \equiv \frac{p_j M_{ij}}{p_i Q_i}$, can be similarly expressed as

$$\omega_{ij} \equiv \frac{p_j M_{ij}}{p_i Q_i} = \mathbb{E}_\nu \left[\omega_{ij} (\nu) \frac{q_i (\nu)}{\mathbb{E}_\nu [q_i (\nu)]} \right] \tag{12}$$

The number of firms N_i is pinned down by the free-entry condition (2), which can be expressed as

$$\frac{\kappa N_i}{p_i Q_i} = 1 - \alpha_i - \sum_{j=1}^{S} \omega_{ij}. \tag{13}$$

Equilibrium To characterize the equilibrium, I take note that despite firm's production functions being convex-concave, the economy features sectoral and aggregate constant returns to scale. This is because I allow for the entry of ex-ante identical entrepreneurs into sectors that produce homogeneous goods: any firm-level profits induced by concavity will be driven down to zero in net of fixed cost, and as a result the number of firms can be viewed as a flexible input at the sector level. Taking out input-output linkages, my sectoral production model is indeed a static version of the dynamic competitive model with entry studied by Hopenhayn (1992), Hopenhayn and Rogerson (1993), and more recently, by Restuccia and Rogerson (2008) and Buera et al. (2011, 2015).

Given input prices $(w, \{p_j\})$, the cost of producing q units of output can be captured by the sectoral cost function, which is the solution to the dual of the entry and profit maximization problem:

$$\mathcal{TC}_i (q; w, \{p_j\}) \equiv \min_{n, \left\{ \ell_i(\nu), \left\{ m_{ij}(\nu) \right\}_j \right\}_\nu} n \left(\kappa_i + \int_\nu \left(w \ell_i (\nu) + \sum_{j=1}^{S} p_j m_{ij} (\nu) \right) d\Phi_i (\nu) \right) \tag{14}$$

$$\text{s.t. } n \int_\nu z_i (\nu) q_i (\nu) d\Phi_i (\nu) \geq q$$

$$\sum_{j \in K_i} p_j m_{ij} \leq W_i (\nu)$$

$$q_i (\nu) = f_i (\ell_i (\nu), m_{i1} (\nu), \dots, m_{iS} (\nu))$$

A direct implication of the constant returns to scale property is that the sectoral cost function is linear in the level of output q. In other words, the sectoral *unit cost of production*, which I write as

$$\mathcal{C}_i (w, \{p_j\}) \equiv \frac{\mathcal{TC}_i (q; w, \{p_j\})}{q}, \tag{15}$$

is a function of only input prices but not output levels. Moreover, because the production function $F(\cdot)$ of the final good also features constant returns to scale, I can write its unit cost function as

$$\mathcal{C}^F (\{p_j\}) \equiv \min_{\{\tilde{Y}_j\}} \sum_j p_j \tilde{Y}_j \quad \text{s.t. } F(\tilde{Y}_j) \geq 1$$

Equilibrium prices $(w, \{p_j\})$ solve the set of equations

$$\mathcal{C}_i\left(w, \{p_j\}\right) = p_i \quad \text{for all } i \tag{16}$$

$$\mathcal{C}^F\left(\{p_j\}\right) = 1, \tag{17}$$

where (17) reflects the normalization that the price of the final good is 1.

Proposition 1. *There exists a unique decentralized equilibrium.*

The intuition for the result is as follows. In this economy, the set of prices $(w, \{p_j\})$ completely pins down equilibrium allocations. First, firm-level allocations are directly pinned down by prices, and solving for equilibrium boils down to deriving total sectoral level of output and inputs. Second, because labor supply is exogenous, the wage rate pins down aggregate consumption as $C = wL$. Third, given that sectoral production features constant returns to scale, when holding input prices constant, the expenditure share on inputs is constant for both intermediate producers and the final producer. Given the level of aggregate consumption, we know the quantities of intermediate goods that go into the production of aggregate consumption. Next, given intermediate expenditure shares, we know the second round quantities of intermediate goods as well as the number of firms in each sector that go into the production of the intermediate goods that are used directly for the production of aggregate consumption. Iterating this logic ad infinitum and sum over quantities of goods at each iteration, we can derive the total input and output levels in each sector. The infinite sum is well defined because labor share is positive in every industry by Assumption 1, and as a result, intermediate shares sum to less than one.

The uniqueness of the decentralized equilibrium therefore depends on the uniqueness of the price vector that satisfies the unit cost equations (16) and (17). My model is nested under the class of generalized Leontief models, and the standard argument of uniqueness for this class of models also applies to this setting (e.g., see Stiglitz 1970, Arrow and Hahn 1971), in which the Jacobian matrix of the mapping that represents the system of unit cost equations has the dominant diagonal property, which ensures the global uniqueness of solution by the classic results of Gale and Nikaido (1960).

2.3 Influence and Sales

We now proceed to better understand how credit constraints affect equilibrium allocations and sectoral sales. Recall ω_{ij} denotes sector i's expenditure share on intermediate good j and, as shown in equation (12), can be re-written as a weighted average of firm-level expenditure shares with weights being each firm's output. I define a similar object based on firm-level elasticities:

$$\sigma_{ij} \equiv \mathbb{E}_\nu\left[\sigma_{ij}(\nu)\frac{q_i(\nu)}{\mathbb{E}_\nu\left[q_i(\nu)\right]}\right].$$

That is, σ_{ij} captures the proportional change in total output of sector i if every firm in sector i expands its use of intermediate input j by 1%. I refer to σ_{ij} as the *sectoral output elasticity* with respect to input j, and indeed it is the elasticity of sectoral unit cost with respect to the price of inputs:

Proposition 2. *In equilibrium,*

$$\sigma_{ij} = \frac{\partial \ln \mathcal{C}_i\left(w, \{p_j\}\right)}{\partial \ln p_j} \quad \textit{for all } i, j.$$

Absent credit constraints, $\sigma_{ij}(\nu) = \omega_{ij}(\nu)$ for all firms, and as a result, sectoral expenditure shares are also equal to sectoral elasticities. On the other hand, when the constraints bind for a positive measure of firms, the sectoral expenditure shares of constrained inputs are distorted downwards relative to the equilibrium elasticities, with $\omega_{ij} < \sigma_{ij}$ for $j \in K_i$. Furthermore, the presence of additional constraints in the cost minimization problem (14) implies that if input prices are held fixed, the unit cost of output is higher when firms in a sector are subject to constraints.

I define the sectoral wedge on input j as the ratio between sectoral expenditure share and average sectoral elasticity:

$$\varphi_{ij} \equiv \frac{\omega_{ij}}{\sigma_{ij}} \leq 1.$$

If every firm in sector i expands its use of constrained input j by 1%, the total increase in sectoral sales would be σ_{ij}% while the cost of using these additional inputs is ω_{ij}% of the sectoral sales. The inverse sectoral wedge on input j, $\left(\varphi_{ij}\right)^{-1}$, can therefore be interpreted as the average private return to expenditure on input j as it captures the ratio between the marginal product and marginal cost of an uniform expansion in the use of input j across all firms in the sector. It can also be written as the average firm-level private returns to expenditure on constrained inputs, $\varphi_i^K(\nu)$ (c.f. Lemma 1), weighted by the level of good j used by each firm:

$$PR_i^j \equiv \left(\varphi_{ij}\right)^{-1} = \mathbb{E}_\nu\left[\left(\varphi_i^K(\nu)\right)^{-1} \frac{m_{ij}(\nu)}{\mathbb{E}_\nu\left[m_{ij}(\nu)\right]}\right] \quad \text{for } j \in K_i.$$

Relatedly, I denote the sectoral average private return to capital inputs as a whole by

$$\begin{aligned} PR_i^K \equiv \left(\bar{\varphi}_i^K\right)^{-1} &= \frac{\sum_{j\in K_i}\sigma_{ij}}{\sum_{j\in K_i}\omega_{ij}} \\ &= \mathbb{E}_\nu\left[\left(\varphi_i^K(\nu)\right)^{-1} \frac{\sum_{j\in K_i} p_j m_{ij}(\nu)}{\mathbb{E}_\nu\left[\sum_{j\in K_i} p_j m_{ij}(\nu)\right]}\right]. \end{aligned} \tag{18}$$

When the production function $f_i(\cdot)$ is homothetic, $\varphi_{ij} = \varphi_i^K$ for all $j \in K_i$.

The sectoral average private return of unconstrained inputs, including labor and intermediate inputs $j \notin K_i$, is equal to one, just as the firm-level counterparts in equation (11):

$$PR_i^\ell = PR_i^j = 1 \quad \text{for } j \notin K_i.$$

Because a smaller fraction of revenue is spent on inputs, a larger fraction must accrue to variable profits and attract firm entry. Indeed, equation (13) reveals that, holding input prices fixed, when firms in a sector are constrained, more firms enter per unit of sectoral output relative to when there are no constraints in the sector. Intuitively, financial constraints manifest themselves at the sector level by creating wedges between the marginal product and marginal cost for both the constrained intermediate inputs and the number of firms that are established. When a sector is constrained, resources are misallocated within the sector, with too many firms in equilibrium, each using too little of the constrained inputs. While the result may seem counterintuitive, it is merely a statement about a local property of the equilibrium and does not imply that discrete changes to the environment, such as removing credit constraints altogether from a sector, would induce fewer firms to be in the new equilibrium. Furthermore, the result is not necessarily at odds with empirical observations: Hsieh and Olken (2014) find that the distributions of manufacturing firm size in India, Indonesia, and Mexico are skewed to the left relative to that in the U.S., with the developing countries having many more small firms relative to medium and large firms.

I now define three important objects that are central to the analysis. Recall that h_i denotes the Hicks-neutral sectoral productivity that is common to all firms in sector i, which is normalized to 1 throughout the exposition. The notation is introduced solely for the purpose of the following definition:

Definition 3. The *influence* vector $\mu' \equiv (\mu_1, \cdots, \mu_S)$ is the elasticity of net aggregate output Y with respect to sector productivity,

$$\mu_i \equiv \frac{d \ln Y}{d \ln h_i}.$$

Definition 4. The *sales* vector $\gamma' \equiv (\gamma_1, \cdots, \gamma_S)$ is the ratio between total sectoral sales and net aggregate output,

$$\gamma_i \equiv \frac{p_i Q_i}{Y}.$$

Definition 5. The *sales gap* vector $\xi' \equiv (\xi_1, \cdots, \xi_S)$ is the element-wise ratio between influence and sales:

$$\xi_i \equiv \frac{\mu_i}{\gamma_i}.$$

The influence vector μ' is a notion of sectoral importance, whereas the sales vector γ' represents the equilibrium size of sectors. The sales gap ξ_i captures the the wedge between sectoral importance and size and is a key object in the policy analysis. To better understand these objects and how credit constraints endogenously affect them, I first go to the specific example $\mathcal{E}$.

Influence and Sales in Example $\mathcal{E}$ The expenditure shares on intermediate goods in sectors 2 and 3 are:

$$\frac{p_1 M_{21}}{p_2 Q_2} = \sigma_2 \varphi_2^K, \quad \frac{p_2 M_{32}}{p_3 Q_3} = \sigma_3 \varphi_3^K, \tag{19}$$

where σ_i is the output elasticity in sector i with respect to intermediate input and $\sigma_i \varphi_i$ is sector i's expenditure share on the constrained intermediate input, with $\varphi_i < 1$ iff the constraint in sector i binds.

In this example, the influence, sales, and sales gap are respectively:

$$\mu' \propto \Big(\sigma_3\sigma_2,\ \sigma_3,\ 1\Big),$$

$$\gamma' \propto \Bigg(\Big(\sigma_3\varphi_3^K\Big)\cdot\Big(\sigma_2\varphi_2^K\Big),\ \sigma_3\cdot\varphi_3^K,\ 1\Bigg),$$

$$\xi' \propto \Bigg(\Big(\varphi_3^K\varphi_2^K\Big)^{-1},\ \Big(\varphi_3^K\Big)^{-1},\ 1\Bigg).$$

I highlight three observations. First, the influence of downstream sector 3 is larger than that of midstream sector 2, which in turn has larger influence than upstream sector 1. This is because the final good is produced directly from the downstream good, and any productivity shock in sector 3 will directly affect the effective aggregate productivity, whereas positive productivity shocks in up- and midstream sectors will affect the effective aggregate productivity only through their indirect effect on the relative price of good 3. A similar intuition applies to more general network structures: the sectors with high influence will be those that heavily supply to the final good either directly or indirectly through other sectors.

The second observation relates to how credit wedges φ_i^K affect sales and the sales gap. In this economy, the entire output of sector 2 is used as inputs by sector 3, hence the total sales of sector 2 relative to those of sector 3 is captured by $\sigma_3\varphi_3^K$, the intermediate expenditure share of sector 3. Similarly, sales of sector 1 relative to sector 2 is simply $\sigma_2\varphi_2^K$. Note that sector 2's relative sales are affected by φ_3^K but not φ_2^K: in other words, it is the credit constraints faced by downstream buyers, not within the sector itself, that affect the relative size of sector 2. Furthermore, sales is most suppressed in upstream sector 1, despite the fact that sector 1 itself is unconstrained. This is because sector 1's size is affected by constraints in both midstream and downstream sectors—an effect that is multiplicative in the sectoral wedges. The further upstream we go, and as we travel through an increasing number of constrained sectors, the higher sales gap we would find of a sector. In equilibrium, the upstream sectors are too small in sales relative to their influence, while the downstream ones are too large.

Lastly, absent credit constraints, $\varphi_2^K = \varphi_3^K = 1$ and influence equals sales. This property holds under my general model and is originally formalized by Hulten (1978).

Influence and Sales in the General Model I now proceed to derive influence and sales in the general model and extend the intuitions to this environment. To find influence and sales in equilibrium, it is convenient to stack the sectoral elasticities and expenditure shares into matrices Σ and Ω:

$$\Sigma \equiv \begin{bmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1M} \\ \sigma_{21} & \sigma_{22} & & \sigma_{2M} \\ \vdots & & \ddots & \vdots \\ \sigma_{M1} & \sigma_{M2} & \cdots & \sigma_{MM} \end{bmatrix}, \quad \Omega \equiv \begin{bmatrix} \omega_{11} & \omega_{12} & \cdots & \omega_{1M} \\ \omega_{21} & \omega_{22} & & \omega_{2M} \\ \vdots & & \ddots & \vdots \\ \omega_{M1} & \omega_{M2} & \cdots & \omega_{MM} \end{bmatrix}.$$

Each row in the matrices represents an *output sector*, while each column represents an *input sector*. That is, σ_{ij}, or the entries on the i-th row and j-th column of the matrix Σ, represents the sectoral output elasticity in sector i of input j. Similarly, ω_{ij} is the share of expenditure on input j as a fraction of the total sales in sector i. For this reason, Ω represents the input-output table of the economy and is directly observable from national accounts. In an economy without any distortions or tax interventions, the output elasticity matrix coincides with the input-output table: $\Omega \equiv \Sigma$. The entries σ_{ij} and ω_{ij} differ precisely because of sectoral distortions such as financial constraints and tax interventions.

Let β' denote the equilibrium vector of expenditure share of the final good producer,

$$\beta' \equiv \left(\frac{p_1 Y_1}{F(Y_1, \cdots, Y_S)}, \cdots, \frac{p_S Y_S}{F(Y_1, \cdots, Y_S)} \right),$$

which is referred to as the vector of *final share*s. Because the final producer is unconstrained, β' also represents the equilibrium vector of final good's output elasticities with respect to inputs, i.e. $\beta_i = \frac{\partial \ln F(Y_1, \cdots, Y_S)}{\partial \ln Y_i}$.

Proposition 3. *In the decentralized equilibrium,*

1. The influence vector μ' equals
$$\mu' = \frac{\beta' (I - \Sigma)^{-1}}{\beta' (I - \Sigma)^{-1} \cdot \alpha},$$
where α' is the vector of sectoral output elasticity with respect to labor.

2. The sales vector γ' equals
$$\gamma' = \frac{\beta' (I - \Omega)^{-1}}{\beta' (I - \Omega)^{-1} \cdot \alpha}.$$

Corollary 1. *(Hulten 1978) Absent credit constraints, $\omega_{ij} = \sigma_{ij}$ for all i, j, and influence equals to sales.*

The fact that influence is equal to sales absent market imperfections is first shown by Hulten (1978) on the class of generalized Leontief models with aggregate constant returns to scale, and it is the basis for using sales to measure sectoral importance in the growth accounting literature. This equivalence holds in my model when there are no credit constraints but is otherwise broken. I now provide the intuition for why this is the case through the lens of the general model.

The object $(I - \Sigma)^{-1} = I + \Sigma + \Sigma^2 + \cdots$ is the Leontief inverse of the sectoral output-elasticity matrix Σ. This object, important in the input-output literature, summarizes how sectoral productivity shocks propagate downstream to other sectors through the infinite hierarchy of cross-sectoral linkages. To understand why influence takes the form in the proposition, consider normalizing all prices by wage rate and hold constant the fixed cost of entry relative to the wage rate at κ / w. An one-percent increase in Hicks-neutral productivity h_j in sector j has the direct effect of lowering output prices in its downstream sector i by σ_{ij} percent, represented by the ij-th entry of the output elasticity matrix Σ. The shock also has a second order effect that lowers output prices for all goods k that use j's output as inputs, which in turn further lowers the prices in sector i. This second order effect is captured by the ij-th entry of the matrix Σ^2, and so on. The ij-th entry in the Leontief inverse matrix $\left[(I - \Sigma)^{-1} \right]_{ij}$ therefore captures the total effect of a productivity shock in sector j on the output price of sector i. These effects then translate into higher aggregate output (or equivalently, lower relative price of the final good to wage rate), reflected by the dot product between the output elasticity of the final good β' and the Leontief inverse $(I - \Sigma)^{-1}$. In sum, the sectors with high influence in an economy are those with high network-adjusted final shares.

The scalar term $\frac{1}{\beta'(I-\Sigma)^{-1}\alpha'}$, which is not present in formulations like Acemoglu et al. (2012), arises from the endogenous entry of firms in my model. As the final good becomes cheaper relative to the wage rate, entry becomes less costly. This attracts to more firms to enter all industries, creating an amplification effect.

The sales vector takes the same form as the influence vector, replacing the elasticity matrix Σ with the expenditure share matrix Ω. To see why sales are constructed with the expenditure share matrix, note that sectoral sales can be written as the infinite sum that consists of 1) its output supplied to produce the final good; 2) its output used by other sectors to produce the final good; 3) its output used by other sectors, which supply to other sectors to produce the final good, and so on:

$$p_j Q_j = p_j C_j + \sum_{i=1}^{S} p_i C_i \omega_{ij} + \sum_{i=1}^{S} p_i C_i \left[\Omega^2\right]_{ij} + \cdots \tag{20}$$

The common denominator in the sales vector reflects the fact that only a fraction of the final good accrues to net aggregate output while the remaining fraction is used to incur the overhead fixed cost of entry.

There are two ways in which financial frictions affect equilibrium allocations. First, as discussed earlier, constraints within a sector lower the effective sectoral productivity by increasing the price of sectoral output when input prices are held constant. This effect travels downstream, serving as a negative productivity shock that increases the price of all downstream goods and eventually the final good.

Second and more central to my analysis, financial frictions also affect the relative sectoral size and distort sales away from influence. Credit constraints suppress equilibrium demand of constrained intermediate inputs, endogenously affecting the equilibrium input-output linkages by reducing the sales of upstream goods that are subject to constraints. Contrary to the downstream travel of productivity shocks, this effect instead travels upstream, as can be seen from equation (20). Credit constraints in sector i reduce the equilibrium sales of good j to sector i. Even if sector j is not constrained, the sector still uses fewer inputs from its own upstream suppliers because it faces less demand for its output, and in turn these upstream suppliers end up with lower sales. In equilibrium, it is the sectors that supply to many constrained sectors, which in turn supply to many constrained sectors, ad infinitum, that have the least equilibrium sales relative to influence.

Baqaee (2015) and Acemoglu et al. (2016) observe that in a production network under Cobb-Douglas technology assumption, productivity shocks travel downstream through input-output linkages from suppliers to buyers, while demand shocks travel upstream. My analysis so far makes two additional contributions to understanding how shocks propagate. First, financial frictions serve both as a productivity shock and a shock to intermediate demand that emanates from the constrained sectors. The productivity shock aspect of financial frictions propagates downstream by lowering aggregate output, while the demand shock aspect propagates upstream and suppresses the relative size of upstream sectors. Second, my analysis shows that these results do not rely on specific functional form assumptions. Demand shocks have no effect on the prices of output or the unit costs of production in a generalized Leontief model with sectoral constant-returns-to-scale[1] and affect equilibrium quantities only through *backward linkages* or, in other words, by traveling upstream. On the other hand, productivity shocks travel only through *forward linkages*, affecting the unit cost of production and equilibrium prices of downstream buyers. Equilibrium prices of upstream sectors are unaffected, and output quantities in upstream sectors change in response to productivity shocks from downstream only through the changes in demand induced by these shocks.

2.4 Industrial Policies

I now proceed to show that the sales gap, i.e. the ratio between sectoral influence and sales, is a sufficient statistic that could guide policy. There is clearly room for policy intervention in this economy. Even without relaxing credit constraints, if a planner could impose firm-level subsidies and taxes on production inputs and profits, first-best allocations can be restored. Specifically, the planner would tax firm profits to reduce entry in constrained sectors while imposing firm-specific subsidies to constrained inputs and undo the wedges imposed by credit constraints. The level of firm-specific subsidies that restore first-best would be such that the Lagrange multiplier on credit constraints is precisely zero. However, to implement such policies successfully in any real-world economies, a benevolent government has to grapple with two difficulties. First, the planner needs to have the fiscal flexibility to tailor subsidies to individual firms within sectors. Second, the planner has to know the exact nature of credit constraints for each firm, which requires not only information on each firm's amount of working capital and trade credit but also knowledge of firm-level productivities. The required information and fiscal flexibility in first-best implementation are luxuries that most policymakers do not have. For this reason, I consider tax instruments that apply to all firms equally within a given sector.

To make progress, I leverage one crucial feature of generalized Leontief models: not only do distortions generated by credit constraints propagate through input-output linkages, but so does the effect of policy interventions. Due to pecuniary externalities from the input-output linkages, subsidizing production upstream lowers the prices of upstream goods, which indirectly relaxes credit constraints downstream and ameliorates cross-sector resource misallocation. This property of production networks leaves room for welfare-improving policy interventions, even when the planner only has access to a limited set of instruments. My main results show that the sales gap exactly captures the ratio between the social and private marginal return of spending resources on sectoral production, starting with the decentralized no-tax equilibrium. Rather than making assumptions about the set of instruments at the planner's disposal and prescribing optimal policies under these assumptions, an exercise I conduct under Cobb-Douglas assumptions, these results instead provide answers to the following question: starting from the decentralized equilibrium without any government intervention, where should the fiscal authority spend the first dollar of its tax budget, among a given set of linear instruments that induce firms to use

[1]This result can be viewed as an application of the famous non-substitution theorem by Samuelson (1951).

more production inputs? The result is especially usable because the theory makes no assumptions about the availability and flexibility of tax instruments: policymakers can use the theory to compute social returns given the respective fiscal constraints they face.

Equilibrium with Taxes The planner is has access to lump-sum tax T on the representative consumer's wage income and a flexible set of linear subsidies $\left\{\tau_i^R, \tau_i^L, \tau_i^1, \ldots, \tau_i^S\right\}_{i=1}^S$ that applies to either sales or production inputs in sector i. The planner also has some real, non-tax expenditure E that is financed by lump-sum tax T. This could capture expenditures on public goods or other forms of public consumption. I introduce E merely for interpretational purposes, and do not explicitly model how the planner and the representative consumer value E. In the presence of sectoral subsidies, the planner has to finance both the real expenditure E and the subsidies by the lump-sum tax, with a budget constraint

$$T = E + \sum_{i=1}^{S}\left(\tau_i^R p_i Q_i + \tau_i^L w L_i + \sum_{j=1}^{S} \tau_i^j p_j M_{ij}\right). \tag{21}$$

The budget constraint of the representative consumer is

$$wL = C + T. \tag{22}$$

The resource constraint of the economy is

$$Y = C + E. \tag{23}$$

In presence of the subsidies, the profit maximization problem for firms in sector i becomes

$$\left(\mathrm{P}_{i,\text{firm}}^{\text{Tax}}\right) \qquad \max_{\{m_{ij}\}_{i=1}^S, \ell} \left(1+\tau_i^R\right) p_i q_i\left(\nu, \ell, \{m_{ij}\}\right) - \frac{w}{1+\tau_i^L}\ell - \sum_{j=1}^{S} \frac{p_j}{1+\tau_i^j} m_{ij}$$

$$\text{s.t.} \sum_{j \in K_i} \frac{p_j}{1+\tau_i^j} m_{ij} \leq W_i(\nu)$$

I modify the definition of an equilibrium to incorporate taxes.

Definition 6. An *equilibrium with taxes* is a collection of subsidies $\left\{\tau_i^R, \tau_i^\ell, \tau_i^1, \ldots, \tau_i^S\right\}_{i=1}^S$, prices $\{p_i\}_{i=1}^S$, wage rate w, measure of firms $\{N_i\}_{i=1}^S$, firm-level allocations $\left\{\ell_i(\nu),\ m_{i1}(\nu),\ \cdots,\ m_{iS},\ q_i(\nu)\right\}_{i=1,\cdots S}$, production inputs for the final good $\{Y_i\}_{i=1}^S$, net aggregate output Y, aggregate labor supply L, aggregate consumption C, lump-sum tax T, and non-tax fiscal expenditure E such that (i) firms in the intermediate sectors solve the constrained profit maximization problem $\left(\mathrm{P}_{i,\text{firm}}^{\text{Tax}}\right)$; (ii) free-entry drives ex-ante profits to zero in all intermediate sectors according to (2); (iii) the final good producer solves (3); (iv) the budget constraint (21) for the planner and (22) for the representative hold; (v) all markets clear such that (4), (5), (6), and (23) hold.

Elasticity of Net Aggregate Output to Subsidies All equilibrium allocations and prices can be written as functions of the exogenous subsidy vector $\vec{\tau}$, lump-sum tax T, and expenditure E. In the following exercise, I start from the no-subsidy equilibrium with $\vec{\tau} = \mathbf{0}$ and some expenditure level $E = T$ balanced by lump-sum tax. I evaluate the change in net aggregate output in response to a marginal increase in a subsidy for a given sector, balanced by a marginal increase in T while holding E constant.

