Pricing Macroeconomic Uncertainty

Francesco Bianchi  Howard Kung  Mikhail Tirskikh
Duke University  London Business School  London Business School
CEPR and NBER

September 2017

Abstract

We construct and estimate a general equilibrium model with a sizable and endogenous term premium to study the interaction between the macroeconomy and the term structure of interest rates. Our estimates show that changes in uncertainty affect both the yield curve and the macroeconomy in significant ways, with large effects on the term premium and the slope of the term structure. As a technical contribution, we present a novel and tractable approach to incorporate the effects of uncertainty in general equilibrium models.

JEL Codes: E31, E52, E62, E63, D83
Keywords: Term Structure, Uncertainty, Markov-switching models, Bayesian methods.

*Emails: francesco.bianchi@duke.edu, hkung@london.edu, mtirskikh@london.edu.
1 Introduction

There is ample empirical evidence documenting the link between the macroeconomy and the term structure of interest rates. Figure 1 illustrates the strong relation between real activity, measured as detrended GDP, and the slope of the nominal yield curve, computed here as the difference between the five-year yields and the one-year yield.\(^1\) The two series are negatively correlated (\(-.49\)): As the economy enters a recession, the slope of the yield curve tends to go up. At the same time, both the macro and asset pricing literatures have recognized the importance of time-varying risks and uncertainty in determining equilibrium outcomes. Recessions tend to coincide with an increase in uncertainty. Similarly, the slope of the nominal yield curve and term premia tend to increase during high uncertainty episodes. In this paper we are interested in studying the link between the macroeconomy, the term structure of interest rates, and uncertainty dynamics. As we will see, uncertainty changes play a key role in explaining term structure dynamics and reconciling the predictability of excess bond returns.

We build and estimate a New Keynesian DSGE model with endogenous bond risk premia. Our model has two important departures from the canonical paradigm (e.g., Christiano et al. (2005)). First, we assume that the representative household has Epstein-Zin recursive preferences. These preferences allow us to disentangle risk aversion from the elasticity of intertemporal substitution. The relative values of these two parameters are important determining the household’s sensitivity to changes in macroeconomic risk. With these preferences, the agent is sensitive to news about expected growth and expected discount rates, which are captured through low-frequency shocks to both productivity growth rates and time discount rates. Second, we model stochastic volatility to standard macroeconomic shocks, such as productivity, monetary policy, marginal efficiency of investment, preferences, using a Markov chain. We quantitatively assess the effect of these second moment shocks in our estimated model.

In order to capture the effects of risk on both the term structure and the macroeconomy, we propose a novel iterative solution procedure to capture the effects of risk on agents’ decisions with rational expectations in a conditionally linear framework. Our approach allows for a characterization of the model solution as a Markov-Switching Vector Autoregression (MS-VAR). The tractability of our solution method facilitates the estimation of the model. Given that the model is conditionally linear, the model can be estimated by a simple modification of the Kalman filter or by using a Gibbs sampling algorithm. As a result, we are able

\(^1\)Detrended GDP is obtained by applying the Holt-Winters filter. Similar results hold if GDP is detrended using an HP filter. In fact, in this case the negative correlation is even larger, around -.59.
to conduct a full estimation of the model by using both macro and term structure variables. Our estimated model produces a good fit to the term structure of interest rates. The model generates an average upward-sloping real and nominal yield curve. With recursive preferences, persistent shocks to the household’s time discount rate generate large swings in marginal utility. In particular, low-frequency discount rate shocks generate a strong negative relation between marginal utility and real interest rates, which makes long real bonds riskier than short ones. A persistent increase in the real short rate erodes the price of a long real bond more than a short real bond when marginal utility is high. Consequently, long real bonds provide less insurance against bad states of the world than short ones, which in equilibrium is reflected in upward-sloping real yield curve. In addition, these discount rate shocks imply a negative relation between inflation and marginal utility. With a Taylor rule, nominal interest rates rise upon an increase in inflation. Following a similar intuition as with the real bonds, a positive link between nominal interest rates and marginal utility translates to an upward-sloping nominal yield curve. Quantitatively, we generate sizable nominal term premia using only moderate levels of risk aversion (less than 10).

Time-varying bond risk premia in the model are attributed to the persistent variation in macroeconomic uncertainty. Furthermore, we find that macroeconomic uncertainty plays a key role in explaining variation in the slope of the yield curve. When computing a decomposition of the overall unconditional variance of the yield curve, changes in uncertainty are found to be more important than the traditional macro disturbances. Furthermore, our model reproduces the predictability of excess bond returns by a linear combination of forward rates (Cochrane-Piazzesi).

