WINNING BY DEFAULT: WHY IS THERE SO LITTLE COMPETITION IN GOVERNMENT PROCUREMENT?

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ABSTRACT. Government procurement contracts generally have a small number of participants, and it is not uncommon for only one seller to be considered. We empirically quantify the factors determining the extent of competition observed in the United States federal procurement data, by developing, identifying, and estimating a principal-agent model in which the buyer exerts costly efforts for a more competitive field and offers a menu of contracts to sellers when choosing a winner. Our structural estimates for the IT and telecommunications service contracts show that relative to the costs of bidding and other costs associated with competitive solicitation, procuring agencies extract a large portion of the informational rents from low-cost sellers by designing contracts to be contingent on outcomes correlated with sellers’ private cost type; and hence the equilibrium number of bids is small. We investigate alternative policies to mandate more competition, and find that unless bidding costs are reduced, stripping procuring agencies of their discretion and their project-specific knowledge would increase the government procurement costs. (JEL H11, H57)
1. Introduction

The market for the United States federal government procurement constitutes about 16 percent of the federal government spending. Despite its vast size, the extent of competition for a procurement contract is not very intense; for example, $241 billion (or 45 percent of the payments for procurement contracts) were paid to contracts that attracted a single bid during FY 2010, based on the Federal Procurement Data System (FPDS). This could potentially be a source of a significant inefficiency in government resource allocation, especially for commercially unavailable, customized goods that do not have readily comparable market price. In this paper, we develop, identify, and estimate a procurement model to empirically quantify the factors determining the extent of competition observed in the data.

To conduct this analysis, we integrate two important institutional features of federal government procurement that have attracted attention from the literature, but have not yet been studied jointly. First, a procuring agency, or a buyer hereafter, chooses the extent to which a contract is competed.\footnote{According to the Federal Acquisition Regulations (FAR, hereafter), authority and responsibility to contract for supplies and services are vested in the government agency head (FAR 1.6). The agency head may establish contracting activities and delegate broad authority to contracting officers. In our analysis, a buyer refers to a government agency head.} Federal regulations permit contracting without providing for full and open competition under broadly defined circumstances.\footnote{FAR 6.2 and 6.3 provide details on these circumstances, including discretionary ones (e.g., limited data rights, patents, copyrights, follow-on contract, and urgency) as well as non-discretionary ones (e.g., statutes and set-asides for small business concerns). We focus on the former type of restrictions in competition.} Such practices are prevalent; for instance, 1.2 million contracts (51 percent) were awarded without full and open competition in FY 2010. Many empirical studies such as Krasnokutskaya and Seim (2011) and Athey, Coey and Levin (2013) estimate the effects of restricting entry or providing preferential treatments to certain bidders on auction outcomes, taking such policies as given. Our paper, on the other hand, studies how competition is endogenously determined, focusing on the buyers’ preferences over the extent of competition and for particular sellers. In this
regard, a growing empirical literature on the behaviors of the buyers in government procurement (Bandiera, Prat and Valletti, 2009; Coviello, Guglielmo and Spagnolo, 2017) is closely related.

Second, the final contract price can differ from, and is often much larger than the initially agreed price. The ex-post price changes may arise either from contingencies specified in the original contract or from ex-post renegotiations on that contract. Ex-post renegotiations and their associated costs have been empirically studied by Gagnepain, Ivaldi and Martimort (2013) and Bajari, Houghton and Tadelis (2014), but the other type of ex-post price changes have received scant attention in the empirical literature, despite the extensive theoretical literature on optimal contracting in procurement (Laffont and Tirole, 1987; McAfee and McMillan, 1987; Riordan and Sappington, 1987). Our paper fills this gap by exploiting a distinctive feature in the FPDS that specifies each of the ex-post contract price and duration changes and the corresponding reason. Federal regulations distinguish two types of ex-post changes, and refer to those related to the original contract as unilateral and those associated with renegotiations as bilateral. Because competitive behavior affects the terms of the initial contract and hence the final contract price, it is important to study the mechanism through which ex-post price changes occur in conjunction with the buyer’s discretion regarding the extent of competition.

Incorporating these two institutional features of federal procurement in the model, we account for the following three notable trends in the data. First, more competition is correlated with higher contract prices, even after controlling for observed contract attributes. In our model, the number of bids is endogenously determined by the buyer’s efforts to solicit bids, which are driven by considerations not part of our data: the distribution of sellers’ costs and the buyer’s costs of intensifying competition.3 The latter costs could result from corruption and capture as well as administrative costs of conducting

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3Our paper is complementary to the empirical literature on endogenous entry in auctions when the bidders pay entry costs (Bajari and Hortacsu, 2003; Hendricks, Pinkse and Porter, 2003; Li, 2005; Li and Zheng, 2009; Susan Athey and Seira, 2011). Our model assumes that the project costs are independently distributed across sellers. However, in common value or affiliated private value auctions, a positive relationship between bids and the number of bidders may arise even in the absence of entry (Bulow and Klemperer, 2002; Pinkse and Tan, 2005).
and evaluating the competition (Bajari and Tadelis, 2001; Bandiera, Prat and Valletti, 2009) and noncontractible quality (Manelli and Vincent, 1995).4

Second, both types of ex-post price changes are frequent and large in size. In FY 2010, unilateral price changes accounted for 39 percent ($209 billion) of the total procurement payments, and bilateral ones for 14 percent ($75 billion). In our model, the buyer offers a menu of contracts when selecting a winner.5 The contracts in the menu lay out contingency plans for unilateral price changes regarding contractible outcomes that are correlated with the seller’s private cost type. Renegotiations or bilateral changes are allowed for design or specification modifications, whose associated costs are independent of the seller type. The ex-post price changes can be costly, potentially due to adaptation costs (Crocker and Reynolds, 1993; Bajari and Tadelis, 2001; Bajari, Houghton and Tadelis, 2014) and/or sellers’ risk aversion (Baron and Besanko, 1987; Laffont and Rochet, 1998; Arve and Martimort, 2016).

Third, unilateral delays are positively correlated with ex-post price increases. This pattern is at odds with time incentive contracts based on moral hazard (Lewis and Bajari, 2011, 2014). Yet it occurs in the equilibrium of our model in which unilateral delays are positively correlated with sellers’ private costs and the buyer uses the delays as a screening mechanism.

We provide sufficient conditions under which the model is nonparametrically identified from our data. In this regard, our paper belongs to the literature on the identification of principal-agent models (Perrigne and Vuong, 2011; Gayle and Miller, 2015). The primitives of the model include the preferences of public officials and sellers’ preferences over payment schedules. We allow continuously distributed unobserved project heterogeneity to stochastically determine seller costs; unobserved heterogeneity has been emphasized in the auction literature (Krasnokutskaya, 2011). Our identification arguments rely on the equilibrium arguments of acquiring information and preparing for bidding can also be included, at least partially, in the buyer’s costs of intensifying competition. The more aggressive the buyer is to help reduce sellers’ participation costs, the more likely will they participate.

5Menus for contracts like those Laffont and Tirole (1993) are not used for construction contracts, and mechanisms other than contract menus such as competitive bidding, reputation, and third-party bonding companies seem to be important in addressing adverse selection problems in procurement (Bajari and Tadelis, 2001; Bajari, McMillan and Tadelis, 2008). In the contracts that we study, competition is not intense, most contractors do not win more than one contract, and performance and payment bonds are not required (FAR 28.103).
relationship between the winning contract and the distribution of seller costs. We exploit variations in the realized number of bids, conditional on the unobserved project heterogeneity.

We focus on contracts for commercially unavailable IT and telecommunications services that were awarded during FY 2004-2012, worth a total of $1.2 billion. We control for observed heterogeneity by focusing on a single sector because procurement contracts differ widely in the scale, scope, and competitive intensity. The estimates of the structural model show that relative to buyers’ costs of intensifying competition, they extract a large portion of the informational rents from low-cost sellers by offering contracts that are contingent on outcomes that are correlated with sellers’ private cost type. If standard first-price auctions were used instead, the number of bids would increase by 0.7 bids on average (47 percent), but the contract price would also increase by $35,800 per project (3 percent). We also find that policies that mandate more competition would decrease the contract price, the expected project cost of the winner, and the efficiency loss associated with ex-post price changes, but these would come at larger costs associated with implementing a formal solicitation procedure and attracting and processing more bids. While our empirical results are specific to IT and telecommunications services, the empirical framework itself can be readily applied to other similar types of contracts.

To reduce potential inefficiency in procurement due to agency problems between taxpayers and the public officials (the buyers in our model), various policies to reduce their discretion could be considered. The federal procurement reforms in mid-1990s, on the other hand, increased flexibility and struck down some of the onerous regulations mainly designed to prevent collusion between sellers and public officials (Kelman, 1990, 2005). Rewinding the clock, we consider policies where centralized, first-price auctions are mandated for all projects and an identical amount of effort is expended to attract and process bids. We find limited support for permitting the use of discretion over the alternative of giving no discretion to public officials.

The rest of the paper is organized as follows. In Section 2, we delineate the institutional setting and the data, and present empirical features that motivate our model, which is described in Section 3. The equilibrium of the model is
presented in Section 4, followed by the identification of the model in Section 5. Section 6 presents the estimation results, and Section 7 provides our empirical answers to the title question of the paper and discusses counterfactual policies. Section 8 concludes.

2. INSTITUTIONAL BACKGROUND AND DATA

The data is drawn from the Federal Procurement Data System (Next Generation), through usaspending.gov. With the exception of a few recent papers (Warren, 2014; Liebman and Mahoney, forthcoming; MacKay, 2016), this data has not been extensively used yet. For each procurement project, we observe the solicitation procedure, the number of bids, the history of price and duration changes, product and service code, contracting agency (e.g., Department of Defense), and the location of contract performance. The purpose of this section is to describe the institutional background and the features of the data that are the most pertinent to our analysis.

We study contracts with specified terms and conditions for commercially unavailable services that were initiated during the fiscal years of 2004–2012. In this paper, we focus on the information technology (IT) and telecommunications service contracts to consider a relatively homogeneous set of contracts in terms of the nature of the provided service. The IT and telecommunications services include, but are not limited to, IT strategy and architecture, programming, cyber security, data entry, backup, broadcasting, storage, and distribution, and telephone/Internet services. The contracts for these services

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6 The alternative type of contract is an indefinite delivery, indefinite quantity contract (basic ordering agreement, blanket purchase agreement, federal supply schedule, government wide acquisition contract, and indefinite delivery contract), based on which delivery orders are made. A large fraction of the government procurement budget is allocated for the definitive contracts and purchase orders, the focus of our analysis. For example, in FY 2010, 47 percent ($253 billion) of the total amount of money that the government was obliged to pay was for these contracts. During the same period, definitive contracts and purchase orders for commercially unavailable goods or services accounted for 39 percent ($210 billion) of the total obligated amount.

7 Specifically, we consider the contracts with a product and service code of Category D3. The federal procurement data system requires that a product and service code be reported for each contract, and the codes are divided into three groups: research and development (R&D), service, and products. Among the service codes, there are 48 categories, and Category D3 is for IT and telecommunications services.
account for $30 billion of the total obligated amount to the federal government in FY 2010.

We further restrict our attention to the contracts that were (i) paid for $300,000–$5 million, (ii) expected to take longer than 30 days to complete, (iii) completed before the end of FY 2014, (iv) not terminated prematurely, and (v) performed within the continental United States. There are in total 2,203 such contracts in the data, costing the government $3.2 billion (CPI-adjusted to 2010 dollars) collectively, as shown in Table 1.

2.1. Competition and Negotiation. Table 1 presents summary statistics of the contracts in our sample by the extent of competition as specified in the data. Full and open competition is the default in the acquisition process, but federal regulations specify the circumstances under which a procuring agency is allowed to limit competition (FAR 6.2 and 6.3). More than two thirds of the contracts in the sample were not fully competed. The reasons stated in the data can be categorized into three cases: (i) unavailable for competition due to domestic statutes or international agreements, (ii) set-aside for small business concerns due to statutory requirements, such as section 8(a) of the Small Business Act and the Historically Underutilized Business Zones Act of 1997, and (iii) discretionary. As for the third category, the detailed reasons include the existence of limited rights in data, patent rights, copyrights, secret processes, or brand (54 percent in our data), follow-on contract (19 percent), urgency (8 percent), and other/unspecified (18 percent).

Even when a contract is competed, having only one bid is quite common (36 percent) and the median number of bids is 2. The number of bids can be affected by the efforts of the buyer to exchange information with potential bidders in advance, via pre-solicitation notices, draft requests for proposals, requests for information, industry conferences, public hearings, market research,

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8The lower threshold of $300,000 is chosen because each acquisition of supplies or services that has an anticipated dollar value between $3,500 and $150,000 is automatically reserved exclusively for small business concerns (see FAR 19.502-2). Because the anticipated payment amount does not always appear in the data, we use a threshold of $300,000 for the actual payment, which is twice of the threshold of the anticipated payment. We exclude the projects performed outside of the continental United States because the cost structure could be very different from those in our sample.
<table>
<thead>
<tr>
<th>Extent of competition</th>
<th>Obs.</th>
<th>Size ($M)</th>
<th>One Bid</th>
<th>Num. Bids</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Mean</td>
<td>SD</td>
<td>Ratio</td>
</tr>
<tr>
<td>No/limited competition</td>
<td>1,631</td>
<td>1.49</td>
<td>1.20</td>
<td>0.93</td>
</tr>
<tr>
<td>Unavailable for competition</td>
<td>796</td>
<td>1.67</td>
<td>1.19</td>
<td>0.98</td>
</tr>
<tr>
<td>Set-aside for small business</td>
<td>183</td>
<td>1.71</td>
<td>1.31</td>
<td>0.44</td>
</tr>
<tr>
<td>Not competed by discretion</td>
<td>652</td>
<td>1.20</td>
<td>1.12</td>
<td>1.00</td>
</tr>
<tr>
<td>Full and open competition</td>
<td>572</td>
<td>1.30</td>
<td>1.10</td>
<td>0.36</td>
</tr>
<tr>
<td>Negotiated acquisition</td>
<td>310</td>
<td>1.38</td>
<td>1.16</td>
<td>0.27</td>
</tr>
<tr>
<td>Sealed-bid auction</td>
<td>9</td>
<td>2.14</td>
<td>1.32</td>
<td>0.78</td>
</tr>
<tr>
<td>Other solicitation procedure</td>
<td>253</td>
<td>1.18</td>
<td>0.99</td>
<td>0.45</td>
</tr>
<tr>
<td>Total</td>
<td>2,203</td>
<td>1.44</td>
<td>1.17</td>
<td>0.78</td>
</tr>
</tbody>
</table>

Notes: This table provides summary statistics of all definitive, commercially unavailable IT and telecommunications service contracts of FY 2004-2012 with a large size ($0.3–5 million) and a long expected duration (≥ 30 days). Size refers to the CPI-adjusted total amount of obligated money to the government per contract as of FY 2014, where CPI of December 2010 is 100.

and one-on-one meetings. Note that these efforts are costly to the buyer, because attracting and evaluating an additional bid or proposal incurs an extra administrative burden.9

Various procedures are used to solicit bids for competed contracts. We partition them into three: negotiated acquisition, sealed-bid auction, and others.10 Table 1 shows that sealed-bid auctions are rarely used; only 9 contracts out of the 572 fully competed ones were auctioned. FAR 6.4 delineates the conditions under which sealed-bid auctions are required. When these conditions are not met, the most prevalent solicitation method is negotiated acquisition, where a buyer issues a request for proposal, upon which interested sellers submit

9Furthermore, there is anecdotal evidence that the risk of receiving a bid protest from losing bidders is not small. Federal Times reported in July 2013 on how bid protests are slowing down procurements. The article quoted Mary Davie, assistant commissioner of the Office of Integrated Technology Services at the General Services Administration: “We build time in our procurement now for protests. We know we are going to get protested.”

10Examples of the other solicitation procedures are architect-engineer, two-step, basic research, and simplified acquisition.
Table 2. Competition and Military Contracts

<table>
<thead>
<tr>
<th></th>
<th>Noncompetitive</th>
<th>One Bid</th>
</tr>
</thead>
<tbody>
<tr>
<td>Military agency</td>
<td>0.144***</td>
<td>0.120***</td>
</tr>
<tr>
<td></td>
<td>(0.0417)</td>
<td>(0.0258)</td>
</tr>
<tr>
<td>Log(expected duration in days)</td>
<td>-0.0367</td>
<td>-0.0263**</td>
</tr>
<tr>
<td></td>
<td>(0.0220)</td>
<td>(0.0126)</td>
</tr>
<tr>
<td>Product &amp; service code FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>State, year, month FE</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
<td>962</td>
<td>962</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.182</td>
<td>0.193</td>
</tr>
</tbody>
</table>

Note: The dependent variables are (i) Noncompetitive, a dummy variable that indicates whether the solicitation was competitive; and (ii) One Bid, a dummy variable that indicates whether only one bid was considered (including noncompetitive and competitive). All contracts in the final sample are included. The standard errors are clustered at the 4-digit product and service code level, and provided in parentheses; *$p < 0.10$, **$p < 0.05$, ***$p < 0.01$. Military agencies include the Departments of Defense, State, and Homeland Security.

their proposals. After receipt of proposals, the buyer negotiates price, project schedule, technical requirements, and the terms of the contract.\textsuperscript{11}

We study discretionary noncompetitive contracts and competitive contracts solicited via negotiated acquisition. We exclude the noncompetitive contracts related to the government statutes in order to study the role of discretion. This produces a final sample of 652 noncompetitive contracts and 310 competitive ones, shown in Table 1.

Based on the final sample, we find that the contracts awarded by the military-related agencies (the Departments of Defense, State, and Homeland Security) tend to be less competitive than others, controlling for the product and service code, the location of the contract performance, and the period of the contract award, as can be seen in Table 2. This may be related to more pre-qualification requirements to work for these agencies, such as security clearance, than others. In our analysis, we control for these two types of government agencies.