Theorem 1. *Starting from a decentralized equilibrium with no subsidies ($\vec{\tau} = \mathbf{0}$) and holding E constant,*

1. *The elasticity of net aggregate output with respect to labor subsidy in sector i is*

$$\left.\frac{d \ln Y}{d \ln \left(1+\tau_i^L\right)}\right|_{\vec{\tau}=\mathbf{0}, \text{holding } E \text{ constant}} = \alpha_i \left(\mu_i - \gamma_i\right).$$

2. *The elasticity of net aggregate output with respect to subsidy to unconstrained input $j \in X_i$ in sector i is*

$$\left.\frac{d \ln Y}{d \ln \left(1+\tau_{ij}\right)}\right|_{\vec{\tau}=\mathbf{0}, \text{holding } E \text{ constant}} = \sigma_{ij} \left(\mu_i - \gamma_i\right).$$

3. *The elasticity of net aggregate output with respect to subsidy to constrained input $j \in K_i$ in sector i is*

$$\left.\frac{d \ln Y}{d \ln \left(1+\tau_{ij}\right)}\right|_{\vec{\tau}=\mathbf{0}, \text{holding } E \text{ constant}} = \sigma_{ij} \mu_i - \omega_{ij} \gamma_i.$$

4. *The elasticity of net aggregate output with respect to sales (revenue) subsidy in sector i is*

$$\left.\frac{d \ln Y}{d \ln \left(1+\tau_i^R\right)}\right|_{\vec{\tau}=\mathbf{0}, \text{holding } E \text{ constant}} = \mu_i - \gamma_i.$$

While the relative sectoral size can be captured by sales under equilibrium production, it is captured by influence under optimal production. Theorem 1 shows that the distance between influence and sales provides a direction in which production efficiency can be improved. To understand these results, consider the effect of a labor subsidy in sector i. Recall that α_i is the labor elasticity, and $\alpha_i \mu_i$ captures the proportional effect on aggregate output if every firm in sector i expands its labor input by 1%, holding labor allocation in every other sector fixed. On the other hand, this exercise violates the resource constraint because the total labor endowment is fixed, and labor has to be scaled back from other sectors in order for L_i to increase. By financing the labor subsidy via the lump-sum tax, labor scales back uniformly across all sectors, including i. The amount of labor that must be scaled back from every sector in order to balance the 1% increase in sector i is captured by the total amount of labor hired by sector i relative to the rest of the economy, i.e. the product between sector i's sales and its labor share, $\alpha_i \gamma_i$. This product in turn captures the negative effect on net aggregate output Y from scaling back labor, and the difference between the two terms is the total effect.

This intuition ignores the fact that resources reallocate endogenously in response to changes in labor allocation, but these re-allocative effects cancel out and have no additional impact on net aggregate output. This is due to the aggregation of the envelope conditions from each firm's optimization problem: although firms face additional credit constraints in their profit maximization problems, their optimization over constrained inputs ensures that envelope condition applies. The intuition is similar on the results for the other subsidies.

Social Return to Tax Dollar Spent on Inputs Consider again starting from the no-subsidy equilibrium with $\vec{\tau} = \mathbf{0}$ and some fiscal expenditure $E = T$ balanced by lump-sum tax. Suppose the planner wants to implement a marginal subsidy τ_i^L to labor in sector i but cannot raise any additional lump-sum tax and must balance the budget by cutting back on fiscal expenditure E. The following object captures the marginal change in total private consumption as a result of cutting back one dollar of fiscal expenditure E and spending it by increasing τ_i^L:

$$SR_i^L \equiv -\frac{dC/d\tau_i^L}{dE/d\tau_i^L}\bigg|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} \tag{24}$$

I refer to this object as the social return to expenditure on labor in sector i. I define the social return to expenditure on intermediate inputs by replacing τ_i^L in equation (24) with τ_i^j:

$$SR_i^j \equiv -\frac{dC/d\tau_i^j}{dE/d\tau_i^j}\bigg|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}}$$

The social return to sectoral expenditure on all inputs can be defined similarly by replacing τ_i^L with τ_i^R, the subsidy to revenue or sales, which affects all inputs uniformly:

$$SR_i^R \equiv -\frac{dC/d\tau_i^R}{dE/d\tau_i^R}\bigg|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}}$$

As the next theorem demonstrates, the social return to expenditures on inputs in a sector is closely related to the sales gap.

Theorem 2. *Starting from a decentralized equilibrium with no subsidies ($\vec{\tau} = \mathbf{0}$) and holding T constant,*

1. *The social return to expenditure on labor in sector i is*

$$SR_i^L = \xi_i.$$

2. *The social return to expenditure on unconstrained input $j \in X_i$ in sector i is*

$$SR_i^j = \xi_i \quad \textit{for } j \in X_i.$$

3. *The social return to expenditure on constrained input $j \in K_i$ in sector i is*

$$SR_i^j = \left(\varphi_{ij}\right)^{-1} \xi_i \quad \text{for } j \in K_i.$$

4. *The social return to a revenue subsidy in sector i is*

$$SR_i^R = \xi_i.$$

Recall that $\left(\varphi_{ij}\right)^{-1}$ can be interpreted as the average sectoral private return to expenditures on constrained input j, which captures the ratio between the marginal product and marginal cost if every firm in sector i expands its use of input j uniformly. Theorem 2 can be restated as:

Corollary 2. *The ratio between social and average private return to sectoral expenditure on inputs satisfies:*

$$\frac{SR_i^L}{PR_i^L} = \frac{SR_i^j}{PR_i^j} \; (\text{for all } j) = \xi_i.$$

The sales gap is a sufficient statistic that captures the ratio between social and private marginal return to expanding the use of production inputs in the sector. The result is especially usable because it answers the following question: if the planner has one dollar of tax budget to spare on providing subsidies to inputs, to which sector and to which input should the planner direct the subsidy?

The intuition behind Theorem 2 and the corollary is similar to that of Theorem 1, in that sectoral size scales with influence under optimal production and with sales under equilibrium production. In a sense, influence locally represents the potential sales vector that would have prevailed had there been no credit constraints. The ratio between the potential and actual sales summarizes the inefficiencies in the production network, and as long as we know the sales gap, knowledge of the underlying frictions in the input-output system becomes irrelevant for welfare analysis.

It is worth emphasizing that sectors with the highest sales gaps are not necessarily sectors in which firms are most constrained; instead, they are sectors that directly or indirectly supply to many constrained sectors. While the most constrained sectors have the highest private return to expenditure on capital goods, the social return might not be high in these sectors.

There are two additional and complementary intuitions through the three-sector example economy $\mathcal{E}$ for why input subsidies applied to higher-sales-gap sectors provide higher social returns to tax dollars. Recall that in the example, the vector of sales gaps according to Theorem 2 is

$$(\xi_1, \;\; \xi_2, \;\; \xi_3) \propto \left(\frac{1}{\varphi_3^K \varphi_2^K}, \;\; \frac{1}{\varphi_3^K}, \;\; 1 \right).$$

The first intuition is that there are prices in credit constraints, and hence pecuniary externalities do not net out in this economy (Greenwald and Stiglitz 1986). In fact, suppliers prices are what show up in buyers' constraints. Subsidizing production for upstream suppliers indirectly relaxes the credit constraints of midstream buyers, who are then able to expand production and further relax the constraints of downstream producers. The higher a sector's sales gap, the greater is this effect. The second intuition relates directly to the allocation of productive resources. Credit constraints from downstream firms reduce demand for goods produced upstream, which in turn reduces the amount of inputs allocated to upstream sectors. The higher a sector's sales gap, the more severe is the misallocation of production input to that sector due to the cumulative rounds of distortions as goods change hands from up- to downstream firms. Corollary 2 points out that the ratio between social and private return to expenditure on inputs is exactly captured by the degree to which sectoral inputs are misallocated due to frictions along input-output linkages.

In Appendix D, I show that Corollary 2 holds true even when policy instruments target only a subset of firms (rather than all firms) within each sector: that is, the sales gap still captures the ratio between social and private marginal returns of tax expenditure for these policy instruments.

Directed Credit Consider a modified economic environment in which credit constraints can be relaxed at a cost. That is, suppose the planner controls an instrument τ_i^C that relaxes the credit constraints faced by firms in sector i according to

$$\sum_{j\in K_i} p_j m_{ij} \leq \left(1+\tau_i^C\right) W_i\left(\nu\right). \tag{25}$$

A firm's problem under the additional working capital is to solve

$$\max_{\left\{m_{ij}\right\}_{j=1}^{S},\ell} p_i q_i\left(\nu,\ell,\left\{m_{ij}\right\}\right) - \sum_{j=1}^{S} p_j m_{ij} - w\ell$$

subject to (25).

In order to deliver any additional credit, the planner has to incur a monitoring cost (in terms of the final good) that is linearly proportional to the amount of additional credit taken up by constrained producers, which can be expressed as

$$DC\left(\left\{\tau_i^C\right\}\right) = \chi \sum_i N_i\left(\left\{\tau\right\}_i^C\right) \mathbb{E}_\nu \left[\max\left\{\left(\sum_{j\in K_i} p_j m_{ij}\left(\nu\right)\right) - W_i\left(\nu\right),0\right\}\right], \tag{26}$$

where the max operator reflects the fact that not all firms are constrained and the linear monitoring cost χ is only incurred on the portion of additional working capital that prevails in equilibrium due to the instruments $\left\{\tau_i^C\right\}$. We modify the budget constraint for the planner as

$$T = E + DC. \tag{27}$$

To understand the effect of directed credit on equilibrium, consider a marginal increase $dW_i\left(\nu\right)$ in working capital available to firm ν starting from the decentralized equilibrium. The gain captured by the firm is

$$d\pi_i(\nu) = dW_i(\nu) \times \left(\varphi_i^K(\nu)^{-1} - 1\right)$$

and the cost of delivering the additional working capital is $\mathbf{1}\left(\varphi_i^K(\nu)^{-1} > 1\right) \cdot \chi \cdot dW_i(\nu)$. Therefore, for a marginal change $d\tau_i^C$ which uniformly relaxes the credit constraints for all firms in sector i, the ratio between the gains captured by firms in sector i and the total monitoring cost is

$$PR_i^C \equiv \chi^{-1} \cdot \mathbb{E}\left[\left(\varphi_i^{-1}(\nu) - 1\right) \frac{W_i(\nu)}{\mathbb{E}\left[W_i(\nu)\mathbf{1}\left(\varphi_i(\nu)^{-1} > 1\right)\right]}\right]. \tag{28}$$

I refer to PR_i^C as *the private return to credit*[2]. On the other hand, the social return to credit is

$$\begin{aligned} SR_i^C &\equiv -\frac{dC/d\tau_i^C}{dE/d\tau_i^C}\bigg|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} \\ &= \frac{dC/d\tau_i^C}{dDC/d\tau_i^C}\bigg|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}}. \end{aligned}$$

We have a result in this environment that is analogous to Corollary 2.

Proposition 4. *The ratio between the social and private return to credit instrument τ_i^C is captured by the sales gap:*

$$\frac{SR_i^C}{PR_i^C} = \xi_i.$$

Proposition 4 points to a powerful intuition: suppose the return to credit is equalized across all firms and all sectors, but firms are still constrained with $\varphi_i(\nu) = \varphi < 1$ for all i and ν. In this case, the social return to working capital would not be the same across sectors. Instead, the social return is highest precisely for the sectors that tend to be upstream and supply, directly or indirectly, to many constrained buyers[3]. This result offers a potential explanation of why industrial policies often direct credit to upstream sectors. More importantly, we see that it might be in the planner's interest to impose restrictions on private credit markets because their operations could compromise the efficient social allocation of working capital.

The result in Proposition 4 assumes a proportional increase of working capital for all firms within a given sector, but a similar result can be obtained if instead the working capital relaxation is uniform in

[2] This formulation of directed credit assumes that the monitoring cost is incurred only for the amount of additional working capital that is taken up by firms. The environment can be modified trivially to accommodate the alternative assumption that a cost of $\tau_i^C W_i(\nu)$ has to be incurred regardless of whether the credit constraint binds for firm ν: simply replace equation (26) with $DC\left(\{\tau_i^C\}\right) = \chi \sum_i N_i \left(\{\tau\}_i^C\right) \mathbb{E}_\nu\left[\tau_i^C W_i(\nu)\right]$ and remove the indicator function $\mathbf{1}\left(\varphi_i(\nu)^{-1} > 1\right)$ from equation (28). Proposition 4 below applies to this modified environment.

[3] This case is analyzed in Appendix C.1, in which I model financial frictions as a linear monitoring cost χ_i and endogenize lending by allowing firms to choose $W_i(\nu)$ freely and pay the monitoring cost as interests. In that environment, when $\chi_i \equiv \chi$ for all i, the marginal private return to credit is equalized across all sectors with $\varphi_i = \frac{1}{1+\chi}$ but the sales gap still captures the ratio between social and private marginal returns to credit and it differs from one in general as long as $\chi > 0$.

levels across firms. To see this, consider instrument $\tau_i^{C'}$ that relaxes the credit constraints according to

$$\sum_{j\in K_i} p_j m_{ij} \leq W_i(\nu) + \tau_i^{C'}.$$

I again make the assumption that the cost of delivering the additional working is proportional to the actual amount taken up by firms as in equation (26). The private return for a marginal change in $\tau_i^{C'}$ is

$$\begin{aligned} PR_i^{C'} &\equiv \frac{\mathbb{E}\left[d\pi_i(\nu)/d\tau_i^{C'}\right]}{\chi Pr\left(\varphi_i(\nu)^{-1} > 1\right)} \\ &= \chi^{-1}\cdot\mathbb{E}\left[\left(\varphi_i^{-1}(\nu) - 1\right)|\varphi_i(\nu)^{-1} > 1\right]. \end{aligned}$$

and the social return is again defined as

$$SR_i^C \equiv -\left.\frac{dC/d\tau_i^C}{dE/d\tau_i^C}\right|_{\vec{\tau}=\mathbf{0},\text{ holding } T \text{ constant}}.$$

Proposition 5. *The ratio between the social and private return to credit instrument $\tau_i^{C'}$ is captured by the sales gap:*

$$\frac{SR_i^{C'}}{PR_i^{C'}} = \xi_i.$$

The Irrelevance of Social Welfare Functions The welfare results in Theorem 2 and Proposition 4 provide guidance as to how the planner should spend the first dollar of the tax budget given a set of feasible instruments. In choosing which tax instrument to use and which sector to subsidize, how the planner marginally trades off between private and public consumption is irrelevant. To see this, let $U(C,E)$ denote the social welfare function. The marginal change in U following an intervention that cuts back E by one dollar to finance a subsidy τ_i is

$$\left.\frac{dU/d\tau_i}{dE/d\tau_i}\right|_{\vec{\tau}=\mathbf{0},\text{ holding } T \text{ constant}} = \frac{\partial U}{\partial E} + \frac{\partial U}{\partial C}\left.\frac{dC/d\tau_i}{dE/d\tau_i}\right|_{\vec{\tau}=\mathbf{0},\text{ holding } T \text{ constant}}.$$

It is immediate apparent that the planner's preference ranking over subsidies is completely captured by the ranking of the social returns.

Optimal Labor Subsidy under Cobb-Douglas Production Functions My next result pertains to optimal (as opposed to marginal) linear subsidies to labor in production. Labor is special relative to other production inputs because it is the only exogenous factor and the only source of net value-added in this economy. With iso-elastic firm-level production functions or Cobb-Douglas sectoral technologies, both the elasticity matrix and the input-output table are stable. In this case, the sales gap captures not only the marginal social return to expanding labor inputs but also the subsidies at the social optimum if the planner can freely choose any level of τ_i^L.

Theorem 3. *Suppose all production functions are iso-elastic, with*

$$f_i(\nu) = \ell(\nu)^{\alpha_i} \prod_{i=1}^{S} m_{ij}(\nu)^{\sigma_{ij}}$$

for firms and

$$F(\{Y_i\}) = \prod_{i=1}^{S} Y_i^{\beta_i}$$

for the final good. The optimal value-added subsidies, i.e. the solution to the planning problem

$$\vec{\tau}^L \equiv \arg\max_{\{\tilde{\tau}_i^L\}} Y\left(\left\{\tilde{\tau}_i^L\right\}\right)$$

satisfies

$$1 + \tau_i^L \propto \xi_i. \tag{29}$$

The result as stated is on the proportionality of $\left(1+\tau_i^L\right)$ because the levels are not pinned down: having access to unrestricted usage of value-added tax is a substitute for lump-sum tax on consumers, or a uniform tax on wages—the planner can always scale $\left(1+\tau_i^L\right)$ by a constant and adjust the lump-sum tax accordingly to balance the budget.

Sales Gap and Hirschmanian Linkages I conclude this theory section with a closed-form formula for the sales gap measure in the general model, and I use this result to place the measure in a historical context and connect it to an early literature that follows from the seminal work of Hirschman (1958).

As discussed, distortions in sales pass through from downstream to upstream sectors through backward demand linkages, and in the three-sector example $\mathcal{E}$, the pass-through is complete: even if midstream sector 2 is unconstrained with $\varphi_2^K = 1$, the sales of upstream sector 1 are still distorted to exactly the same degree as those of sector 2 relative to their influence. This is because midstream is the sole buyer of upstream good in that example. Under more general network structures, the pass-through of sales gap from sector i (buyer) to sector j (seller) depends on the importance of i as a buyer of sector j's output. To capture these notions, let

$$\hat{\omega}_{ij} \equiv \frac{p_j M_{ij}}{p_j Q_j} = \omega_{ij}\frac{\gamma_i}{\gamma_j}, \quad \hat{\sigma}_{ij} \equiv \sigma_{ij}\frac{\gamma_i}{\gamma_j}.$$

Recall ω_{ij} captures the expenditure share of sector i on good j, or the *equilibrium importance* of j as a supplier for i. On the other hand, $\hat{\omega}_{ij}$ captures the share of good j that is used by sector i as a fraction of total output of sector j. In other words, $\hat{\omega}_{ij}$ captures the *equilibrium importance* of sector i as a consumer of good j. Similarly, σ_{ij} and $\hat{\sigma}_{ij}$ respectively capture the *counterfactual unconstrained (or technological) importance* of j as a supplier for i and i as a consumer for j. To interpret these measures in another way, recall that an input-output table contains pair-wise flow of value between industries. The input-output coefficient matrix Ω is obtained by dividing IO table entries by the output of using industry, whereas $\hat{\Omega}$ is obtained by dividing entries by the output of supplying industry, and similarly for Σ and $\hat{\Sigma}$.

Proposition 6. *In equilibrium,*

$$\xi' \propto \mathbf{1}' \left(I - \hat{\Omega}\right) \left(I - \hat{\Sigma}\right)^{-1}.$$

Hirschman (1958) argues that industrial policies should target and promote the economic sectors with the strongest linkages. Following his work, there is a literature that aims to develop measures of Hirschmanian linkages and use these measures to study economic policies, including Chenery and Watanabe (1958), Rasmussen (1965), Yotopoulos and Nugent (1973), and many others. All measures proposed and debated during that period are ad-hoc and without microfoundations. One notable measure by Jones (1976) is later used for applied policy work:

$$\delta_i^{Jones} \equiv \left(\mathbf{1}' \left(I - \hat{\Sigma}\right)^{-1}\right)_i.$$

δ_i^{Jones} is proposed as a measure of the "forward linkages" for industry i and is supposed to capture the extent to which downstream industries could benefit from output gains in industry i. Noland (2004) finds that this measure strongly explains which sectors were promoted during South Korea's "Heavy-Chemical Industry" drive in the 1970s. It turns out that δ_i^{Jones} is closely related to sales gap under a particular assumption of how credit constraints affect sectoral production. Specifically, suppose credit constraints in the economy create a constant wedge φ between output elasticities and expenditure shares for all industries and all intermediate inputs such that $\omega_{ij} = \sigma_{ij}\varphi$ for all i, j. This assumption corresponds to a setup in which all intermediate inputs are constrained in every sector, and the allocation of working capital equalizes the sectoral average private marginal returns to capital inputs. In this scenario,

$$\xi \propto const + \delta^{Jones}. \tag{30}$$

That is, under the assumption that $\omega_{ij} = \sigma_{ij}\varphi$ for all i and j, the sales gap measure is an affine transformation of δ^{Jones}.

Empirics

I now turn to examining whether the sufficient statistic I have developed can explain interventions implemented by developing countries. The two essential ingredients in constructing the sales gap measure are sectoral sales and influence. Sectoral sales is directly observable from national accounts, but influence requires estimation. If TFP shocks are directly observable, sectoral influence can be estimated from a regression of aggregate output on the TFP shocks. On the other hand, TFP shocks are almost never observed and are often obtained as residuals from estimations of aggregate production functions. For this reason, I choose an indirect route to estimating influence by first estimating the matrix of production elasticities, i.e. the unconstrained input-output table, and then computing the influence vector using the Leontief-inverse formulae provided in early sections. I perform two distinct empirical exercises that recover the sales gap measure via this route, and I examine the correlation between a sector's sales gap and plausible proxy measures of government interventions therein.

3 Structural Analysis: China

In this section I conduct empirical analysis in the context of China. Because it has a government with deep fiscal capacity and a strong legacy of state intervention in production due to its socialist roots, China is a particularly interesting setting in which to apply my theoretical results. I estimate production elasticities using firm-level production data from year 2007 of China's Annual Industrial Survey (AIS) for 66 manufacturing industries and construct the sales gap measure by applying these estimates to the 2007 Chinese input-output table. I show that private firms in sectors with higher sales gaps tend to receive more external loans and pay lower interest rates, and that the sectoral presence of SOEs is heavily directed towards sectors with larger sales gaps rather than sectors that are the most constrained or have the highest private return to capital.

History of SOEs in China SOEs play two important roles in this part of my empirical analysis. First, I treat the presence of SOEs as a measure of government intervention into a sector, as they can be an indirect vehicle for the state to subsidize production when direct production subsidies are difficult to implement due to practical obstacles[4]. Second, I exploit SOEs in the estimation strategy by assuming that, conditional on being in a sector, they are price-taking profit maximizers that are unconstrained but potentially less productive than their private-owned counterparts, as in Song et al. (2011). In what follows, I provide a very brief overview of Chinese SOE's institutional setting to defend this assumption[5].

The Chinese manufacturing sector was dominated by SOEs before the late 1990s. Under a "dual track" system, private and state-owned firms coexisted but were subject to vastly different market regulations. SOEs faced price controls and production quotas, and most importantly, profit maximization was not their objective. Managers were typically ranking Communist party officials whose compensation was heavily regulated. Moreover, the government's top priorities include promoting social stability and avoiding layoffs, and loss-making SOEs were kept alive by loan injections and bank bailouts. The lack of exit and market selection of more productive firms further distorted SOEs' economic incentives.

Major waves of SOE reform began in the late 1990s, with the explicit objective of letting SOEs compete with private firms through market mechanisms while keeping state control. The Chinese government retracted its commitment to stable employment, and a large number of failing SOEs were closed down or privatized through management buyouts. While the overall SOE share in the manufacturing sector steadily declined between 1998 and 2007, surviving SOEs are positively selected on both size and productivity. As a result of the reforms, many of the larger SOEs became corporatized and are now publicly traded (though the state retains controlling shares), successful market participants. In fact, many scholars of the Chinese economy argue that the SOE reforms were intended to turn SOEs into profit-maximizing entities (Hsieh and Song 2015). Because my empirical setting focuses on the industrial production of year 2007, well into the post-reform period, it may therefore be reasonable to model SOEs as profit max-

[4]Indeed this is a popular view among economic historians on Taiwan's industrial policies in 1960s, where public enterprises were heavily involved in the manufacturing sector (Hernandez 2004).

[5]Detailed discussions of the history of SOEs and their role in the Chinese economy can be found in Naughton (2006), Brandt et al. (2008), Zhu (2012), Storesletten and Zilibotti (2014), and Hsieh and Song (2015).

imizers.

There is also well-documented evidence that SOEs in China receive easy access to credit from state-owned Chinese banks (Boyreau-Debray and Wei 2005), while private firms face significant financial constraints (Allen et al. 2005), especially for capital goods purchases (Dollar and Wei 2007, Riedel et al. 2007). Song et al. (2011, section I.C) provide a systematic compilation of related evidence.

Technological Assumptions A significant data challenge is that there is no firm-level data on input use by sector of origin. Instead, the available firm-level production surveys (AIS) categorize inputs into labor, capital, and intermediate materials, as do most similar manufacturing data sets for other countries. To make progress despite the data limitations, I partition industries into those that produce "capital goods" and those that produce "intermediate materials." I further assume that firm production functions are separable in these two goods bundles and are homogeneous within each bundle. This assumption ensures that expenditures on each bundle of intermediate goods are comparable across firms within the same industry, as the ratio of expenditure between any two goods within a bundle is the same across firms within the same industry. Formally,

Assumption 2. *Intermediate goods can be partitioned into mutually exclusive categories K and X such that production functions in all industries are homothetically separable in the two groups of industries. That is, for all industries $i = 1, \cdots S$, there exist continuously differentiable functions $k_i : \mathbb{R}_{\geq 0}^{|K|} \to \mathbb{R}_{\geq 0}$ and $x_i : \mathbb{R}_{\geq 0}^{|X|} \to \mathbb{R}_{\geq 0}$ that are homogeneous of degree one, such that*

$$f_i\left(\ell, m_1, \cdots, m_S\right) = f_i\left(\ell, k_i\left(\left\{m_j\right\}_{j \in K}\right), x_i\left(\left\{m_j\right\}_{j \in X}\right)\right).$$

Each industry is assigned to be a producer of capital goods iff more than 5% of its total output in 2007 is used for "gross capital formation", which is a measure that captures the value of goods that is un-depreciated in the accounting year and is to be used at a future time. Out of 66 industries, 20 are assigned as capital good producers, and 46 are assigned as material good producers. Our results are qualitatively insensitive to alternative cutoff rules.

Under the homogeneity assumptions on $k_i(\cdot)$ and $x_i(\cdot)$, there exist industry-specific price indices p_i^K and p_i^X for the bundle of capital and material goods, respectively:

$$p_i^K \equiv \min_{\left\{m_j\right\}_{j \in K}} \sum p_j m_j \quad \text{s.t. } k_i\left(\left\{m_j\right\}_{j \in K}\right) \geq 1$$

$$p_i^X \equiv \min_{\left\{m_j\right\}_{j \in X}} \sum p_j m_j \quad \text{s.t. } x_i\left(\left\{m_j\right\}_{j \in X}\right) \geq 1$$

The *unconstrained* profit maximization problem of SOEs can be re-written as

$$\max_{\ell, k, x} p_i z_i(\nu) f_i(\ell, k, x) - w\ell - p_i^K k - p_i^X x,$$

while private firms solve the same problem but are subject to the credit constraints. Motivated by the evidence that private firms in China obtain significantly fewer bank loans when financing capital investments (Dollar and Wei 2007, Riedel et al. 2007) and have substantially lower capital-output and capital-labor ratios than SOEs (Song et al. 2011), I specify that only capital goods are subject to credit constraints, while labor and intermediate materials are not:

$$p_i^K k \leq W_i(\nu). \tag{31}$$

The fact that capital inputs are subject to the constraint implies, as shown in the previous section, that there will be a firm-level wedge between the expenditure share on capital and output elasticity for private firms. In Appendix F.2 I conduct a specification test for this assumption[6].

To rationalize the residuals in estimation, I introduce an ex-post and multiplicative productivity shock $\epsilon_i(\nu)$ that affects output but is observable to firms only after input choices are made. Formally, the productivity $z_i(\nu)$ that affects input choices can be written as the product between a component that is known to the firm when making input choices, $\tilde{z}_i(\nu)$, and the expectation over the ex-post productivity shock:

$$z_i(\nu) = \tilde{z}_i(\nu)\,\mathbb{E}_\nu\left[\exp\left(\epsilon_i(\nu)\right)\right]$$

The observable firm revenue $r_i(\nu)$ can be written as

$$p_i q_i(\nu) = p_i z_i(\nu) \frac{\exp\left(\epsilon_i(\nu)\right)}{\mathbb{E}_\nu\left[\exp\left(\epsilon_i(\nu)\right)\right]} f_i\left(\ell_i(\nu), k_i(\nu), m_i(\nu)\right). \tag{32}$$

The term $\frac{\exp(\epsilon_i(\nu))}{\mathbb{E}_\nu[\exp(\epsilon_i(\nu))]}$ can be equivalently interpreted as a multiplicative measurement error in the firm's revenue. In what follows, I choose a normalization of $\epsilon_i(\nu)$ such that $\mathbb{E}_\nu\left[\exp\left(\epsilon_i(\nu)\right)\right] = 1$. I assume the distribution of the ex-post shocks (or measurement errors) is independent of both the firm's status and any other variable in the firm's information set when making input choices to ensure that the ex-post shocks are not correlated with input choices.

Assumption 3. *$\epsilon_i(\nu)$ is independent of $z_i(\nu)$, $W_i(\nu)$, and the firm's status (SOE or private).*

3.1 Identification

Identification of Output Elasticity with Respect to Capital I now lay out the identification strategy for firm-level output elasticity with respect to the bundle of constrained inputs, or capital K. For this subsection, I temporarily drop industry subscript i for the ease of notation and focus the exposition on a single industry. The same argument applies to all industries.

The main difficulty in estimating features of production functions comes from the transmission bias:

[6]Specifically, I show that the estimated wedges between firm-level expenditure shares and output elasticity are much higher for capital inputs than for labor and material inputs, and the average wedge across industries on labor and material inputs is close to one, which is the unconstrained benchmark.

the Hicks-neutral productivity shock $z(\nu)$ endogenously affects both firm-level output and the observable inputs choices, and therefore a regression of firm-level output on inputs yields biased estimates of output elasticities. To get around this problem, I adapt the strategy proposed by Gandhi et al. (2016) to my static setting. The key insight from Gandhi et al. (2016) is that for unconstrained firms, the first-order conditions with respect to inputs provide additional information that enables us to purge the endogeneity generated by Hicks-neutral productivity $z(\nu)$. More specifically, under the assumption that SOEs are unconstrained, an SOE's first-order condition with respect to capital is

$$p^K = pz(\nu)\frac{\partial}{\partial k}f(\ell(\nu), k(\nu), x(\nu)). \tag{33}$$

Multiplying both sides of (33) by $k(\nu)$, dividing by revenue using (32), and then taking logs,

$$\ln s^K(\nu) = \ln \sigma^K(\ell(\nu), k(\nu), x(\nu)) - \epsilon(\nu), \tag{34}$$

where $s^K(\nu) \equiv \frac{p^K k(\nu)}{r(\nu)}$ is the capital share and $\sigma^K(\nu) \equiv \frac{\partial}{\partial \ln k}\ln f(\ell(\nu), k(\nu), x(\nu))$ is the output elasticity with respect to capital goods. Given that $s^K(\nu)$, $w\ell(\nu)$, $p^K k(\nu)$, and $p^X x(\nu)$ are all observable and that the ex-post productivity shock $\epsilon(\nu)$ is independent of the input expenditures,

Proposition 7. *The output elasticity with respect to the bundle of capital goods $\sigma^K(\cdot)$ is non-parametrically identified from SOE production data. The distribution function $F^\epsilon(\cdot)$ of the ex-post productivity shock $\epsilon(\nu)$ is also non-parametrically identified.*

Identification of Return to Capital for Private Firms Next, I take $\sigma^K(\cdot)$ as a known function of input expenditures and outline how this information can be used to identify the distribution of firm-level returns to capital for the private firms. Taking the first-order condition with respect to capital inputs and multiplying it by $k(\nu)/pq(\nu)$ again derives an equation relating the private firm's expenditure share on capital goods to output elasticities:

$$s^K(\nu) = \sigma^K(\nu)\varphi^K(\nu)e^{-\epsilon(\nu)},$$

The left-hand side is again directly observable, and $\sigma^K(\nu)$ is a known function of observable variables thanks to the previous identification result. Therefore for any given firm ν,

$$\left(\varphi^K(\nu)\right)^{-1}e^{\epsilon(\nu)} = \frac{\sigma^K(\nu)}{s^K(\nu)}$$

and the distribution function of $\left(\varphi^K(\nu)\right)^{-1}\epsilon(\nu)$ is identified.

While I cannot separately recover firm-level wedges $\left(\varphi^K(\nu)\right)^{-1}$ and the ex-post productivity shock $\epsilon(\nu)$ for each individual firm ν, I can indeed recover distributional properties of $\varphi^M(\nu)$. Specifically, because the distribution function of both $\ln\varphi^M(\nu) - \epsilon(\nu)$ and $\epsilon(\nu)$ are identified, I can apply the method of deconvolution (Chen et al. 2011) and recover the distribution function of $\varphi^M(\nu)$ such that:

Proposition 8. *The distribution function $F^{\varphi^M}(\cdot)$ of φ^M is non-parametrically identified.*

3.2 Estimation

In this subsection I outline an estimation procedure to construct the sales gap measure ξ based on the identification results from the previous subsection.

From the AIS micro data, I observe firm revenue, costs of labor inputs, cost of intermediate materials, and the book value of total firm-level capital stock. Firm production in the real world is dynamic in nature, and one can interpret the one-shot production game after entry in the static model as representing the steady-state of a dynamic production game. In order to map the dynamic real-world data to the static production model, one should adopt a "flow" capital expenditure measure that corresponds to the gross capital depreciation during the production period. Reliable measures of firm-level capital depreciation are unavailable in the Chinese production data, but fortunately, identifying output elasticities with respect to capital only requires the econometrician to observe a constant multiple of the capital expenditure, which can be proxied by the book value of capital stock variable as long as I assume firms within an industry have the same depreciation rate.

Formally, for each firm ν in industry i, I separately observe firm-level revenue $r_i(\nu) \equiv p_i q_i(\nu)$ and expenditures on labor $w\ell_i(\nu)$, on the bundle of commodity goods $p_i^X x_i(\nu)$, and a constant multiple of the expenditures on capital goods $c_i \cdot p_i^K k_i(\nu)$, where the industry-specific constant c_i is the inverse of the depreciation rate. The identification results in the previous subsection specify $\sigma^K(\nu)$ as a function of input *quantities*. On the other hand, given that the depreciation rate and input prices are industry wide, the observed variables are simply multiplicative transformations of input quantities, and $\sigma^K(\nu)$ can be re-written as a function of the observed variables via a simple change of variable[7]. To avoid carrying redundant notations, I simply write $\ell_i(\nu)$, $x_i(\nu)$, $k_i(\nu)$ as observed variables and omit the multiplicative constants.

For each industry i, I parametrize $\sigma_i^K(\cdot)$ as a second-order polynomial in logs of the input expenditures:

$$\begin{aligned} \ln s_i^K(\nu) \quad = \quad & \log\Big(\eta_i^1 + \eta_i^2 \cdot \log \ell_i(\nu) + \eta_i^3 \cdot \log k_i(\nu) + \eta_i^4 \cdot \log x_i(\nu) \\ & + \eta_i^5 \cdot (\log \ell_i(\nu))^2 + \eta_i^6 \cdot (\log k_i(\nu))^2 + \eta_i^7 \cdot (\log x_i(\nu))^2 \Big) - \epsilon_i(\nu) \end{aligned} \tag{35}$$

I estimate the model on the sample of SOEs for each industry i[8], using GMM with moment conditions $\mathbb{E}_\nu[\exp(\epsilon_i(\nu))] = 1$ (our chosen normalization of $\epsilon_i(\nu)$) and $Cov\Big(\epsilon_i(\nu), v_i(\nu)\Big) = 0$ for $v \in$

[7] That is, I can define a revenue production function by simply scaling the quantity variables by constants (p, w, cp^K, p^X) :

$$f_R\left(w\ell(\nu), cp^K k(\nu), p^X x(\nu)\right) \equiv pf(\ell(\nu), k(\nu), x(\nu)) \quad \text{with} \quad \frac{\partial \ln f_R(\nu)}{\partial \ln(cp^K k(\nu))} = \frac{\partial \ln f(\nu)}{\partial \ln(k(\nu))}.$$

[8] Appendix Table F.7 provides summary statistics on the number of SOEs in each industry.

$\left\{\log \ell, \log k, \log x, (\log \ell)^2, (\log k)^2, (\log x)^2\right\}$. In Appendix F.2 I show that my results are qualitatively similar if I simply parametrize $\sigma_i^K(\cdot)$ as an industry-specific constant, which corresponds to production functions with constant output elasticity of capital inputs.

Equipped with estimates $\widehat{\sigma_i^K(\nu)}$, I obtain the residuals for private firms:

$$\widehat{res}_i(\nu) \equiv \frac{\widehat{\sigma_i^K}(\nu;\hat{\eta}_i)}{s_i^K(\nu)} = \hat{\varphi}_i^K(\nu)^{-1}\hat{\epsilon}_i(\nu). \tag{36}$$

The average (weighted) private return to capital inputs in industry i can be written as

$$\begin{aligned}\left(\bar{\varphi}_i^K\right)^{-1} &= \mathbb{E}_\nu\left[\varphi_i^K(\nu)^{-1}\frac{k_i(\nu)}{\mathbb{E}_\nu(k_i(\nu))}\right] \\ &= \frac{\mathbb{E}_\nu\left[\varphi_i^K(\nu)^{-1}\epsilon_i(\nu)k_i(\nu)\right]}{\mathbb{E}_\nu[\epsilon(\nu)]\,\mathbb{E}_\nu[k_i(\nu)]},\end{aligned} \tag{37}$$

where the second line follows by independence in Assumption 3.

I estimate $\left(\bar{\varphi}_i^K\right)^{-1}$ by forming estimates of the three expectations separately using an empirical Bayes procedure (Morris 1983) that exploits cross-industry information. In particular, $\mathbb{E}_\nu\left[k_i(\nu)\right]$ and $\mathbb{E}_\nu\left[\varphi_i^K(\nu)^{-1}\epsilon_i(\nu)k_i(\nu)\right]$ are estimated from private firms' capital stock $k_i(\nu)$ and estimation residuals $\widehat{res}_i(\nu)$, whereas $\mathbb{E}_\nu\left[\epsilon(\nu)\right]$ is estimated using the residuals for the SOEs. I discuss the procedure in detail in Appendix E. I also estimate an unweighted average private return to capital for each industry, $\mathbb{E}_\nu\left[\left(\varphi_i^K(\nu)\right)^{-1}\right]$, via a similar procedure.

Next, I use the estimates $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ to construct the input-output elasticity matrix. Because only capital goods are subject to credit constraints, those entries in the *observed* input-output matrix, which represent expenditure shares on intermediate materials, do reflect actual elasticities, while to obtain elasticity with respect to capital goods I multiply the observed expenditure shares with the input-using industry's average private return:

$$\widehat{\Sigma_{ij}} = \begin{cases}\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}\Omega_{ij} & \text{if } j \in K \\ \Omega_{ij} & \text{if } j \in X,\end{cases} \tag{38}$$

where recall Σ denotes the elasticity matrix and Ω denotes the matrix of expenditure shares or the observed input-output table. Lastly, I construct the industry level sales gap measure $\widehat{\xi}_i$ by

$$\widehat{\xi}_i = \hat{\mu}_i/\gamma_i, \tag{39}$$

where

$$\hat{\mu}' = \frac{\beta'\left(I-\widehat{\Sigma}\right)^{-1}}{\beta'\left(I-\widehat{\Sigma}\right)^{-1}\alpha}, \quad \gamma' = \frac{\beta'(I-\Omega)^{-1}}{\beta'(I-\Omega)^{-1}\alpha}, \tag{40}$$

and as guided by my theory (Corollary 2), industry level social return to expenditure on capital is defined as

$$\widehat{SR}_i^K = \widehat{\xi}_i \times \widehat{\left(\bar{\varphi}_i^K\right)^{-1}}. \tag{41}$$

3.3 Data

The firm-level variables come from year 2007 of China's Annual Industrial Survey (AIS), which is a firm-level manufacturing survey that includes private firms with sales above 5 million RMB as well as all SOEs. To make the two sets of firms comparable, I drop SOEs with sales below the 5 million RMB cutoff. The 2007 Chinese input-output table comes from the national accounts published by China's National Bureau of Statistics. Following the methodology described by Hsieh and Song (2015), throughout this section I use the term SOE to refer both to firms that are legally registered as state-owned and to firms whose controlling shareholder is the state. Below, I elaborate on three details of how the conceptual framework is mapped to actual data and estimation.

The first issue pertains to the construction of the sales vector γ. Rather than using actual sales directly observed from national accounts for my analysis, I use the measure constructed from the Leontief inverse of the input-output table, with the final share vector β measured as

$$\beta_i \equiv \frac{\text{Private and public consumption of good } i}{\text{Total priv. and public consumption of all goods}}. \tag{42}$$

In a closed and static economy, sectoral sales directly observed from national accounts are mechanically proportional, by construction, to the measure I adopt. On the other hand, real-world production involves both dynamic accumulation of capital and inventory as well as imports and exports, hence the observed total sectoral sales are equivalent to that constructed based on the Leontief inverse method if the final shares are modified to incorporate net sectoral output that is used for dynamic accumulation and trade. I exclude these components of sectoral output from the final share measure in equation (42) because they are not relevant to my theoretical model, and I use the same final share vector to construct both influence and sales vector according to (40). I show in Appendix Table F.1 that my results are robust to using influence and sales measures constructed with alternative final share vectors that include net exports.

The second issue relates to constructing net value-added share α and how I deal with accounting profits and the total consumption of fixed capital accumulated in previous accounting years. Together with wage payments, these entries are recorded in input-output tables as being part of each industry's gross value-added. In my model, there are accounting profits but no economic profits: the accounting profits cover the fixed cost of entry. For this reason, I apply the same treatment to profits in the model and take the net value-added of an industry as being the recorded wage,

$$\alpha_i \equiv \frac{Wage_i}{Output_i}. \tag{43}$$

The recorded value for the total consumption of previously accumulated fixed capital is proportionally added to the value of capital inputs used for production. These recorded entries for capital depreciation constitute a small fraction of gross value-added, averaging 12% for the manufacturing sectors and 13% for the economy as a whole, and account for an even smaller fraction of total output (2.5% on average for the manufacturing sector). As a result, adjusting for the value of capital goods used for production has little impact on the results.

The third issue pertains to how I combine the micro-estimates from the AIS data with the Chinese input-output table when constructing the sales gap measure. The input-output table records the flow of value across manufacturing as well as primary and tertiary sectors, while the AIS micro data cover only manufacturing industries under a different and more disaggregated industrial classification code. I manually create the concordance between manufacturing industries in the two data sources, merging industries when necessary, with a final combined data set that includes 66 distinct manufacturing industries for which wedges can be recovered from the AIS data. My empirical strategy does not recover the wedges for the primary and tertiary sectors due to the lack of micro data, and I report results in the main text based on the conservative assumption that firms in these sectors are not constrained. In Appendix Table F.2 I show that my results are robust to dropping the primary and tertiary sectors altogether and performing the Leontief inverse on a partial input-output table with the manufacturing sectors only. I also winsorize the estimates of $\left(\bar{\varphi}_i^K\right)^{-1}$ below one to reflect the theoretical lower bound, as $\left(\bar{\varphi}_i^K\right)^{-1}$ should be equal to one if firms in industry i are unconstrained and greater than one if a positive measure of firms are constrained. This winsorization affects only 7 out of the 66 manufacturing industries, thus indicating that private firms in most industries face constraints in their capital choices, and the fact that these 7 industries have wedge estimates less than one could be due to sampling error.

3.4 Results

Private and Social Return to Capital Table 1 provides summary statistics for the estimated wedges, social return, and sales gap measures. Column (1) corresponds to estimates of the *unweighted* sectoral average of firm-level wedges, $\mathbb{E}\left[\varphi_i^K(\nu)^{-1}\right]$. Across sectors, an average firm has gross private rate of return to capital of 1.2 or net rate of return of 20%, which is in-line with other estimates from the literature (e.g. Bai et al. 2014). Column (2) corresponds to estimates of $\left(\bar{\varphi}_i^K\right)^{-1}$, the sectoral average private return to capital inputs *weighted by the amount of capital used by each firm*. This weighted average return is the relevant object in our model (in equation 18 and 38) and can be written as the sum between the unweighted average return $\mathbb{E}\left[\varphi_i^K(\nu)^{-1}\right]$ and the covariance between the level of and return to capital inputs:

$$\begin{aligned}\left(\bar{\varphi}_i^K\right)^{-1} &\equiv \mathbb{E}_i\left[\left(\varphi_i^K(\nu)\right)^{-1}\frac{k_i(\nu)}{\mathbb{E}_i\left(k_i(\nu)\right)}\right]\\ &= \mathbb{E}_i\left[\left(\varphi_i^K(\nu)\right)^{-1}\right]+Cov_i\left[\left(\varphi_i^K(\nu)\right)^{-1},\frac{k_i(\nu)}{\mathbb{E}_i\left(k_i(\nu)\right)}\right].\end{aligned}$$

The estimates $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ averages to 1.34 across sectors and is higher than the unweighted average $\mathbb{E}_i\left[\left(\varphi_i^K(\nu)\right)^{-1}\right]$ in every industry, thereby implying that the covariance term is positive and that firms using more capital inputs also tend to have higher marginal return to additional capital inputs. This empirical finding implies that while more productive firms are allocated more credit ($z_i(\nu)$ and $W_i(\nu)$ are positively correlated), these firms should receive even more working capital in order for marginal return of capital inputs to equalize across firms within each sector. This finding is consistent with the evidence in Hsieh and Klenow (2009) that larger firms in China tend to have higher returns to capital.

The network effect of the sectoral credit constraints is large: while the $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ averages to 1.34, or 34% net private return to capital, the network work inefficiency magnifies the average gross social return to capital inputs to 1.46. Much of this large difference of 12 percentage points is due to a heavier right tail induced by the network adjustment in constructing the social return (equation 41), as illustrated by Figure 2, which shows the density distributions of $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ and $\widehat{SR}_i^K$ with dashed-grey and solid-black lines, respectively.

Table 1: Summary Statistics

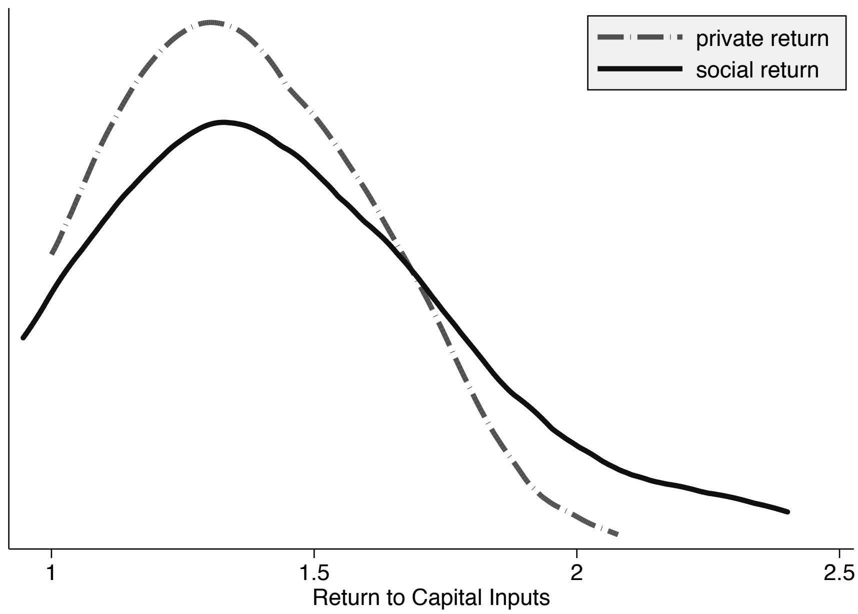
	$\mathbb{E}\left[\widehat{\varphi_i^K(\nu)^{-1}}\right]$	$\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$	$\widehat{\xi}_i$	$\widehat{SR}_i^K$
Mean	1.20	1.34	1.08	1.46
St. dev	0.19	0.24	0.17	0.37

Notes: the four columns respectively show summary statistics for the following industry-level variables: 1) unweighted average of firm-level marginal returns to capital inputs in the industry; 2) "private return to capital inputs", i.e. average firm-level marginal returns to capital inputs, weighted by the capital stock of each firm; 3) the sectoral sales gap; 4) the social return to capital inputs.

Sectoral Sales Gap Table 2 lists the top and bottom 15 manufacturing industries ranked by their sales gap measures. Unsurprisingly, many of the industries characterized as capital goods producers, such as “industrial furnace and boiler” and “rail equipment”, appear on the Top 15 list, i.e. top sales gaps. This is because these industries *produce* goods that are directly subject to credit constraints, which distort their sales below optimal production level. What is perhaps less obvious is the set of metal industries that are also on this Top 15 list, such as the smelting, stamping, and rolling of iron, steel, and ferrous and non-ferrous alloy. Because these industries are not classified as capital goods by the selection criterion, the purchase of these goods are not directly subject to credit constraints. The reason that these goods are on the Top 15 list is because of network propagation of financial distortions: the metal products are extensively used as intermediate materials that go into the production of capital goods, mostly machines and equipment. Credit constraints distort the sales of capital goods downwards, which in turn distorts the equilibrium sales of the metal products through upstream propagation of the demand distortion. As a result, manufacturers of these unconstrained inputs end up with high sales gaps, and the economy could benefit from subsidizing their production.

On the other side of the table, light industries—those that produce food and textiles—dominate the Bottom 15 list. These industries are downstream from the capital goods and are arguably also the most

Figure 2: Density of sectoral private return $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ and social return $\widehat{SR}_i^K$



downstream in the network structure: although they use capital goods for production, these industries' output is used more for direct consumption rather than industrial production, and, in particular, their output is not heavily used for the production of capital goods. While private firms in these industries do face credit constraints that lower their effective productivity, the constraints do not impose large sales gaps, and, in aggregate, these are the sectors that are too large relative to optimum. Lastly, the set of industries that are not on these partial lists includes chemical industries and a set of industries that produce non-metallic materials such as rubber, lime glass, and plastic products. Despite the fact that "upstreamness" is not uniquely defined in the context of a complete network in which every industry purchases at least some inputs from every other, we tend to think of these as the midstream industries: these products are used more as intermediate inputs to manufacturing rather than for final consumption, but, on the other hand, the production of these goods requires more upstream machineries than the extent to which these goods are used as inputs for upstream production.

Debt-To-Capital Ratio and Interest Rates I next examine the correlation between sales gap and plausible measures of government intervention. Though the manufacturing sector in China was largely driven by market forces in 2007, the credit market remained predominantly state-controlled, with the government holding direct ownership of the largest commercial banks. Targeted and subsidized credit through the banking sector to both SOEs and private manufacturing firms played an important policy role (Aghion et al. 2015). While detailed firm-level data on bank loans is unavailable, firms in the AIS do report their total interest payments and total liabilities. Based on these variables, I derive proxy measures of credit market interventions. I define the Debt-To-Capital ratio (D/K) for firm ν in industry i

Table 2: Top and Bottom 15 Industries Ranked by Sales Gap

Top 15	Bottom 15
Industrial furnace and boiler	Misc. food products
Metal cutting machinery	Meat processing
Misc. general-purpose machinery	Medicine manufacturing
Auto manufacturing	Sugar making
Electrical machinery	Liquor and alcoholic drinks
Fabricated metal products	Clothing and footwear manufacturing
Ferroalloy smelting	Stationery manufacturing
Iron smelting	Household chemical products
Misc. special-purpose equipment	Knit textiles
Steel smelting	Consumer electronics
Misc. electrical equipment	Leather, fur, and down products
Steel rolling and stamping	Vegetable oil manufacturing
Non-ferrous metal rolling and stamping	Seafood processing
Rail equipment	Wool weaving and printing
Non-ferrous metal smelting	Cotton and polyester weaving and printing

Notes: The table ranks 66 manufacturing sectors by their sectoral sales gaps and shows the top and bottom 15 sectors.

as

$$(D/K)_i(\nu) \equiv \frac{\text{Total Liabilities}_i(\nu)}{\text{Total Capital Inputs}_i(\nu)}$$

and I define firm-level interest rate as

$$IntRate_i(\nu) \equiv 100 \times \frac{\text{Total Interest Payment}_i(\nu)}{\text{Total Liabilities}_i(\nu)}. \tag{44}$$

Holding private marginal return to capital $\left(\varphi_i^K\right)^{-1}$ constant, the social return to capital inputs is higher in sectors with higher sales gaps. Welfare-enhancing policies should direct more working capital towards these sectors, potentially through a combination of targeted and subsidized lending. While sectors that have higher average D/K ratios across firms can be interpreted through the model as those that tend to receive more external loans[9], interest rates do not have a natural counterpart in the model because credit rationing is common in lending markets in China and interest rates do not clear the credit market. Nevertheless, lower average sectoral interest rates can be viewed as evidence of subsidized credit to the sector.

Table 3 compares the mean of these two measures of credit market intervention between private firms and SOEs. Columns (1) and (3) respectively regress the firm-level D/K ratio and interest rates on a dummy that captures whether the firm is state-owned, and columns (2) and (4) control for sector fixed effects for the respective outcome variable. The results reveal that SOEs receive significantly more favorable access to credit markets and are on average 9.1 percentage points higher in debt-to-capital

[9] The total working capital $W_i(\nu)$ can be modeled as the sum of entrepreneurial wealth $EW_i(\nu)$ and external liability $D_i(\nu)$ and we have $(D/K)_i(\nu) = \frac{D_i(\nu)}{W_i(\nu)}$. When $EW_i(\nu)$ do not vary systematically across sectors, firms in sectors with higher average D/K ratio tend to receive more external loans.

ratio, compared to a baseline of 54 percentage points for the private firms. SOEs also pay interest rates that are 2.1 percentage points lower relative to a baseline of 4.3 percentage points paid by their private counterparts[10]. These results are consistent with the assumption that SOEs are not subject to credit constraints when making production choices.