\textsuperscript{11}Although the contract type is included in solicitation (FAR 16.105), it is treated as a matter for negotiation (FAR 16.103), considering various factors such as price competition, price and cost analysis, and type and complexity of the project (FAR 16.104).
2.2. **Unilateral versus Bilateral Changes.** The contract price at the time of the award, *base price*, can be different from the actual price at the end of the contract, *final price*. We define the base price as the total amount of money that the government is obliged to pay at the beginning of the contract; the final price as the sum of all payments. The final price is typically higher than the base price, but not always.

Similarly, the duration of a contract may change ex-post. The *base duration* is the difference between the expected completion date and the starting date as in the initial contract. The *final duration* is the difference of the expected completion date of the last contract action and the starting date of the initial contract action. A *delay* is then the difference between the final duration and the base duration.

FAR 43.1 defines two types of ex-post price and duration changes: *bilateral* or *unilateral*. A bilateral change must be signed by both the seller and the buyer. They are used to make negotiated adjustments resulting from ex-post agreements modifying the terms of contracts. A unilateral change, on the other hand, requires the approval of the buyer alone, following the predetermined terms of a contract. We partition each record of ex-post price and duration changes into bilateral or unilateral ones, based on the stated reasons for modification. There are twenty, including additional work, supplemental agreement for work within scope, change order, definitization of letter contracts, and definitization of change orders. We consider price and duration changes due to these five reasons as bilateral, while the other remaining reasons, such as an administrative action, an exercise of an option, and close-out, as unilateral.\(^\text{12}\)

Table 3 shows that both types of price changes are frequent and considerable in size. Among the contracts in the sample, 58 percent of them underwent unilateral ones, and 37 percent experienced bilateral ones. The average amounts

\(^\text{12}\)The full list of the remaining fifteen categories of reasons for modification are: C. Funding Only Action, E. Terminate for Default, F. Terminate for Convenience, G. Exercise an Option, J. Novation Agreement, K. Close Out, M. Other Administrative Action, N. Legal Contract Cancellation, R. Re-representation of Non-Novated Merger/Acquisition, S. Change Procurement Instrument Identifier, T. Transfer Action, V. Vendor DUNS Change, W. Vendor Address Change, X. Terminate for Cause. Note that we drop contracts that were terminated prematurely or canceled.
Table 3. Price and Duration by Competition

<table>
<thead>
<tr>
<th></th>
<th>All (962)</th>
<th>Not Competed (652)</th>
<th>Competed 1 Bid (83)</th>
<th>Competed 2+ Bids (227)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of observations</td>
<td>962</td>
<td>652</td>
<td>83</td>
<td>227</td>
</tr>
<tr>
<td>Final contract price ($M)</td>
<td>1.26 (1.13)</td>
<td>1.20 (1.12)</td>
<td>1.20 (1.11)</td>
<td>1.45 (1.17)</td>
</tr>
<tr>
<td>Contracts with price changes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unilateral</td>
<td>0.58</td>
<td>0.58</td>
<td>0.60</td>
<td>0.54</td>
</tr>
<tr>
<td>Bilateral</td>
<td>0.37</td>
<td>0.38</td>
<td>0.35</td>
<td>0.35</td>
</tr>
<tr>
<td>Amount of price changes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unilateral</td>
<td>0.36 (0.66)</td>
<td>0.35 (0.63)</td>
<td>0.35 (0.62)</td>
<td>0.39 (0.74)</td>
</tr>
<tr>
<td>Bilateral</td>
<td>0.18 (0.51)</td>
<td>0.19 (0.51)</td>
<td>0.18 (0.45)</td>
<td>0.16 (0.53)</td>
</tr>
<tr>
<td>Total duration (years)</td>
<td>2.16 (1.77)</td>
<td>2.08 (1.74)</td>
<td>2.55 (1.92)</td>
<td>2.24 (1.80)</td>
</tr>
<tr>
<td>Length of duration changes</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unilateral</td>
<td>0.69 (1.28)</td>
<td>0.63 (1.23)</td>
<td>0.96 (1.48)</td>
<td>0.77 (1.39)</td>
</tr>
<tr>
<td>Bilateral</td>
<td>0.30 (0.85)</td>
<td>0.32 (0.85)</td>
<td>0.27 (0.73)</td>
<td>0.27 (0.90)</td>
</tr>
</tbody>
</table>

Notes: All contracts in our final sample are included in this table, and standard deviations are provided in parentheses. Size refers to the CPI-adjusted total amount of obligated money to the government per contract as of FY 2014, where CPI of December 2010 is 100. Bilateral changes are associated with additional work, supplemental agreement for work within scope, change order, and definitization of letter contracts or change orders, and unilateral changes are associated with other reasons such as an administrative action, an exercise of an option, and close-out.

of ex-post price changes per contract are $362,000 (unilateral) and $182,000 (bilateral) in 2010 dollars, which in sum amount to $544,000, or 43 percent of the average final price of a contract in the sample.

The ex-post price changes occur regardless of the contract type as stated in the data. In particular, although firm fixed price contracts are supposed to make the contractor take full responsibility for the performance costs and resulting profit or loss (FAR 16), 54 percent of them in our sample experienced a unilateral price change and 34 percent of them experienced a bilateral price change. These frequencies of an ex-post price change are slightly less than the counterparts of other contracts in the data. Furthermore, the average amount of ex-post price changes, conditional on having one, are not statistically different among the firm-fixed contracts and the other contracts. For these reasons, we ignore the stated contract type in our analysis and focus on the ex-post, realized price and duration changes.
Table 4. Relationship between Price Changes and Delays

<table>
<thead>
<tr>
<th>Unilateral price change</th>
<th>Bilateral price change</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Unilateral delay (years)(^a)</td>
<td>201.3*** 189.8***</td>
</tr>
<tr>
<td></td>
<td>(23.44) (28.80)</td>
</tr>
<tr>
<td>Bilateral delay (years)(^b)</td>
<td>21.60 41.98**</td>
</tr>
<tr>
<td></td>
<td>(16.87) (17.71)</td>
</tr>
<tr>
<td>Base price and duration</td>
<td>No Yes No Yes</td>
</tr>
<tr>
<td>Product &amp; service code FE</td>
<td>Yes Yes Yes Yes</td>
</tr>
<tr>
<td>Agency, state, year, month FE</td>
<td>No Yes No Yes</td>
</tr>
<tr>
<td>N</td>
<td>962 962 962 962</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.208 0.378 0.262 0.466</td>
</tr>
</tbody>
</table>

Note: The dependent variable in specifications (1) and (2) is the amount of the unilateral price change in thousand dollars; and the counterpart in specifications (3) and (4) is the amount of the bilateral price change in thousand dollars.  
\(^a\) Unilateral delay refers to the sum of all delays associated with reasons other than additional work, supplemental agreement for work within scope, change order, and definitization of letter contracts or changer orders.  
\(^b\) Bilateral delay refers to the sum of all delay associated with the aforementioned five reasons. All contracts in the final sample are included; standard errors are clustered at the 4-digit product and service code level, and provided in parentheses; \(*p < 0.10, **p < 0.05, ***p < 0.01.\)

2.3. Price and Duration. Table 4 shows the statistical relationships between the ex-post price changes and delays. The results of the regressions of delays and other project attributes on the amount of the unilateral (bilateral) price changes are presented in Columns (1) and (2) (Columns (3) and (4)). There are a few notable trends manifest in the table. First, longer delays are associated with larger ex-post price increases for both bilateral and unilateral changes. Second, bilateral price changes are slightly more responsive to the length of the delays than unilateral ones. One-year delays due to unilateral actions are associated with an increase in the unilateral price change of about $189,800-201,300, depending on the specifications; and one-year bilateral delays are associated with a larger increase in the bilateral price change at the point estimates, about $265,900-275,800. It seems as if contracts reward delays, which is at odds with a standard moral hazard model which would predict that tardiness is penalized.
Table 5. Competition and Price

<table>
<thead>
<tr>
<th></th>
<th>Log (Price)</th>
<th>Bilateral ΔPrice</th>
<th>Unilateral ΔPrice</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Competitive</td>
<td>0.256***</td>
<td>0.047</td>
<td>73.40*</td>
</tr>
<tr>
<td></td>
<td>(0.093)</td>
<td>(0.104)</td>
<td>(39.77)</td>
</tr>
<tr>
<td>Log (num. of bids)</td>
<td>0.213***</td>
<td>34.06</td>
<td>53.91</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(52.54)</td>
<td></td>
</tr>
<tr>
<td>Base duration</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Fixed effects†</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>N</td>
<td>962</td>
<td>962</td>
<td>962</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.297</td>
<td>0.309</td>
<td>0.324</td>
</tr>
</tbody>
</table>

Note: The dependent variable for (1) and (2) is the log of the total contract price, the CPI-adjusted total amount of obligated money to the government per contract as of FY 2014. The dependent variable for (3) and (4) is the total ex-post unilateral price change in thousand dollars, and that for (5) and (6) is the total ex-post bilateral price change in the same unit. All contracts in the final sample are included; standard errors are clustered at the product and service code level, and provided in parentheses; *p < 0.10, **p < 0.05, ***p < 0.01.
†: 4-digit product and service codes, procuring agency, state of the contract performance, year of award, and month of the award, respectively.

2.4. Competition and Price. We find that more competition is associated with higher contract prices. Table 3 shows that the average contract price does not decrease as competition intensifies from no competition to more than two bids. Furthermore, the frequency and the extent of the price and duration changes do not seem to vary with the extent of competition. These findings persist even after controlling for observed heterogeneity of each contract as shown in Table 5. This pattern is inconsistent with the equilibrium of standard auction models, which predicts that procurement price falls as the number of bids increases.

Repeated interactions occur infrequently overall. Table 6 presents the summary statistics of our sample by the seller’s history of winning contracts. To allow that the reputation of a seller may also be built from similar contracts to those in our sample, we look at all contracts with definitive terms and conditions for IT and telecommunications services with a final price at least $300,000, initiated during the period of our study. There are in total 8,199
Table 6. Non-repeat vs. Repeat Sellers

<table>
<thead>
<tr>
<th></th>
<th>Num. Sellers</th>
<th>Num. Contracts</th>
<th>Competed</th>
<th>Num. Bids</th>
<th>Size ($M)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-repeat sellers</td>
<td>284</td>
<td>284</td>
<td>0.33</td>
<td>2.38</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.54)</td>
<td>(0.65)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Repeat sellers (≤ 10)</td>
<td>282</td>
<td>405</td>
<td>0.28</td>
<td>1.69</td>
<td>1.26</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.10)</td>
<td>(0.54)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Repeat sellers (&gt; 10)</td>
<td>52</td>
<td>273</td>
<td>0.37</td>
<td>2.57</td>
<td>1.36</td>
</tr>
<tr>
<td></td>
<td>(0.03)</td>
<td>(0.40)</td>
<td>(0.74)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>618</td>
<td>962</td>
<td>0.32</td>
<td>2.14</td>
<td>1.26</td>
</tr>
</tbody>
</table>

Notes: We divide the contracts in our sample into three categories based on the seller’s history of winning any of the definitive contracts for IT and telecommunications contracts with a contract size greater than or equal to $300,000 (8,199 contracts in total): non-repeat sellers, repeat sellers with 2–10 contracts, and those with more than 10. Size refers to the CPI-adjusted total amount of obligated money to the government per contract as of FY 2014, where CPI of December 2010 is 100. The numbers in parentheses are standard errors.

contracts, which were performed by 3,244 unique sellers collectively.\(^{13}\) The 962 contracts in our sample were performed by 618 sellers, and 52 (8 percent) of them won more than ten of the 8,199 contracts. Table 6 shows that the contracts won by these sellers are on average more likely to be competed and tend to have more bids. These trends do not support the hypothesis that the sellers who win multiple contracts face less competition. This implies that discretionary restrictions in competition may not be associated with dynamic incentive schemes.\(^{14}\)

\(^{13}\)To identify a unique seller, we use its parent company’s DUNS Number. A DUNS number is a unique nine-digit identification number for each physical location of a business, and is required for all businesses to register with the federal government for contracts or grants. For example, there are 45 unique DUNS numbers that are associated with AT&T as a parent company, and we treat them as one seller in Table 6.

\(^{14}\)This is consistent with the findings of the US Government Accountability Office in its report to the Congress in April 2009 (GAO-09-374). Based on their analysis of 62 contract solicitations from FY 2007 and 2008 and meetings with 121 contracting officials, the authors of the report conclude that better performance information is needed to support agency contract award decisions. The contracting officials’ reluctance to rely more on past performance was found to be due, in part, to their skepticism about the reliability of information and difficulty assessing relevance to specific acquisition.
Furthermore, we observe the number of losing bids for each competitive contract, but we do not observe their identities. This limits our capacity to study the possibility of collusion and reputation. However, most sellers win only one contract during the period of study (Table 6), and the contracts in our sample tend to appear irregularly in terms of size and requirements. These features make it difficult for sellers to maintain a collusive relationship (Porter and Zona, 1993). Although the data are not suitable to studying inter-temporal incentives, we partially accommodate long-term relationships of buyer-seller pairs through buyer preferences for no competition.

3. Model

The institutional features and data trends highlighted in the previous section are guideposts for developing a model that explains why there is so little competition in government procurement. This section exposits our model.

3.1. Total Cost to a Buyer. The rules described in Section 2 on solicitation delegate responsibility to the buyer for deciding whether she will permit competition or not. This is a choice variable in our model. We denote by \( \eta \) a cost to the buyer of choosing a competitive process rather than simply selecting a default seller. We do not assume that \( \eta \) reflects a social cost, but do allow for the possibility that it might.

Should she permit competition, the second choice confronting the buyer in our model is the extent of soliciting extra bids. The various activities mentioned in Section 2.1 testify to the range of instruments available to the buyers for publicizing the request for proposals. In our model we denote the level of effort by \( \lambda \in \mathbb{R}^+ \), which is the arrival rate of a Poisson probability distribution for the number of bids exceeding one. The greater the number of bids, denoted by \( n \), the higher the administrative costs of attracting and processing bids, denoted by \( \kappa(n) \). We assume that \( \kappa(n) \) is positive, increasing and convex in \( n \).\(^{15}\)

\(^{15}\)The buyer’s costs of soliciting extra bids may include sellers’ costs of acquiring information and preparing for bidding in the sense that more efforts of the buyer can reduce sellers’ costs. Our model, however, assumes that sellers’ participation costs are independent of the cost of the project and its quality. Hence procurement of research and innovation is beyond the scope of the model because sellers’ (unverifiable) efforts prior to bidding may affect the
Section 2.2 distinguishes between the base price, which we now denote by \( p \), and the final price, which we express as \( p + \Delta \). Thus the total cost of the project to the buyer is:

\[
p + \Delta + \kappa(n) + \eta \quad \text{if the project is competed with } n \text{ bids},
p + \Delta + \kappa(1) \quad \text{if the project is not competed}.
\]

3.2. Payoff to a Selected Seller. In the model there are two types of sellers, low and high cost. The total cost of completing a given procurement project is the sum of the deterministic cost, \( \alpha \in \mathbb{R}^+ \) for a low-cost seller and \( \alpha + \beta > \alpha \) for a high-cost seller, and ex-post stochastic cost change, denoted by \( \epsilon \in \mathbb{R} \).

Liquidity concerns, or costs of working capital, lead sellers to discount \( \Delta \) and enlarge \( \epsilon \).\(^{16}\) Thus the payoff from contract \((p, \Delta)\) and realized value of \( \epsilon \) is:

\[
p + \psi(\Delta - \epsilon) - \alpha \quad \text{if the seller is low cost},
p + \psi(\Delta - \epsilon) - \alpha - \beta \quad \text{if the seller is high cost},
\]

where \( \psi(\cdot) \) is a continuous real-valued function defined on \( \mathbb{R} \), with \( \psi(0) = 0, \psi'(0) = 1, \psi' > 0, \) and \( \psi'' < 0 \).

3.3. Timeline and Information. Figure 1 represents the timeline of the model. When a project is realized, the buyer decides whether to hold competitive solicitation, and if so, she also determines the level of effort to attract and process bids, \( \lambda \), which stochastically determines \( n \), the realized number of participating bidders.

Given the number of bids, the buyer offers menu of contracts, each defined by its base price \( p \) plus a probability distribution over its ex-post price change \( \Delta \). She announces her preference ordering over the items on the menu. Then

\(^{16}\)Arve and Martimort (2016), for example, explicitly models firms’ risk aversion in a procurement context. See pages 3240–3241 for their justifications, including imperfect risk management or diversification, bankruptcy or auditing cost of issuing debt, liquidity constraints, nonlinear tax systems, and internal agency problems.
sellers simultaneously select a contract from the menu. Last, the buyer chooses a winner following her preference ordering.

After the project is completed, contractible, stochastic outcomes are revealed to both parties, and the final payment is made. The outcomes consist of ex-post cost changes \( (\epsilon) \) and delays unrelated to the cost changes, denoted by \( s \). The key difference between the two outcomes is that \( \epsilon \) is distributed independently of seller’s type, while \( s \) is not. Let \( F(s) \) denote the cumulative distribution function of \( s \) for the low-cost sellers; the counterpart for the high-cost ones is \( \overline{F}(s) \). We assume both functions are differentiable with densities \( f(s) \) and \( \overline{f}(s) \), respectively.

The seller cost type is hidden information, known to the seller only. The procure knows the type distribution: a project specific parameter \( \pi \in [0,1] \) denotes the proportion of the low-cost sellers in the population. We assume that \( s \) is informative but imperfect: \( F(s) \) and \( \overline{F}(s) \) are defined on common support denoted by \( S \), but \( F(s) \neq \overline{F}(s) \) for some \( s \in S \).

3.4. Ex-post Price Changes and Liquidity Constraints. The description of the agreement between buyer and winning seller given in Section 2.2 distinguishes between unilateral ex-post price changes versus bilateral ones. Bilateral price changes may result from ex-post design or specification modification such as additional work, which is unrelated to private information of sellers. In our model the buyer is risk neutral, but the winning seller has liquidity concerns characterized by \( \psi(q) \). It is straightforward to show that
since $\epsilon$ is revealed to both the buyer and the winning seller, it is optimal to fully insure him against $\epsilon$ on a cost-plus basis, so that bilateral price changes track this insurance against ex-post cost changes observed by both parties.

Unilateral price changes may depend on contract outcomes related to sellers’ private information on cost, such as the length of unilateral delays. Netting out the insurance from bilateral costs changes, the resulting contract is a schedule determining the base price and unilateral price changes. Accordingly we define $q \equiv \Delta - \epsilon$ and express a contract on the menu in terms of a base price $p$ and a probability distribution for $q$, recognizing that the total ex-post price adjustment is simply $\Delta = q + \epsilon$. This is consistent with the institutional feature, described in Section 2.2 that unilateral price changes arise following the initial contract, while bilateral changes occur via renegotiation.

We also assume there exists a maximal penalty the buyer can impose on sellers, denoted by $M \in \mathcal{R}^-$, such that $q \geq M$. In theory, the maximal penalty finesses situations where it might otherwise be optimal to impose an extremely steep penalty on a low-cost winner in the event of a very unlikely outcome for a high-cost seller to achieve an outcome very close to first best. In practice, $M$ reflects limited liability and bankruptcy constraints of sellers.

4. Equilibrium

It is optimal for the buyer to offer a menu of two contracts: a preferred fixed price contract that depends on the number of bids; and a variable contract that does not depend on the number of bids but does depend on the informative contract outcome, $s$. Presented with an optimally designed contract menu, sellers truthfully reveal their cost type through their contract selection: low-cost sellers choose the fixed contract and high-cost ones submit the variable contract. When feasible, the buyer selects a seller submitting the fixed contract. In the case of a tie, the buyer randomly selects a winner. The extent of competition is chosen to minimize the expected total cost. This section characterizes and illustrates the optimal menu of contracts for given a number of bids, and then shows how the optimal extent of competition is derived.

\footnote{We ignore offers on the menu that are never accepted because there is no record of them in the data, and in the simultaneous game of selecting a contract from the menu, such offers cannot affect the equilibrium selection.}
4.1. **Contract Menu.** Denote the number of bids by \( n \in \{1, 2, \ldots \} \), the price of the fixed contract in the menu by \( p_n \), the base price of the variable contract by \( \overline{p} \), and the variable component by \( q(s) \). Since sellers reveal their type in equilibrium through their choice of a contract, and the probability that a low-cost seller bids is \( 1 - (1 - \pi)^n \), the expected transfer from the buyer to the winning seller is:

\[
[1 - (1 - \pi)^n] p_n + (1 - \pi)^n \left[ \overline{p} + \int q(s)f(s)ds \right].
\]  

(1)

Appealing to the revelation principle, the buyer is limited to choosing \( p_n \) and \( (\overline{p}, q(s)) \) subject to three constraints: (i) individual rationality (IR) conditions inducing both seller types to bid if presented with an opportunity to do so; (ii) incentive compatibility (IC) conditions inducing them to reveal their true type; and (iii) a limited liability condition restricting the range of \( q(s) \). The IR constraints for the two types are:

\[
p_n \geq \alpha \quad \text{and} \quad \overline{p} + \int \psi[q(s)]f(s)ds \geq \alpha + \beta.
\]

To derive the IC constraint for the low-cost type, we first compute the winning probability if he chooses the fixed contract when the other bidders follow their equilibrium strategy, denoted by \( \overline{\phi}_n \):

\[
\overline{\phi}_n \equiv \sum_{k=0}^{n-1} \binom{n-1}{k} \frac{\pi^k(1-\pi)^{n-1-k}}{k+1} = \frac{1}{n\pi} \sum_{j=1}^{n} \binom{n}{j} \pi^j(1-\pi)^{n-j} = \frac{1 - (1 - \pi)^n}{n\pi}.
\]

(2)

If the bidder chooses the variable contract instead, the probability of winning is:

\[
\overline{\phi}_n \equiv \frac{(1 - \pi)^{n-1}}{n}.
\]

(3)

Thus a low-cost seller prefers \( p_n \) to \( (\overline{p}, q(s)) \) if and only if:

\[
\phi_n \{p_n - \alpha\} \geq \overline{\phi}_n \{\overline{p} + \int \psi[q(s)]f(s)ds - \alpha\}.
\]

The IC condition for a high-cost seller can be similarly defined.

To characterize the optimal menu, four additional pieces of notation are helpful. Let \( h : \mathcal{R}^+ \to \mathcal{R} \) denote the inverse of the first derivative of \( \psi(q) \); that is \( h[\psi'(q)] \equiv q \). Let \( l(s) \equiv f(s)/\overline{f}(s) \) denote the likelihood ratio of
the probability density function of the low-cost type's \( s \) to the high-cost density. Define a threshold likelihood ratio associated with the limited liability condition by:

\[
\tilde{l}(\pi) \equiv \frac{1}{\pi} - \frac{1 - \pi}{\pi \psi'(M)}.
\]  

(4)

A lemma in the Appendix proves there is at most one root in \( \pi \in (0, 1) \) to the expression:

\[
\beta - \int_{l(s) < \tilde{l}(\pi)} \psi \left( h \left( \frac{1 - \pi}{1 - \pi l(s)} \right) \right) [1 - l(s)] \bar{f}(s) ds - \psi(M) \int_{l(s) \geq \tilde{l}(\pi)} [1 - l(s)] \bar{f}(s) ds.
\]

We denote the root by \( \tilde{\pi} \) when it exists, and otherwise set \( \tilde{\pi} = 1 \).

**Theorem 4.1.** The minimal number of items on an optimal menu is two. All optimal menus induce a separating equilibrium amongst the sellers: low-cost sellers submit fixed contracts and high-cost sellers submit variable contracts. The optimal menu containing two items is uniquely defined by the price of the fixed contract:

\[
P_n = \alpha + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left( \beta - \int \psi[q(s)] [1 - l(s)] \bar{f}(s) ds \right),
\]  

(5)

the base price of the variable contract:

\[
P = \alpha + \beta - \int \psi[q(s)] \bar{f}(s) ds,
\]  

(6)

and the price adjustment schedule:

\[
q(s) = \begin{cases} 
 h \left( \frac{1 - \min\{\pi, \tilde{\pi}\}}{1 - \min\{\pi, \tilde{\pi}\} l(s)} \right) & \text{if } l(s) \leq \tilde{l}(\min\{\pi, \tilde{\pi}\}), \\
 M & \text{if } l(s) > \tilde{l}(\min\{\pi, \tilde{\pi}\}).
\end{cases}
\]  

(7)

Intuitively, two contracts suffice to minimize costs because there are only two seller types, and for the remainder of the paper we focus on that uniquely defined menu. In the interior solution to this optimization problem \( P_n \) and \( P \) are found by solving two equations characterizing the IR constraint for high-cost sellers and the IC constraint for low-cost sellers, both of which hold with equality, and minimizing the resulting expression for the buyer’s cost with respect to \( q \) for each \( s \). When \( \pi > \tilde{\pi} \) it is optimal for the buyer to extract all the rent from the low cost seller by offering a fixed contract at \( \alpha \); in this case
the IR constraint for the low-cost type binds at optimum. When $l(s) > \bar{l}(\pi, M)$ the buyer is restricted to charging the maximal penalty to sellers selecting the variable contract because of limited liability and bankruptcy considerations.

Figure 2 illustrates the optimal menu of contracts for an example in which the distribution of $s$ for the low-cost type is $\text{Gamma}(1, 1.5)$, the counterpart for the high-cost type is $\text{Gamma}(1, 2)$. We set the cost parameters to $\alpha = 1000$, $\beta = 500$, and $\mathbb{E}(\epsilon) = 0$, the penalty $M$ is high enough to be non-binding and the costs of liquidity are modeled as:

$$
\psi(q) = -\psi_0 e^{-q/\psi_0} + \psi_0,
$$

where $\psi_0 = 2500$.

The solid line in Panel (A) in Figure 2 represents the likelihood ratio $l(s)$, while the two dotted lines in the panel show $q(s)$, the price adjustment schedule of the variable contract for two values of the ratio of the low-cost type in the population, $\pi = 1/3$ and $\pi = 1/2$. Since $h(1) = 0$ and its derivative is negative, it follows from (7) that $q(s) \geq 0$ as $l(s) \leq 1$ with $q(s) = 0$ if and only if $l(s) = 1$.

From (6) and (7), neither $\bar{p}$ and $q(s)$ depend on $n$, and hence the variable price contract depends on $\pi$ and $l(s)$ but not on the number of bidders and consequently the expected cost does not either.\(^{18}\) This is because the IR condition for the high-cost type is satisfied with equality. Consequently the expected cost of a variable contract does not depend on $n$. It is straightforward to prove that a higher $\pi$ is associated with a steeper price adjustment schedule as illustrated in Panel (A).\(^{19}\) Since a steeper price schedule is associated with a more volatile variable contract, in order to meet their certainty equivalent payment of $\alpha + \beta$ determined by the IR constraint, high-cost sellers must be

---

\(^\text{18}\)This result is similar to McAfee and McMillan (1987), Laffont and Tirole (1987), and Riordan and Sappington (1987), where the distortions due to information asymmetry are invariant to the number of bids, though expected distortions and profits decline with the number of bids.

\(^\text{19}\)In the interior solution $q(s)$ also depends on $\pi$ so we can write $q(s; \pi)$ for $q(s)$. Partially differentiating $q(s; \pi)$ with respect to $\pi$ establishes that if $l(s) < 1$ then $q(s; \pi)$ is increasing in $\pi$ and if $l(s) > 1$ then $q(s; \pi)$ is decreasing in $\pi$. See Lemma 5.1 in the next section.
Figure 2. Optimal Contracts and Expected Transfer

(A) Optimal Price Adjustments as a Function of the Informative Contract Outcome

(B) Expected Transfer by Contract Type

Notes: The above graphs show the optimal variable contracts and the expected transfer by contract type for an example case with \( \alpha = 1000, \beta = 500, \) and \( E(\epsilon) = 0. \) The distribution of the informative contract outcome, \( s, \) for the low-cost sellers is \( \text{Gamma}(1, 1.5) \) and the counterpart for the high-cost sellers is \( \text{Gamma}(1, 2). \)
paid a higher risk premium, defined as:

\[ r \equiv \int \{q(s) - \psi[q(s)]\} \bar{f}(s)ds \] (9)

As Panel (B) of Figure 2 shows, in our example the risk premium increases from 15 to 68 when \( \pi \) increases from 1/3 to 1/2 due to the steeper variable component.

Panel (B) also depicts the expected transfer by contract type as a function of the number of bids for two different values of \( \pi \). Differentiating (5), the fixed cost contract declines with the number of bidders, converging to \( \alpha \), almost achieved with only handful of bidders in the example depicted in Panel (B).

If \( s \) is uninformative, meaning \( f(s) = \bar{f}(s) \) for all \( s \in S \), the optimal menu reduces to a menu of two fixed contracts. This menu consists of one preferred contract, \( \alpha + \pi (1 - \pi)^{n-1} \beta / [1 - (1 - \pi)^n] \) and a default contract of \( \alpha + \beta \), which is selected only if no bidder chooses the preferred lower price contract. For the example when \( \pi = 1/3 \), these prices are 1500 and the highest downward sloping piecewise-linear line with the 1500 intercept (the red dotted line in the graph).

Using \( s \) and replacing fixed price of \( \alpha + \beta \) with a variable price contract enables the buyer to reduce the fixed price contract offered to low-cost producers without violating their IC constraint. In the example when \( \pi = 1/3 \) the fixed price contract designed for the low-cost seller shifts down to 1439 when there is only one bidder. From (5), the expected amount extracted is:

\[ \pi (1 - \pi)^{n-1} \frac{1}{1 - (1 - \pi)^n} \left( \int \psi[q(s)] [l(s) - 1] \bar{f}(s)ds \right) \equiv \pi (1 - \pi)^{n-1} \frac{1}{1 - (1 - \pi)^n} \gamma \] (10)

where \( \gamma \) is the expected amount of rent extracted from a low-cost seller when there is only one bid. As \( \pi \) increases the probability of settling with a high-cost seller falls, inducing the buyer to raise \( r \) and \( \gamma \).

The buyer balances the gains of extracting rent from the low-cost type with the losses of the risk premium she pays to the high-cost type. Substituting (10) into (5), and (9) into (6) and the resulting expressions into (1) yields the expected transfer to a winning seller given \( n \) bids:

\[ T(n) = \alpha + (1 - \pi)^{n-1} [\beta - \pi \gamma + (1 - \pi)r] \equiv \alpha + (1 - \pi)^{n-1} [\beta + \Gamma] \] (11)
where $\Gamma$ is the expected net benefit of using the informative contract outcome where there is only one bidder. Equation (11) captures a basic intuition permeating through our analysis: in her quest to extract rent from the low-cost type when faced with the constraint of having to accept a high-cost type as a last resort, the buyer uses contract outcomes to discriminate between different cost types as a partial substitute for attracting more bidders.

4.2. Extent of Competition. Having solved the optimal menu of contracts and the expected transfer to a winning seller given a number of bids, we can now derive the expected total cost of competed procurement with effort $\lambda$, denoted by $U(\lambda)$. Recall that $\lambda$ is the arrival rate of extra bids, denoted by $j$ in the equation below, which follows a Poisson process.

$$U(\lambda) \equiv \sum_{j=0}^{\infty} \frac{\lambda^j e^{-\lambda}}{j!} [T(j+1) + \kappa(j+1)] + \eta$$

$$= \alpha + e^{-\lambda \pi}(\beta + \Gamma) + \mathbb{E}[\kappa(j+1)|\lambda] + \eta.$$  

The expected total cost of non-competed procurement, denoted by $U_0$, is:

$$U_0 = \alpha + \beta + \Gamma + \kappa(1).$$

The buyer chooses to hold a competitive solicitation if and only if $U_0 \geq \min_\lambda U(\lambda)$. Because $U(\lambda)$ is convex, it attains a global minimum at its unique stationary point, denoted by $\lambda^*$. If $\lambda^* \leq 0$, then the choice reduces to the sign of $\eta$. Alternatively if $\lambda^* > 0$, then a competitive solicitation is chosen if and only if:

$$\eta \leq (1 - e^{-\lambda^* \pi})(\beta + \Gamma) + \kappa(1) - \mathbb{E}[\kappa(j+1)|\lambda^*].$$  

5. Identification

Our data comprises: whether the contract was competed, which we denote by setting $c = 1$, or not (setting $c = 0$); how many bids were tendered, $n$; whether the winning bid is a variable contract, denoted by setting $v = 1$, or not (setting $v = 0$); the informative contract outcome $s$ for both contract types; the fixed price $p_n$ if the winning contract is a fixed contract; and the base price $p$ and the ex-post unilateral price change $q$ if the winning contract is variable. We also observe price changes arising from bilateral price changes,
\(\epsilon\). Thus an observation in the data set, which we denote by \(I\), is defined as:

\[ i \equiv \{c, n, v, s, v\bar{p}, vq, (1 - v)p_n, \epsilon\} \]

We assume each \(i \in I\) is generated by an independent draw of \((\pi, s, \epsilon)\).

Under the null hypothesis that \(\pi\) is constant conditional on observed project characteristics, this parameter is identified off the proportion of variable contracts \((1 - \pi)^n\) for any given \(n \in \{1, 2, \ldots\}\). Thus \(\pi\) is over-identified from variation in \(n\). However, this null hypothesis is rejected. Accordingly we treat \(\pi\) as an unobserved project specific continuous variable filtering through the equilibrium and complicating identification.

After conditioning on the number of bidders, the contract type, and other observed characteristics, we allow costs to vary with \(\pi\) as well. Thus \(\alpha, \beta, \kappa(n)\) are now expressed as \(\alpha(\pi), \beta(\pi)\) and \(\kappa(n, \pi)\). Similarly we make explicit the dependence of \(p_n, \bar{p}\), and \(q(s)\) on \(\pi\) by writing \(p_n(\pi, \bar{p}(\pi))\), and \(q(s, \pi)\) respectively. The primitives of the econometric structure therefore comprise: the distribution of the proportion of the low-cost type, \(F_\pi(\pi) \in F_\pi\); the liquidity cost function, \(\psi(q) \in \Psi\); project costs, \(\alpha(\pi) \in A\) and \(\beta(\pi) \in B\); the distribution function of \(s\), \(F(s) \in F\) and \(\overline{F}(s) \in \overline{F}\); the bid processing cost function \(\kappa(n, \pi) \in \mathcal{K}\); and the distribution function of \(\eta\), denoted by \(F_\eta(\eta) \in F_\eta\). It is convenient to partition both the primitives and the identification analysis into those determining sellers' costs, namely:

\[ \theta_1 \in \Theta_1 \equiv F_\pi \times \Psi \times A \times B \times \overline{F} \times \overline{F} \]

and those determining the buyer’s preferences, that is: \(\theta_2 \in \Theta_2 \equiv F_\eta \times \mathcal{K}\).

To preserve tractability, our empirical analysis makes the following two assumptions about the unobserved variables:

\textbf{A1}: \(s \perp (\pi, \eta)\) and \(\eta \perp \pi\).

\textbf{A2}: \(F_\pi(\pi)\) is strictly increasing for all \(\pi \in \Pi\).