Tables 4 and 5 explore the correlation between sales gap and the two measures of credit market interventions using the following firm-level regression:

$$Outcome_i(\nu) \quad = \quad \delta^1 + \delta^2 \times \widehat{\xi}_i + \delta^3 \times \widehat{\left(\bar{\varphi}_i^K\right)^{-1}} + Controls_i + \epsilon_i(\nu).$$

Table 4 shows the outcome variable Debt-To-Capital ratio and Table 5 shows the firm-level interest rate. Columns (1) through (4) in both tables show results estimated on the sample of private firms, whereas the results in columns (5) through (8) are based on the sample of SOEs. All standard errors are clustered conservatively at the industry level.

A consistent pattern that emerges from Tables 4 and 5 is that private firms in sectors with higher sales gaps tend to have higher D/K ratios and tend to pay lower interest rates, suggesting that private firms in these sectors receive favorable access to credit markets. Moreover, neither of these intervention measures correlates with the average sectoral wedge $\left(\bar{\varphi}_i{}^K\right)^{-1}$, suggesting that it is the network inefficiency measure (sales gap) rather than the private marginal return to capital that predicts the cross-sector variation in credit market interventions.

The preferred specification in column (4) of both tables includes controls for firm-level capital intensities as well as the fraction of sectoral output that is used to form capital stock for future production. The former variable (K/Y) measures how intensively capital is used for production, while the latter ($CapForm$) measures the capital content of sectoral output and is the basis on which sectors are categorized as producing capital or material goods (c.f. Assumption 2). These controls are introduced to partially address the concern that D/K ratio might be mechanically driven by sectoral characteristics. While firms with higher capital intensities tend to have better access to external loans and receive lower interest rates, these control variables do not remove the correlation between sales gap and credit intervention measures: private firms in sectors with one percentage point higher sales gaps tend to have 0.0924 percentage point higher D/K ratio and tend to receive interest rates that are 0.0247 percentage point lower. Given that the standard deviation in the average sectoral D/K ratio and interest rate are 4.67 and 1.53 percentage points, respectively, these effects are quantitatively large: when comparing across sectors, those that are one standard deviation higher in sales gaps (24 percentage point as in Table 1) tend to have average D/K ratios that are 0.47 standard deviations higher and average interest rates that are 0.39 standard deviation lower.

Columns (5) through (8) in Tables 4 and 5 replicate the first four columns but are estimated using SOEs instead of private firms. While SOEs tend to have significantly better access to credit markets (as

[10]For interpretational purposes, I treat firms that report zero interest payments as missing data in the main text; all results that involve the interest rate variable are based on the subsample of firms that report positive total interest payments, which corresponds to 58% of firms in the full sample. In Appendix F.1, I show that my results are robust to using the entire sample of firms and interpreting firms with no recorded interest payment as those facing zero interest rate.

Table 3: SOEs have higher Debt-To-Capital ratios and pay lower interest rates

	$(D/K)_i(\nu)$		$IntRate_i(\nu)$	
	(1)	(2)	(3)	(4)
$\mathbf{1}(SOE_i(\nu))$	0.0912***	0.0937***	-2.125***	-2.016***
	(0.00227)	(0.00228	(0.0751)	(0.0743)
Constant	0.544***	-	4.252***	-
	(0.000505)	-	(0.0167)	-
Sector Fixed Effects	No	Yes	No	Yes
Obs.	296784	296784	172275	172275
adj. R^2	0.005	0.027	0.005	0.052

Notes: The table compares the firm-level Debt-to-Capital ratio (columns 1 and 2) and interest rate (columns 3 and 4) of private firms and SOEs. Columns (2) and (4) adds sector fixed effects to columns (1) and (3), respectively. Columns (1) and (2) drop outlier firms with Debt-To-Capital ratios that are either negative or above the 99th percentile, and columns (3) and (4) drop firms with interest rate that is either negative or above the 99th percentile.

in Table 3), the results in these columns of Table 5 show that the negative correlation between sales gap and interest rates is smaller in magnitude for the SOEs than that for the private firms. Furthermore, Table 4 reveals that SOEs' D/K ratios do not significantly correlate with the sectoral sales gap, unlike the private firms. This finding rules out the story that the correlation between sales gap and D/K ratio for private firms in each sector is driven by unobserved sectoral characteristics that determine firms' reliance on external debt, thus lending further credibility to our interpretation that more external loans are being directed to private firms in sectors with higher sales gaps.

The Sectoral Presence of SOEs I next examine the sectoral presence of SOEs. While they are not explicitly present in the theoretical model, SOEs can be seen as an indirect vehicle for the state to subsidize production when direct production subsidies are difficult to implement due to practical obstacles. A positive correlation between the presence of SOEs and the sectoral sales gap is therefore consistent with the hypothesis that SOEs have been placed strategically to expand sectoral production. I capture the presence of SOEs via the share of an industry's total wage payment that is contributed by SOEs:

$$SOEshr_i \equiv \frac{\text{Total wages paid by SOEs in industry } i}{\text{Total wages paid by all firms in industry } i}. \tag{45}$$

I refer to $SOEshr$ as the value-added share of SOEs because labor is the only source of net value-added in the model. In Appendix Table F.5 I show that our results are robust to using SOE's share of gross value-added (which includes wage payments as well as capital depreciation and variable profits) or total revenue to capture SOE presence.

Figure 3 plots the relationship between $SOEshr$ and sales gap for the 66 manufacturing industries,

Table 4: Private firms in sectors with high sales gaps receive more external loans

	Outcome Variable: Debt-To-Capital Ratio							
	Sample: Private Firms				Sample: SOEs			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\widehat{\xi}_i$	0.0711***	0.0705***	0.0710***	0.0924***	0.0600*	0.0567	0.0670**	0.0407
	(0.0225)	(0.0233)	(0.0237)	(0.0253)	(0.0354)	(0.0355)	(0.0335)	(0.0353)
$\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$		0.00779	0.00847	0.0105		0.0343	0.0316	0.0319
		(0.0258)	(0.0263)	(0.0255)		(0.0329)	(0.0327)	(0.0323)
$(K/Y)_i\,(\nu)$			0.0108***	0.0112***			0.000302***	0.000296***
			(0.00311)	(0.00305)			(0.000110)	(0.000108)
$CapForm_i$				-0.0388**				0.00400
				(0.0172)				(0.0247)
Constant	0.465***	0.455***	0.445***	0.421***	0.568***	0.525***	0.509***	0.534***
	(0.0288)	(0.0400)	(0.0415)	(0.0433)	(0.0469)	(0.0668)	(0.0638)	(0.0633)
Obs.	282126	282126	279060	279060	14658	14658	14211	14211
adj. R^2	0.003	0.003	0.004	0.004	0.001	0.002	0.003	0.003

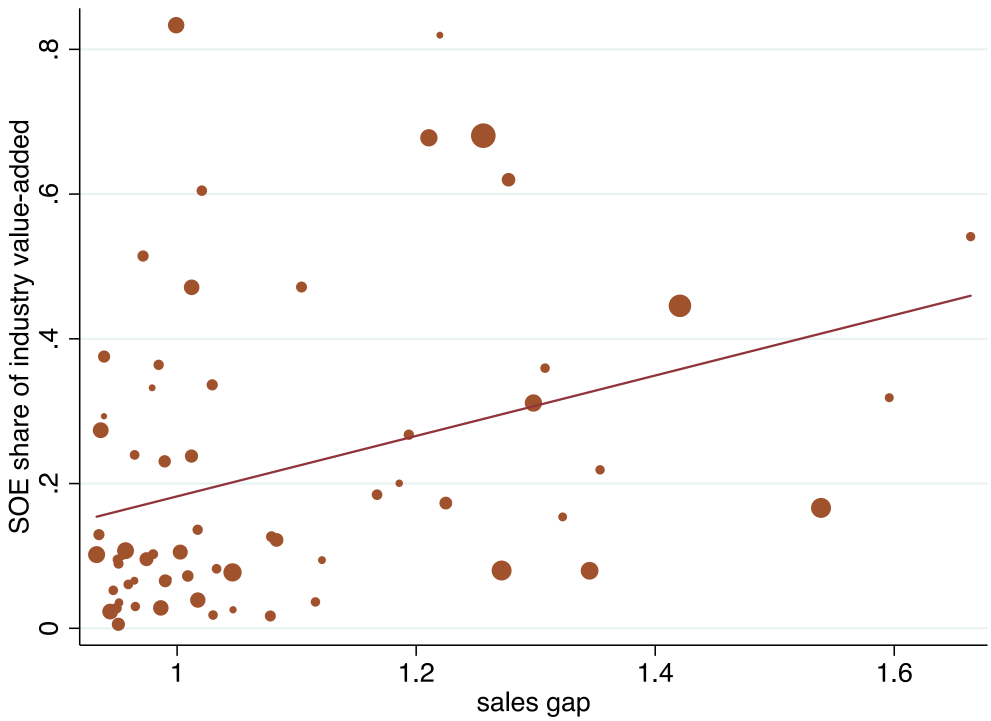
Notes: The table examines the correlation between firm-level Debt-to-Capital ratio and sectoral sales gap. Columns (1) through (4) are based on the sample of private firms while columns (5) through (8) are based on the sample of SOEs. $\widehat{\xi}_i$ is the sales gap measure, as in equation (39). $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ is the sectoral private return to capital inputs, as defined in equation (37). $(K/Y)_i(\nu)$ is the firm-level capital intensity, defined as the ratio between capital stock and firm revenue. $CapForm_i$ is the fraction of sectoral output that is unused in the accounting year and is to be used at a future time. All specifications drop outlier firms with Debt-to-Capital ratios that are either negative or above the 99th percentile. Columns (3), (4), (7), and (8) also drop outlier firms with capital intensities that are either negative or above the 99th percentile. Standard errors in parentheses are clustered at the industry level.

Table 5: Private firms in sectors with higher sales gaps pay lower interest rates

	Outcome Variable: Interest Rate							
	Sample: Private Firms				Sample: SOEs			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\widehat{\xi}_i$	-2.824***	-2.890***	-2.900***	-2.471***	-1.785***	-1.878***	-1.901***	-0.998***
	(0.817)	(0.814)	(0.789)	(0.806)	(0.296)	(0.286)	(0.282)	(0.253)
$\widehat{(\bar{\varphi}_i^K)^{-1}}$		0.832	0.733	0.765		0.890***	0.825***	0.814***
		(0.788)	(0.789)	(0.809)		(0.277)	(0.263)	(0.228)
$(K/Y)_i\,(\nu)$			-1.117***	-1.110***			-0.0872***	-0.0828***
			(0.179)	(0.178)			(0.0153)	(0.0148)
$CapForm_i$				-0.785				-1.407***
				(0.593)				(0.247)
Constant	7.384***	6.335***	7.470**	7.013***	4.136***	3.041***	3.317***	2.475***
	(1.043)	(1.030)	(1.077)	(1.136)	(0.364)	(0.380)	(0.373)	(0.295)
Obs.	163783	163783	162143	162143	8492	8492	8407	8407
adj. R^2	0.006	0.007	0.026	0.027	0.020	0.024	0.032	0.039

Notes: The table examines the correlation between firm-level interest rates (as defined in 44) and sectoral sales gaps. Columns (1) through (4) are based on the sample of private firms while columns (5) through (8) are based on the sample of SOEs. $\widehat{\xi}_i$ is the sales gap measure, as in equation (39). $\widehat{(\bar{\varphi}_i^K)^{-1}}$ is the sectoral private return to capital inputs, as defined in equation (37). $(K/Y)_i(\nu)$ is the firm-level capital intensity, defined as the ratio between capital stock and firm revenue. $CapForm_i$ is the fraction of sectoral output that is unused in the accounting year and is to be used at a future time. All specifications drop firms with either zero interest payment or interest rates that are above the 99th percentile. Columns (3), (4), (7), and (8) also drop outlier firms with capital intensities that are either negative or above the 99th percentile. Standard errors in parentheses are clustered at the industry level. Appendix Table F.4 replicates results in this table without dropping firms with zero interest payments.

Figure 3: Industries with higher sales gaps have more SOEs



in which the size of each point in the figure reflects the total value-added of each industry. The fitted line has a significantly positive slope: SOEs constitute a higher share of industry value-added when industries have higher sales gaps. The positive relationship is robust to using SOEs' revenue share as the outcome variable. Table 6 illustrates the same relationship between *SOEshr* and sales gap using linear regressions. Column (1) represents the same information as in figure 3, with the positive coefficient on sales gap being the slope of the fitted line in the figure. That is, sectors with one percentage point higher sales gaps tend to have 0.418 percentage point higher SOE share of sectoral value-added. Column (2) examines the relationship between *SOEshr* and $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$, the sectoral private return to capital goods, while column (3) regresses *SOEshr* on both $\widehat{\xi}$ and $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$. These two specifications show that *SOEshr* is largely uncorrelated with the sectoral private return and that its correlation with the sales gap $\widehat{\xi}$ remains significant and unaffected after controlling for $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$. These results imply that SOEs do not have greater presence in sectors that are themselves very constrained, lending empirical support to the theory that it is indeed the network inefficiency captured by the sufficient statistic $\widehat{\xi}$ rather than the within-sector inefficiency that determines SOE presence. These findings echo the results in Tables 4 and 5.

Columns (4) and (5) of Table 6 refine the regression specification in column (3) by progressively adding sector-level control variables. The coefficient on sales gap remains largely unchanged in column (4), which controls for the capital intensity averaged across firms in the sector. Column (5) controls for $CapForm_i$, the fraction of sectoral output that is used to form future capital stock. While the coefficient on sales gap becomes smaller in magnitude for this specification, it remains marginally significant with p-value less than 10%, and it is statistically indistinguishable from the coefficients in columns (1) through (4). Recall that *CapForm* is the measure based on which we categorize industries into capital goods producers and material goods makers, and in columns (6) and (7) I investigate which of these two industry groups drives the positive relationship between sectoral SOE presence and sales gap measure. Column (6) performs the specification in column (5) on the subsample of industries that produce material goods, and column (7) does the same on the capital goods industries. The results in these specifications show that the correlation between *SOEshr* and the sales gap $\widehat{\xi}$ is not driven by the set of capital goods producers; instead, it is driven by the variation of sales gap within the group of material goods producers. These are industries whose output is not directly subject to credit constraints, and the variation in their sales gap is solely driven by the indirect network effect of financial frictions.

Taken together, my results in this section show that distortions in input-output linkages among the Chinese manufacturing sectors are quantitatively important, as the sales gap causes the social return to capital to be on average 12 percentage points higher than the private return. I find that private firms in sectors with higher sales gaps tend to receive more external loans and pay lower interest rates, and that Chinese SOEs are heavily directed towards sectors with larger sales gaps rather than sectors that are most constrained or have the highest private return to capital. My results suggest these interventions can be potentially welfare-enhancing because such policies effectively subsidize upstream industries (or those with large sales gaps) and as a result, these policies could address pecuniary externalities and ameliorate inefficiencies due to credit constraints faced by downstream producers. While I do not argue that my model captures the decision-making process of Chinese policymakers, my findings do allow for a positive reappraisal of the selective state interventions in the Chinese manufacturing sectors and provide a counterpoint to the prevailing view (e.g. Song et al 2011) that SOEs are a sign of sectoral inefficiency.

4 Cross Country Analysis

> Each of the industries in this combined [input-output] table has its own peculiar input requirements, characteristic of that industry not only in the United States and in Europe but also wherever it happens to be in operation. The recipe for satisfying the appetite of a blast furnace, a cement kiln, or a thermoelectric power station will be the same in India or Peru as it is, say, in Italy or California. In a sense the input-coefficient matrix derived from the U.S.-European input-output table represents a complete cookbook of modern technology. It constitutes, without doubt, the structure of a fully developed economy insofar as development has proceeded anywhere today. An underdeveloped economy can now be defined as underdeveloped to the extend that it lacks the working parts of this system.
>
> — Leontief, "The Structure of Development", 1963.

I now turn to the second empirical exercise, in which I construct the sales gap measure for a set of developing countries using a panel of cross-country input-output tables. I then examine the country-level correlation between the sales gap and a proxy measure of sectoral production tax rates.

Table 6: SOEs have higher value-added shares in industries with high sales gaps but not in those that are constrained or have high private returns to spending on capital.

	SOE share of industry value-added						
	All industries					Producers of material goods	Producers of capital goods
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
$\widehat{\xi}_i$	0.418***		0.417***	0.417***	0.299*	0.943**	0.112
	(0.143)		(0.145)	(0.145)	(0.173)	(0.275)	(0.251)
$\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$		0.0373	0.00671	-0.0114	0.0121	-0.0433	-0.0272
		(0.106)	(0.101)	(0.103)	(0.104)	(0.111)	(0.265)
$\overline{(K/Y)}_i$				-0.00133	-0.00136	0.196*	-0.000744
				(0.00142)	(0.00141)	(0.112)	(0.00154)
$CapForm_i$					0.211	-4.315	0.641**
					(0.171)	(8.136)	(0.252)
Constant	-0.236	0.168	-0.243	-0.215	-0.137	-0.878**	-0.0334
	(0.157)	(0.144)	(0.198)	(0.200)	(0.209)	(0.352)	(0.345)
Obs.	66	66	66	66	66	46	20
adj. R^2	0.104	-0.014	0.090	0.088	0.096	0.157	0.218

Notes: The table examines the correlation between sector-level SOE share (as defined in 45) and the sectoral sales gap. Columns (1) through (5) are based on the sample of all 66 sectors; column (6) is based on the subsample of 46 sectors that produce material goods; column (7) is based on the subsample of 20 sectors that produce capital goods. $\widehat{\xi}_i$ is the sales gap measure, as in equation (39). $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ is the sectoral private return to capital inputs, as defined in equation (37). $\overline{(K/Y)}_i$ is the average capital intensity of firms in the sector. $CapForm_i$ is the fraction of sectoral output that is unused in the accounting year and is to be used at a future time. Appendix Table F.5 replicates these results by using alternative measures of SOE share.

4.1 Data Source

The main data source for my cross-country analysis is the national IO tables from OECD TiVA data set, which includes the national IO tables of 60 countries from 1995 to 2011. For the purpose of this analysis, I categorize countries into three groups according to IMF's World Economic Outlook (IMF 2015): 24 more developed countries (MDC), 13 graduated developing countries (GDC), and 23 less developed countries (LDC). I sometimes refer to the combined MDCs and GDCs as *developed countries*. Table 11 provides a list of countries in the data set.

The input-output tables in the data are based on each individual country's Supply and Use Table (SUT), which measures the flow of goods among industries, typically at a relatively disaggregated level (for example, the previous structural exercise with Chinese data is based on an input-output table with 135 sectors). Despite the fact that most of these countries' statistical departments notionally follow United Nations System of National Accounts (SNA) guidelines, in practice the accounting standards and practices vary across countries, so their SUTs and IO tables are not directly comparable. Researchers on the OECD TiVA project took on the painstaking job of harmonizing the national SUTs into IO tables that are comparable across countries with 33 aggregated sectors, listed in Table 12, of which 15 are manufacturing industries. By my best judgement, the mapping of sectors in the data set remains imperfect. The main problem comes with the sectors "renting of machinery and equipment" and "machinery and equipment"; in some countries the former sector's output is an order of magnitude smaller than the latter, while the pattern reverses for other countries. It is only after these two sectors are merged that their total combined output becomes comparable across countries. For this reason, I merge these two sectors label the combination "machinery and equipment" for my analysis.

There are two other data repositories for cross-country IO tables, namely the World Input-Output Database (WIOD) and Global Trade Analysis Project (GTAP). WIOD is constructed in a similar way as the OECD data, starting from the national SUTs and harmonized to match national account statistics. I do not use WIOD because it includes few developing countries, which are the focus of my analysis. The GTAP, on the other hand, is a primarily a database on bilateral trade information. Although in principle one can extract IO tables from GTAP under a more disaggregated industrial classification and for a wider range of countries, the data are constructed primarily for a different purpose and rely more heavily on imputation. As a result, the data quality of the IO tables extracted from GTAP is lower than the OECD data. For this reason, I do not use GTAP in this analysis.

4.2 Sales Gap

I take this section's opening quotation from Leontief (1963) seriously and infer the elasticity matrix and sales gap measure for the set of *developing* countries based on the observed input-output tables from the set of *developed* countries. Recall Σ denotes the input-output elasticity matrix, Ω denotes the observed input-output table, and β denotes the vector of final shares defined as (42). In what follows, I use subscript c to refer to countries, t to refer to years, and i, j to refer to sectors.

Assumption 4. *I make the following technology assumptions throughout this section.*

1. *Firms have Cobb-Douglas production technologies such that β and Σ are exogenous and do not change in response to allocations.*

2. *Firms in developed countries (MDCs and GDCs) are unconstrained and $\Sigma_{ct} = \Omega_{ct}$ for all t and $c \in MDC \cup GDC$.*

3. *For all t and $c \in MDC \cup GDC$, Σ is a function of observable country characteristics X_{ct}, with entries $\sigma_{ct,ij}(X_{ct};\theta)$ parametrized by θ.*

Under Assumption 4, I can predict the unconstrained input-output table of the developing countries using their country characteristics and the observed Σ's for MDCs and GDCs. I implement the strategy by regressing entries of the Leontief inverse of Σ_{ct} on the log-population for the set of developed countries, allowing for entry-year-specific constants and slopes. Specifically, I perform

$$(I-\Sigma_{ct})^{-1}_{ij} = \theta^0_{t,ij} + \theta^1_{t,ij} \cdot \ln Pop_{ct} + \epsilon_{ct,ij} \tag{46}$$

on $c \in MDC \cup GDC$ to form estimates $\widehat{\theta^0_{t,ij}}$ and $\widehat{\theta^1_{t,ij}}$. I then construct

$$\widehat{(I-\Sigma_{ct})^{-1}}_{ij} \equiv \widehat{\theta^0_{t,ij}} + \widehat{\theta^1_{t,ij}} \cdot \ln Pop_{ct} \tag{47}$$

for $c \in LDC$. I impute the influence vector μ and sales gap measure ξ as

$$\widehat{\mu'_{ct}} \equiv \frac{\beta'_{ct}\widehat{(I-\Sigma_{ct})^{-1}}}{\beta'_{ct}\widehat{(I-\Sigma_{ct})^{-1}} \cdot \alpha_{ct}} \tag{48}$$

$$\widehat{\xi_{ct,i}} \equiv \frac{\widehat{\mu_{ct,i}}}{\gamma_{ct,i}}, \tag{49}$$

where γ'_{ct} the sales vector computed according to (40) and β'_{ct} again reflects the vector of consumptions shares as in (42). For completeness, I also construct $\widehat{\xi}$ for $c \in MDC \cup GDC$ according to (47), (48), and (49). This imputed measure reflects differences in the predicted and actual input-output tables for these countries and it is used as a placebo check for some of my empirical specifications.

Population is included as a control variable since the size of an economy could determine its underlying production technologies, and I conservatively use population as the only characteristic to predict the input-output technology in order to avoid adopting variables that correlate with and potentially affect financial development or income levels. My results in this section are robust to either using population density rather than levels as the predictor or using only the year-entry fixed effects $\theta^0_{t,ij}$ to predict the Leontief inverse in equations (46) and (47). The results are also robust to the alternative assumption that only firms in MDCs (but not GDCs) are unconstrained.

The assumption that undistorted production technologies can be imputed based on data from developed countries is a popular approach adopted by other studies that conduct cross-country comparison of industries, including Rajan and Zingales (1998) and Hsieh and Klenow (2009). In a study more related to mine, Bartelme and Gorodnichenko (2015) also assume that wealthy nations' IO tables represent undistorted technologies and impute that for developing countries in some of their analysis.

I now discuss the reduced form patterns of sales gap across countries. First as a validation check, in table 7 I list the 5 top and bottom industries, ranked by sales gap, among the 15 manufacturing industries in the OECD data for China in 2007. On the left side, with the exception of "food, beverages, and tobacco", a puzzling outlier, the industries with high sales gaps belong to the heavy manufacturing category, which produces either capital goods or materials used for capital goods production, and it is the same set of industries that are found to have high sales gaps in the previous structural exercise based on micro data. The set of manufacturing industries that have low sales gap, which are listed on the right side of the table, consists mostly of light manufacturing industries (with the exception of the

Table 7: Top and Bottom 5 Manufacturing Industries Ranked by Sales Gap in China

Top 5 Sales Gap	Bottom 5 Sales Gap
Fabricated metal products	Wood products
Misc. transport equipment	Rubber and plastic products
Food, beverages, and tobacco	Coke and petroleum products
Motor vehicles, trailers	Electrical machinery and apparatus
Machinery and equipment	Textiles, leather, footwear

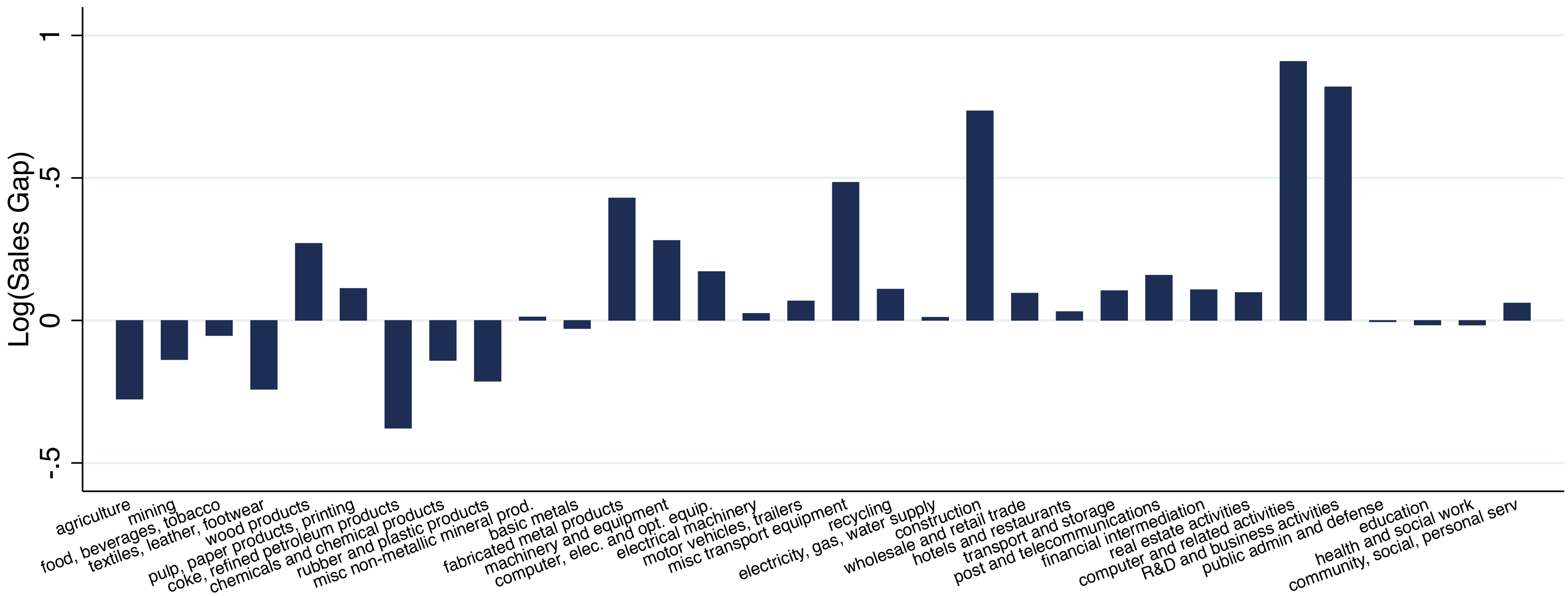
Notes: The table ranks the 15 manufacturing sectors in the OECD data by their sectoral sales gaps in China and shows the top and bottom 5 sectors.

"electrical machinery and apparatus" industry). Despite the fact that these two exercises are performed under very different set of assumptions and on distinct data at different levels of aggregation, there is a consistent broad pattern—that heavy manufacturing industries in China have higher sales gaps than light manufacturing industries—which lends credibility to the empirical results.

Figure 4 plots the log-sales gap by sector averaged for the LDCs for year 2011, the latest year in the OECD data. The sectors are labeled on the horizontal axis and arranged according to the underlying ISIC rev 4 industry codes, from primary and light manufacturing industries on the left to heavy manufacturing and tertiary or service industries on the right. A taller bar on the figure represents a higher sales gap, with a bar above (or below) zero indicating that the industry size is too small (or large) relative to its potential sales. The broad pattern in this figure reveals that as a group, developing countries tend to have lower sales gaps in primary and light secondary industries, including agriculture, mining, and the manufacturing of consumer goods such as "textiles, leather, and footwear", "basic chemicals", and "non-metallic mineral products". The LDCs tend to have higher sales gaps in the tertiary sectors and heavy manufacturing industries, such as those that produce "fabricated metal products" and "machinery and equipment", as well as businesses that provide computer services (which includes software companies) and businesses that support R&D and business activities (including legal and consulting companies).

Independent of the theoretical importance of sales gap in the model, the pattern in figure 4 is striking because both influence and sales measures are constructed using each country's own final shares β_{ct}. The difference between sectoral influence and sales, encapsulated in the sales gap measure shown in the figure, reflects only the differences between the observed input-output tables and the ones predicted based on developed countries' IO tables. It is well known that richer countries tend to have larger tertiary sectors while poorer countries tend to have a bigger share of their economies in primary sectors and light industries, but the conventional wisdom is that such differences in relative sectoral size across countries represent either variations in specialization or non-homothetic preferences, both of which boil down to differences in the final shares across countries (Comin et al. 2016, Buera and Kaboski 2012). This figure shows that this argument is incomplete: some variation in relative sectoral size across countries with different income levels reflects differences in their underlying input-output linkages across sectors.

Figure 4: Average Sectoral Log(Sales Gap) For LDCs



4.3 Sectoral Production Subsidies

Even though industrial policies are pervasive in developing countries, they are difficult to quantify with data because they almost always are implemented via a multiple explicit and implicit policy supports on many margins that could affect sectoral production. In this cross-country exercise, I turn to a measure of sectoral net (of subsidies) production taxes, recorded as part of the national input-output tables, which are available for 53 of the 60 countries in the dataset. These net taxes are net transfers to fiscal authorities incurred during production, and broadly include all miscellaneous indirect taxes and subsidies that are not commodity taxes (i.e. sales tax and value-added tax). According to the System of National Accounting (United Nations Department of Economic and Social Affairs, 1999), the national account construction standard that most countries in the data notionally follow, these net taxes should include payroll taxes (and subsidies), stamp duties, taxes for business licenses, taxes on energy use and the use of automobiles, property taxes, pollution taxes, and any monetary grant paid by government agencies to private businesses, etc.. The tax rate measure created from this data is indeed a noisy measure for two reasons. First, the tax data do not distinguish among the exact margins on which the taxes or subsidies are levied, and nor do I model some of the finer details of real-world production, such as business licensing. Second, the exact underlying taxes that are aggregated to this measure differ across countries not only because tax systems are not completely comparable, particularly among developing countries, but also because countries follow the SNA guidelines to varying degrees. One major outlier is China, whose ratio of taxes to value-added is significantly higher than that of other countries. The reason is that, unlike that of other countries, the Chinese SUT compilation aggregates all indirect net taxes to this measure, including sectoral value-added tax, sales tax, and business tax (which is sales tax paid by businesses rather than consumers) in addition to the taxes outlined by the SNA standard. In the analysis below, I abstract away from the heterogeneous channels through which sectoral polices can be implemented. Instead, I take a naive approach and map the data to the model by simply assuming all taxes recorded in this variable are levied on labor, the source of net value-added in the model. The measure of sectoral subsidies is constructed according to

$$1 + \tau^L_{c,i} \equiv \frac{\text{Wage Payment}_{c,i}}{\text{Wage Payment}_{c,i} + \text{Net Producton Taxes}_{c,i}}. \tag{50}$$

The measure $1 + \tau^L_{c,i}$ is constructed such that $1 + \tau^L_{c,i} > 1$ represents a subsidy and < 1 represents a tax. In what follows, the words "subsidy" and "tax" are used interchangeably, thereby recognizing that one is the inverse of the other. Results are reported based on dropping 1% tail on either end of the subsidy measure, and the results are robust to winsorizing instead of trimming the tails. To partially address the measurement error induced by varied accounting practices, all of the reported regression results in this section conservatively include country-by-year fixed effects, thus purging systematic differences in accounting standards across countries and time. My results are robust to excluding China.

4.4 Results

Recall again that the sales gap measure captures the ratio between social and private marginal return to expenditure on production inputs, and if network inefficiencies are of concern to fiscal authorities when designing tax policies, sectors with high sales gap should have higher subsidies $1 + \tau_i^L$ (or lower taxes). To check whether this pattern holds true in the data, we perform regressions of the form

$$Log\left(1 + \tau_{ct,i}^L\right) = \eta_0 \cdot Log\left(\widehat{\xi}_{ct,i}\right) + \zeta_{ct} + \delta_i + \epsilon_{ct,i}, \tag{51}$$

where I regress log production subsidies on log sales gap for different subsamples of countries, controlling for a full set of sector fixed effects as well as country-by-year interacted fixed effects. The sector fixed effects are introduced to eliminate the sectoral characteristics that could otherwise create variations in tax rates absent network inefficiencies; for example, when certain sectors are taxed more heavily due to pollution and other externalities. The country-by-year fixed effects are used to purge systematic differences in tax rates across countries and time, reflecting not only cross-country and temporal variation in fiscal capacity and tax optimality but also the varied accounting standards. Introducing country-by-year interacted fixed effects does not affect the interpretation because my theory suggests that if the subsidies are indeed levied on net value-added margin and are rebated back to consumers as a lump-sum transfer, multiplying subsidies across all sectors by the same constant (which translates into adding a constant in logs) does not affect allocations and it is only the relative subsidies across sectors that matter for a given country at a given time.

I perform regression (51) on the sample of 27 sectors rather than the full set of 33 sectors. I exclude the public sectors "public admin and defense; social security", "education", and "health and social work" because these are less relevant to my theory. I exclude the "agriculture" and "coke, refined petroleum products and nuclear fuel" sectors because the wage payments recorded in these two sectors are orders of magnitude too small relative to sectoral output for a number of countries and years, likely due to different accounting practices in these sectors relative to others, and, consequently, a significant number of observations in these two sectors lie outside of the 1% tail in the subsidy measure and get dropped as the tails are trimmed. My results are robust to include any or all five of the omitted sectors.

My results are shown in Table 8 with standard errors clustered conservatively at the country level, recognizing the potential correlations of estimation residuals within each country across sectors and time. Column (1) estimates regression (51) on the sample of developing countries (LDCs). The coefficient on the sales gap measures is positive and highly significant, thereby indicating that, on average and across the set of developing countries in my sample, sectors with higher sales gap tend to have higher subsidies or lower tax rates, which is consistent with the interpretation that when designing tax policies, fiscal authorities in some of these countries recognize the importance of network inefficiencies that distort the relative sector size. Column (2) estimates (51) based on the sample of countries in $GDC \cup MDC$, for which the imputed sales gap measure reflects differences between the predicted and actual input-output tables for these countries. This specification is performed as a placebo check: if firms in developed countries are truly unconstrained, there is no reason for their sectoral production tax rates to correlate with

$Log\left(\widehat{\xi}\right)$, which can be interpreted as estimation error. Indeed, the coefficient on $Log\left(\widehat{\xi}\right)$ is much closer to and statistically indistinguishable from zero.

Columns (3) through (5) report results from estimating (51) using an alternative measure of sales gap $\widehat{\xi}^{MDC}$, calculated by modifying Assumption 4 and assuming that only firms in More Developed Countries are unconstrained. Specifically, the measure is constructed by estimating equation (46) using only MDCs and excluding GDCs such as Hong Kong, Singapore, South Korea, and Taiwan. These three columns are then estimated on the sample of LDCs, GDCs, and MDCs, respectively. The coefficient on $Log\left(\widehat{\xi}^{MDC}\right)$ for the LDC sample is significant and statistically indistinguishable from the coefficient in column (1), while those in columns (4) and (5) are indistinguishable from zero. Note that these close-to-zero coefficients are not mechanical: for example, results in column (4) imply that the sectoral tax rates in GDCs do not correlate with the sales gap measures based on differences between their IO tables and the IO tables of the MDCs, thereby indicating that financial constraints are not a first-order concern in designing tax policies in the GDCs.

Table 8: On average, LDCs have higher subsidies in sectors with larger sales gap

	$Log\left(1+\tau^L\right)$				
	LDC	GDC+MDC	LDC	GDC	MDC
	(1)	(2)	(3)	(4)	(5)
$Log\left(\widehat{\xi}\right)$	0.0125**	0.00154			
	(0.00580)	(0.00307)			
$Log\left(\widehat{\xi}^{MDC}\right)$			0.0131**	0.00263	0.00244
			(0.00578)	(0.00472)	(0.00595)
Obs.	8278	14446	8278	5461	8985
adj. R^2	0.557	0.481	0.557	0.426	0.544

Notes: The table examines the correlation between sectoral sales gap and sectoral net production subsidies, as defined in (50), for a panel of countries between 1995 and 2011. Columns (1) and (3) are based on the sample of less developed countries; column (2) is based on the sample of graduated developing countries and more developed countries; column (4) is based on the sample of graduated developing countries; lastly, column (5) is based on the sample of more developed countries. $Log\left(\widehat{\xi}\right)$ is the sectoral sales gap measure constructed under the assumption that firms in both GDCs and MDCs are unconstrained, whereas $Log\left(\widehat{\xi}^{MDC}\right)$ is the sectoral sales gap measure constructed under the assumption that firms in MDCs are unconstrained. All specifications drop outlier sectors for which the net production subsidy measure lies below the 1st or above the 99th percentiles. All specifications include country-by-year and sector fixed effects. Standard errors in parentheses are clustered at the country level.

Next, I look within the LDCs and examine whether the correlation between sales gap and production subsidies is more prominent for any particular subset of countries. First, I break the countries into four groups: developing countries in 1) Asia, 2) Latin America, 3) Eastern Europe and the Middle East, and 4) Africa. There are 6 countries in each of the first 3 categories, and 2 countries in the African group. I then perform a regression pulling all 4 groups, using group dummies interacted with $Log\left(\widehat{\xi}\right)$ to capture different elasticity of tax rates for each group. The exact specification is as follows:

$$
\begin{aligned}
Log\left(1+\tau_{ct,i}^{L}\right) \quad = \quad & \beta_1 \cdot \mathbf{1}\left(\text{Asia}\right) \cdot Log\left(\widehat{\xi}_{ct,i}\right) \\
& +\beta_2 \cdot \mathbf{1}\left(\text{South America}\right) \cdot Log\left(\widehat{\xi}_{ct,i}\right) \\
& +\beta_3 \cdot \mathbf{1}\left(\text{Eastern Europe \& Middle East}\right) \cdot Log\left(\widehat{\xi}_{ct,i}\right) \\
& +\beta_3 \cdot \mathbf{1}\left(\text{Africa}\right) \cdot Log\left(\widehat{\xi}_{ct,i}\right) \\
& +\alpha_{ct} + \delta_i + \epsilon_{ct,i}.
\end{aligned} \tag{52}
$$

The results are reported in table 9. The coefficient on $Log\left(\widehat{\xi}\right)$ is significantly different from zero only for the Asian developing countries, and the coefficients are actually slightly negative, though indistinguishable from zero, for the other three groups of countries. The result is striking: the set of Asian developing countries for which we have tax data includes China, Indonesia, Malaysia, Philippines, Thailand, and Vietnam, a group that as a whole has better economic performance over the past decade than the developing countries in the other groups. What is perhaps even more striking occurs when I re-estimate (52) and use country-specific dummies interacted with sales gaps; the coefficients and standard errors are in table 10. Good economic performers such as China, Vietnam, and Thailand have significantly positive coefficients while poor performers such as Turkey and Tunisia have significantly negative coefficients. The coefficient even picks up some variations within the Latin America region: Chile, a country with relatively good economic performance in the last decade, has a positive coefficient, while countries that suffered from weaker growth have negative coefficients.

There are two caveats in interpreting the results. First, the reduced form evidence presented in this section is on the slope of sectoral subsidies as a function of sales gap: for some countries, sectors with higher sales gap tend to have higher subsidies or lower taxes. This does not imply that these countries get the level of subsidies right: in fact, in order to fully address network inefficiencies, for many countries the subsidies need to be orders of magnitude higher than they currently are. This is hardly surprising, as sales gap is an imputed measure and is potentially noisy, thus creating attenuation bias in the estimation and weakening the coefficients towards zero. Moreover, as discussed in the theory section, tax implementation in the real world faces a wealth of practical constraints that limit the scope of intervention, and it is precisely these practical constraints and limitations that make my marginal intervention results valuable.

Second and most importantly, I emphasize that the production subsidy measure adopted in this cross-country analysis is only a noisy measure of actual subsidies and state interventions, and I take caution in interpreting the results too strongly or causally. Nevertheless, the consistent pattern that emerges from the results in this section is indicative of the importance of the network inefficiencies that my theory highlights, and my empirical findings are consistent with the hypothesis that governments in countries with strong economic performance understand the network distortions and are adopting policies to address them.

Table 9: On average, Asian LDCs have higher subsidies in sectors with larger sales gap

	$Log\left(1+\tau^{L}\right)$
$\mathbf{1}\,(\text{Asia}) \times Log\left(\widehat{\xi}\right)$	0.0374***
	(0.0118)
$\mathbf{1}\,(\text{South America}) \times Log\left(\widehat{\xi}\right)$	-0.00784
	(0.0118)
$\mathbf{1}\,(\text{Eastern Europe \& Middle East})$	-0.00347
$\times Log\left(\widehat{\xi}\right)$	(0.0116)
$\mathbf{1}\,(\text{Africa}) \times Log\left(\widehat{\xi}\right)$	-0.00881
	(0.0158)
Obs.	8278
adj. R^2	0.562

Notes: The table examines the correlation between the sectoral sales gap and sectoral net production subsidies, as defined in (50), for a set of less developed countries between 1995 and 2011. $Log\left(\widehat{\xi}\right)$ is the sectoral sales gap measure constructed under the assumption that firms in both GDCs and MDCs are unconstrained. All specifications drop outlier sectors for which the net production subsidy measure lies below the 1st or above the 99th percentile. All specifications include country-by-year and sector fixed effects. Standard errors in parentheses are clustered at the country level.

5 Conclusion

In this paper I construct a model of a production network in which firms purchase intermediate goods from each other in the presence of credit constraints. I show that these constraints distort input choices, reducing demand for upstream goods and creating a wedge between the influence and sales of upstream sectors. I further show that the ratio between the influence and sales, which I define as the sectoral sales gap, is a sufficient statistic for inefficiencies in a network and it captures the ratio between social and private marginal return to spending resources on production inputs and credit.

I conduct two distinct empirical exercises in which I estimate the sales gap measure and examine its correlations with proxy measures of government interventions into the sector. In the context of China, I estimate the sales gap of manufacturing sectors based on firm-level production data. I find that private firms in sectors with higher sales gaps tend to receive more external loans and pay lower interest rates, and that the sectoral presence of Chinese SOEs is heavily directed towards sectors with larger sales gaps rather than sectors that are most constrained or have the highest private return to capital. My theory shows that these interventions can be welfare-enhancing because they effectively subsidize upstream industries (or those with large sales gaps), and, as a result, these policies could address pecuniary externalities and ameliorate inefficiencies due to credit constraints faced by downstream producers. My findings therefore allow for a positive reappraisal of China's selective state interventions.

My second empirical exercise uses a panel of cross-country input-output tables to impute sales gaps for developing countries. I show that, for developing countries in Asia, the sectoral sales gap measure correlates with a measure of sectoral production subsidies, while the pattern is absent or even reversed in developing countries from other continents, which on average have had worse economic performances in recent years than their Asian counterparts. These results are consistent with the hypothesis that governments in countries with strong economic performance understand network distortions and are adopting policies to address them.

The model I present in this paper is static in nature and assumes credit constraints are imposed exogenously. A natural question to ask is whether constraints would persist if agents can save. In a related line of inquiry, I answer this question by studying a multi-sector growth model with inter-sectoral linkages and credit constraints. Entrepreneurs rationally make consumption and saving decisions, understanding that sector-specific capital stock is used both as a factor of production and also as a storage of value to serve as collateral for purchasing constrained inputs. My analysis suggests that endogenous saving is not sufficient for the economy to grow out of credit constraints. The economy features a unique equilibrium with many steady states or poverty traps, each with different levels of sectoral capital and output. The reason for stagnant economic development is demand externality: the return to saving in one sector depends on the future demand for its output, which in turn depends on the credit constraints in downstream sectors and the size of their capital stock. In a stagnant economy, capital stock is low across many sectors, and all sectors in the economy have low incentives to save. My model thus provides a dynamic, credit-based microfoundation of the "big push" theory of Rosenstein-Rodan (1943). Although it is commonly held that a "big push" environment requires large and sustained government-led investment across many sectors to take a country out of stagnation, I study development policy in this environment and show that temporary government intervention in the bottleneck sectors is sufficient to place the economy back on the path of development. The optimal development path might feature long periods of unbalanced growth as hypothesized by Hirschman (1958).

Table 10: Regression of sectoral subsidies on sales gap, by country

		Asia			
China	0.0870***	Indonesia	0.0160**	Malaysia	-0.00675
	(0.0108)		(0.00589)		(0.0104)
Philippines	0.0303***	Thailand	0.0663***	Vietnam	0.0181***
	(0.00579)		(0.00561)		(0.00534)
		Latin America			
Argentina	-0.0229**	Brazil	-0.0324**	Chile	0.0280**
	(0.00923)		(0.0124)		(0.00894)
Colombia	-0.0747***	Costa Rica	-0.00314	Mexico	0.00876
	(0.0108)		(0.00665)		(0.00896)
		Eastern Europe & Middle East			
Bulgaria	-0.0133	Croatia	-0.00352	Hungary	-0.00952
	(0.00814)		(0.00879)		(0.0117)
Poland	-0.0186	Saudi Arabia	0.0121	Turkey	-0.0212**
	(0.0151)		(0.00709)		(0.00826)
		Africa			
Tunisia	-0.0633***	South Africa	0.00314		
	(0.0109)		(0.00640)		

Notes: The table reports coefficients from regressing sectoral net production subsidies, as defined in equation 50, on country-specific dummy variables interacted with sectoral sales gaps, based on a sample of developing countries between 1995 and 2011. The reported coefficients reflect country-specific slopes of net sectoral subsidies on sales gaps. All specifications drop outlier sectors for which the net production subsidy measure lies below the 1st or above the 99th percentiles. All specifications include country-by-year and sector fixed effects. Standard errors in parentheses are clustered at the country level.

Table 11: Countries in the OECD IO Table Dataset

More Developed Countries (MDC)

Australia	Finland	Italy	Portugal
Austria	France	Japan	Spain
Belgium	Germany	Luxembourg	Sweden
Brunei Darussalam*	Greece	Netherlands	Switzerland*
Canada	Iceland*	New Zealand	United Kingdom
Denmark	Ireland	Norway	United States

Graduated Developing Countries (GDC)

Hong Kong (China)	Israel	Republic of Korea	Taiwan
Cyprus*	Latvia	Singapore	
Czech Republic	Lithuania	Slovakia	
Estonia	Malta	Slovenia	

Less Developed Countries (LDC)

Argentina	Colombia	Malaysia	South Africa
Bulgaria	Costa Rica	Mexico	Thailand
Brazil	Croatia	Philippines	Tunisia
Cambodia*	Hungary	Poland	Turkey
Chile	India*	Russia*	Vietnam
China	Indonesia	Saudi Arabia	

* There are no data on production taxes for these countries.

Table 12: Non-Public Sectors in the OECD IO Table Dataset

33 Sectors in the OECD IO Tables
Agriculture
Mining
Food products, beverages, and tobacco
Textiles, leather, footwear
Wood products
Pulp, paper products, printing
Coke, refined petroleum products and nuclear fuel
Chemicals and chemical products
Rubber and plastic products
Other non-metallic mineral products
Basic metals
Fabricated metal products
Machinery and equipment
Computer, electronic and optical equipment
Electrical machinery and apparatus
Motor vehicles, trailers
Other transport equipment
Recycling
Electricity, gas and water supply
Construction
Wholesale and retail trade
Hotels and restaurants
Transport and storage
Post and telecommunications
Financial intermediation
Real estate activities
Renting of machinery and equipment*
Computer and related activities
R&D and business activities
Public admin and defense; social security
Education
Health and social work
Other community, social, and personal service

* This sector is merged with "Machinery and equipment" sector for all of the analysis in this section.

References

Acemoglu, Daron, Ufuk Akcigit, and William Kerr (2016). Networks and the Macroeconomy: An Empirical Exploration. *NBER Macroeconomics Annual*.

Acemoglu, Daron, Vasco Carvalho, Asuman Ozdaglar, and Alireza Tahbaz-Salehi (2012). The Network Origins of Aggregate Fluctuations. *Econometrica*.

Aghion, Philippe, Jing Cai, Mathias Dewatripont, Luorsha Du, Ann Harrison, and Patrick Legros (2015). Industrial Policy and Competition. *American Economic Journal: Macroeconomics*.

Ahmad, Ehtisham and Nicholas Stern (1984). The Theory of Reform and Indian Indirect Taxes. *Journal of Public Economics*.

Ahmad, Ehtisham and Nicholas Stern (1991). *The Theory and Practice of Tax Reform in Developing Countries*. Cambridge: Cambridge University Press.

Altinoglu, Levent (2015). The Origins of Aggregate Fluctuations in a Credit Network Economy. *Working Paper*.

Amsden, Alice (1989). *Asia's Next Giant: South Korea and Late Industrialization*. Oxford University Press.

Antras, Pol, Davin Chor, Thibault Fally, and Russell Hillberry (2012). Measuring the Upstreamness of Production and Trade Flows. *American Economic Review Papers and Proceedings*.

Arrow, Kenneth (1951). Alternative proof of the substitution theorem for Leontief models in the general case. *Activity Analysis of Production and Allocation, ed. T.C. Koopmans.*.

Arrow, Kenneth and Frank Hahn (1971). *General Competitive Analysis*. North Holland.

Bai, Chong-En, Chang-Tai Hsieh, and Zheng Michael Song (2014). Crony Capitalism with Chinese Characteristics. *Working paper*.

Banerjee, Abhijit and Benjamin Moll (2010). Why does misallocation persist? *American Economic Journal: Macroeconomics*.

Banerjee, Abhijit and Esther Duflo (2005). Growth Theory Through the Lens of Development Economics. *Handbook of Economic Growth*.

Baqaee, David (2015). Labor Intensity in an Interconnected Economy. *Working Paper*.

Baqaee, David (2016). Cascading Failures in Production Networks. *Working Paper*.

Bartelme, Dominick and Yuriy Gorodnichenko (2015). Linkages and Economic Development. *NBER Working Paper*.

Bigio, Saki and Jennifer La'O (2016). Financial Frictions in Production Networks. *NBER Working Paper*.

Boyreau-Debray, Genevieve and Shang-Jin Wei (2005). Pitfalls of a State-Dominated Financial System: The Case of China. *NBER Working Paper*.

Brandt, Loren and Thomas G. Rawski (2008). China's Great Economic Transformation: Edited by Loren Brandt and Thomas G. Rawski.

Buera, Francisco and Joseph Kaboski (2012). The Rise of the Service Economy. *American Economic Review*.

Buera, Francisco, Joseph Kaboski, and Yongseok Shin (2011). Finance and Development: A Tale of Two Sectors. *American Economic Review*.

Buera, Francisco, Joseph Kaboski, and Yongseok Shin (2015). Entrepreneurship and Financial Frictions: A Macrodevelopment Perspective. *Annual Review of Economics*.

Chen, Xiaohong, Han Hong, and Denis Nekipelov (2011). Nonlinear Models of Measurement Errors. *Journal of Economic Literature*.

Chenery, Hollis and Tsunehiko Watanabe (1958). International Comparisons of the Structure of Production. *Econometrica*.

Chenery, Hollis, Sherman Robinson, and Moshe Syrquin (1986). *Industrialization and Growth: A Comparative Study, Edited by Hollis Chenery, Sherman Robinson and Moshe Syrquin*. Oxford University Press.

Cohen, Alex (2016). Constraints on Labor and Land and the Return to Microcredit and Microinsurance. *Working Paper*.

Comin, Diego, Danial Lashkari, and Marti Mestieri (2016). Structural Change with Long-run Income and Price Effects. *Working Paper*.

De Mel, Suresh, David McKenzie, and Christopher Woodruff (2016). Labor Drops: Experimental Evidence on the Return to Additional Labor in Microenterprises. *NBER Working Paper 23005*.

Deaton, Angus (1987). The Theory of Taxation for Developing Countries. *Econometric Issues for Tax Design in Developing Countries*.

Dervis, Kemal, Jaime de Melo, and Sherman Robinson (1982). *General Equilibrium Models for Development Policy*. The Press Syndicate of the University of Cambridge.

Dollar, David and Shang-Jin Wei (2007). Das (Wasted) Kapital: Firm Ownership and Investment Efficiency in China. *IMF Working Paper*.