We also simplify the analysis by restricting the parameter space so that an interior solution invariably attains, meaning neither the IR constraint for the low-cost type nor the maximal penalty constraint bind. Therefore, instead of (7), the following first order condition characterizes \(q(s, \pi)\) for any \((s, \pi) \in\)
\( S \times \Pi: \)
\[
\psi'[q(s)][1 - \pi l(s)] = 1 - \pi. \tag{12}
\]

In addition we assume that as the proportion of the low-cost type increases, the expected cost of the project to either type declines.

**A3:** \( \Pi \subset (0, \min\{\bar{\pi}, 1\}) \), and \( l(s) \leq \bar{l}(\pi, M) \) for all \((s, \pi) \in S \times \Pi\).

**A4:** \( \alpha(\pi) \) and \( \beta(\pi) \) are nonincreasing in \( \pi \).

**A5:** \( \alpha(\pi) \) and \( \beta(\pi) \) satisfy the following inequality for all \( \pi \):
\[
\frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + \frac{h' [\psi'(M)] [\psi'(M)]^2 [1 - \psi'(M)]}{\pi (1 - \pi)} < 0.
\]

The distributions of the informative contract outcome, \( F(s) \) and \( \overline{F}(s) \), are directly identified off the data on the observed \( s \) of fixed and variable contracts. Identifying the remaining components of the model proceeds as follows. We begin with a preliminary lemma showing that the absolute value of the variable component is increasing in the proportion of low-cost sellers \( \pi \), the base price is declining in \( \pi \), and the fixed contract price declines in \( \pi \) for each number of bids, \( n \).

**Lemma 5.1.** (i) If A3 holds then \( \partial |q(s; \pi)| / \partial \pi > 0 \). (ii) If A3 and A5 hold then \( \partial p(\pi) / \partial \pi < 0 \). (iii) If A3 and A4 hold then \( \partial p_n(\pi) / \partial \pi < 0 \) for all \( n \in \{1, 2, \ldots \} \).

In the following, we assume that A1 through A5 are satisfied.

### 5.1. Sellers’ Costs.

Identification of \( \psi(q) \) is based on the rate at which the ex-post price change in the variable contract, \( q \), decreases as the likelihood ratio \( l \) increases for any given \( \pi \), i.e., an equation derived from totally differentiating (12) with respect to \( q \) and \( l \). Since \( s \) only enters the optimal contract only through the likelihood ratio we can summarize outcomes of variable contracts in terms of \((\overline{p}, q, l)\) rather than \((\overline{p}, q, s)\), where \( l = l(s) \). Since \( \pi \) is unobserved, (12) is redefined in terms of \( \overline{p} \), which under our assumptions is monotone in \( \pi \); we write \( \pi = \pi(\overline{p}) \) and rearrange the first order condition to define:
\[
l^*(\overline{p}, q) \equiv \frac{1}{\pi(\overline{p})} - \frac{1 - \pi(\overline{p})}{\pi(\overline{p})} \psi'(q). \tag{13}
\]
Lemma 5.2. For all variable contract outcomes \((\overline{p}, q, s)\) such that \(l(s) = l\), \((p, q, l) = (\overline{p}, q, l^*(q, \overline{p}))\) holds. For all \((p, q)\) such that \(l^*(p, q) \neq 1\), the following equation holds:

\[
\psi''(q) = \left[\frac{1 - \psi'(q)}{1 - l^*(\overline{p}, q)}\right] \psi'(q) \frac{\partial l^*(\overline{p}, q)}{\partial q}.
\]  

(14)

We assume conditions on \(\psi(q)\) that guarantee \(l^*(\overline{p}, q)\) defined in (13) is uniformly Lipschitz continuous in \(q\). Then the Picard–Lindelöf theorem applies, proving the differential equation (14) has a unique solution given the normalizing constant \(\psi'(0) = 1\). It now follows that \(\psi(q)\) is solved from the other normalizing constant for the liquidity cost function, that \(\psi(0) = 0\). Since \(l^*(\overline{p}, q)\) is identified off variable contract outcomes \((\overline{p}, q, l)\), so is \(\psi(q)\).

Since \(\psi'(q)\) is identified, \(\pi\) corresponding to each variable contract \((\overline{p}, q, s)\), denoted by \(\pi_{q,s}\), is identified from the first order condition (12) by:

\[
\pi_{q,s} = \frac{1 - \psi'[q(s)]}{1 - l(s) \psi'[q(s)]}.
\]

Noting that \(\pi_{q,s}\) can be interpreted as a random draw from the \(f_{\pi|c,n,v}(\pi|c,n,0)\) probability density, it now follows that \(f_{\pi|c,n,v}(\pi|c,n,0)\) is identified. Let us define the odds ratio related to contract types conditional on \((c, n)\) as:

\[
\varphi_{c,n} = \frac{\Pr(v = 1|c,n)}{\Pr(v = 0|c,n)}.
\]

We show in the appendix that \(f_{\pi|c,n,v}(\pi|c,n,0)\) is linked to \(f_{\pi|c,n,v}(\pi|c,n,1)\) by the conditional probability of a high-cost seller winning given \(\pi\) and \(n\) and \(\varphi_{c,n}\) as follows:

\[
f_{\pi|c,n,v}(\pi|c,n,0) = \varphi_{c,n} \frac{[1 - (1 - \pi)^n]}{(1 - \pi)^n} f_{\pi|c,n,v}(\pi|c,n,1),
\]

for any \((\pi, c, n) \in \Pi \times \{0,1\} \times \mathcal{N}\).

Lemma 5.3. \(f_{\pi|c,n,v}(\pi|c,n,v)\) is identified for any \((\pi, c, n, v)\).

We identify project costs, \(\alpha(\pi)\) and \(\beta(\pi)\), by exploiting two identified mappings: (i) the mapping from \((l, q)\) to \(p\), denoted by \(p^*(l, q)\); and (ii) the mapping from \((\pi, c)\) to \(p_n^*\) for any given \(n\), denoted by \(p_s^*\). The existence and the derivation of the former mapping is guaranteed from (13), and the counterparts of the latter mapping from Lemma 5.1 (iii), showing that \(p_n^*\) is
monotone (decreasing) in \( \pi \). Let \( G_{p_n|c} (\cdot | c) \) denote the cumulative distribution function for \( p_n \) conditional on \( c \in \{0, 1\} \). Then the monotonicity guarantees that for any \( (\pi, c) \in \Pi \times \{0, 1\} \),

\[
p^*_n (\pi, c) \equiv G_{p_n|c}^{-1} \left( \int_\pi^{\pi_{\text{max}}} f_{p|c,n,v} (x | c, n, 0) \, dx \right) |c\).
\]

To identify \( \alpha(\pi) \) and \( \beta(\pi) \) we substitute \( p^*_n (\pi, c) \) for \( p_n \) and \( p^*_1 (q, s) \) for \( \overline{p} \) in (5) and (6) for any \( s \) and any \( n \in \{2, 3, \ldots\} \), rearrange the resulting expressions and substitute out \( q \) using (12) to obtain:

\[
\begin{align*}
\alpha(\pi) &= \frac{1 - (1 - \pi)^n}{1 - (1 - \pi)^{n-1} p^*_n (\pi, c) - \pi (1 - \pi)^{n-1} p^*_1 (\pi, c)} - \frac{1 - (1 - \pi)^n}{1 - (1 - \pi)^{n-1} p^*_1 (\pi, c)}, \\
\beta(\pi) &= p^* \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right], s \right) + \int \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(t)} \right] \right) \overline{f}(x) \, dx - \alpha(\pi).
\end{align*}
\]

Taken together, the three lemmas in this section provide the critical arguments for establishing the identification of the seller side of the model.

**Theorem 5.1.** \( \theta_1 \in \Theta_1 \) is identified from \( i \in I \).

In fact, the seller sellerside of the model, \( \theta_1 \), is over-identified. First, the optimal setting of \( p^*_1 \) does not depend on whether competition is restricted or not, \( p^*_1 (\pi, 0) = p^*_1 (\pi, 1) \). Second, varying \( n \in \{2, 3, \ldots\} \) in (15) yields testable restrictions. Third, setting \( n = 1 \) in (5) and (6) and substituting \( p^*_1 (q, s) \) for \( p^*_1 (\pi, c) \) for \( p^*_1 \) yields:

\[
\int \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(t)} \right] \right) \overline{f}(t) \, dt = p^*_1 (\pi, c) - \overline{p} \left[ \frac{1 - \pi}{1 - \pi l(s)}, s \right].
\]

Varying \( \pi \) in the above equation provides further over-identifying information for \( \psi(q) \).

**5.2. Buyer’s Costs.** Note that Lemma 5.3 provides that the distribution of the number of bids conditional on \( \pi \) is identified. Given this, the bid cost function \( \kappa(n, \pi) \) in a competitive solicitation is partially identified, and \( F_\eta(\eta) \) is partially identified from buyer choices, through variation in \( \pi \) transmitted through the identified costs to both parties.

To illustrate this point, consider a simple case where \( \kappa(\cdot, \pi) \) is linear in \( n \); i.e., \( \kappa(n, \pi) = \kappa_0(\pi)n \). Under this specification, there is a closed-form solution
for the the optimal arrival rate of extra bids for a project with \( \pi \), denoted by \( \lambda^*(\pi) \):

\[
\lambda^*(\pi) = \max \left\{ 0, \frac{1}{\pi} \ln \left[ \frac{\pi (\beta(\pi) + \Gamma(\pi))}{\kappa_0(\pi)} \right] \right\},
\]

where \( \Gamma(\pi) \) is to express the dependence of \( \Gamma \), defined in (11), on \( \pi \). Then a competitive solicitation for a project with \((\pi, \eta)\) is preferred if and only if

\[
\eta \leq \Omega(\pi) \equiv (1 - e^{-\lambda^*(\pi)\pi}) [\beta(\pi) + \Gamma(\pi)] - \kappa_0(\pi) \lambda^*(\pi).
\]

Note that \( \lambda^*(\pi) \) is identified by:

\[
\lambda^*(\pi) = \sum_{n=1}^{\infty} \frac{n f_{\pi,n|c}(\pi, n - 1 | 1)}{f_{\pi|c}(\pi | 1)}.
\]

If \( \lambda^*(\pi) = 0 \), then only a lower bound of \( \kappa_0(\pi) \) is identified; otherwise,

\[
\kappa_0(\pi) = \pi [\beta(\pi) + \Gamma(\pi)] \exp \left[ -\pi \lambda^*(\pi) \right].
\]

By exploiting the variation in \( \pi \), which is guaranteed by A1, we identify \( F_\eta(\eta) \) for \( \eta \in \{ \Omega(\pi) : \lambda^*(\pi) > 0 \} \) and \( \eta = 0 \). Specifically, if \( \lambda^*(\pi) > 0 \), then \( \Pr(c = 0 | \pi) = F_{\eta}[\Omega(\pi)] \); otherwise, \( \Pr(c = 0 | \pi) = F_{\eta}(0) \).

In the estimated specification of the model, the bid processing cost is non-linear in the number of bids:

\[
\kappa(n, \pi) = (\kappa_1 + \kappa_2 \pi) (n - 1) + (\kappa_3 + \kappa_4 \pi) (n - 1)^2. \tag{16}
\]

The first order condition for interior optimality simplifies to:

\[
\kappa_1 + \pi \kappa_2 + [1 + 2 \lambda^*(\pi)] \kappa_3 + \pi [1 + 2 \lambda^*(\pi)] \kappa_4 = \pi \exp \left[ -\pi \lambda^*(\pi) \right] \left[ \beta(\pi) + \Gamma(\pi) \right].
\]

Given the mappings \( \lambda(\pi) \), \( \beta(\pi) \) and \( \Gamma(\pi) \), identified by our previous arguments, the vector of coefficients \( (\kappa_1, \kappa_2, \kappa_3, \kappa_4) \) solves a system of linear equations for different values of \( \pi \). The rank condition sufficient to identify \( (\kappa_0, \kappa_1, \kappa_2, \kappa_3) \) can be checked directly in any given application.

6. Estimation Results

6.1. Definition of the Variables. We constructed the observed variables in the model from the data as follows. Contract \( i \) is restricted \( (c_i = 0) \) if there is
Contracts are fixed \( (v_i = 0) \) or variable \( (v_i = 1) \) depending on whether or not there is a unilateral modification.\(^{21}\) For fixed contracts, the difference in the final and the base prices is denoted by \( \epsilon_i \). For variable contracts, the base price is \( \bar{p}_i \), the sum of all changes in the price related to unilateral modifications is \( q_i \), and the difference between the final price and \( \bar{p}_i + q_i \) is \( \epsilon_i \). We consider the length of delay related to unilateral modifications, divided by the base duration of the contract, as an observed informative contract outcome, \( s_i \). If the final duration is shorter than the expected duration then \( s_i = 0 \).

### 6.2. Parameterization and Estimation

The proof of identification yields an approach to estimating the model nonparametrically. However nonparametric estimation of the model is infeasible with our sample size. Instead we impose a parametric functional form, and estimate its primitives using a simulated GMM estimator with moment conditions that are motivated by the identification arguments: the joint probabilities regarding entry restrictions, number of bids, and contract type; moments of the joint distribution of \( s \) and contract type; and the quantiles of contract prices conditional on contract type and number of bids. The appendix gives further details about estimation.

The primitives of the model are parameterized as follows. We assume that the distribution of \( \pi \) is given by \( \text{Beta}(\zeta_1, \zeta_2) \) on \((0, 1)\), that \( \psi(q) \) takes the parametric form of (8) with \( \psi_0 > 0 \), and that the maximal penalty is a constant denoted by \( \delta_0 \). The direct cost of competitive solicitation, \( \eta \), is assumed to follow \( N(\mu_\eta, \sigma_\eta) \) for non-military contracts and \( N(\mu_\eta^m, \sigma_\eta^m) \) for military contracts. The parametric assumption for the bid costs \( \kappa(n, \pi) \) is represented in (16).

The distribution of \( s \) for the low-cost sellers is:

\[
F_s(s) = \begin{cases} 
\rho & \text{if } s = 0, \\
(1 - \rho)G(s) & \text{if } s > 0,
\end{cases}
\]

\(^{20}\)Contracts with more than one bid categorized in the data as noncompetitive are treated as competitively solicited.

\(^{21}\)Note that our categorization of contract types does not coincide with the nomenclature of fixed or cost plus contracts. See Section 2.3 for our discussion on the firm fixed fixed price contracts frequently experiencing both types of ex-post price changes.
where $G(s)$ is the CDF of a Gamma distribution with shape parameter $\alpha_s > 0$ and scale parameter $\beta_s > 0$. The counterpart for the high-cost sellers, $\overline{F}_s(s)$, is similarly defined with $\overline{p}$ and $\overline{G}(s)$, where $G(s)$ is the CDF of a gamma distribution with shape parameter $\alpha_s$ and scale parameter $\beta_s$.

Costs depend on whether or not the project is for military agencies, denoted by a binary variable, $m \in \{0, 1\}$. Given $m$, we assume the cost for low-cost sellers, $\alpha$, is linear in $\pi$, and the cost differential $\beta$ is a fraction of $\alpha$:

$$\alpha(\pi, m) = \alpha_1 + \alpha_2 m + \alpha_3 \pi,$$

and

$$\beta(\pi, m) = [\beta_1 (1 - m) + \beta_2 m] \alpha(\pi, m).$$

We assume $\alpha_3 \leq 0$, so that monotonicity results of Lemma 5.1 apply.\textsuperscript{22} Finally $\epsilon$ is equal to bilateral price change, which we directly observe in the data and is independent of $\pi$ and $\eta$ by assumption. We estimate the distribution of $\epsilon$ conditional on $m$ without parametric assumptions.

6.3. Parameter Estimates. Using the estimated parameters, we simulate the data and calculate key moments displayed in Table 7. The table shows the actual and predicted moments regarding the extent of competition, the contract types, and the contract prices. The predicted moments are based on a simulation of 5,000 observations using the estimated parameters. The overall fit of the simulated data to the actual data is good in both level and trend.

The parameter estimates and their estimated asymptotic standard errors are presented in Table 8. The 95 percent confidence intervals for our estimates of the mean direct cost of competitive solicitation ($\mu_\eta$ for non-military contracts and $\mu_\eta^m$ for military ones) are $[11,200, 29,900]$ for non-military contracts, and $[18,800, 48,400]$ for military contracts. The cost of competitive solicitation for military contracts is greater than the counterpart for non-military ones by $13,100$ with asymptotic error $6,900$, a statistically significant difference that partially explains why there is less competition for military contracts than non-military ones.