Dupor, Bill (1999). Aggregation and Irrelevance in Multi-Sector Models. *Journal of Monetary Economics*.

Farhi, Emmanuel and Ivan Werning (2013). A Theory of Macroprudential Policies in the Presence of Nominal Rigidities. *Working Paper*.

Fisman, Raymond (2001). Trade Credit and Productive Efficiency in Developing Countries. *World Development*.

Franklin, Allen, Jun Qian, and Meijun Qian (2005). Law, Finance, and Economic Growth in China. *Journal of Financial Economics*.

Gale, David and Hukakane Nikaido (1965). The Jacobian Matrix and Global Univalence of Mappings. *Mathematische Annalen*.

Gandhi, Amit, Salvador Navarro, and David Rivers (2016). On the Identification of Production Functions: How Heterogeneous is Productivity. *Working Paper*.

Geanakoplos, John and H.M. Polemarchakis (1986). Existence, regularity and constrained suboptimality of competitive allocations when the asset market is incomplete. *Starrett, D., Heller, W.P., Starr, R.M. (Eds.), Uncertainty, Information and Communication: Essays in Honor of K.J. Arrow, vol. III*.

Greenwald, Bruce and Joseph Stiglitz (1986). Externalities in Economies with Imperfect Information and Incomplete Markets. *The Quarterly Journal of Economics*.

Hernandez, Zenaida (2004). Industrial Policy in East Asia: in Search for Lessons. *World Development Report Background Papers*.

Hirschman, Albert O. (1958). *The Strategy of Economic Development*. Yale University Press.

Hopenhayn, Hugo (1992). Entry, Exit, and firm Dynamics in Long Run Equilibrium. *Econometrica*.

Hopenhayn, Hugo and Richard Rogerson (1993). Job Turnover and Policy Evaluation: A General Equilibrium Analysis. *Journal of Political Economy*.

Horvath, Michael (1998). Cyclicality and Sectoral Linkages: Aggregate Fluctuations from Independent Sectoral Shocks. *Review of Economic Dynamics*.

Horvath, Michael (2000). Sectoral Shocks and Aggregate Fluctuations. *Journal of Monetary Economics*.

Hsieh, Chang-Tai and Benjamin Olken (2014). The Missing "Missing Middle". *Journal of Economic Perspectives*.

Hsieh, Chang-Tai and Peter Klenow (2009). Misallocation and Manufacturing TFP in China and India. *Quarterly Journal of Economics*.

Hsieh, Chang-Tai and Zheng Michael Song (2015). Grasp the Large, Let Go of the Small. *Brookings Papers in Economic Activity*.

Hulten, Charles (1978). Growth Accounting with Intermediate Inputs. *Review of Economic Studies*.

IMF (2015). World Economic Outlook.

Jeong, Hyeok and Robert M. Townsend (2005). Sources of TFP Growth: Occupational Choice and Financial Deepening. *IEPR Working Paper No. 05.28*.

Jones, Chad, j. . A. (2013). Misallocation, Economic Growth, and Input-Output Economics.

Jones, Leroy P. (1976). The Measurement of Hirschmanian Linkages. *The Quarterly Journal of Economics*.

Kilenthong, Weerachart T and Robert M. Townsend (2016). A Market Based Solution for Fire Sales and Other Pecuniary Externalities. *Working Paper*.

Korinek, Anton and Alp Simsek (2016). Liquidity Trap and Excessive Leverage. *American Economic Review*.

Leontief, Wassily (1963). The Structure of Development. *Scientific American*.

Li, Xi, Xuewen Liu, and Yong Wang (2015). A Model of China's State Capitalism. *HKUST IEMS Working Paper*.

Lipsey, R. G. and Kelvin Lancaster (1956). The General Theory of Second Best. *The Review of Economic Studies*.

Long, John and Charles Plosser (1983). Real Business Cycles. *Journal of Political Economy*.

Morris, Carl N. (2011). Parametric Empirical Bayes Inference: Theory and Applications. *Journal of the American Statistical Association*.

Naughton, Barry J. (2006). *The Chinese Economy: Transitions and Growth*. The MIT Press.

Noland, Marcus (2004). Selective Intervention and Growth: The Case of Korea. *Empirical Methods in International Trade*.

Pack, Howard, j. . T. (2000). Industrial Policy: Growth Elixir or Poison?

Pack, Howard and Larry E. Westphal (1986). Industrial Strategy and Technological Change: Theory versus Reality. *Journal of Development Economics*.

Page, John (1994). The East Asian Miracle: Four Lessons for Development Policy. *NBER Macroeconomics Annual*.

Rasmussen, Poul NÃÿrregaard (1965). *Studies in Intersectoral Relations*. Amsterdam: North-Holland.

Restuccia, Diego and Richard Rogerson (2008). Policy Distortions and Aggregate Productivity with Heterogeneous Establishment. *Review of Economic Dynamics*.

Riedel, James, Jing Jin, and Jian Gao (2007). *How China grows: investment, Finance, and Reform*. Princeton and Oxford: Princeton University Press.

Robinson, Sherman (1989). Multisector Models. *Handbook of Development Economics*.

Rodrik, Dani (2004). Industrial Policy For The Twenty First Century. *Working Paper*.

Rodrik, Dani (2008). Normalizing Industrial Policy. *Commission on Growth and Development Working Paper No. 3*.

Rosenstein-Rodan, Paul (1943). Problems of Industrialization of Eastern and South-Eastern Europe. *The Economic Journal*.

Rotemberg, Martin (2014). Equilibrium Effects of Firm Subsidies. *Working Paper*.

Samuelson, Paul A. (1951). Abstract of a theorem concerning substitutability in open Leontief models. *Activity Analysis of Production and Allocation, ed. T.C. Koopmans.*.

Shea, John (2002). Complementarities and Comovements. *Journal of Money, Credit and Banking*.

Shultz, T. Paul (1982). Effective Protection and the Distribution of Personal Income in Colombia. *Trade and Employment in Developing Countries: Factor Supply and Substitution, Edited by Anne O. Krueger*.

Song, Zheng Michael, Kjetil Storesletten, and Fabrizio Zilibotti (2011). Growing Like China. *American Economic Review*.

Stiglitz, Joseph (1970). Non-Substitution Theorems with Durable Capital Goods. *Review of Economic Studies*.

Storesletten, Kjetil and Fabrizio Zilibotti (2014). China's Great Convergence and Beyond. *Annual Review of Economics*.

United Nations Department of Economic and Social Affiars (1999). *Handbook of Input-Output Table Compilation and Analysis*.

Wade, Robert (1989). What Can Economics Learn from East Asian Success? *The Annals of the American Academy of Political and Social Science*.

Wade, Robert (1990). *Governing the Market: Economic Theory and The Role of Government in East Asian Industrialization*. Princeton.

Westphal, Larry E. (1990). Industrial Policy in an Export-Propelled Economy: Lessons from South Korea's Experience. *Journal of Economic Perspective*.

Woo-Cumings, Meredith (2001). Miracle as Prologue : the State and the Reform of the Corporate Sector in Korea. *Rethinking the East Asian Miracle*.

Xiaodong, Zhu (2012). Understanding China's Growth: Past, Present, and Future. *Journal of Economic Perspectives*.

Yotopoulos, Pan and Jeffrey B. Nugent (1973). A Balanced-Growth Version of the Linkage Hypothesis: A Test. *The Quarterly Journal of Economics*.

Appendix (Work In Progress)

I use boldface Roman alphabets and greek letters without subscripts to represent column vectors (e.g. $\mathbf{p} \equiv (p_1, \cdots, p_S)^T$ and $\theta \equiv (\theta_1, \cdots, \theta_S)^T$), and let $\mathbf{1}$ be a column vector of ones.

A Proofs

In this section of the appendix I provide proofs for results in the model without Cobb-Douglas assumptions. Proof of Theorem 3 is deferred to Appendix B.2, after we provide closed form solutions of the equilibrium for the Cobb-Douglas case in Appendix B.1.

A.1 Proof of Proposition 2

Let $\pi_i\left(\nu; C_i, w, \{p_j\}\right)$ be the solution to the profit maximization problem ($\mathbf{P_{firm}}$) for firm ν in industry i, where C_i is the price at which firm ν sells its output and w and $\{p_j\}$ are the cost of inputs. Note that $C_i = p_i$ in equilibrium, but the sectoral unit cost $C_i(\cdot)$ can be *defined* as a function of p_i as well as the prices of other production inputs as in (15). Totally differentiating π_i and applying the Envelope theorem yields:

$$d\pi_i(\nu) = C_i q_i(\nu)\left(d\ln C_i - \alpha_i(\nu)\, d\ln w - \sum_j \sigma_{ij}(\nu)\, d\ln p_j\right). \tag{A.1}$$

which implies

$$\frac{\partial \pi_i(\nu)}{\partial \ln C_i} = C_i q_i(\nu),$$
$$\frac{\partial \pi_i(\nu)}{\partial \ln w} = -\alpha_i(\nu)\, C_i q_i(\nu),$$
$$\frac{\partial \pi_i(\nu)}{\partial \ln p_j} = -\sigma_{ij}(\nu)\, C_i q_i(\nu).$$

The free-entry condition defines an implicit function of C_i and the input prices w and $\{p_j\}$:

$$\kappa = \int \pi_i\left(\nu; C_i\left(w, \{p_j\}\right), w, \{p_j\}\right) d\Phi_i(\nu)$$

Totally differentiating the free-entry condition, we get

$$0 \quad = \quad \int q_i(\nu)\left(d\ln C_i - \alpha_i(\nu)\, d\ln w - \sum_j \sigma_{ij}(\nu)\, d\ln p_j\right) d\Phi_i(\nu)$$

Applying the Implicit Function Theorem, we have

$$\begin{aligned}\frac{\partial \ln C_i}{\partial \ln p_j} &= \int \sigma_{ij}(\nu) \frac{q_i(\nu)}{\int q_i(\nu)\, d\Phi_i(\nu)} d\Phi_i(\nu) \\ &= \mathbb{E}_\nu \left[\sigma_{ij}(\nu) \frac{q_i(\nu)}{\mathbb{E}_\nu [q_i(\nu)]}\right].\end{aligned}$$

We can similarly find the elasticity of the sectoral unit-cost function with respect to the wage rate, which we will use later in this appendix:

$$\alpha_i \equiv \mathbb{E}_\nu \left[\alpha_i(\nu) \frac{q_i(\nu)}{\mathbb{E}_\nu [q_i(\nu)]}\right] = \frac{\partial \ln C_i}{\partial \ln w}. \tag{A.2}$$

A.2 Proof of Proposition 3

Influence Recall from (16) and (17) that the equilibrium price $\{p_j\}$ vector is the fixed point to the system of equations

$$p_i = C_i\left(w, \{p_j\}; h_i\right)$$

with the normalization

$$1 = C^F\left(\{p_j\}\right).$$

Totally differentiating, we have

$$d\ln p_i = -d\ln h_i + \alpha_i d\ln w + \sum \sigma_{ij} d\ln p_j$$

$$0 = \sum \beta_i d\ln p_i$$

where the first equation follows from Proposition 2 and equation (A.2). Re-writing these equations using matrix notation, we have

$$\begin{aligned}d\ln \mathbf{p} &= -d\ln \mathbf{h} + \alpha d\ln w + \Sigma d\ln \mathbf{p} \\ &= (I-\Sigma)^{-1}\left(-d\ln \mathbf{h} + \alpha d\ln w\right)\end{aligned}$$

and

$$0 = \beta' d\ln \mathbf{p}$$

where $\alpha' \equiv (\alpha_1, \cdots, \alpha_S)$ is the vector of sectoral average labor share. The consumer budget constraint and the resource constraint implies $wL = Y$ hence

$$\begin{aligned}d\ln Y &= d\ln w \\ &= \frac{\beta'(I-\Sigma)^{-1}}{\beta'(I-\Sigma)^{-1}\alpha} \cdot d\ln \mathbf{h}\end{aligned}$$

which proves the result.

Sales The market clearing condition for good j is

$$Q_j = Y_j + \sum_i M_{ij}$$

Multiplying by p_j/Y and using the fact that $\gamma_j \equiv \frac{p_j Q_j}{Y}$ and $\beta_j \equiv \frac{p_j Y_j}{F(Y_1,\cdots,Y_S)}$, we obtain

$$\gamma_j = c \cdot \beta_j + \sum_i \omega_{ij}\gamma_i$$

or in matrix notation,

$$\gamma' = c \cdot \beta' \left(I - \Omega\right)^{-1}$$

where $c \equiv \frac{F(Y_1,\cdots,Y_S)}{Y}$ is a scalar. To figure out the value for c, note that the total wage payment in industry i can be written as

$$wL_i = \alpha_i p_i Q_i$$

or

$$wL = \sum_i wL_i = \left(\sum_i \alpha_i \gamma_i\right) Y.$$

The consumer budget constraint and the resource constraint imply $wL = Y$, or

$$\sum_i \gamma_i \alpha_i = 1.$$

The constant c can be therefore found as

$$c = \frac{1}{\beta' \left(I - \Omega\right)^{-1}}.$$

A.3 Proof of Theorem 1

In presence of taxes, equation (A.1) can be modified as

$$\begin{aligned} d\pi_i(\nu) &= C_i q_i(\nu)\left(d\ln C_i - \alpha_i(\nu)\, d\ln w - \sum_j \sigma_{ij}(\nu)\, d\ln p_j\right) \\ &\quad + C_i q_i(\nu)\left(d\ln\left(1+\tau_i^R\right) + \alpha_i(\nu)\, d\ln\left(1+\tau_i^L\right) + \sum_j \sigma_{ij}(\nu) \ln\left(1+\tau_i^j\right)\right). \end{aligned} \tag{A.3}$$

Exploiting (A.3) and applying the same argument as in the proof for Proposition 2, we have

$$\begin{cases} \frac{\partial \ln C_i\left(w,\left\{p_j\right\},\tau_i^R,\tau_i^L,\left\{\tau_i^j\right\}\right)}{\partial \ln\left(1+\tau_i^R\right)} = -1, \\ \frac{\partial \ln C_i\left(w,\left\{p_j\right\},\tau_i^R,\tau_i^L,\left\{\tau_i^j\right\}\right)}{\partial \ln\left(1+\tau_i^L\right)} = -\alpha_i, \\ \frac{\partial \ln C_i\left(w,\left\{p_j\right\},\tau_i^R,\tau_i^L,\left\{\tau_i^j\right\}\right)}{\partial \ln\left(1+\tau_i^j\right)} = -\sigma_{ij}. \end{cases} \tag{A.4}$$

Similar to the proof of Proposition 3, for each subsidy $\tau_i \in \left\{\tau_i^R, \tau_i^L, \left\{\tau_i^j\right\}\right\}$ we can totally differentiate the sectoral unit-cost functions and obtain a system of equations

$$d\ln p_j = \alpha_j d\ln w + \sum \sigma_{jk} d\ln p_k + \frac{\partial \ln C_j}{\partial \ln(1+\tau_i)} d\ln(1+\tau_i)$$

$$0 = \sum \beta_j d\ln p_j$$

in which $\frac{\partial \ln C_j}{\partial \ln(1+\tau_i)} = 0$ for all $i \neq j$. Manipulating the equations and using matrix notations, we have

$$\begin{aligned} \frac{d\ln w}{d\ln(1+\tau_i)} &= -\frac{\beta'(I-\Sigma)^{-1}}{\beta'(I-\Sigma)^{-1}\alpha}\left(\frac{\partial \ln \mathbf{C}}{\partial \ln(1+\tau_i)}\right) \\ &= -\mu_i \frac{\partial \ln C_i}{\partial \ln(1+\tau_i)}. \end{aligned}$$

Lastly, from the budget constraints for the consumer and the planner

$$wL = C + T$$

$$T = E + \sum_{i=1}^{S}\left(\tau_i^R p_i Q_i + \tau_i^L w L_i + \sum_{j=1}^{S} \tau_i^j p_j M_{ij}\right)$$

as well as the resource constraint of the economy

$$Y = C + E$$

we obtain

$$wL = Y + \sum_{i=1}^{S}\left(\tau_i^R p_i Q_i + \tau_i^L w L_i + \sum_{j=1}^{S} \tau_i^j p_j M_{ij}\right)$$

which implies

$$\left.\frac{d\ln Y}{d\ln(1+\tau_i)}\right|_{\vec{\tau}=\mathbf{0},\text{ holding } E \text{ constant}} = \left.\left(\frac{d\ln w}{d\ln(1+\tau_i)} - \frac{dT/d\tau_i}{Y}\right)\right|_{\vec{\tau}=\mathbf{0},\text{ holding } E \text{ constant}}.$$

Using (A.4) and the fact that sectoral expenditure shares follow

$$wL_i = \alpha_i p_i Q_i, \quad p_j M_{ij} = \omega_{ij} p_i Q_i,$$

we have

$$\begin{cases} \left.\frac{d\ln Y}{d\ln\left(1+\tau_i^R\right)}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } E \text{ constant}} & = \mu_i - \gamma_i \\ \left.\frac{d\ln Y}{d\ln\left(1+\tau_i^L\right)}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } E \text{ constant}} & = \alpha_i\left(\mu_i - \gamma_i\right) \\ \left.\frac{d\ln Y}{d\ln\left(1+\tau_i^j\right)}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } E \text{ constant}} & = \sigma_{ij}\mu_i - \omega_{ij}\gamma_i, \end{cases} \tag{A.5}$$

which proves parts 1, 3, and 4 of the theorem. Part 2 follows part 3 and the observation that $\sigma_{ij} = \omega_{ij}$ for all unconstrained intermediate inputs $j \notin K_i$.

A.4 Proof of Theorem 2

From the resource constraint we derive that for any subsidy $\tau_i \in \left\{\tau_i^R, \tau_i^L, \left\{\tau_i^j\right\}\right\}$,

$$\frac{dY}{d\ln\left(1+\tau_i\right)} = \frac{dC}{d\ln\left(1+\tau_i\right)} + \frac{dE}{d\ln\left(1+\tau_i\right)}. \tag{A.6}$$

Starting from $\vec{\tau} = \mathbf{0}$ and holding the lump sum tax T constant, we have

$$\begin{cases} \left.\frac{dE}{d\ln\left(1+\tau_i^R\right)}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} & = -\gamma_i Y \\ \left.\frac{dE}{d\ln\left(1+\tau_i^L\right)}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} & = -\alpha_i\gamma_i Y \\ \left.\frac{dE}{d\ln\left(1+\tau_i^j\right)}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} & = -\omega_{ij}\gamma_i Y. \end{cases}$$

Combining with (A.5) and (A.6), this implies

$$\begin{cases} \left.\frac{dC}{d\ln\left(1+\tau_i^R\right)}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} & = \mu_i Y \\ \left.\frac{dC}{d\ln\left(1+\tau_i^L\right)}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} & = \alpha_i\mu_i Y \\ \left.\frac{dE}{d\ln\left(1+\tau_i^j\right)}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} & = \sigma_{ij}\mu_i Y. \end{cases}$$

The theorem follows from these two sets of equations and the fact that $d\ln\left(1+\tau_i\right)\big|_{\vec{\tau}=\mathbf{0}} = d\tau_i$.

A.5 Proof of Propositions 4 and 5

Following a similar procedure as in the proof for Proposition 2, we can show

$$\left.\frac{\partial \ln C_i}{\partial \ln\left(1+\tau_i^C\right)}\right|_{\vec{\tau}=\mathbf{0}} = -\mathbb{E}_\nu\left[\left(\varphi_i^K(\nu)^{-1}-1\right)\frac{W_i(\nu)}{\mathbb{E}_\nu\left[p_i q_i(\nu)\right]}\right].$$

We then use a similar procedure as in the proof for Theorem 1 to show

$$\left.\frac{d\ln w}{d\ln\left(1+\tau_i^C\right)}\right|_{\vec{\tau}=\mathbf{0}} = \mu_i \times \mathbb{E}_\nu\left[\left(\varphi_i^K(\nu)^{-1}-1\right)\frac{W_i(\nu)}{\mathbb{E}_\nu\left[p_i q_i(\nu)\right]}\right].$$

Lastly, differentiating (26) with respect to τ_i^C and using planner's budget constrain (27), we have

$$\left.\frac{dE}{d\tau_i^C}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} = -\gamma_i Y\cdot\chi\frac{\mathbb{E}\left[W_i(\nu)\mathbf{1}\left(\varphi_i(\nu)^{-1}>1\right)\right]}{\mathbb{E}\left[p_i q_i(\nu)\right]}.$$

The social return to credit SR_i^C can be found similarly as in the proof of Theorem 2:

$$\begin{aligned}
SR_i^C &\equiv -\left.\frac{dC/d\tau_i^C}{dE/d\tau_i^C}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} \\
&= -\left.\frac{d\ln w/d\ln\left(1+\tau_i^C\right)}{\left(dE/d\tau_i^C\right)/Y}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} \\
&= \xi_i\cdot\chi^{-1}\cdot\mathbb{E}\left[\left(\varphi_i^{-1}(\nu)-1\right)\frac{W_i(\nu)}{\mathbb{E}\left[W_i(\nu)\mathbf{1}\left(\varphi_i(\nu)^{-1}>1\right)\right]}\right],
\end{aligned}$$

which proves Proposition 4 as we note that $PR_i^C \equiv \chi^{-1}\cdot\mathbb{E}\left[\left(\varphi_i^{-1}(\nu)-1\right)\frac{W_i(\nu)}{\mathbb{E}\left[W_i(\nu)\mathbf{1}\left(\varphi_i(\nu)^{-1}>1\right)\right]}\right]$.

To prove Proposition 5, we follow the same steps to show

$$\left.\frac{d\ln w}{d\tau_i^{C'}}\right|_{\vec{\tau}=\mathbf{0}} = \mu_i\frac{\mathbb{E}\left[\varphi_i(\nu)^{-1}-1\right]}{\mathbb{E}\left[p_i q_i(\nu)\right]}$$

and

$$\left.\frac{dE}{d\tau_i^{C'}}\right|_{\vec{\tau}=\mathbf{0}} = -\gamma_i\frac{\chi Y Pr\left(\varphi_i(\nu)<1\right)}{\mathbb{E}\left[p_i q_i(\nu)\right]}.$$

The social return is thus

$$\begin{aligned}
SR_i^{C'} &= -\left.\frac{d\ln w/d\tau_i^{C'}}{\left(dE/d\tau_i^C\right)/Y}\right|_{\vec{\tau}=\mathbf{0},\,\text{holding } T \text{ constant}} \\
&= \xi_i\cdot\chi^{-1}\cdot\mathbb{E}\left[\varphi_i^{-1}(\nu)-1\middle|\varphi_i(\nu)<1\right],
\end{aligned}$$

as desired.

A.6 Proof of Proposition 6 and Derivation of Equation (30)

From Proposition 3 we have

$$\mu' \propto \beta' (I-\Sigma)^{-1}, \quad \gamma' \propto \beta' (I-\Omega)^{-1}$$

which implies that for some constants c_1 and c_2,

$$c_1\mu' - c_2\gamma' \quad = \quad c_1\mu'\Sigma - c_2\gamma'\Omega.$$

Writing out the j-th equation of the system above and dividing by $c_2\gamma_j$, we have

$$\begin{aligned} \frac{c_1\mu_j - c_2\gamma_j}{c_2\gamma_j} &= \sum_i \left(\sigma_{ij}\frac{c_1\mu_i - c_2\gamma_i}{c_2\gamma_j} + \frac{\gamma_i}{\gamma_j}(\sigma_{ij}-\omega_{ij})\right) \\ &= \sum_i \left(\hat{\sigma}_{ij}\frac{c_1\mu_i - c_2\gamma_i}{c_2\gamma_i} + (\hat{\sigma}_{ij}-\hat{\omega}_{ij})\right) \end{aligned}$$

where the second equality follows from $\hat{\omega}_{ij} = \omega_{ij}\frac{\gamma_i}{\gamma_j}$ and $\hat{\sigma}_{ij} = \sigma_{ij}\frac{\gamma_i}{\gamma_j}$. Stacking the equations using matrix notations, we have

$$\frac{c_1}{c_2}\xi' - \mathbf{1}' = \mathbf{1}'\left(\hat{\Sigma}-\hat{\Omega}\right)\left(I-\hat{\Sigma}\right)^{-1} \tag{A.7}$$

$$\begin{aligned} \implies \xi' &\propto \mathbf{1}'\left(I + \hat{\Sigma}\left(I-\hat{\Sigma}\right)^{-1} - \hat{B}\left(I-\hat{\Sigma}\right)^{-1}\right) \\ &= \mathbf{1}'\left(I-\hat{\Omega}\right)\left(I-\hat{\Sigma}\right)^{-1}. \end{aligned}$$

To obtain equation (30), note that the assumption $\omega_{ij} = \sigma_{ij}\varphi$ implies $\hat{\Omega} = \hat{\Sigma}\varphi$ hence equation (A.7) implies

$$\begin{aligned} \xi' &\propto \mathbf{1}' + \mathbf{1}'\left(\hat{\Sigma}-\hat{\Omega}\right)\left(I-\hat{\Sigma}\right)^{-1} \\ &= \varphi\mathbf{1}' + (1-\varphi)\mathbf{1}'I + (1-\varphi)\mathbf{1}'\hat{\Sigma}\left(I-\hat{\Sigma}\right)^{-1} \\ &= \varphi + (1-\varphi)\mathbf{1}'\left(I-\hat{\Sigma}\right)^{-1} \\ &\propto const + \left(\delta^{Jones}\right)'. \end{aligned}$$

B Cobb-Douglas Production Functions

B.1 Cobb-Douglas Fully Solved

In this subsection of the appendix I setup the model with Cobb-Douglas production function and fully solve for the equilibrium. I then prove Theorem 3 in the next subsection. For simplicity, I exposit without firm-level heterogeneity, which can be added with minor modifications in notations.

Setup Potential entrepreneurs pay fixed cost κ_i units of the final good to acquire a production function

$$q_i = z_i \ell_i^{\alpha_i} \prod_j m_{ij}^{\sigma_{ij}}$$

and working capital W_i with constraint

$$\sum_{j \in K_i} p_j m_{ij} \leq W_i.$$

The final good is produced according to

$$F = \prod_i Y_i^{\beta_i}.$$

Equilibrium An equilibrium is the a of allocation and prices such that 1) all producers maximize profits taking prices and constraints as given, 2) free-entry drives ex-ante profits to zero, and 3) all markets clear.