\textsuperscript{22}This inequality was not imposed in estimation, but as Table 8 shows, the restriction is satisfied by our unconstrained estimate of $\alpha_3$, which is statistically significant.
Table 7. Model Fit

<table>
<thead>
<tr>
<th>Probability of</th>
<th>All Data</th>
<th>Model</th>
<th>Non-military Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>No competitive solicitation</td>
<td>0.6778</td>
<td>0.7374</td>
<td>0.6341</td>
<td>0.6876</td>
</tr>
<tr>
<td>1 bid given competitive solicitation</td>
<td>0.2677</td>
<td>0.3062</td>
<td>0.2500</td>
<td>0.2913</td>
</tr>
<tr>
<td>2 bids or less given competitive solicitation</td>
<td>0.4258</td>
<td>0.5209</td>
<td>0.3664</td>
<td>0.5058</td>
</tr>
<tr>
<td>5 bids or less given competitive solicitation</td>
<td>0.8516</td>
<td>0.9162</td>
<td>0.8534</td>
<td>0.9173</td>
</tr>
<tr>
<td>Fixed contracts given no competition</td>
<td>0.4156</td>
<td>0.4307</td>
<td>0.4254</td>
<td>0.4443</td>
</tr>
<tr>
<td>Fixed contracts given 1 bid</td>
<td>0.4419</td>
<td>0.5788</td>
<td>0.4655</td>
<td>0.5933</td>
</tr>
<tr>
<td>Fixed contracts given 2 bids or less</td>
<td>0.3976</td>
<td>0.4254</td>
<td>0.4310</td>
<td>0.4224</td>
</tr>
<tr>
<td>Fixed contracts given 5 bids or less</td>
<td>0.4091</td>
<td>0.4561</td>
<td>0.4000</td>
<td>0.4620</td>
</tr>
</tbody>
</table>

Average transfer ($M) of fixed contracts
| Given no competitive solicitation | 0.9722 | 0.9422 | 0.9027 | 0.8754 |
| Given competitive solicitation | 1.2551 | 1.2491 | 1.2183 | 1.2199 |

Average transfer ($M) of variable contracts
| Given no competitive solicitation | 1.3590 | 1.2838 | 1.2311 | 1.2031 |
| Given competitive solicitation | 1.1984 | 1.1855 | 1.1193 | 1.1631 |

Table 9 displays project and bidding costs, plus the expected loss to the buyer from soliciting competitive bids, for nonmilitary and military procurement projects evaluated at the median value of the ratio of the low-cost sellers (π), 0.38.23 The expected value of ε, or bilateral price changes, is $139,100 per non-military contract and $265,200 per military contract. Overall, 85 percent of total project costs are attributable to the predetermined components (α and β) and 15 percent to the stochastic component (ε). In expectation, low-cost sellers spend $1.02 (1.18) million in expectation, the sum of α and the expected value of ε, to complete a procurement project for non-military (military) agencies, and high-cost sellers an extra $0.27 (0.24) million. This cost differential represents the most a buyer can save from contracting with a low-cost seller. Relative to this differential the bidding cost of having two bids instead of one is 19 percent (22 percent), while the loss the seller incurs from competitive solicitation is lower, 8 percent (14 percent) for non-military (military) contracts.

23Note that project costs and bid costs vary with the ratio of the low-cost sellers (π), which we treat as the contract-level unobserved heterogeneity.
### Table 8. Parameter Estimates

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low project cost ($M)</td>
<td>$\alpha_1$</td>
<td>1.7178</td>
<td>$\alpha_2$</td>
<td>0.0267</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0698)</td>
<td></td>
<td>(0.0251)</td>
</tr>
<tr>
<td></td>
<td>$\alpha_3$</td>
<td>-2.1699</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1536)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Project cost differential ($M)</td>
<td>$\beta_1$</td>
<td>0.3069</td>
<td>$\beta_2$</td>
<td>0.2583</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0325)</td>
<td></td>
<td>(0.0333)</td>
</tr>
<tr>
<td>Bidding cost function ($M)</td>
<td>$\kappa_1$</td>
<td>-0.0105</td>
<td>$\kappa_2$</td>
<td>0.1530</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0023)</td>
<td></td>
<td>(0.0261)</td>
</tr>
<tr>
<td></td>
<td>$\kappa_3$</td>
<td>-0.0009</td>
<td>$\kappa_4$</td>
<td>0.0124</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0004)</td>
<td></td>
<td>(0.0030)</td>
</tr>
<tr>
<td>Maximum penalty ($M)</td>
<td>$\delta_0$</td>
<td>-0.0100</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1695)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Liquidity cost function ($M)</td>
<td>$\psi_0$</td>
<td>10.4020</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>(7.5244)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Distribution of $\pi$</td>
<td>$\zeta_1$</td>
<td>6.7445</td>
<td>$\zeta_2$</td>
<td>10.7797</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.4396)</td>
<td></td>
<td>(0.8946)</td>
</tr>
<tr>
<td>Distribution of $\eta$ ($M)</td>
<td>$\mu_\eta$</td>
<td>-0.0205</td>
<td>$\sigma_\eta$</td>
<td>0.0150</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0048)</td>
<td></td>
<td>(0.0042)</td>
</tr>
<tr>
<td></td>
<td>$\mu_\eta^m$</td>
<td>-0.0336</td>
<td>$\sigma_\eta^m$</td>
<td>0.0237</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0076)</td>
<td></td>
<td>(0.0033)</td>
</tr>
<tr>
<td>Distribution of $s$</td>
<td>$\rho$</td>
<td>0.9167</td>
<td>$\overline{\rho}$</td>
<td>0.3520</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.0155)</td>
<td></td>
<td>(0.0221)</td>
</tr>
<tr>
<td></td>
<td>$\alpha_s$</td>
<td>1.7828</td>
<td>$\beta_s$</td>
<td>0.2881</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.4853)</td>
<td></td>
<td>(0.0715)</td>
</tr>
<tr>
<td></td>
<td>$\overline{\alpha}_s$</td>
<td>0.8240</td>
<td>$\overline{\beta}_s$</td>
<td>2.8523</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.1739)</td>
<td></td>
<td>(0.7617)</td>
</tr>
</tbody>
</table>

**Notes**: Numbers in parentheses are asymptotic standard errors.

Note that the cost differential between low-cost and high-cost sellers is slightly higher for non-military contracts than for military ones. In terms of the parameter estimates of $\beta_0$ (for non-military contracts) and $\beta_1$ (for military ones), both of which measure the ratio of the cost differential to the cost of low-cost sellers for the respective contract category, the percentage difference is 4.8 with estimated asymptotic standard error 2.3 (statistically significant at the 4 percent level). This is another reason why military contracts are less competitive than non-military ones.
Table 9. Estimated Cost Components for Median Contracts

<table>
<thead>
<tr>
<th></th>
<th>Non-military</th>
<th></th>
<th>Military</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Estimate</td>
<td>SE</td>
<td>Estimate</td>
<td>SE</td>
</tr>
<tr>
<td>Project cost for low-cost sellers, $\alpha(\pi_{med}, m)$</td>
<td>884.1</td>
<td>40.4</td>
<td>910.8</td>
<td>45.8</td>
</tr>
<tr>
<td>Project cost difference, $\beta(\pi_{med}, m)$</td>
<td>271.3</td>
<td>32.1</td>
<td>235.3</td>
<td>32.1</td>
</tr>
<tr>
<td>Ex-post cost changes, $E(\epsilon</td>
<td>m)$</td>
<td>139.1</td>
<td>15.8</td>
<td>265.2</td>
</tr>
<tr>
<td>Bidding cost with two bidders, $\kappa(\pi_{med}, 2)$</td>
<td>52.1</td>
<td>8.9</td>
<td>52.1</td>
<td>8.9</td>
</tr>
<tr>
<td>Cost of competitive solicitation, $E(\eta</td>
<td>m)$</td>
<td>20.5</td>
<td>4.8</td>
<td>33.6</td>
</tr>
</tbody>
</table>

Notes: Given that the project costs and bidding costs vary with $\pi$, the numbers in this table are evaluated at the unconditional median value of $\pi_{med}$, 0.38. Furthermore, since the bidding costs depend on the total number of bids, we evaluate such costs at two bids.

Figure 3 illustrates the estimated distribution of the ratio of the low-cost sellers ($\pi$) conditional on solicitation (competitive or not). The unconditional estimated median value of $\pi$ is 0.38, the 5th percentile of $\pi$ is 0.21, and the 95th percentile is 0.59. Controlling for military status, the estimated distribution of $\pi$ for noncompeted contracts first order stochastically dominates the distribution of competed contracts. This finding illustrates the importance of accounting for the endogeneity of the extent of competition. All else equal, prices rise when fewer sellers compete, but Table 5 shows this is not the case, and Figure 3 shows buyers are less likely to promote competition when the probability of any given seller being cost-efficient rises. In our analysis this offsetting factor is created by bidding costs that increase with $\pi$, as both $\kappa_2$ and $\kappa_4$ are positive and significant, making competition more attractive when $\pi$ is low. Similarly our estimates of the cost differential (as transmitted through $\alpha_3$) decline with $\pi$, reducing the benefit of competitive solicitation.

7. Why So Little Competition?

7.1. Contract Negotiations and Informational Rent. Section 4 demonstrates how negotiating contract terms helps a buyer extract informational rent from low-cost sellers. Figure 4 illustrates the estimated cost savings from contract negotiations when compared to a first-price sealed-bid auction in our empirical application, for a non-military contract with the unconditional median ratio of low-cost sellers ($\pi = 0.38$). Panel (A) shows the expected transfer
conditional on a given number of bids, under contract negotiations and first-price sealed bid auctions, respectively. We remark the expected transfer under contract negotiations with one bidder is similar to the expected transfer under a first-price auction with two bidders, a pattern that persists in the range of the number of bidders in the graph. Consequently the marginal benefit of an extra bidder is lower when negotiations are permitted. Panel (B) shows that, given competitive solicitation, the optimal number of bids is less than 2.5 for negotiations and over 3 for auctions.

To quantify the extent to which contract negotiations reduce the expected transfer from the buyer to the winning seller, we compare the current regime with a policy where unilateral price changes are not permitted and a first-price auction is used when there is competitive solicitation. The comparison is presented in the columns labeled Current and (3) of Table 10. Appealing

\footnote{Except where indicated below, we assume that the model primitives, including the distribution of π, the project cost functions, and bidding cost functions, are invariant. To allow that the incentives for potential sellers to participate may change, one counterfactual policy (Policy (6) in Table 10) incorporates a change in bidding costs.}
Figure 4. The Value of Contract Negotiations

Notes: Panel (A) compares contract negotiations with first-price auctions. It shows the expected transfer net of the minimum project cost for a non-military contract with the median ratio of low-cost sellers ($\pi = 0.38$) conditional on the number of bids. The error bars represent 95 percent confidence intervals. In Panel (B), we define the marginal benefit of a bid as the decline in the expected transfer from increasing the number of bidders by one.
TABLE 10. Effects of Counterfactual Policies

<table>
<thead>
<tr>
<th>(Costs in $ thousand)</th>
<th>Negotiation</th>
<th>First-Price Auction</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Current</td>
<td>(1)</td>
</tr>
<tr>
<td>Number of bids</td>
<td>1.5</td>
<td>+0.3</td>
</tr>
<tr>
<td>Transfer</td>
<td>1,209.5</td>
<td>-16.8</td>
</tr>
<tr>
<td>Cost components</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Project</td>
<td>1,201.9</td>
<td>-16.9</td>
</tr>
<tr>
<td>B. Liquidity</td>
<td>3.4</td>
<td>-0.5</td>
</tr>
<tr>
<td>C. Bidding</td>
<td>16.2</td>
<td>+14.3</td>
</tr>
<tr>
<td>D. Comp. solicitation</td>
<td>2.5</td>
<td>+22.5</td>
</tr>
<tr>
<td>Aggregate costs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>A+B</td>
<td>1,205.3</td>
<td>-17.3</td>
</tr>
<tr>
<td>A+B+C</td>
<td>1,221.5</td>
<td>-3.0</td>
</tr>
<tr>
<td>A+B+C+D</td>
<td>1,223.9</td>
<td>+19.5</td>
</tr>
</tbody>
</table>

Note: Under Policies (1) and (2), there are contract negotiations, but competitive solicitation is mandated. Policy (1) requires no minimum number of bids; Policy (2) requires at least two bids. Under Policies (3) through (6), first-price sealed bid auctions are used. Policy (3) gives the buyer the discretion to choose the extent of competition. Policies (4) through (6) mandate soliciting competitive bids. In Policy (5), \( \lambda = 1.06 \) for all contracts. Under Policy (6), the bidding costs \( \kappa \) coefficients are halved and \( \lambda = 2.48 \).

to Lemma 8.3 in the Appendix, the expected transfer from conducting a first-price auction for a project of type \( \pi \) with \( n \) bidders is:

\[
\alpha + (1 - \pi)^{n-1} \beta + E[\epsilon].
\]  

Under Policy (3), the buyer chooses the optimal \( \lambda \), the average number of bids, given project type \( \pi \). On average we find that the number of bids under first-price auctions is 0.7 greater. Despite more bids, the expected transfer under first-price auctions is $35,800 higher per project on average, amounting to 3 percent of the expected transfer per project under the current regime ($1.2 million). Even more striking are the results displayed in Column (4) of Table 10: if competitive solicitation was mandatory, the expected transfer would still rise, on average, by $12,000.

7.2. Benefits and Costs of Competition. The expected transfer from buyer to seller is just one of several dimensions to evaluate policy. Another dimension is the expected project cost, based on \( \alpha (\pi, m) + \epsilon \) (for low-cost sellers) and \( \alpha (\pi, m) + \beta (\pi, m) + \epsilon \) (for high-cost sellers). The liquidity cost associated
with a variable contract, which arises in our model because screening by the buyer creates a distortion in risk sharing, is defined as:

\[
\int \{ \psi[q(s, \pi, m)] - q(s, \pi, m) \} f(s) \, ds.
\]

Bidding costs, \( \kappa(\pi, n) \), add yet another dimension to costs. Last is \( \eta \), which is a mirror image of the direct benefit to the seller from not soliciting competitive bids and awarding the project to the default seller. Her reasons for doing so are unobserved by econometricians; in principle they could range from her concerns about noncontractible project quality of the other potential sellers, to increased administrative costs of conducting a competitive bidding process, to bribery and corruption stemming from favoritism. Table 10 summarizes how these cost components are affected by changing procurement policy. For example, Columns (3) and (4) show that reverting to a first price auction reduces project costs (because there are more bidders), eliminates liquidity costs (because there is no screening) and increase bidding costs.

To isolate the effects of greater competition, we compare how the cost components would change if the seller must solicit competitive bids on all contracts. We consider the effects of two counterfactual policies; under Policy (1), competitive solicitation is mandatory for all projects, but buyers have discretion in choosing their effort to attract more bids. Our findings are summarized in Column (1) of Table 10: the size of the expected transfer to the sellers would decrease $16,900 per project, or 1 percent of the average expected transfer under the current regime.

Policy (2) requires at least two bids; the buyer chooses how much effort to expend to attract a greater number of bids than two. Thus if no effort is expended, there are two bids (rather than just one), so effort generates two or more bids. Under the policy proposal, the buyer’s expected total cost for a project of type \( \pi \) is:

\[
\sum_{j=0}^{\infty} \frac{\lambda^j e^{-\lambda}}{j!} \{ T(\pi, j + 2) + \kappa(\pi, j + 2) \} + \eta,
\]

\[25\text{Note that because } \psi(q) > q \text{ for all } q \neq 0 \text{ and } \psi(0) = 0, \text{ the liquidity cost is always positive.}\]
where $\lambda$ is the expected number of extra bids, $T(\pi, n)$ denotes the expected transfer given the optimal contracts associated with type $\pi$ and $n$ bids, and $\kappa(\pi, n)$ denotes the bidding cost. Buyers optimally choose $\lambda$ for each project type $\pi$. Referring to Column (2), this policy would decrease the expected transfer by $45,700$ per project on average, or 4 percent of the average expected transfer under the current regime.

These two alternative policies not only decrease the size of the expected transfer, but also decrease project costs and liquidity costs, on average by $17,300$ and $46,000$ per project, respectively. The reason is that increasing competition makes the selection of a low-cost seller more likely, thus reduce the project costs; since variable contracts are only incurred when high-cost sellers are selected, expected liquidity costs also fall. However, more competition comes at higher bidding costs, as well as the cost of soliciting competitive bids. Table 10 shows that most of the aforementioned cost savings associated with more competition are offset by an increase in the bidding cost and competitive solicitation cost. The increases of the bidding cost only ($14,300$ and $50,100$) eclipse the gain from selecting more low-cost sellers.

Making welfare comparisons across policies hinges on the nature of the bidding cost and the competitive solicitation cost. Suppose the bidding cost reflects frictions in the market that are beyond the scope of a single buyer’s responsibility (Kelman, 1990, 2005), and the cost of competitive solicitation is mostly social waste (perhaps due to collusion with a certain seller). Then we might conclude that Policy (1) is welfare-improving while Policy (2) is not because the former decreases total procurement costs ($A + B + C$ in the table) by $3,000$ per project while the latter will increase the total procurement cost by $4,100$ per project on average. However, if the cost of soliciting competitive bids represents a legitimate social cost (for example due to higher noncontractible quality of the default seller), then both counterfactual policies are suboptimal.

In measuring government inefficiency, Bandiera, Prat and Valletti (2009) propose a distinction between active waste and passive waste. The dichotomy lies in that the public decision maker (the buyer in our context) benefits from
the former (e.g. corruption) but she does not from the latter (e.g. administrative burden). If we assume that the buyers’ cost of soliciting competitive bids is associated with their own private benefits, then active waste in our model is the expected increase in transfers from not soliciting bids on every contract. Policies (1) and (2), mandating competitive solicitation, reduce the expected transfer in this sector by 1 and 4 percent respectively. On the other hand, Bandiera, Prat and Valletti (2009) estimates that the Italian governments pay for procuring standard goods up to 11 percent more than what they would have paid absent active waste; and Di Tella and Schargrodsky (2003) estimate that procurement officers in Argentinian hospitals overprice by 10 percent. Compared to these estimates, our figures are smaller, which quite possibly reflect differences in law enforcement across countries.\(^\text{26}\)

With respect to passive waste, we further assume that the bidding costs do not benefit the buyers. Then passive waste in our model is the difference between the expected transfer under the current policy and the expected project cost of the low-cost sellers. Without passive waste, the buyer can draw bids until she finds a low-cost seller, which leaves the seller no informational rent. Based on our estimates, the average passive waste is $145,100 per contract; i.e., the expected transfer is 14 percent higher due to passive waste. This is smaller than the estimates in Bandiera, Prat and Valletti (2009), 15-43 percent, but our result is consistent with their findings that passive waste accounts for a large portion of total estimated government waste.

7.3. The Value of Discretion. To assess the importance of discretion, we consider counterfactual policies in which the buyer promotes each procurement project with the same intensity, and unilateral price changes are not permitted. Policy (5) strips buyers of all their discretion; they cannot use information about project type and cannot negotiate on contract terms either. Given that the expected transfer under a first-price auction is represented by (17), if \(\lambda\), the average number of bids, is chosen without regard to the value of \(\pi\), then

\(^{26}\)Transparency International’s Corruption Perceptions Index annually ranks countries by the perceived levels of corruption on a scale of 100 (very clean) to 0 (highly corrupt) using expert assessments and opinion surveys. In 2016 US scored 74, Italy scored 47, and Argentina, 36.
the constrained optimum level of solicitation, $\lambda^*$, solves:

$$\min_{\lambda \geq 0} \int \left\{ \sum_{j=0}^{\infty} \frac{\lambda^j e^{-\lambda}}{j!} \left[ \alpha + (1 - \pi)^j \beta + \kappa(\pi, j + 1) \right] \right\} F'_\pi(v) dv.$$ 

Our parameter estimates yield $\lambda^* = 1.06$, or 2.06 bids per procurement auction on average.

Column (5) in Table 10 presents the outcomes under such a policy. Expected transfers increase by $30,500, bidding costs rise by $46,000, project costs fall by $27,200, and liquidity costs are eliminated. The only measure of aggregate costs that declines is project and liquidity costs (A + B in the table). Moreover, referring to the (A + B + C) row in Table 10, the current regime and the first four counterfactual policies all have lower costs than under Policy (5). Thus if bidding costs represent a resource cost but $\eta$ is does not (such as a bribe), then aggregate costs are (A + B + C), and giving the buyer discretion to condition on project type confers positive value.

An important reason for limiting discretion is that it diminishes the demand for collecting and processing information, represented by bidding costs in our model. Accordingly Policy (6) removes all discretion, as in Policy (5), but halves bidding costs as well. Due to lower bidding costs, the optimal $\lambda^*$ in Policy (6) increases to 2.48, generating more competition. Under this scenario, the expected transfer decreases by $36,400 per project, and the total procurement cost including the project, the liquidity, and the bidding cost, also decreases by $7,700 per project. Nevertheless Policy (6) cannot be justified unless there are grounds to believe that the costs of soliciting competitive bids constitute active waste, rather than more legitimate concerns.

8. Conclusion

This paper is an empirical analysis of government procurement auctions that seeks to explain the small number of bids and the features of the winning contracts. The institutional setting and data patterns in our application motivate the parsimonious principal-agent model we develop, identify, and estimate. Winning bids are positively correlated with the number of bids, even after controlling for observed project characteristics; in our model the number of
sellers who bid is endogenous, and the buyer’s efforts to solicit bids are driven by considerations not part of the data. Contracts are subject to both unilateral (by the buyer alone) and bilateral (involving both parties) modifications; in our model the seller internalizes outcomes that are correlated with the seller’s cost type by using unilateral modifications as a screening mechanism, and resorts to bilateral adjustments for project modifications when costs are independent of the seller’s type. Delays are positively correlated with price adjustments arising from unilateral modifications; in our model the timing outcomes of unilateral adjustments of the winning contract are positively correlated with seller costs.

In our application of optimal contracting, the primary reason why procuring agencies use their discretion to restrict competition or to expend little effort to attract greater numbers of sellers is that they can extract much of the rent from low cost sellers by offering a menu of contracts that induces sellers to reveal private information about their own costs. This reduces the marginal value from attracting a large bidding pool, reduces the equilibrium probability of opting for competitive solicitation and the equilibrium number of bids. It explains why there is so little competition in government procurement. Our estimates of liquidity costs associated with screening and nonlinear pricing are low compared to the costs procuring agencies incur from soliciting bids.

Recognizing that the objectives of the procuring agencies do not necessarily follow either voter or taxpayer priorities, we develop several distinct measures of procurement costs to evaluate counterfactual policies. If soliciting competitive bids on every project in our IT/telecommunications service contract sample was mandated, then we estimate the average number of bids per project would rise from 1.5 to 1.8 and that transfers to sellers and also project costs would fall by about 1 to 4 percent; according to our estimates, the reasons why procuring agencies are not always soliciting competitive bids are attributable to the costs they incur from soliciting and processing multiple bids. Depending on the factors that comprise these costs, taxpayers and voters might discount them as waste, and prefer that all contracts to be competed. To the extent that the costs of soliciting and processing bids are unavoidable consequences
from enlarging the competitive pool, our estimates show that allowing procuring agencies to exercise some discretion when setting the level of competition by using their knowledge of project type can reduce procurement costs, even if they are simultaneously engaging in rent-seeking behavior.

References


Laffont, Jean-Jacques, and Jean-Charles Rochet. 1998. “Regulation of


Appendix A. Proofs

A.1. Proof of Theorem 4.1. First we establish that the menu described in the theorem is well defined, by proving in Lemma 8.1 below that $\tilde{\pi}$ is well defined. Then we state and prove six additional lemmas used in the proof of Theorem 4.1 before concluding this appendix with a proof to the theorem itself.

Lemma 8.1. Define the real valued mapping from $\pi \in [0, 1]$ as:

$$H(\pi) \equiv \int_{l(s)<\tilde{l}(\pi)} \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) [1 - l(s)] f(s) ds$$

$$+ \psi(M) \int_{l(s)\geq\tilde{l}(\pi)} [1 - l(s)] f(s) ds,$$

Either $\beta > H(\pi)$ for all $\pi \in [0, 1]$ in which case we define $\tilde{\pi} = 1$, or there exists a unique value $\pi \in (0, 1]$ solving $\beta = H(\pi)$, which we denote by $\tilde{\pi}$.

Proof. Rewrite $H(\pi)$ as:

$$H(\pi) = \int \tilde{H}(\pi, s) f(s) ds,$$

where:

$$\tilde{H}(\pi, s) \equiv \begin{cases} \psi \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) [1 - l(s)] & \text{if } l(s) < \tilde{l}(\pi), \\ \psi(M) [1 - l(s)] & \text{otherwise}. \end{cases}$$
If \( l(s) \geq \tilde{l}(\pi) \) then \( \partial \tilde{H}(\pi, s)/\partial \pi = 0 \). Otherwise:

\[
\frac{\partial}{\partial \pi} \tilde{H}(\pi, s) = -\psi' \left( h \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \right) h' \left( \frac{1 - \pi}{1 - \pi l(s)} \right) \frac{[l(s) - 1]^2}{[1 - \pi l(s)]^2} > 0.
\]

Taking the expectation of \( \tilde{H}(\pi, s) \) with respect to \( s \) proves \( H(\pi) \) is strictly increasing in \( \pi \). From (19) and (4), \( H(0) = 0 \). Hence \( \beta > H(0) \). Therefore \( \beta > H(\pi) \) for all \( \pi \in (0, 1] \) or there exists a unique \( \pi \in (0, 1] \) solving \( \beta = H(\pi) \).

**Lemma 8.2.** The optimal menu includes a fixed contract.

**Proof.** The proof is by a contradiction argument. Suppose to the contrary that every contract on the menu is a variable contract. Then a variable contract is offered to the low-cost seller, which we denote by \( \{p^{(l)}, q^{(l)}(s)\} \). The seller obtains expected utility of:

\[
p^{(l)} + \int \psi[q^{(l)}(s)]f(s)ds \equiv p'.
\]

Define the fixed contract of \( p' + \delta \) where \( \delta \) satisfies the inequalities:

\[
0 < \delta < \int \{\psi \left[ q^{(l)}(s) \right] - q^{(l)}(s) \} f(s)ds \equiv \tilde{R}^{(l)}.
\]

The low-cost seller prefers a fixed contract of \( p' + \delta \) for all \( \delta > 0 \) to \( \{p^{(l)}, q^{(l)}(s)\} \) because the former yields a higher expected utility. The expected transfer from the buyer is lower because \( \tilde{R}^{(l)} \), the risk premium associated with the variable contract \( \{p^{(l)}, q^{(l)}(s)\} \), exceeds \( \delta \). There are two cases to consider.

First, suppose \( p' < \alpha + \beta \). We consider adding \( p' + \delta \) to the menu of variable contracts. Let this contract have the same precedence as \( \{p^{(l)}, q^{(l)}(s)\} \). The high-cost seller would not select \( p' + \delta \) because his IR constraint would be violated; he follows the submission strategy he would follow if \( p' + \delta \) is not on the menu. The low-cost seller would switch from \( \{p^{(l)}, q^{(l)}(s)\} \) to \( p' + \delta \). Hence the buyers expected costs are reduced whenever a low-cost seller previously selecting \( \{p^{(l)}, q^{(l)}(s)\} \) wins.

Second suppose \( p' \geq \alpha + \beta \). We consider replacing the whole menu with a single fixed contract of \( \alpha + \beta \) offered to both types. Under the original menu both types are paid a risk premium and both receive a certainty equivalent of
at least $\alpha + \beta$. Thus a uniform fixed contract of $\alpha + \beta$ is cheaper for the buyer, and trivially satisfies the IC and IR constraints for both types. \hfill \Box

**Lemma 8.3.** It is not optimal to offer only one contract if $n > 1$. If the buyer is constrained to offer fixed contracts only, then it is optimal to offer two contracts, namely $\alpha + \beta$, and:

$$p^{(l)}_n = \alpha + \pi(1 - \pi)^{n-1} \frac{1 - (1 - \pi)^n}{1 - (1 - \pi)^n} \beta$$

(19)

where the lower price takes precedence over the higher one. When $n = 1$ the menu defined by $\alpha + \beta$ and (19) collapses to the optimal one contract menu, $\alpha + \beta$.

**Proof.** The last statement in the lemma is verified by setting $n = 1$ in (19), noting that by Lemma 8.2 the optimal one contract menu is a fixed contract, and that to meet the high-cost seller IR constraint, the price must must be at least $\alpha + \beta$.

The remainder of the proof is in three parts: deriving the optimal fixed contracts when only two are permitted on the menu, showing the optimal fixed contract menu includes more than one item if $n > 1$, and showing that offering more than two items is redundant.

First, consider a menu of two fixed contracts, $\{p^{(l)}, p^{(h)}\}$, where the former is directed towards the low-cost seller and the latter towards the high-cost seller. The IR constraints require $p^{(h)} \geq \alpha + \beta$ and $p^{(l)} \geq \alpha$; the IC constraint for the low-cost seller is:

$$\phi_n (p^{(l)} - \alpha) \geq \overline{\phi}_n (p^{(h)} - \alpha),$$

(20)

where $\phi_n$ and $\overline{\phi}_n$ denote the subjective probability that a seller selecting $p^{(l)}$ wins and its counterpart for selecting $p^{(h)}$, as defined in (2) and (3). The buyer’s objective is to minimize:

$$[1 - (1 - \pi)^n] p^{(l)} + (1 - \pi)^n p^{(h)}$$

with respect to $p^{(l)}$ and $p^{(h)}$ subject to the two IR constraints and (20). The menu characterized in the lemma solves this constrained optimization problem, and also satisfies the remaining IC constraint for the high-cost seller.
Second, if only one item is offered on the menu, then to meet both IR and IC constraints the total expected transfer is at least \( \alpha + \beta \). The two fixed contracts defining menu referred to in the lemma satisfy both IR and IC constraints and yield a total expected transfer of:

\[
T^F_n \equiv [1 - (1 - \pi)^n] p^{(l)}_n + (1 - \pi)^n (\alpha + \beta) = \alpha + (1 - \pi)^{n-1} \beta < \alpha + \beta,
\]
for any \( n > 1 \) and \( \pi \in (0, 1) \).

Third, suppose multiple fixed contracts are offered to low-cost sellers, which we now denote by \( \{p^{(l_1)}_n, p^{(l_2)}_n, \ldots\} \). Also let \( \Phi^{(i)}_n \) denote the probability of winning the contract by bidding \( p^{(l_i)}_n \). Since low-cost sellers are indifferent between submitting these fixed contracts, \( \Phi^{(i)}_n p^{(l_i)}_n = \Phi^{(j)}_n p^{(l_j)}_n \) for all \( i, j \).

Conditional on a low-cost seller winning the contract, the expected transfer from the seller is

\[
P^{(l_0)}_n \equiv \int \psi(q^{(l_0)}_n(s)) f(s) ds \leq p^{(l_0)}_n + \int q^{(l_0)}_n(s) f(s) ds \equiv \tilde{P}^{(l_0)}_n,
\]
where \( \tilde{P}^{(l_0)}_n \) is the expected transfer to a low-cost seller from submitting the \( i \)th contract. When \( i \) is a variable contract, \( q^{(l_i)}_n(s) \neq 0 \) for some \( s \in S \) and appealing to the assumption that \( \psi(q) < q \) for any \( q \neq 0 \) and \( \psi(0) = 0 \), it follows that \( p^{(l_i)}_n < \tilde{P}^{(l_i)}_n \). Following analogous notation define \( \mathcal{H} \) as the set of contracts offered to high-cost sellers. Note that \( \mathcal{L} \cap \mathcal{H} \) can be nonempty.

Suppose there exists a variable contract \( i \in \mathcal{L} \) satisfying the inequality \( p^{(l_i)}_n < \alpha + \beta \). If \( i \in \mathcal{L} \) and \( i \notin \mathcal{H} \) we consider replacing it with a fixed contract of \( p^{(l_i)}_n \); if \( i \in \mathcal{L} \) and \( i \in \mathcal{H} \) we add the fixed contract of \( p^{(l_i)}_n \). The new contract is

**Lemma 8.4.** It is not optimal to offer variable contracts to low-cost sellers.

**Proof.** The proof is by contradiction argument. Suppose a set of multiple contracts \( \mathcal{L} \equiv \{p^{(l_1)}_n, q^{(l_i)}_n(s) : i = 1, 2, \ldots\} \) is offered to low-cost sellers in the conjectured equilibrium, where \( \mathcal{L} \) includes at least one variable contract. We define the certainty equivalent of each contract \( i \in \mathcal{L} \) as \( p^{(l_i)}_n \), defined:

\[
\tilde{P}^{(l_i)}_n \equiv p^{(l_i)}_n + \int \psi(q^{(l_i)}_n(s)) f(s) ds \leq p^{(l_i)}_n + \int q^{(l_i)}_n(s) f(s) ds \equiv \tilde{P}^{(l_i)}_n,
\]
where \( \tilde{P}^{(l_i)}_n \) is the expected transfer to a low-cost seller from submitting the \( i \)th contract. When \( i \) is a variable contract, \( q^{(l_i)}_n(s) \neq 0 \) for some \( s \in S \) and appealing to the assumption that \( \psi(q) < q \) for any \( q \neq 0 \) and \( \psi(0) = 0 \), it follows that \( p^{(l_i)}_n < \tilde{P}^{(l_i)}_n \). Following analogous notation define \( \mathcal{H} \) as the set of contracts offered to high-cost sellers. Note that \( \mathcal{L} \cap \mathcal{H} \) can be nonempty.
designated to have the same precedence as the $i^{th}$ contract (the buyer randomly selecting amongst sellers submitting either contract when $i \in \mathcal{H}$). High-cost sellers do not select the fixed contract of $p''_i$ since $p''_i < \alpha + \beta$ violating their IR constraint. The new fixed contract yields the same level of expected utility as $i$ to low-cost sellers and therefore satisfies their IR and the IC constraints. Since low-cost sellers are indifferent, it is a best response for them to choose the fixed cost contract $p''_i$ rather than $i$ (when the latter is still available). Therefore replacing or adding a fixed contract $p''_i$ is feasible, does not change the submission behavior of high-cost sellers, but reduces the buyer’s expected transfer (since $p''_i < \tilde{p}_i'\prime$). Hence offering a variable contract with a certainty equivalent less than $\alpha + \beta$ to low-cost sellers is not optimal.

Suppose there exists at least one contract in $\mathcal{L}$ for which $p''_i \geq \alpha + \beta$. We consider replacing all such contracts in $\mathcal{L}$ with a fixed contract of $\alpha + \beta$, while retaining the others. We also replace $\mathcal{H}$ with $\alpha + \beta$. Note that $\alpha + \beta$ is a lower bound of the expected transfer to high-cost sellers to satisfy their IR condition. Because the retained contracts in $\mathcal{L}$ are fixed contracts less than $\alpha + \beta$ by the above argument, both the IR and the IC constraints for high-cost sellers are satisfied. Furthermore, the IR and the IC conditions for the low-cost sellers are trivially satisfied and hence this modified menu is feasible. Note that $\alpha + \beta$ is less desirable to low-cost sellers than the contracts that have been withdrawn from $\mathcal{L}$. Therefore low-cost sellers would not decrease the probability of choosing the retained contracts. Thus the distribution of expected transfers associated with the contract for the modified contracts is first order stochastically dominated by the distribution when there are variable contracts offered to low-cost sellers with $p''_i \geq \alpha + \beta$, and this completes the proof. \hfill \square

**Lemma 8.5.** The menu defined in Lemma 8.3 is not optimal.

**Proof.** Our proof is by constructing a feasible, alternative menu of contracts, a fixed contract $\tilde{p}_n$ and a variable contract $\{\tilde{p}, \tilde{q}(s)\}$, which is less costly than $T_n^F$ (defined in Lemma 8.3) in expectation.