Let small-case letters denote firm-level variables and let capital letters denote sectoral and aggregate variables, and let N_i be the number of firms that enter sector i in equilibrium. The market clearing conditions are

$$Q_j = Y_j + \sum_i M_{ij} \quad \text{for all } j$$

$$F = Y + \sum_i \kappa_i N_i$$

where

$$Q_i = N_i q_i, \quad L_i = N_i \ell_i, \quad M_{ij} = N_i m_{ij}.$$

Firm Allocations Profit maximization implies

$$w\ell_i = \alpha_i p_i q_i$$

$$p_j m_{ij} = \begin{cases} \sigma_{ij} p_i q_i & \text{for } j \notin K_i \\ \frac{\sigma_{ij}}{\sum_{j \in K_i} \sigma_{ij}} \min\left\{\sum_{j \in K_i} \sigma_{ij} p_i q_i, W_i\right\} & \text{for } j \in K_i. \end{cases}$$

Sectoral Allocations Free-entry implies

$$\kappa_i = \left(1 - \alpha_i - \sum_{j \notin K_i} \sigma_{ij}\right) p_i q_i - \min\left\{\sum_{j \in K_i} \sigma_{ij} p_i q_i, W_i\right\}$$

which implies that constraints bind in sector i if and only if

$$\frac{W_i}{\kappa_i} < \frac{\sum_{j \in K_i} \sigma_{ij}}{1 - \alpha_i - \sum_j \sigma_{ij}}.$$

Let

$$\varphi_i \equiv \min\left\{1,\ \frac{W_i}{W_i+\kappa_i}\frac{\left(1-\alpha_i-\sum_{j\notin K_i}\sigma_{ij}\right)}{\sum_{j\in K_i}\sigma_{ij}}\right\}$$

and we can rewrite the firm-level allocation of constrained inputs as

$$p_j m_{ij} = \sigma_{ij}\varphi_i p_i q_i \quad \text{for } j \in K_i.$$

To obtains sectoral allocations, we multiply both sides of the firm-level allocation equation by N_i to get

$$wL_i = \alpha_i p_i Q_i$$

$$p_j M_{ij} = \begin{cases} \sigma_{ij} p_i Q_i & \text{for } j \notin K_i \\ \sigma_{ij}\varphi_i p_i Q_i & \text{for } j \in K_i \end{cases}$$

$$\kappa_i N_i = \left(1-\alpha_i-\sum_{j\notin K_i}\sigma_{ij}-\varphi_i\sum_{j\in K_i}\sigma_{ij}\right)p_i Q_i$$

Aggregate Allocations Define $\gamma_i \equiv \frac{p_i Q_i}{Y}$ as the sales vector. Labor market clearing implies

$$L_i = \frac{\alpha_i\gamma_i}{\sum_j \alpha_j\gamma_j}L.$$

Using the fact that $wL = Y$, we have $\sum wL_i = \sum \alpha_i\gamma_i Y = Y$ which implies the denominator in the equation above is 1 and that

$$L_i = \alpha_i\gamma_i L. \tag{B.1}$$

The sectoral allocation for intermediate goods can be re-written as

$$M_{ij} = \begin{cases} \sigma_{ij}\frac{\gamma_i}{\gamma_j}Q_j & \text{for } j \notin K_i \\ \sigma_{ij}\varphi_i\frac{\gamma_i}{\gamma_j}Q_j & \text{for } j \in K_i \end{cases}$$

We can solve for aggregate allocations by substituting sectoral allocations into production functions.

The sectoral production function for sector i can be aggregated from firm-level production functions as

$$\begin{aligned} Q_i &= N_i q_i \\ &= N_i z_i \ell_i^{\alpha_i} \prod_j m_{ij}^{\sigma_{ij}} \\ &= z_i N_i^{\left(1-\alpha_i-\sum_j \sigma_{ij}\right)} L_i^{\alpha_i} \prod_j M_{ij}^{\sigma_{ij}} \\ &= z_i \left(\frac{1-\alpha_i-\sum_{j\notin K_i}\sigma_{ij} - \varphi_i \sum_{j\in K_i}\sigma_{ij}}{\kappa_i} \gamma_i Y \right)^{\left(1-\alpha_i-\sum_j \sigma_{ij}\right)} \\ &\quad \times (\alpha_i \gamma_i L)^{\alpha_i} \prod_j \left(\sigma_{ij} \frac{\gamma_i}{\gamma_j} Q_j \right)^{\sigma_{ij}} \times \prod_{j\in K_i} \varphi_i^{\sigma_{ij}} \end{aligned}$$

Taking logs and move $\ln \gamma_i$ to the left-hand-side,

$$\begin{aligned} \ln Q_i - \ln \gamma_i &= \ln \omega_i + \ln z_i + \left(1-\alpha_i-\sum_j \sigma_{ij}\right) \ln Y + \alpha_i \ln L + \sum \sigma_{ij} \left(\ln Q_j - \ln \gamma_j\right) \\ &\quad + \left(1-\alpha_i-\sum_j \sigma_{ij}\right) \ln \left(\frac{1-\alpha_i-\sum_{j\notin K_i}\sigma_{ij} - \varphi_i \sum_{j\in K_i}\sigma_{ij}}{1-\alpha_i-\sum_j \sigma_{ij}} \right) + \sum_{j\in K_i} \sigma_{ij} \ln \varphi_i \end{aligned}$$

where $c_i \equiv \left(1-\alpha_i-\sum_j \sigma_{ij}\right) \ln \left(1-\alpha_i-\sum_j \sigma_{ij}\right) + \alpha_i \ln \alpha_i + \sum_j \sigma_{ij} \ln \sigma_{ij}$. To economize notation, let

$$\phi_i \equiv \left(\frac{1-\alpha_i-\sum_{j\notin K_i}\sigma_{ij} - \varphi_i \sum_{j\in K_i}\sigma_{ij}}{1-\alpha_i-\sum_j \sigma_{ij}} \right)$$

$$\theta_i \equiv \left(1-\alpha_i-\sum_j \sigma_{ij}\right)$$

$$\sigma_i^K \equiv \sum_{j\in K_i} \sigma_{ij}.$$

We can then write

$$\begin{aligned} \ln Q_i - \ln \gamma_i &= \ln c_i + \ln z_i + \theta_i \ln Y + \alpha_i \ln L + \sum \sigma_{ij} \left(\ln Q_j - \ln \gamma_j\right) \\ &\quad + \theta_i \ln \phi_i + \sigma_i^K \ln \varphi_i. \end{aligned} \tag{B.2}$$

Note that when credit constraints do not bind in the sector, we have $\phi_i = \varphi_i = 1$.

Profit-maximization by the final good producer implies

$$p_i Y_i = \beta_i F$$

The sales vector can therefore be re-written as

$$\begin{aligned}\gamma_i &= \frac{p_i Q_i}{Y} \\ &= \frac{Q_i}{Y_i}\frac{\beta_i F}{Y}\end{aligned}$$

Thus

$$Y_i = \frac{Q_i}{\gamma_i}\frac{\beta_i F}{Y}$$

The net aggregate output can be found by using the production function of the final good:

$$\begin{aligned}\ln Y &= \ln F + (\ln Y - \ln F) \\ &= \beta' (\ln \mathbf{Q} - \ln \gamma + \ln \beta + \ln F - \ln Y) + \ln Y - \ln F \\ &= \beta' (\ln \mathbf{Q} - \ln \gamma + \ln \beta)\end{aligned}$$

and from equation (B.2) we have

$$\ln \mathbf{Q} - \ln \gamma = (I - \Sigma)^{-1} \left(\ln \mathbf{c} + \ln \mathbf{z} + \theta \ln Y + \alpha \ln L + \theta \ln \phi + \sigma^K \ln \varphi \right).$$

Hence

$$\begin{aligned}\ln Y &= \beta' \ln \beta + \beta' (I - \Sigma)^{-1} \left(\ln \mathbf{c} + \ln \mathbf{z} + \theta \ln Y + \alpha \ln L + \theta \ln \phi + \sigma^K \ln \varphi \right) \\ &= \frac{\beta' \ln \beta + \beta' (I - \Sigma)^{-1}}{1 - \beta' (I - \Sigma)^{-1} \theta} \left(\ln \mathbf{c} + \ln \mathbf{z} + \alpha \ln L + \theta \ln \phi + \sigma^K \ln \varphi \right)\end{aligned}$$

Given that

$$\theta_i = 1 - \alpha_i - \sum_j \sigma_{ij}$$

we have

$$\begin{aligned}\beta' (I - \Sigma)^{-1} (\theta + \alpha) &= \beta' (I - \Sigma)^{-1} (\mathbf{1} - \Sigma \mathbf{1}) \\ &= \beta' (I - \Sigma)^{-1} (I - \Sigma) \mathbf{1} \\ &= 1.\end{aligned}$$

Thus we can write $\ln Y$ as

$$\ln Y = \frac{\beta' \left(\ln \beta + (I - \Sigma)^{-1} \ln \mathbf{c} \right)}{\beta' (I - \Sigma)^{-1} \alpha} + \frac{\beta' (I - \Sigma)^{-1}}{\beta' (I - \Sigma)^{-1} \alpha} \left(\ln \mathbf{z} + \theta \ln \phi + \sigma^K \ln \varphi \right) + \ln L$$

The first additive term is a scalar that depends on the Cobb-Douglas coefficients.

We can immediately see that (1) there is aggregate constant returns to scale as the net aggregate consumption is linear in L; (2) $\frac{\beta'(I-\Sigma)^{-1}}{\beta'(I-\Sigma)^{-1}\alpha}$ is the influence vector μ_i; and (3) credit constraints lower effective sectoral productivity by a factor of $\phi_i^{\theta_i}\varphi_i^{\sigma_i^K}$, and they lower aggregate productivity by $\left(\phi_i^{\theta_i}\varphi_i^{\sigma_i^K}\right)^{\mu_i}$: that is, the sectoral productivity shock is transmitted to affect aggregate productivity via the influence of the sector.

Once we know Y, solving for sectoral allocations and prices boils down to solving for the sales vector γ, which can be found from the market clearing conditions:

$$p_j Q_j = p_j Y_j + \sum_i p_j M_{ij}$$

$$\iff \gamma_j = \beta_j \frac{F}{Y} + \sum \omega_{ij}\gamma_i \tag{B.3}$$

where

$$\omega_{ij} = \begin{cases} \sigma_{ij} & \text{if } j \notin K_i \\ \varphi_i \sigma_{ij} & \text{otherwise.} \end{cases}$$

and the matrix $\Omega \equiv \left[\omega_{ij}\right]$ is exactly the input-output table. Stacking (B.3) into matrix notation, we have

$$\begin{aligned} \gamma' &= \beta' \frac{F}{Y} + \gamma'\Omega \\ &= \frac{F}{Y}\beta' \left(I - \Omega\right)^{-1} \\ &= \frac{\beta' \left(I - \Omega\right)^{-1}}{\beta' \left(I - \Omega\right)^{-1} \alpha} \end{aligned}$$

where the third equality follows from the fact that $\gamma'\alpha = 1$ as in (B.1).

B.2 Proof of Theorem 3

Consider labor subsidies $\left\{\tau_i^L\right\}$. The sectoral allocation of labor, intermediate inputs, and number of firms can be re-written as

$$L_i = \frac{\left(1+\tau_i^L\right)\gamma_i\alpha_i}{\sum_j \left(1+\tau_j^L\right)\gamma_j\alpha_j} L \tag{B.4}$$

$$M_{ij} = \begin{cases} \sigma_{ij}\frac{\gamma_i}{\gamma_j} Q_j & \text{for } j \notin K_i \\ \sigma_{ij}\varphi_i \frac{\gamma_i}{\gamma_j} Q_j & \text{for } j \in K_i \end{cases} \tag{B.5}$$

$$\kappa_i N_i = \left(1 - \alpha_i - \sum_{j\notin K_i} \sigma_{ij} - \varphi_i \sum_{j \in K_i} \sigma_{ij}\right)\gamma_i Y$$

Following the derivation in Appendix B.1, we can write total output as

$$\ln Y = const + \sum \mu_i \alpha_i \ln\left(1+\tau_i^L\right) - \ln\left(\sum_j \left(1+\tau_j^L\right)\gamma_j\alpha_j\right)$$

The planner's problem is therefore to solve

$$\max_{\{\tau_i^L\}} \sum \mu_i \alpha_i \ln \left(1+\tau_i^L\right) - \ln \left(\sum_j \left(1+\tau_j^L\right) \gamma_j \alpha_j \right)$$

Note in particular that we do not need to impose the budget constraint for the maximization problem for two reasons. First, the lump sum tax T does not affect allocations and can be chosen to satisfy the budget constraint after $\left\{\tau_i^L\right\}$ are pinned down as the solution to the maximization problem. Second, it should be clear from the objective function that it is the proportionality of $\left(1+\tau_i^L\right)$ across sectors that matters, rather than the levels of subsidies. As a result, for any given lump sum tax T, the planner can always rescale the subsidies by a constant and balance his budget without affecting allocation. It is therefore without loss of generality to set $T = 0$.

The first-order condition with respect to τ_i^L is

$$\frac{\mu_i \alpha_i}{1+\tau_i^L} = \frac{\gamma_i \alpha_i}{\sum_j \left(1+\tau_j^L\right) \gamma_j \alpha_j}$$

which implies that the optimal labor subsidies follow

$$1+\tau_i^L \propto \frac{\mu_i}{\gamma_i}.$$

C Robustness: Theory

C.1 Alternative Specification of Financial Frictions: Credit Delivery With Linear Monitoring Cost

In this subsection of the appendix, I outline a version of the model with financial frictions taking the form of a linear monitoring cost, which is reminiscent of the implicit wedge formulation in Hsieh and Klenow (2009) and Jones (2013). I abstract away from firm-heterogeneity and entry for expositional clarity, but these modeling elements can be added with notational changes that are conceptually simple. Indeed, it can be shown that this formulation of the model is isomorphic to the model in the main text.

There is a representative consumer who supplies labor inelastically and consumes a unique final good produced competitively according to a CRTS production function $F\left(\{Y_i\}\right)$. There are S intermediate sectors, each is occupied with a *representative producer with CRTS production technology*

$$Q_i = h_i F_i \left(L_i, \{M_{ij}\}\right).$$

Each intermediate producer faces a working capital constraint

$$\sum_{j \in K_i} p_j M_{ij} \leq W_i$$

where $K_i \subseteq \{1, \cdots, S\}$ is the set of constrained inputs and W_i is an *endogenous* amount of working capital that is available to the producer in sector i.

We introduce a representative financial institution (lender) to the model, who extends working capital to producers and incur a linear cost χ_i in terms of the final good for every unit of working capital that is given to producer i. The lender behaves competitively and makes zero profit, charging a flat interest rate of χ_i to producer i. Producer i therefore solves

$$\max_{W_i, L_i, \{M_{ij}\}} p_i F_i \left(L_i, \{M_{ij}\}\right) - wL_i - \sum_j p_j M_{ij} - \chi_i W_i$$

$$\text{s.t.} \sum_{j \in K_i} p_j M_{ij} \leq W_i.$$

The equilibrium allocation in sector i follows

$$wL_i = \alpha_i p_i Q_i$$

$$p_j M_{ij} = \begin{cases} \sigma_{ij} p_i Q_i & \text{if } j \notin K_i \\ \frac{1}{1+\chi_i} \sigma_{ij} p_i Q_i & \text{if } j \in K_i \end{cases}$$

$$\chi_i W_i = \frac{\chi_i}{1+\chi_i} \sum_{j \in K_i} \sigma_{ij} p_i Q_i$$

Like in our main text, let Y be the equilibrium net aggregate output and let the expenditure share matrix be $\Omega = \left[\omega_{ij}\right]$ and the elasticity matrix be $\Sigma = \left[\sigma_{ij}\right]$. The influence $\left(\frac{d \ln Y}{d \ln h_i}\right)$ and sales vectors $\left(\frac{p_i Q_i}{Y}\right)$ in this economy are respectively

$$\mu' = \beta' \left(I - \Sigma\right)^{-1}, \quad \gamma' = \frac{\beta' \left(I - \Omega\right)^{-1}}{\beta' \left(I - \Omega\right)^{-1} \alpha}$$

and the resource constraint in this economy is

$$Y = F\left(\{Y_i\}\right) - \sum_i \chi_i W_i.$$

This economy is indeed constrained inefficient despite the fact that credit is competitively allocated by the representative lender. The intuition is similar to that provided in the main text: when downstream producers are constrained, there is a wedge between the marginal social and private return to sectoral spending, either on production inputs or on working capital, and this wedge is exactly captured by the sales gap. To see this, consider labor subsidies τ_i^L, under which the sectoral allocation of labor, intermediate inputs, and monitoring cost respectively follow

$$L_i = \frac{\left(1 + \tau_i^L\right) \alpha_i \gamma_i}{\sum \left(1 + \tau_j^L\right) \alpha_j \gamma_j} L$$

$$M_{ij} = \begin{cases} \sigma_{ij}\frac{\gamma_i}{\gamma_j}Q_j & \text{for } j \notin K_i \\ \frac{1}{1+\chi_i}\sigma_{ij}\frac{\gamma_i}{\gamma_j}Q_j & \text{for } j \in K_i \end{cases}$$

$$\chi_i W_i = \frac{\chi_i}{1+\chi_i}\left(\sum_{j\in K_i}\sigma_{ij}\right)\gamma_i Y.$$

By totally differentiating production function $F_i(\cdot)$, we get

$$\begin{aligned} d\ln Q_i &= \frac{d\ln F_i}{d\ln L_i}d\ln L_i + \sum_j \frac{d\ln F_i}{d\ln M_{ij}}d\ln M_{ij} \\ &= \alpha_i\left(d\ln\alpha_i + d\ln\left(1+\tau_i^L\right) + d\ln\gamma_i - d\ln\left(\sum_j\left(1+\tau_j^L\right)\gamma_j\alpha_j\right)\right) \\ &\quad + \sum_j \sigma_{ij}\left(d\ln\sigma_{ij} + d\ln\gamma_i + d\ln Q_j - d\ln\gamma_i\right). \end{aligned}$$

Recognizing that $\alpha_i d\ln\alpha_i + \sum_j \sigma_{ij}d\ln\sigma_{ij} = 0$ due to $F(\cdot)$ being CRTS and stacking the equations using matrix notations, we have

$$d\ln\mathbf{Q} - d\ln\gamma = (I-\Sigma)^{-1}\left(\alpha\circ\left(d\ln\left(1+\tau^L\right) - d\ln\left(\sum_j\left(1+\tau_j^L\right)\gamma_j\alpha_j\right)\right)\right)$$

where $\circ$ represents the Kronecker product. From the fact that $\frac{p_iQ_i}{Y} = \gamma_i$ and $\frac{p_iY_i}{F}$, we get

$$\begin{aligned} d\ln Y &= \beta' d\ln\beta' + \beta'\left(d\ln\mathbf{Q} - d\ln\gamma\right) \\ &= \mu\circ\alpha\circ d\ln\left(1+\tau^L\right) - d\ln\left(\sum_j\left(1+\tau_j^L\right)\gamma_j\alpha_j\right). \end{aligned}$$

where we have used the fact that $F(\cdot)$ is CRTS hence $\beta' d\ln\beta' = 0$. From here and applying similar arguments as in the proof for Theorem 2, we can derive that

$$SR_i^L \equiv -\frac{dC/d\tau_i^L}{dE/d\tau_i^L}\bigg|_{\vec{\tau}=\mathbf{0},\,\text{holding } T\text{ constant}} = \frac{\mu_i}{\gamma_i}.$$

Results that parallel other parts of Theorems 1 and 2 as well as Corollary 2 and Proposition 4 follow from analogous derivations.

C.2 Alternative Specification of Financial Frictions: Input-Specific Requirement for Upfront Payment

Consider a more general form of credit constraints:

$$\sum_{j\in S}\eta_{ij}(\nu)\,p_j m_{ij} \le W_i(\nu) \quad \text{for } \eta_{ij}(\nu)\in[0,1]. \tag{C.1}$$

where $\eta_{ij}(\nu)$ parametrizes the fractional cost for input j that must be paid upfront out of working capital $W_i(\nu)$ by firm ν in sector i. The constraint formulation (1) in the main text corresponds to a special case in which $\eta_{ij}(\nu) = 0$ for all $j \notin K \subseteq S$ and $\eta_{ij}(\nu) = 1$ otherwise.

Under the more general formulation of constraints in (C.1), firm ν's problem becomes

$$\pi_i(\nu) = \max_{\ell, \{m_{ij}\}_{j=1}^{S}} p_i q_i\left(\nu, \ell, \{m_{ij}\}\right) - \sum_{j=1}^{S} p_j m_{ij} - w\ell \quad \text{subject to (C.1)}$$

The firm's first-order conditions are:

$$w\ell_i(\nu) = \alpha_i(\nu)\, p_i q_i(\nu)$$

$$\left(1 + \lambda_i(\nu)\, \eta_{ij}(\nu)\right) p_j m_{ij}(\nu) = \sigma_{ij}(\nu)\, p_i q_i(\nu)$$

where $\lambda_i(\nu)$ is the Lagrange multiplier on the credit constraint and also the highest interest rate that firm ν is willing to pay to obtain additional working capital. Under the more general formulation, we no longer have a firm-specific wedge between elasticity and expenditure share on constrained inputs; instead, the wedges are firm-input specific, with

$$\left(1 + \lambda_i(\nu)\, \eta_{ij}(\nu)\right) = \frac{\sigma_{ij}(\nu)}{s_{ij}(\nu)}. \tag{C.2}$$

Nevertheless, our main theoretical results survive under (C.1). To see this, we follow the proof of Proposition 2 and derive the semi-elasticity of firm ν's profit with respect to input and output prices. To distinguish between the effect of a change in the output price and a change in the cost of input produced by the same industry i, we again use C_i to denote output price and p_i to denote the cost of input i, noting that $C_i = p_i$ in equilibrium. We have

$$\frac{\partial \pi_i(\nu)}{\partial \ln C_i} = C_i q_i(\nu),$$

$$\frac{\partial \pi_i(\nu)}{\partial \ln p_j} = -\alpha_i(\nu)\, C_i q_i(\nu).$$

By applying the Implicit Function Theorem on the free-entry condition in sector i, we can verify that Proposotion 2 holds under the more general formulation of credit constraints:

$$\frac{\partial \ln C_i}{\partial \ln p_j} = \sigma_{ij} \equiv \mathbb{E}_\nu\left[\sigma_{ij}(\nu)\, \frac{q_i(\nu)}{\mathbb{E}_\nu[q_i(\nu)]}\right], \quad \frac{\partial \ln C_i}{\partial \ln w} = \alpha_i \equiv \mathbb{E}_\nu\left[\alpha_i(\nu)\, \frac{q_i(\nu)}{\mathbb{E}_\nu[q_i(\nu)]}\right].$$

Theorems 1, 2, Corollary 2, and Propositions 4 and 5 follow similarly, with the private return to tax instrument τ_i^j, defined as the ratio between private marginal product and marginal cost for a marginal change in τ_i^j starting from the decentralized equilibrium, still being $PR_i^j = \frac{\sigma_{ij}}{\omega_{ij}},$. Note that due to the presence of firm-input specific wedges as in (C.2), the private return to τ_i^j can no longer be written as a weighted average of firm-specific wedges.

Proposition 6 and equation (30) hold true as they are independent of the microfoundation for the credit constraints.

C.3 Alternative Specification of Financial Frictions: Pledgeable Revenue

We can generalize the constraints (C.1) even further and enable firms to obtain additional working capital by pledging a fraction δ_i of firm revenue:

$$\sum_{j\in S}\eta_{ij}p_j m_{ij}\le W_i+\delta_i p_i q_i \quad \text{for } \eta_j,\delta_i\in[0,1]\,. \tag{C.3}$$

This constraint formulation in (C.3) nests the specification in Bigio and La'O (2016), who specify all production inputs, including labor, are subject to the constraint with $\eta_{ij}=1$ for all $j\in S$ and $W_i=0$ for all i (note again that whether labor is constrained or not does not affect our main results: we can restate Theorems 1 and 2 by introducing a sectoral labor wedge):

$$w\ell_i+\sum_j p_j m_{ij}\le \delta_i p_i q_i.$$

Proposition 2, a key property in establishing our main results in Theorems 1 and 2, holds under constraints (C.3) only when the within-sector heterogeneity across firms is removed: all firms from sector i obtain the same productivity z_i, working capital W_i, revenue pledgebility δ_i, and input-specific requirement for upfront payment η_{ij}. To see this, consider firm's problem under the more general constraint (C.3) without heterogeneity:

$$\pi_i\left(C_i,w,\{p_j\}\right)\equiv\max_{\ell_i,m_{ij}} C_i q_i-w\ell_i-\sum_j p_j m_{ij}$$

$$\text{s.t. } q_i=z_i f\left(\ell_i,\{m_{ij}\}\right)$$

$$\sum_{j\in S}\eta_{ij}p_j m_{ij}\le W_i+\delta_i p_i q_i$$

where recall that we use C_i to denote output price of good i in order to distinguish it with the input price p_i: the two objects are equal in equilibrium, but the sectoral unit cost $C_i(\cdot)$ is *defined* as a function of p_i and prices of other production inputs. Let λ_i be the Lagrange multiplier on the credit constraint, we have

$$\frac{\partial\pi_i}{\partial\ln C_i}=C_i q_i\left(1+\lambda_i\delta_i\right)$$

$$\frac{\partial\pi_i}{\partial\ln p_j}=-\sigma_{ij}C_i q_i\left(1+\lambda_i\delta_i\right).$$

From here we can again derive $\frac{d\ln C_i}{d\ln p_j}=\sigma_{ij}$ as in Proposition 2 by applying Implicit Function Theorem on the free-entry condition for sector i. Theorems 1, 2, Corollary 2, and Propositions 4 and 5 follow analogously.

On the other hand, in presence of within-sector heterogeneity, the elasticity of unit cost of production with respect to input prices is

$$\begin{aligned}\frac{\partial \ln C_i}{\partial \ln p_j} &= \int \sigma_{ij}(\nu) \frac{q_i(\nu)}{\int q_i(\nu)\, d\Phi_i(\nu)} d\Phi_i(\nu) \\ &= \mathbb{E}_\nu \left[\sigma_{ij}(\nu) \frac{q_i(\nu)(1+\lambda_i(\nu)\delta_i(\nu))}{\mathbb{E}_\nu [q_i(\nu)(1+\lambda_i(\nu)\delta_i(\nu))]} \right]\end{aligned}$$

and Proposition 2 fails to hold except in the knife-edge case in which $\lambda_i(\nu)\delta_i(\nu)$ is the same across all firms within sector i.