For some $\epsilon > 0$, define $S \equiv \{s : \tilde{f}(s) - f(s) > \epsilon\}$. Let the probability that an outcome $s$ is in $S$ conditional on that the seller is low-cost as $\gamma_1$ and that conditional on that the seller is high-cost as $\gamma_2$. By assumption $\tilde{F}(s) \neq F(s)$
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for some outcome \( s \) in the support. Hence there exists \( \epsilon > 0 \) such that \( \gamma_1 \neq 0 \) and \( \gamma_2 \neq 0 \). Note that \( \gamma_2 > \gamma_1 \). For any \( \delta > 0 \) choose \( \mu(\delta) \) for a variable contract in which \( \tilde{p} = \alpha + \beta \) and:

\[
\tilde{q}(s) = \begin{cases} 
\delta & \text{if } s \in S, \\
\mu(\delta) & \text{if } s \notin S,
\end{cases}
\]

where:

\[
\gamma_2 \psi(\delta) + (1 - \gamma_2) \psi[\mu(\delta)] = 0. \tag{21}
\]

Note that the above equation implies that \( \mu(\delta) < 0 \) for \( \delta > 0 \) with \( \lim_{\delta \to 0} \mu(\delta) = 0 \). Because \( \psi(q) \) is strictly increasing, \( \mu(\delta) \) is uniquely defined by the equation:

\[
\mu(\delta) = \psi^{-1} \left[ \frac{-\gamma_2}{1 - \gamma_2} \psi(\delta) \right],
\]

and is twice differentiable with:

\[
\mu'(\delta) = \frac{-\gamma_2 \psi'(\delta)}{1 - \gamma_2 \psi'[\mu(\delta)]}.
\]

The fixed contract, \( \tilde{p}_n \), is:

\[
\tilde{p}_n = \alpha + \pi \left(1 - \pi\right)^{n-1} \left\{ \beta + \gamma_1 (1 - \gamma_1) \psi[\mu(\delta)] \right\}
\]

\[
= \alpha + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left\{ \beta + \gamma_1 \psi(\delta) - (1 - \gamma_1) \left( \frac{\gamma_2}{1 - \gamma_2} \right) \psi(\delta) \right\}
\]

\[
= \alpha + \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left\{ \beta + \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} \psi(\delta) \right\},
\]

where the second equality is derived from (21). This proposed menu, consisting of the preferred \( \tilde{p}_n \) for the low-cost sellers and the less preferred \( \{\tilde{p}, \tilde{q}(s)\} \) for the high-cost ones, is feasible for sufficiently small \( \delta > 0 \). This is because (i) the IC constraint is satisfied with equality by the low-cost sellers and strict inequality by the high-cost sellers since \( \gamma_1 < \gamma_2 \); and (ii) the IR constraint is satisfied with equality by the high-cost sellers and strict inequality by the low-cost sellers as long as \( \delta > 0 \) is small enough.
The expected transfer from the buyer, denoted by $\tilde{T}(\delta)$ to indicate its dependence on the choice of $\delta$, is:

$$[1 - (1 - \pi)^n] \tilde{\rho}_n + (1 - \pi)^n \{ \tilde{\rho} + \gamma_2 \delta + (1 - \gamma_2) \mu(\delta) \}$$

$$= \alpha + \pi (1 - \pi)^{n-1} \left\{ \beta + \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} \psi(\delta) \right\} + (1 - \pi)^n \left\{ \beta + \gamma_2 \delta + (1 - \gamma_2) \mu(\delta) \right\}$$

$$= \alpha + (1 - \pi)^{n-1} \left\{ \beta + \pi \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} \psi(\delta) + (1 - \pi) \left[ \gamma_2 \delta + (1 - \gamma_2) \mu(\delta) \right] \right\}.$$ 

Note that $\tilde{T}(0) = T_n^F$, and the proof is done by showing that $\tilde{T}(\delta)$ is decreasing in the neighborhood of $\delta = 0$. Differentiating $\tilde{T}(\delta)$ with respect to $\delta$ yields:

$$\tilde{T}'(\delta) = (1 - \pi)^{n-1} \left\{ \pi \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} \psi'(\delta) + (1 - \pi) \left[ \gamma_2 - \gamma_2 \frac{\psi'(\delta)}{\psi'[\mu(\delta)]} \right] \right\},$$

and accordingly, $\tilde{T}'(0) = (1 - \pi)^{n-1} \pi \frac{\gamma_1 - \gamma_2}{1 - \gamma_2} < 0$, which completes the proof. □

**Lemma 8.6.** It is not optimal to offer fixed contracts to high-cost sellers.

**Proof.** The proof is by contradiction argument. To satisfy the IR constraint, every fixed price contract offered to high-cost sellers is bounded below by $\alpha + \beta$. Furthermore fixed contracts greater than $\alpha + \beta$ are not offered to high-cost sellers in conjunction with a fixed contract contract of $\alpha + \beta$, because the expected net value to the high-cost seller from submitting $\alpha + \beta$ is zero, strictly less than the expected net value from submitting a fixed contract with a high price. Hence there are only two cases to consider: first, only one fixed contract, $\alpha + \beta$ is offered to high-cost sellers; second, a fixed contract with a price greater than $\alpha + \beta$ is offered to them.

First, consider a menu which includes only one fixed contract of $\alpha + \beta$ offered to high-cost sellers. We now show the menu cannot include a variable contract. Lemma 8.4 shows that low-cost sellers are only offered fixed contracts. To induce a high-cost seller to select a variable contract, the buyer must offer a risk premium, thus increasing the expected transfer above $\alpha + \beta$. However the optimal fixed contract menu is given in Lemma 8.3, and this is more costly than the menu given in Lemma 8.5 which includes a variable contract for high-cost sellers. Therefore the menu does not include only one fixed contract of $\alpha + \beta$ offered to high-cost sellers.
Second, consider a menu which includes a fixed contracts offered to high-cost sellers with a price greater than $\alpha + \beta$. Appealing to Lemma 8.4, every variable contract on the menu is only offered to high-cost sellers; furthermore by the IR constraint and the concavity of $\psi(q)$, the expected transfer of every such contract exceeds $\alpha + \beta$. Suppose we consider an alternative menu by withdrawing all fixed contracts above $\alpha + \beta$ and all variable contracts from the original menu, adding a single fixed contract of $\alpha + \beta$, and retaining every other fixed contract below $\alpha + \beta$ (if there are any on the original menu). Comparing the distribution of expected transfers under the original menu versus the alternative menu, we now show that the former first order stochastically dominates the latter, hence has a higher mean, and thus falsifies the contradictory premise negating the lemma.

Presented with the alternative menu all high-cost sellers select $\alpha + \beta$, since every other fixed contract does not satisfy their IR constraint. If all contracts previously offered to low-cost sellers were less than $\alpha + \beta$, then the choice probabilities of low-cost sellers remain unchanged (since low-cost sellers rejected higher priced fixed contracts on the original menu). Alternatively suppose low-cost sellers were offered a contract of $p''' > \alpha + \beta$ on the original menu, which is now withdrawn. Because the $\alpha + \beta$ fixed contract is less desirable than the $p'''$ contract, the probability that a low-cost seller chooses each of the remaining fixed contracts below $\alpha + \beta$ does not decrease. Therefore, first order stochastic dominance is established by noting that the probability of choosing every fixed contract below $\alpha + \beta$ does not fall and the remaining mass is concentrated at $\alpha + \beta$ (instead of the higher expected transfer values of the withdrawn contracts on the original menu). □

Lemma 8.7. The optimal menu of contracts consisting of one fixed contract and one variable contract is unique and is defined by (5), (6) and (7).

Proof. Suppose the buyer is constrained to offer a menu of two contracts consisting of one fixed contract and one variable contract. Then by Lemma ??, the fixed contract, denoted by $\mathbb{p}_n$, must be accepted by the low-cost sellers only and the variable one, denoted by $\{\overline{p}, q(s)\}$, by the high-cost sellers only.
Given this, the buyer’s problem is to minimize the expected transfer:

$$[1 - (1 - \pi)^n] p_n + (1 - \pi)^n \left[ \bar{p} + \int q(s) f(s) ds \right].$$

subject to the IR and IC constraints and the limited liability constraint. A necessary condition of the optimal menu is that the IR constraint the high-cost seller holds with equality (otherwise the base price $\bar{p}$ could be further reduced, reducing the price and strengthening the IC constraint for the low-cost seller). Solving for $\bar{p}$ yields (6). The IC constraint for the low-cost seller is:

$$\phi_n (p_n - \alpha) \geq \bar{\phi}_n \{ \bar{p} + \int \psi(q) f(s) ds - \alpha \},$$

Substituting for $\bar{p}$ using (6) and appealing to the definitions of $\bar{\phi}_n$ and $\underline{\phi}_n$ as in (2) and (3):

$$p_n \geq \alpha + \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left( \beta - \int \psi[q(s)]\{1 - l(s)\} \bar{f}(s) ds \right).$$

Note that the IC constraint for the high-cost sellers is satisfied with strict inequality at the optimum (i.e., $\underline{\phi}_n (p_n) < \alpha + \beta$) by Lemma ?? Therefore, at least the IR or IC constraints for the low-cost sellers must bind at the optimum. Otherwise, the price of the fixed contract could be reduced, to the buyer’s benefit.

Case 1: IC binds but IR does not  
Solving for $p_n$ from the IC constraint, and substituting the resulting expressions for $\underline{p}_n$ and $\bar{p}$, obtained from (6) and (5), into the expected total cost for the buyer or (22), we obtain:

$$[1 - (1 - \pi)^n] \left\{ \alpha + \frac{\pi(1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \left( \beta - \int \psi[q(s)]\{1 - l(s)\} \bar{f}(s) ds \right) \right\}$$

$$+(1 - \pi)^n \left\{ \alpha + \beta + \int \{ q(s) - \psi[q(s)] \} \bar{f}(s) ds \right\}$$

$$= \alpha + (1 - \pi)^{n-1} \left( \beta + \int [(1 - \pi)(q(s) - \psi[q(s)]) - \pi \psi[q(s)]\{1 - l(s)\}] \bar{f}(s) ds \right).$$
The (scaled) Lagrangian for the cost minimization problem can now be expressed as:

\[ L = \int [(1 - \pi - \varpi)\{q(s) - \psi[q(s)]\} - \pi\psi[q(s)]\{1 - l(s)\} - \nu_1(s)(q(s) - M)]\overline{f}(s)ds, \]

where \( \nu_1(s) \geq 0 \) denotes the Kuhn Tucker multiplier for the limited liability constraint \( q(s) \geq M \). The first order condition for \( q(s) \) is:

\[ (1 - \pi)(1 - \varpi'[q(s)]) - \pi\varpi'[q(s)]\{1 - l(s)\} - \nu_1(s) = 0. \]

Rearranging terms we obtain:

\[ \psi'[q(s)] = \frac{1 - \pi - \nu_1(s)}{1 - \pi l(s)}. \] (24)

If \( l(s) < \bar{l}(\pi) \), then \( q(s) = h\left[\frac{1 - \pi}{1 - \pi l(s)}\right] > M \) and \( \nu_1(s) = 0 \) solve (24). If \( l(s) \geq \bar{l}(\pi) \), then \( \nu_1(s) > 0 \) and \( q(s) = M \) solve (24).

**Case 2: Does IR but not IC bind?** When IR binds, \( \underline{p}_n = \alpha \). Substituting for \( \underline{p}_n \) and \( \overline{p} \) using (6), the expected total transfer (22) simplifies to:

\[ \alpha + (1 - \pi)^n \left\{ \beta + \int \{q(s) - \psi[q(s)]\}\overline{f}(s)ds \right\}. \] (25)

Substituting for \( \underline{p}_n \) in (23) yields:

\[ \beta \leq \int \psi[q(s)]\{1 - l(s)\}\overline{f}(s)ds. \] (26)

Let \( \nu_1(s) \geq 0 \) denote the Kuhn Tucker multiplier for the limited liability constraint \( q(s) \geq M \). If IR binds but IC does not, then the first order condition for the Kuhn Tucker formulation is:

\[ 1 - \psi'[q(s)] = \nu_1(s). \]

If \( q(s) > M \), then the complementary slackness condition requires \( \nu_1(s) = 0 \), and hence \( 1 = \psi'[q(s)] \) implying \( q(s) = 0 \). Therefore, either \( q(s) = M \) or \( q(s) = 0 \). Let us define \( S_M \) as the set of contract outcomes such that \( q(s) = M \) and let \( \mu \) denote \( \Pr(s \in S_M) \). Note that for any \( \mu \in [0, 1] \), both IR constraints and the IC constraint for the high-cost seller are satisfied. The total expected
transfer can now be written as:

\[ \alpha + (1 - \pi)^n \{ \beta + [M - \psi(M)]\mu \}. \]  

(27)

By inspection (27) is increasing in \( \mu \), while setting \( \mu = 0 \) does not satisfy the IC condition for the low-cost seller, or (26). This implies that when both IR constraints bind, the IC for the low-cost seller must bind.

**Case 3: Both IR and IC bind** If (26) holds with equality, the (scaled) Lagrangian for the minimization problem can be written as:

\[
L = \int (q(s) - \psi[q(s)])f(s)ds - \int \kappa_1(s) [q(s) - M]f(s)ds \\
+ \kappa_2 \left\{ \beta - \int \psi[q(s)][1 - l(s)]f(s)ds \right\},
\]

where \( \kappa_1(s) \) denotes the Kuhn Tucker multiplier for the limited liability constraint and \( \kappa_2 \) denotes the counterpart for (26). The first order condition with respect to \( q(s) \) is:

\[
1 - \psi'[q(s)] - \kappa_1(s) - \kappa_2 \psi'[q(s)][1 - l(s)] = 0.
\]

This can be expressed:

\[
\psi'[q(s)] = \frac{1 - \psi'_1(s)}{1 + \psi'_2[1 - l(s)]}.
\]

(28)

It is straightforward to show that the solution is continuous at \( \tilde{\pi} \), as defined in Lemma 8.1, by marginally perturbing the constraint set. Therefore the solution to \( q(s) \) at \( \tilde{\pi} \) solves both first order conditions (24) and (28). Matching them up we deduce \( \kappa_2 = \tilde{\pi} / (1 - \tilde{\pi}) \). Substituting for \( \kappa_2 \) in (28) it now follows that the solution for \( q(s) \) by setting \( \pi = \tilde{\pi} \) in (7).

Summarizing, the IC for the low-cost seller always binds at the optimum, but the IR constraint for the low-cost seller may not. It immediately follows from (19) and (5) that if \( \beta > H(\pi) \) then \( \underline{p}_n > \alpha \), but if \( \beta \leq H(\pi) \) then \( \underline{p}_n = \alpha \). Appealing to Lemma 8.1, we conclude if \( \pi < \tilde{\pi} \), then \( \beta > H(\pi) \) and Case 1 applies; otherwise \( \pi \geq \tilde{\pi} \), then \( \beta \leq H(\pi) \) and Case 3 applies. By construction the menu is uniquely defined. \( \square \)
Proof of Theorem 4.1. By Lemma 8.4 it is not optimal to offer low-cost sellers a variable contracts, and by Lemma 8.6 it is not optimal to offer high-cost sellers fixed contracts. This proves the second statement in the theorem.

Lemma 8.7 characterizes the unique optimal menu of contracts when it consists of one fixed contract directed to low-cost sellers and one variable contract directed to high-cost sellers. Appealing to Lemmas 8.4 and 8.6 proves the third statement of the theorem.

Lemma 8.5 shows that a menu with a variable contract is less costly than the optimal two fixed priced contract menu given in Lemma 8.3, which in turn is less costly than menus containing only one contract. This proves that an optimal menu contains at least two contracts. We now show additional contracts are redundant. Noting that the low-cost seller is offered a fixed contract, exactly the same arguments used in the proof of Lemma 8.3 apply here. Therefore offering several fixed contracts to the low-cost seller does not reduce the expected transfer. Also every variable contract offered to the high-cost seller must individually satisfy the IR constraint with equality, and also deter the low-cost from accepting it. If any two such variable contracts on the menu do not generate the same expected transfer to the high-cost seller, then offering the more expensive variable contract is suboptimal. This proves the first statement of the theorem. □

A.2. Proof of Lemma 5.1. If \( A3 \) holds, then \( q(s) \) satisfies the first order condition, (7). Rewriting (7) while replacing \( q(s) \) by \( q(s, \pi) \) to emphasize the dependence of \( q \) on \( \pi \),

\[
\psi'[q(s, \pi)] [1 - \pi l(s)] = 1 - \pi.
\]

Note that \( q(s, \pi) = 0 \) if \( l(s) = 1 \) and \( q(s, \pi) > 0 \) if \( l(s) < 1 \). Similarly \( q(s, \pi) < 0 \) if \( l(s) > 1 \). Totally differentiating the first order condition with respect to \( \pi \) yields:

\[
\psi''[q(s, \pi)] \frac{\partial q(s, \pi)}{\partial \pi} [1 - \pi l(s)] - \psi'[q(s, \pi)] l(s) = -1.
\]

Rearranging to make \( \frac{\partial q(s, \pi)}{\partial \pi} \) the subject of the equation gives:

\[
\frac{\partial q(s, \pi)}{\partial \pi} = \frac{l(s) - 1}{\psi''[q(s, \pi)] [1 - \pi l(s)]^2}.
\]
Noting $\psi''(q) < 0$ it follows that $\partial q(s, \pi) / \partial \pi > 0$ when $l(s) < 1$ and $\partial q(s, \pi) / \partial \pi < 0$ when $l(s) > 1$. Therefore,
\[
\frac{\partial q(s, \pi)}{\partial \pi} > 0 \text{ if } q(s, \pi) > 0, \\
= 0 \text{ if } q(s, \pi) = 0, \\
< 0 \text{ if } q(s, \pi) < 0.
\]
as was to be proved.

Rewriting (5) while making the dependence of $p_n, q(s), \alpha,$ and $\beta$ on $\pi$:
\[
p_n(\pi) = \alpha(\pi) + \pi(1 - \pi)^{n-1} \left[ \beta(\pi) - \int \psi(q(s, \pi))[f(s) - f(s)] ds \right].
\]
To show that $p_n'(\pi) < 0$ we consider the two expressions involving $\pi$ separately. First:
\[
\frac{\partial}{\partial \pi} \ln \left[ \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^n} \right] = \frac{1 - n\pi - (1 - \pi)^n}{\pi (1 - \pi) [1 - (1 - \pi)^n]}
\]
Note that the derivative is zero at $n = 1$ and that at $n = 2$ is $-\pi^2$, which is negative. Now suppose it is negative for all $n \in \{2, \ldots, n_0\}$. Then for $n_0 + 1$ the denominator is clearly positive and the numerator is:
\[
1 - (n_0 + 1) \pi - (1 - \pi)(1 - \pi)^{n_0} < \pi (1 - \pi)^{n_0} - \pi < 0.
\]
The first inequality follows from an induction hypothesis, and the second one from the inequalities $0 < \pi < 1$. Therefore $\pi (1 - \pi)^{n-1} / \pi (1 - \pi)^{n-1}$ is decreasing in $\pi$ for all $n > 1$.