D Policy Instruments that Target Firms rather than Sectors

Propositions 4 and 5 consider policy instruments that effectively relax credit constraints by cost supplying working capital only to firms that have binding constraints, rather than to all firms, and the results are that the ratio between social and private marginal returns to expenditure on working capital is captured by the sectoral sales gap. Our result on sectoral input subsidies (Corollary 2) can analogously be generalized to tax instruments that apply to a subset of firms rather than to all firms. To see this, consider subsidies $\left\{\tau_i^j(\nu)\right\}_{\nu\in\mathcal{F}_i}$ applied to input j for a subset of firms $\mathcal{F}_i$ in sector i. To capture marginal changes to these firm-specific subsidies, we parametrize

$$\tau_i^j(\nu) \equiv f_i^j(\nu;\tau)$$

with $f_i^j(\nu;0) = 0$ for all ν and we assume $f_i^j(\cdot)$ is differentiable in τ. The private return to a marginal change in τ, defined as the ratio between total marginal product captured by firms and the total marginal cost of expending inputs following a marginal change in τ, is

$$PR \equiv \frac{\mathbb{E}_{\nu\in\mathcal{F}_i}\left[\sigma_{ij}(\nu)\, q_i(\nu)\, df_i^L(\nu;\tau)/d\tau\right]}{\mathbb{E}_{\nu\in\mathcal{F}_i}\left[s_{ij}(\nu)\, q_i(\nu)\, df_i^L(\nu;\tau)/d\tau\right]}.$$

On the other hand, the social marginal return, which can be found by following the procedure in the proofs for Theorems 1 and 2, is

$$SR \equiv \xi_i \times PR.$$

E Shrinkage Estimator in Section 3

This section of the appendix describes the shrinkages procedure used to estimate $\mathbb{E}_\nu\left[\epsilon_i(\nu)\right]$, $\mathbb{E}_\nu\left[k_i(\nu)\right]$, and $\mathbb{E}_\nu\left[\varphi_i^K(\nu)^{-1}\epsilon_i(\nu)\, k_i(\nu)\right]$ in equation (37) of section 3.2. Recall that equation (35) is estimated on the sample of SOEs to obtain parameter estimates $\widehat{\eta}_i$ and residuals $\widehat{\epsilon}_i(\nu)$. I then use the estimates $\widehat{\eta}_i$ to obtain elasticity $\widehat{\sigma_i^K}(\nu;\widehat{\eta}_i)$ for private firms and recover residuals $\widehat{res}_i(\nu)$, which can be interpreted as the product between the private wedge and ex-post TFP shock according to equation (36). As a result, the product between $\widehat{res}_i(\nu)$ and $k_i(\nu)$ is an estimate of $\varphi_i^K(\nu)^{-1}\epsilon_i(\nu)k_i(\nu)$.

For each of $x\in\{\widehat{res}\cdot k,k,\widehat{\epsilon}\}$ I separately apply an empirical Bayes (Morris 1983) procedure to estimate $\mathbb{E}_\nu\left[x_i(\nu)\right]$, exploiting the cross-industry information in the sample. Specifically, I assume $x_i(\nu)$ follows a log-normal distribution with industry-specific means θ_i and standard deviation ζ, and the θ_i's are drawn from a parent Normal distribution with mean μ and standard deviation τ:

$$\ln x_i(\nu)\left|\theta_i\sim N(\theta_i,\eta),\quad \theta_i\sim N(\mu,\tau).\right. \tag{E.1}$$

All parameters $\left(\{\theta_i\}_{i=1}^S,\eta,\mu,\tau\right)$ are unknown, and the end goal is to estimate θ_i and η in order to form estimates of $\mathbb{E}\left[x_i(\nu)\right]=\exp\left(\theta_i+\frac{\eta^2}{2}\right)$. I form estimates of $\widehat{\eta}$ directly as

$$\widehat{\eta}=\frac{1}{\sum N_i-S}\sum_i\sum_\nu\left(x_i(\nu)-\bar{x}_i\right)^2$$

where N_i is the number of firms in sector i and $\bar{x}_i\equiv\frac{\sum_\nu x_i(\nu)}{N_i}$ is the sample average of $x_i(\nu)$ in the sector. I apply empirical Bayes shrinkage procedure to estimate the industry-specific means θ_i's as follows.

Under the hierarchical Normality assumptions (E.1),

$$\bar{x}_i\sim N\left(\mu,\frac{\widehat{\eta}}{N_i}+\tau\right)$$

Let $\hat{\tau}$ be an estimate of τ and let $w_i\equiv\frac{1}{\frac{\widehat{\eta}}{N_i}+\widehat{\tau}}$. We form estimates of $\widehat{\mu}$ by weighting the sample mean of each industry:

$$\widehat{\mu}=\frac{\sum_i\bar{x}_i w_i}{\sum_i w_i}. \tag{E.2}$$

On the other hand, we can form the estimate $\widehat{\tau}$ as a function of $\widehat{\mu}$:

$$\widehat{\tau}=\frac{\sum_i w_i\left\{\left(\frac{S}{S-1}\right)(\bar{x}_i-\widehat{\mu})^2-\frac{\widehat{\eta}}{N_i}\right\}}{\sum_i w_i} \tag{E.3}$$

The 2-vector $(\widehat{\mu},\widehat{\tau})$ is solved as the fixed point of this pair of functions (E.2) and (E.3). Estimates of θ_i is formed by shrinking the sample mean $\bar{x}_i$ towards $\hat{\mu}$,

$$\widehat{\theta}_i=\left(1-\widehat{B}_i\right)\bar{x}_i+\widehat{B}_i\hat{\mu},$$

with weights $\widehat{B}_i$ being the relative precision of the naive sample mean estimator (to the precision of $\hat{\mu}$, the estimator for the mean of the upper distribution), appropriately adjusted for the degrees of freedom:

$$\widehat{B}_i=\frac{S-3}{S-1}\frac{\frac{\widehat{\eta}}{N_i}}{\frac{\widehat{\eta}}{N_i}+\widehat{\tau}}.$$

Lastly, we form the estimate $\widehat{\mathbb{E}}_\nu [x_i(\nu)]$ for $x \in \{\widehat{res} \cdot k, k, \widehat{\epsilon}\}$ as

$$\widehat{\mathbb{E}}_\nu [x_i(\nu)] = \exp\left(\widehat{\theta}_i + \widehat{\eta}/2\right).$$

F Robustness: Empirical Results in Section 3

F.1 Alternative Specifications

Sales Gap The sales gap measure used in the analysis of section 3 is constructed from the final share β, the observed Chinese input-output table Ω, and the estimated average sectoral wedge $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$, according to equations (38), (39), and (40). The results reported in the main text are based on two assumptions that go into the construction of these objects from data. First, β is constructed using only private and public consumption according to (42) and it excludes sectoral net exports from the final demand. Second, $\widehat{\Sigma}$ is constructed from Ω and $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$, and while the input-output table Ω includes both manufacturing and non-manufacturing sectors, our empirical strategy only recovers the sectoral wedges for the manufacturing sector. In the main text I assume that producers in the non-manufacturing sectors are unconstrained with $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}} = 1$ when constructing $\widehat{\Sigma}$ according to (38). Our main results are robust to using alternative ways to construct the sales gap measure.

First, recognizing that net exports is an important component of final demand and that it might have played a role for policy design in China, I define β^{NX} to include net exports and construct alternative sales gap measure $\widehat{\xi^{NX}}$. Specifically, I define

$$\beta_i^{NX} \equiv \frac{\text{Private and public consumption + net export of good } i}{\text{Total priv. and public consumption + net export of all goods}}.$$

and construct the sales vector γ^{NX}, influence vector $\widehat{\mu^{NX}}$, and sales gap vector $\widehat{\xi^{NX}}$ by replacing β with β^{NX} in (38), (39), and (40).

Second, to overcome the lack of data on $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ for non-manufacturing sectors, I construct a partial input-output table that records input coefficients only for the manufacturing sectors. Specifically, let M denote the set of manufacturing sectors. I define $\Omega^M \equiv \left[\omega_{ij}^M\right]_{i,j\in M}$ as the partial input-output table, with

$$\omega_{ij}^M = \frac{\omega_{ij}}{1 - \sum_{j\notin M} \omega_{ij}}$$

and I re-define labor share as

$$\alpha_i^M = \frac{\alpha_i}{1 - \sum_{j\notin M} \omega_{ij}}.$$

That is, I scale up the expenditure shares on labor and manufacturing inputs so that the sum of variable profits and the total expenditure on these inputs add up to one, excluding the expenditures on non-manufacturing goods. I then proceed to construct the elasticity matrix $\widehat{\Sigma^M}$ based on the partial input-output table, which is in turn used to build the alternative sales gap measure, $\widehat{\xi^M}$.

Our main results are that private firms in sectors with higher sales gaps tend to receive more external loans and pay lower interest rates, and that the sectoral presence of Chinese SOEs is heavily directed towards sectors with larger sales gaps. These results are reflected in columns (4) and (8) of Tables 4 and 5 as well as columns (2) and (4) of Table 6. Appendix Tables F.1 and F.2 show that these results are robust to using the alternative sales gap measures, by respectively replicating our main specifications using $\widehat{\xi^{NX}}$ and $\widehat{\xi^M}$ instead of $\widehat{\xi}$ as the sales gap measure.

Interest Rate The two measures of credit market interventions are constructed based on firm-level total liabilities and interest payments made in 2007. While almost all firms in the sample report a positive amount of liability, only 58% report positive interest payments. The results reported in Tables 3 and 5 are based on on the subsample of firms that report positive total interest payments. In Appendix Table F.4 I replicate these results and show that they are robust to using the entire sample of firms.

SOE Share Results reported in Table 6 uses SOE's share of total wage payments as the left-hand-side variable. Appendix Table F.5 replicates columns (4), (6), and (7) of Table 6 using SOE's share of gross value-added and share of revenue as the left-hand-side variable. The results are robust to using either of the alternative measures of SOE share: there is a positive relationship between SOE presence and sectoral sales gap, and the relationship is driven by variations within the subset of sectors that produce material goods.

Dropping Joint-Venture and Foreign Firms The sample of manufacturing firms used for analysis in the main text includes joint ventures between domestic and foreign entities as well as a small sample of firms for which foreign entities hold controlling stakes. I construct alternative sectoral wedge $\widehat{\left(\bar{\varphi}_i^{K,NF}\right)^{-1}}$ and sales gap $\widehat{\xi_i^{NF}}$ by dropping all firms with >10% of equity held by foreign entities, including those from Hong Kong, Macau, and Taiwan. Appendix Table F.6 replicates key specifications in the main text using these measures on the restricted sample of firms.

F.2 Robustness of Estimator

The estimation of industry-specific output elasticities $\sigma_i^K(\cdot)$ boils down to performing industry-specific regressions (35) on the sample of SOEs, while the estimation of industry-specific wedges $\bar{\varphi}_i^K$ relies on applying the elasticity function estimates to the sample of private firms. Appendix Table F.7 provides summary statistics on the number of SOEs and private firms in each of the 66 manufacturing industries on which estimation is performed.

A Specification Test for Credit Constraints A key assumption in my analysis is that credit constraints take the form in (31) and that private firms face credit constraints only on capital inputs but not on labor and intermediate materials for production. If labor and material inputs are also subject to credit constraints, there would be wedges that distorts expenditure shares on these inputs from their output elasticities, and we have to account for these wedges when constructing the input-output elasticity matrix $\widehat{\Sigma}$. The following exercise can be a specification test for this assumption.

Under the maintained assumption that SOEs are unconstrained, we can relate SOE's expenditure shares on labor and material inputs to their respective output elasticities according to equation (34). I therefore estimate the analogue of equation (35), replacing the variable on the left-hand-side to be log-expenditure share on labor and on material inputs, to separately estimate the output elasticities with respect to these inputs. I then form estimates of the hypothetical "labor wedge" and "material wedge" for the private firms using analogous procedures as in equations (36) and (37). If private firms are indeed unconstrained on these inputs, the estimated wedges should be equal to one for all industries. Appendix Table F.8 shows the result of this exercise by replicating part of Table 1. For both of these inputs, the unweighted average sectoral wedges have mean around one across industries while the weighted average sectoral wedges have mean around 1.05. Relative to the wedges on capital inputs, which average to 1.2 for the unweighted and 1.34 for the weighted, the hypothetical wedges on both labor and material inputs are much closer to one and have smaller dispersions around their respective means.

Parametrizing $\sigma_i^K(\cdot)$ as an Industry-Specific Constant In the main text, the output elasticity of capital inputs $\sigma_i^K(\cdot)$ is parametrized as a second-order polynomial of the input expenditures according to equation (35). In Appendix Table F.9 I replicate my main results (columns 4 and 8 of Tables 4 and 5 as well as columns 2 and 4 of Table 6) by parametrizing $\sigma_i^K(\cdot)$ as an industry-specific constant, which corresponds to production functions with a constant output elasticity of capital inputs.

Table F.1: Replication of main results in section 3 using $\widehat{\xi^{NX}}$ as the sales gap measure

Outcome variable	Debt-To-Capital ratio		Interest rate		SOE share of industry value-added	
	Private	SOE	Private	SOE		
	(1)	(2)	(3)	(4)	(5)	(6)
$\widehat{\xi_i^{NX}}$	0.104***	0.0257	-2.674***	-1.116***	0.528***	0.417**
	(0.0292)	(0.0414)	(0.973)	(0.273)	(0.162)	(0.206)
$\widehat{(\bar{\varphi}_i^K)^{-1}}$	0.00428	0.0309	0.903	0.901***	-0.0130	-0.00829
	(0.0274)	(0.0331)	(0.877)	(0.0828)	(0.100)	(0.105)
$(K/Y)_i$	0.0110***	0.000297***	-1.104***	-0.0828***		-0.00123
	(0.00305)	(0.000108)	(0.178)	(0.0149)		(0.00140)
$CapForm_i$	-0.0416**	0.0498*	-0.798	-1.351***		0.152
	(0.0189)	(0.0262)	(0.678)	(0.262)		(0.179)
Constant	0.421***	0.551***	6.929***	2.442***	-0.318	-0.217
	(0.0412)	(0.0606)	(1.137)	(0.291)	(0.201)	(0.221)
Obs.	279060	14211	162143	8407	66	66
adj. R^2	0.004	0.003	0.026	0.039	0.119	0.111

Notes: The table replicates key results from Tables 4, 5, and 6 of the main text, replacing the sales gap measure $\widehat{\xi}_i$ used in the main text with $\widehat{\xi_i^{NX}}$ as defined in F.1. $\widehat{(\bar{\varphi}_i^K)^{-1}}$ is the sectoral private return to capital inputs as defined in equation (37). For columns (1) through (4), $(K/Y)_i$ is the firm-level capital intensity, defined as the ratio between capital stock and firm revenue. For columns (5) and (6), $(K/Y)_i$ is the average capital intensity of firms in the sector. $CapForm_i$ is the fraction of sectoral output that is unused in the accounting year and is to be used at a future time. Columns (1) and (2) drop outlier firms with Debt-to-Capital ratio that is either negative or above the 99th percentile. Columns (3) and (4) drop outlier firms with interest rate that is either negative or above the 99th percentile. All specifications drop outlier firms with capital intensity that is either negative or above the 99th percentile. Standard errors in parentheses are clustered at the industry level for columns (1) through (4).

Table F.2: Replication of main results in section 3 using $\widehat{\xi}^M$ as the sales gap measure

Outcome variable	Debt-To-Capital ratio		Interest rate		SOE share of industry value-added	
	Private	SOE	Private	SOE		
	(1)	(2)	(3)	(4)	(5)	(6)
$\widehat{\xi}_i^M$	0.0617***	0.0305	-1.656***	0.785***	0.322***	0.262**
	(0.0183)	(0.0258)	(0.604)	(0.206)	(0.0971)	(0.117)
$\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$	0.0112	0.0317	0.744	0.819***	-0.000169	-0.00133
	(0.0252)	(0.0321)	(0.804)	(0.232)	(0.0993)	(0.103)
$(K/Y)_i$	0.0110***	0.000296***	-1.106***	-0.0828***		-0.00146
	(0.00303)	(0.000108)	(0.178)	(0.0149)		(0.00139)
$CapForm_i$	-0.0308**	0.0396	-1.007	-1.369***		0.163
	(0.0216)	(0.0240)	(0.662)	(0.244)		(0.169)
Constant	0.453***	0.544***	6.173***	2.245***	-0.135	-0.0774
	(0.0380)	(0.0576)	(1.000)	(0.273)	(0.162)	(0.167)
Obs.	279060	14211	162143	8407	66	66
adj. R^2	0.004	0.003	0.026	0.040	0.123	0.124

Notes: The table replicates key results from Tables 4, 5, and 6 of the main text, replacing the sales gap measure $\widehat{\xi}_i$ used in the main text with $\widehat{\xi}_i^M$ as defined in F.1. $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ is the sectoral private return to capital inputs as defined in equation (37). For columns (1) through (4), $(K/Y)_i$ is the firm-level capital intensity, defined as the ratio between capital stock and firm revenue. For columns (5) and (6), $(K/Y)_i$ is the average capital intensity of firms in the sector. $CapForm_i$ is the fraction of sectoral output that is unused in the accounting year and is to be used at a future time. Columns (1) and (2) drop outlier firms with Debt-to-Capital ratio that is either negative or above the 99th percentile. Columns (3) and (4) drop outlier firms with interest rate that is either negative or above the 99th percentile. All specifications drop outlier firms with capital intensity that is either negative or above the 99th percentile. Standard errors in parentheses are clustered at the industry level for columns (1) through (4).

Table F.3: Replication of Tables 3 to include the sample of firms that report zero interest payments

	$IntRate_i(\nu)$	
	(1)	(2)
$\mathbf{1}(SOE_i(\nu))$	-1.203***	-1.251***
	(0.0520)	(0.0516)
Constant	2.638***	-
	(0.0111)	-
Sector Fixed Effects	No	Yes
Obs.	276596	276596
adj. R^2	0.002	0.04

Notes: The table replicates columns (3) and (4) of Table 3 by including firms that report zero interest payments, which were dropped from the analysis reported in the main text. I drop outlier firms with interest rate that is either negative or above the 99th percentile.

Table F.4: Replication of Tables 5 to include the sample of firms that report zero interest payments

	Outcome Variable: Interest Rate							
	Sample: Private Firms				Sample: SOEs			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\widehat{\xi}_i$	-1.888***	-1.945***	-1.954***	-1.465**	-1.163***	-1.213***	-1.280***	-0.566***
	(0.637)	(0.635)	(0.638)	(0.658)	(0.249)	(0.244)	(0.247)	(0.258)
$\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$		0.745	0.695	0.741		0.556**	0.578**	0.573**
		(0.641)	(0.651)	(0.669)		(0.243)	(0.248)	(0.228)
$(K/Y)_i(\nu)$			-0.596***	0.588***			-0.000993	-0.000823
			(0.114)	(0.113)			(0.000569)	(0.000522)
$CapForm_i$				-0.891*				-1.101***
				(0.459)				(0.222)
Constant	4.736***	3.795***	4.384**	3.852***	2.743***	2.049***	2.134***	1.469***
	(0.814)	(0.843)	(0.884)	(0.926)	(0.308)	(0.347)	(0.357)	(0.312)
Obs.	264005	264005	261089	261089	12591	12591	12179	12179
adj. R^2	0.004	0.005	0.012	0.012	0.010	0.012	0.013	0.018

Notes: The table replicates Table 5 by including firms that report zero interest payments, which were dropped from the analysis reported in the main text. All specifications drop outlier firms with interest rate that is either negative or above the 99th percentile. Columns (3), (4), (7), and (8) also drop outlier firms capital intensity that is either negative or above the 99th percentile. Standard errors in parentheses are clustered at the industry level.

Table F.5: Replication of Table 6 using SOE share of industry gross value-added and industry revenue

	SOE share of industry gross value-added			SOE share of industry revenue		
Sample	All industries	Producers of material goods	Producers of capital goods	All industries	Producers of material goods	Producers of capital goods
	(1)	(2)	(3)	(4)	(5)	(6)
$\widehat{\xi}_i$	0.365***	0.623**	0.163	0.359***	0.633**	0.147
	(0.125)	(0.235)	(0.242)	(0.129)	(0.245)	(0.252)
$\widehat{(\bar{\varphi}_i^K)^{-1}}$	-0.0542	-0.0473	-0.0608	-0.0300	-0.0290	-0.0319
	(0.0886)	(0.0953)	(0.256)	(0.0918)	(0.0991)	(0.266)
$\overline{(K/Y)_i}$	-0.00115	0.170*	-0.000829	-0.00120	0.159	-0.000814
	(0.00122)	(0.0958)	(0.00149)	(0.00127)	(0.0997)	(0.00155)
$CapForm_i$		0.951	0.468*		0.719	0.493*
		(6.963)	(0.244)		(7.243)	(0.253)
Constant	-0.139	-0.570*	-0.0205	-0.162	-0.589*	-0.0486
	(0.172)	(0.301)	(0.334)	(0.178)	(0.313)	(0.346)
Obs.	66	46	20	66	46	20
adj. R^2	0.091	0.079	0.112	0.080	0.066	0.113

Notes: The table replicates selected specifications from Table 6 using alternative measures of SOE share. Columns (1) through (3) use SOE's share of sectoral gross value-added as the outcome variable, whereas columns (4) through (6) use SOE's share of sectoral total revenue as the outcome variable.

Table F.6: Replication of main results in section 3, dropping joint-ventures and firms with foreign ownership

Outcome variable	Debt-To-Capital ratio		Interest rate		SOE share of industry value-added	
	Private	SOE	Private	SOE		
	(1)	(2)	(3)	(4)	(5)	(6)
$\widehat{\xi}_i$	0.0902***	0.0300	-2.713***	-1.055***	0.407***	0.305*
	(0.0291)	(0.0336)	(0.832)	(0.266)	(0.138)	(0.165)
$\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$	0.0109	0.0295	0.692	0.775***	-0.0635	-0.0654
	(0.0315)	(0.0337)	(0.878)	(0.237)	(0.102)	(0.105)
$(K/Y)_i\ (\nu)$	0.0210***	0.000282***	-1.218***	-0.0887***		-0.00157
	(0.00354)	(0.0000994)	(0.195)	(0.0164)		(0.00141)
$CapForm_i$	-0.0324	0.0435	-0.731	-1.383***		0.193
	(0.0191)	(0.0243)	(0.636)	(0.243)		(0.170)
Constant	0.424***	0.561***	7.699***	2.589***	-0.140	-0.0394
	(0.0499)	(0.0643)	(1.131)	(0.281)	(0.191)	(0.202)
Obs.	224541	12714	146924	7556	66	66
adj. R^2	0.008	0.002	0.028	0.041	0.096	0.101

Notes: The table replicates key results from Tables 4, 5, and 6 of the main text, dropping firms with $>$ 10% equity held by foreign entities from the analsis. $\widehat{\xi}_i$ is the sales gap measure as in equation (39). $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ is the sectoral private return to capital inputs as defined in equation (37). For columns (1) through (4), $(K/Y)_i$ is the firm-level capital intensity, defined as the ratio between capital stock and firm revenue. For columns (5) and (6), $(K/Y)_i$ is the average capital intensity of firms in the sector. $CapForm_i$ is the fraction of sectoral output that is unused in the accounting year and is to be used at a future time. Columns (1) and (2) drop outlier firms with Debt-to-Capital ratio that is either negative or above the 99th percentile. Columns (3) and (4) drop outlier firms with interest rate that is either negative or above the 99th percentile. All specifications drop outlier firms with capital intensity that is either negative or above the 99th percentile. Standard errors in parentheses are clustered at the industry level for columns (1) through (4).

Table F.7: Number of firms in each industry by firm type

	Mean	St. Dev	Median	Min	Max
Number of SOEs in each industry	212	202	135	25	885
Number of private firms in each industry	4272	4301	2733	222	20557

Notes: The table provides summary statistics on the number of SOEs and private firms in each industry.

Table F.8: Specification test: hypothetical wedges on labor and material inputs

	$\mathbb{E}\left[\widehat{\varphi_i^L(\nu)^{-1}}\right]$	$\widehat{\left(\bar{\varphi}_i^L\right)^{-1}}$	$\mathbb{E}\left[\widehat{\varphi_i^X(\nu)^{-1}}\right]$	$\widehat{\left(\bar{\varphi}_i^X\right)^{-1}}$
Mean	0.9982	1.0512	0.9971	1.0524
St. dev	0.078	0.086	0.003	0.003

Notes: The table provides summary statistics for the hypothetical wedges on labor and material inputs and it serves as a specification test of the assumption that only capital goods are subject to credit constraints: the hypothetical wedges on labor and material inputs should be zero for all firms in all sectors. Columns (1) and (3) respectively correspond to the unweighted sectoral average of firm-level wedges on labor and material inputs. Columns (2) and (4) corresponds to the industry average labor and material wedge respectively weighted by the amount of labor and material inputs used by each firm.

Table F.9: Replication of main results in section 3, assuming firm-production functions have constant output elasticity for capital inputs

Outcome variable	Debt-To-Capital ratio		Interest rate		SOE share of industry value-added	
	Private	SOE	Private	SOE		
	(1)	(2)	(3)	(4)	(5)	(6)
$\widehat{\xi}_i$	0.135***	0.0471	-3.736***	-1.255***	0.510***	0.348
	(0.0314)	(0.0419)	(1.158)	(0.471)	(0.174)	(0.119)
$\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$	-0.0335	0.00514	1.357	0.0568***	-0.0611	-0.0737
	(0.0322)	(0.0369)	(1.183)	(0.305)	(0.103)	(0.498)
$(K/Y)_i\ (\nu)$	0.0108***	0.000295***	-1.101***	-0.0838***		-0.00125
	(0.00299)	(0.000106)	(0.180)	(0.0149)		(0.380)
$CapForm_i$	-0.0356	0.0445	-0.773	-1.416***		0.217
	(0.0185)	(0.0230)	(0.608)	(0.340)		(0.246)
Constant	0.434***	0.564***	7.599***	3.754***	-0.253	-0.0783
	(0.0558)	(0.0678)	(2.197)	(0.679)	(0.259)	(0.777)
Obs.	279060	14211	162143	8407	66	66
adj. R^2	0.005	0.002	0.028	0.036	0.094	0.096

Notes: The table replicates key results from Tables 4, 5, and 6 of the main text, parametrizing output elasticity with respect to capital as industry-specific constants. $\widehat{\xi}_i$ is the sales gap measure. $\widehat{\left(\bar{\varphi}_i^K\right)^{-1}}$ is the sectoral private return to capital inputs as defined in equation (37). For columns (1) through (4), $(K/Y)_i$ is the firm-level capital intensity, defined as the ratio between capital stock and firm revenue. For columns (5) and (6), $(K/Y)_i$ is the average capital intensity of firms in the sector. $CapForm_i$ is the fraction of sectoral output that is unused in the accounting year and is to be used at a future time. Columns (1) and (2) drop outlier firms with Debt-to-Capital ratio that is either negative or above the 99th percentile. Columns (3) and (4) drop outlier firms with interest rate that is either negative or above the 99th percentile. All specifications drop outlier firms with capital intensity that is either negative or above the 99th percentile. Standard errors in parentheses are clustered at the industry level for columns (1) through (4).