Second, we note that:
\[
\frac{\partial}{\partial \pi} \int \psi[q(s, \pi)] [1 - l(s)] \bar{f}(s) ds = \int \psi'[q(s, \pi)] \frac{\partial q(s, \pi)}{\partial \pi} [1 - l(s)] \bar{f}(s) ds \\
= \int (1 - \pi) \frac{\partial q(s, \pi)}{\partial \pi} \left[ \frac{1 - l(s)}{1 - \pi l(s)} \right] \bar{f}(s) ds \\
= \int \frac{(\pi - 1) [1 - l(s)]^2}{\psi''[q(s, \pi)] [1 - \pi l(s)]^2} \bar{f}(s) ds > 0.
\]
The second equality follows from using the first order condition to substitute out \( \psi' \{ q(s, \pi) \} \). Note that we can use the first order condition because A3 holds. The third equality results from the expression we derived for \( \partial q(s, \pi) / \partial \pi \). The last inequality appeals to the concavity of \( \psi(q) \). Finally note that since the participation constraint is satisfied with an inequality for the low-cost seller under A3.

\[
\beta(\pi) - \int \psi[q(s, \pi)] \left[ 1 - l(s) \right] f(s) ds > 0,
\]

for all \( \pi \in \Pi \). Hence, if \( \alpha(\pi) \) and \( \beta(\pi) \) are nonincreasing in \( \pi \), as assumed in A4, the following inequality holds as claimed.

\[
\frac{\partial}{\partial \pi} \bar{p}(\pi) = \alpha'(\pi) + \frac{\partial \beta}{\partial \pi} \left[ \frac{\pi (1 - \pi)^{n-1}}{1 - (1 - \pi)^{n}} \right] \left\{ \beta(\pi) - \int \psi[q(s, \pi)] \left[ 1 - l(s) \right] f(s) ds \right\} + \frac{\pi (1 - \pi)^{n}}{1 - (1 - \pi)^{n}} \left\{ \beta'(\pi) - \int \psi''[q(s, \pi)] \left[ 1 - \pi l(s) \right]^{2} f(s) ds \right\} < 0.
\]

Recall that

\[
\bar{p} = \alpha + \beta - \int \psi[q(s)] f(s) ds.
\]

Differentiating with respect to \( \pi \) yields:

\[
\frac{\partial \bar{p}}{\partial \pi} = \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} - \int \psi'[q(s)] \frac{\partial q(s; \pi)}{\partial \pi} f(s) ds
\]

\[
= \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} - \int h' \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \frac{(1 - \pi) l(s) - [1 - \pi l(s)]}{1 - \pi l(s)} f(s) ds
\]

\[
= \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + \int h' \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \frac{1}{1 - \pi l(s)} f(s) ds.
\]

First note that the integral is negative for \( l(s) < 1 \) and positive for \( l(s) > 1 \) because \( h'(\cdot) < 0 \) and \( \pi l(s) < 1 \) by A3. Therefore,

\[
\frac{\partial \bar{p}}{\partial \pi} < \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + \int_{l(s) > 1} h' \left[ \frac{1 - \pi}{1 - \pi l(s)} \right] \frac{(1 - \pi) [1 - l(s)]}{1 - \pi l(s) [1 - \pi l(s)]^{3}} f(s) ds.
\]

We define \( m(\pi, l) \) by

\[
m(\pi, l) \equiv h' \left[ \frac{1 - \pi}{1 - \pi l} \right] \frac{1 - l}{[1 - \pi l]^{4}}.
\]
The derivative of $m(\pi, l)$ with respect to the second argument is positive if $h'' > 0$ and $l > 1$.

\[
\frac{\partial m(\pi, l)}{\partial l} = h'' \left[ \frac{1 - \pi}{1 - \pi l} \right] \frac{(1 - \pi)^2 [l - 1] \pi}{[1 - \pi l]^5} - h' \left[ \frac{1 - \pi}{1 - \pi l} \right] \frac{(1 - \pi)}{[1 - \pi l]^3} + 3h' \left[ \frac{1 - \pi}{1 - \pi l} \right] \frac{\pi (1 - \pi) [1 - l]}{[1 - \pi l]^4},
\]

which can be rewritten as:

\[
\frac{\partial m(\pi, l)}{\partial l} = h'' \left[ \frac{1 - \pi}{1 - \pi l} \right] \frac{(1 - \pi)^2 [l - 1] \pi}{[1 - \pi l]^5} + h' \left[ \frac{1 - \pi}{1 - \pi l} \right] (1 - \pi) \frac{2\pi (1 - l) + \pi - 1}{[1 - \pi l]^4}.
\]

Therefore,

\[
\frac{\partial \bar{p}}{\partial \pi} < \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + h' \left[ \frac{1 - \pi}{1 - \pi l(\pi, M)} \right] \frac{(1 - \pi) \left[ 1 - \tilde{l}(\pi, M) \right]}{[1 - \pi \tilde{l}(\pi, M)]^3}. \tag{29}
\]

Using the definition of $\tilde{l}(\pi, M) \equiv \frac{1}{\pi} - \frac{1 - \pi}{\pi \psi'(M)}$, we have

\[
1 - \tilde{l}(\pi, M) = 1 - \frac{1}{\pi} + \frac{1 - \pi}{\pi \psi'(M)} = \frac{\pi \psi'(M) - \psi'(M) + (1 - \pi)}{\pi \psi'(M)} = \frac{(1 - \pi)(1 - \psi'(M))}{\pi \psi'(M)},
\]

\[
1 - \pi \tilde{l}(\pi, M) = 1 - 1 + \frac{1 - \pi}{\psi'(M)} = \frac{1 - \pi}{\psi'(M)}.
\]

Using these, we simplify the RHS of (29) as:

\[
\frac{\partial \bar{p}}{\partial \pi} < \frac{\partial \alpha}{\partial \pi} + \frac{\partial \beta}{\partial \pi} + \frac{h' [\psi'(M)][\psi'(M)]^2 [1 - \psi'(M)]}{\pi (1 - \pi)}.
\]

Therefore, $\frac{\partial \bar{p}}{\partial \pi} < 0$.

**A.3. Proof of Lemma 5.2.** Appealing to A5, $\bar{p}$ is monotone decreasing in $\pi$, implying the existence of a mapping $\pi(\bar{p})$ such that $l^* (q, \bar{p})$ defined in (13) satisfies:

\[
\psi'(q) = \frac{1 - \pi}{1 - \pi l^*(q, \bar{p})}.
\]
Making $\pi$ the subject we obtain:

$$\pi = \frac{1 - \psi'(q)}{1 - l^*(q, \tilde{p}) \psi'(q)}.$$

Differentiating with respect to $q$ holding $\pi$ and $p$ constant yields:

$$\psi''(q) = \frac{\pi}{1 - \pi l^*(q, \tilde{p})} \psi'(q) \frac{\partial l^*(q, \tilde{p})}{\partial q}.$$

Using these two equations we substitute $\pi$ out to obtain (14).

A.4. Proof of Lemma 5.3. The joint probability that the contract type is fixed and $\pi \leq \pi^*$ can be expressed as:

$$\Pr \{\pi \leq \pi^*, v = 0 \mid c, n\} = F_{\pi|c,n,v} (\pi^* \mid c, n, 0) \Pr (v = 0 \mid c, n)$$

$$= \int_{\pi = \pi^*}^{\pi^*} f_{\pi|c,n} (\pi \mid c, n) [1 - (1 - \pi)^n] d\pi. \quad (30)$$

Taking the derivative with respect to $\pi^*$ yields:

$$f_{\pi|c,n,v} (\pi^* \mid c, n, 0) \Pr (v = 0 \mid c, n) = f_{\pi|c,n} (\pi^* \mid c, n) [1 - (1 - \pi^*)^n]. \quad (31)$$

Similarly:

$$\Pr \{\pi \leq \pi^*, v = 1 \mid c, n\} = F_{\pi|c,n,v} (\pi^* \mid n, v = 1) \Pr (v = 1 \mid c, n)$$

$$= \int_{\pi = \pi^*}^{\pi^*} f_{\pi|c,n} (\pi \mid c, n) (1 - \pi)^n d\pi,$$

and taking the derivative with respect to $\pi^*$ yields:

$$f_{\pi|c,n,v} (\pi^* \mid c, n, 1) \Pr (v = 1 \mid c, n) = f_{\pi|c,n} (\pi^* \mid c, n) (1 - \pi^*)^n. \quad (32)$$

Rearranging the quotient of (31) and (32) to make $f_{\pi|c,n,v} (\pi^* \mid c, n, v = 0)$ the subject of the resulting equation, and relabeling $\pi^*$ as $x$, we obtain:

$$f_{\pi|c,n,v} (x \mid c, n, 0) = \varphi_{c,n} \frac{1 - (1 - x)^n}{(1 - x)^n} f_{\pi|c,n,v} (x \mid c, n, 1), \quad (33)$$

where $\varphi_{c,n} \equiv \Pr \{v = 1 \mid c, n\} / \Pr \{v = 0 \mid c, n\}$, as defined in the text.

Appendix B. Estimation and Counterfactual Analyses

B.1. Optimal Competition under the Parametric Specification. Under the specification on $\kappa(\pi, n)$ in (16), the expected total cost of competed
procurement with effort $\lambda$, denoted by $U(\pi, \lambda)$, is:

$$U(\pi, \lambda) = \alpha(\pi) + \exp^{-\lambda \pi}[\beta(\pi) + \Gamma(\pi)] + \mathbb{E}[\kappa(\pi, j + 1)|\lambda] + \eta$$

$$= \alpha(\pi) + \exp^{-\lambda \pi}[\beta(\pi) + \Gamma(\pi)] + \tilde{\kappa}_1(\pi) \lambda + \tilde{\kappa}_2(\pi) \lambda(1 + \lambda) + \eta,$$

where $\tilde{\kappa}_1(\pi) \equiv \kappa_1 + \kappa_2 \pi$ and $\tilde{\kappa}_2(\pi) \equiv \kappa_3 + \kappa_4 \pi$.

Taking the first order condition:

$$\pi \exp^{-\lambda \pi}[\beta(\pi) + \Gamma(\pi)] = \tilde{\kappa}_1(\pi) + \tilde{\kappa}_2(\pi)(1 + 2 \lambda).$$

(34)

Because the LHS is decreasing in $\lambda$ while the RHS is increasing in $\lambda$, there exists a unique solution to the above equation for any given $\pi$, denoted by $\lambda^*(\pi)$. In our estimation, we numerically solve for $\lambda^*(\pi)$ for each $\pi$.

Given $\lambda^*(\pi)$, it is optimal for the buyer to hold a competitive solicitation if and only if

$$U[\pi, \lambda^*(\pi)] \leq U_0(\pi).$$

The above inequality can be rewritten as:

$$\eta \leq (1 - e^{-\lambda^*(\pi) \pi}[\beta(\pi) + \Gamma(\pi)]) - \tilde{\kappa}_1 \lambda^*(\pi) - \tilde{\kappa}_2 \lambda^*(\pi)[1 + \lambda^*(\pi)].$$

(35)

Equations (5), (6), (7), (34), and (35) characterize the equilibrium contracts and competition. Given the parameters of the model and these equations, we simulate the model.

B.2. Simulated GMM Estimator. Let us denote the vector of the parameters of the model by $\theta$. Our estimator minimizes a weighted sum of squared distances:

$$g_n(\theta)'W g_n(\theta), \text{ with } g_n(\theta) = \frac{1}{n} \sum_{t=1}^{n} g(w_t; \theta),$$

where $W$ is a symmetric positive-definite weighting matrix. The $g(w_t; \theta)$ vector is associated with 40 moment conditions: (i) 17 moment conditions on competition, contract type, and contract price for all projects, (ii) the same 17 moment conditions for non-military projects, and (iii) 6 moment conditions on the distribution of $s$, or the standardized delay. The 17 moment conditions consist of $\Pr(c_i = 0)$, $\Pr(n_i = 1|c_i = 1)$, $\Pr(n_i \leq 2|c_i = 1)$, $\Pr(n_i \leq 5|c_i = 1)$, $\Pr(v_i = 0|c_i = 0)$, $\Pr(v_i = 0|c_i = 1)$, $\Pr(v_i = 0|c_i = 1, n_i = 1)$, $\Pr(v_i = 0|c_i = 1, n_i = 0)$, $\Pr(v_i = 0|c_i = 1, n_i = 2)$, $\Pr(v_i = 0|c_i = 1, n_i = 3)$, $\Pr(v_i = 0|c_i = 1, n_i = 4)$, $\Pr(v_i = 0|c_i = 1, n_i = 5)$, $\Pr(v_i = 0|c_i = 1, n_i = 6)$, $\Pr(v_i = 0|c_i = 1, n_i = 7)$, $\Pr(v_i = 0|c_i = 1, n_i = 8)$, $\Pr(v_i = 0|c_i = 1, n_i = 9)$, $\Pr(v_i = 0|c_i = 1, n_i = 10)$, $\Pr(v_i = 0|c_i = 1, n_i = 11)$, $\Pr(v_i = 0|c_i = 1, n_i = 12)$, $\Pr(v_i = 0|c_i = 1, n_i = 13)$, $\Pr(v_i = 0|c_i = 1, n_i = 14)$, $\Pr(v_i = 0|c_i = 1, n_i = 15)$, $\Pr(v_i = 0|c_i = 1, n_i = 16)$, $\Pr(v_i = 0|c_i = 1, n_i = 17)$, $\Pr(v_i = 0|c_i = 1, n_i = 18)$, $\Pr(v_i = 0|c_i = 1, n_i = 19)$, $\Pr(v_i = 0|c_i = 1, n_i = 20)$, $\Pr(v_i = 0|c_i = 1, n_i = 21)$, $\Pr(v_i = 0|c_i = 1, n_i = 22)$, $\Pr(v_i = 0|c_i = 1, n_i = 23)$, $\Pr(v_i = 0|c_i = 1, n_i = 24)$, $\Pr(v_i = 0|c_i = 1, n_i = 25)$, $\Pr(v_i = 0|c_i = 1, n_i = 26)$, $\Pr(v_i = 0|c_i = 1, n_i = 27)$, $\Pr(v_i = 0|c_i = 1, n_i = 28)$, $\Pr(v_i = 0|c_i = 1, n_i = 29)$, $\Pr(v_i = 0|c_i = 1, n_i = 30)$, $\Pr(v_i = 0|c_i = 1, n_i = 31)$, $\Pr(v_i = 0|c_i = 1, n_i = 32)$, $\Pr(v_i = 0|c_i = 1, n_i = 33)$, $\Pr(v_i = 0|c_i = 1, n_i = 34)$, $\Pr(v_i = 0|c_i = 1, n_i = 35)$, $\Pr(v_i = 0|c_i = 1, n_i = 36)$, $\Pr(v_i = 0|c_i = 1, n_i = 37)$, $\Pr(v_i = 0|c_i = 1, n_i = 38)$, $\Pr(v_i = 0|c_i = 1, n_i = 39)$, $\Pr(v_i = 0|c_i = 1, n_i = 40)$. 
$0|c_i = 1, n_i \leq 2$, \( \Pr(v_i = 0|c_i = 1, n_i \leq 5) \), \( \mathbb{E}[n_i] \), \( \mathbb{E}[p_i|c_i = c, v_i = v] \) for \((c, v) \in \{0, 1\} \times \{0, 1\}\) where \( p_i \equiv p_i(1 - v_i) + (p_i + q_i)v_i \), \( \mathbb{E}[p_i|c_i = 1, n_i = 1] \), \( \mathbb{E}[p_i|c_i = 1, n_i \leq 2] \), and \( \mathbb{E}[p_i|c_i = 1, n_i \leq 5] \). The 6 moment conditions on the distribution of the standardized delay are \( \Pr(s_i = 0, v_i = 0) \), \( \Pr(s_i = 0, v_i = 1) \), \( \Pr[s_i(1 - v_i)] \), \( \Pr[s_i^2(1 - v_i)] \), \( \Pr[s_i v_i] \), and \( \Pr[s_i^2 v_i] \). Note that the moments as a function of \( \theta \) are calculated using simulation. In our estimation, the simulation size is 5,000.

We use the two-step procedure to obtain the efficient simulated GMM estimator. We start with a positive definite weighting matrix and obtain a first-step estimator, denoted by \( \tilde{\theta}_n \). The asymptotic variance of \( \sqrt{n}g_n(\theta_0) \), \( S \), is estimated by:

\[
\hat{S} \equiv \frac{1}{n} \sum_t g(w_t, \tilde{\theta}_n)g(w_t; \tilde{\theta}_n)'.
\]

Then we re-estimate the parameters using the optimal weighting matrix \( \hat{S}^{-1} \). Let us denote this efficient simulated GMM estimator by \( \hat{\theta}_n \).

Under certain regularity conditions, the efficient simulated GMM estimator is asymptotically normally distributed. A consistent estimator of the asymptotic variance of \( \sqrt{n}(\hat{\theta}_n - \theta_0) \) is:

\[
\left( \frac{\partial g_n(\hat{\theta}_n)}{\partial \theta} \right)' \hat{S}^{-1} \left( \frac{\partial g_n(\hat{\theta}_n)}{\partial \theta} \right)^{-1}.
\]

Since the moments are calculated by simulation, we use a numerical derivative of \( g_n(\theta) \) to estimate the asymptotic variance of the estimator.