We find that periods of high uncertainty lead to a decline in GDP and an increase in the slope of the term structure. This pattern is in line with what documented above, even if the movement in GDP is relatively modest when compared to the shift in the slope of the term
structure. We also document the differential effect of uncertainty shocks depending on the source of the shock. Increases in the volatility of preference shocks have a large effect on the term structure and lead to a decline in GDP. However, due to the precautionary motive, the decline in GDP is mostly due to a decline in consumption, while investment slightly increases. On the other hand, we find that increases in uncertainty about future TFP produce negative responses to consumption, investment, employment hours, and output, as observed in the data, and also to an increase in the slope of the yield curve.

The magnitude of capital adjustment costs and the degree of sticky prices are important to generate the aforementioned co-movement between the macroeconomic variables in response to the uncertainty shocks to TFP. In contrast, with second moment shocks to monetary policy, time discount rates, and the marginal efficiency of investment, the precautionary savings motive dominates, which leads to a negative response to consumption but positive responses to investment, employment, and hours. In this paper, we trace out the sources of macroeconomic uncertainty and investigate the transmission channels for uncertainty shocks to bond prices and the aggregate economy in a general equilibrium model with production. Quantitatively, the estimated effect of uncertainty shocks in our model is sizable, and an important source of business cycle fluctuations.

Our analysis of uncertainty shocks on the macroeconomy builds on Bloom (2009), Fernández-Villaverde et al. (2011), and Basu and Bundick (2012). More broadly, this paper relates to New Keynesian models that examine the term structure of interest rates (Rudebusch and Swanson (2012), Palomino (2012), Dew-Becker (2014), van Binsbergen et al. (2012), Ang and Bekaert (2002), Bekaert et al. (2010), Baele et al. (2015), Bikbov and Chernov (2013), Kung (2014), and Corhay et al. (2017)), the estimation of asset pricing models with Bayesian methods (Schorfheide et al. (2014)), and the effects of inflation uncertainty and the term structure of interest rates (Wright (2011)).

With respect to these contributions, the current project innovates by specifying and estimating a fully micro-founded New-Keynesian DSGE model to capture the response of macroeconomic dynamics and asset prices to changes in uncertainty and standard macroeconomic shocks. Our approach allows risk to affect not only asset prices, but also the policy functions controlling the macroeconomic variables. Assets are priced in a way that is consistent with agents’ behavior in the model. In this respect, our work is related to Bianchi et al. (2017) that estimate a business cycle model that delivers a positive equity premium because of ambiguity averse agents. However, there are a series of important differences in this paper. First, agents have rational expectations: Premia arise because of risk, not

\footnote{Previous contributions that study the term structure of interest rates in a NK environment use calibration or price assets ex-post, using the NK model to derive macro dynamics.}
because of ambiguity aversion. Second, we study the term structure. Third, we work in a new-Keynesian environment as opposed to a real business cycle framework. Finally, we propose an iterative procedure to capture the effects of uncertainty on asset prices and the macroeconomy while at the same time using standard solution packages.

2 Capturing the Effects of Uncertainty

Our goal is to study the effects of uncertainty on both asset prices and the macroeconomy. The literature presents a series of papers that propose to use a log-linearization with a risk adjustment to solve for asset prices in DSGE models, for example Backus et al. (2010), Uhlig (2010), Malkhozov (2014). The main difference with respect to these contributions consists of how we solve the model. Instead of using a method based on perturbations of the policy functions, we solve a system of linear expectational difference equations augmented with an iterative procedure meant to capture a risk-adjustment component. This procedure allows us to solve rational expectation models in which uncertainty is controlled by a Markov-switching process by using solution methods that have been developed for log-linear approximations. Furthermore, some of the methods proposed in the literature use a risk-adjustment step only in pricing assets (for example, this is approach taken in Uhlig (2010)). Instead, our approach allows risk to affect not only asset prices, but also the policy functions controlling the macroeconomic variables. This is crucial to study the effects of uncertainty on the macroeconomy.

To describe the approach, let’s consider a model from Kaltenbrunner and Lochstoer (2010) and augment it with Markov-switching volatility regimes. In the model households have Epstein-Zin utility function:

\[
V_t = \{(1 - \beta)C_t^{1-1/\psi} + \beta CU_t^{1-1/\psi}\}^{\frac{1}{1-\psi}}
\]

\[
CU_t = E_t[V_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}
\]

The production function is given by:

\[
Y_t = (Z_t L_t)^{1-\alpha} \overline{K}_{t-1}^\alpha
\]

where \(Z_t\) is Total Factor Productivity (TFP), \(L_t\) is an exogenous labor input, and \(\overline{K}_{t-1}^\alpha\) is the amount of capital accumulated from the previous period. TFP growth evolves according to the process \(\Delta \log (Z_t) = \mu + x_t\), where \(x_t = \sigma_x \varepsilon_{x,t}\) is an \(iid\) process with volatility depending on the regime \(\xi_t\). The regime \(\xi_t\), in turn, follows a Markov-switching process with transition
matrix $H$, where the elements on the diagonal, $p_{11}$ and $p_{22}$, capture the persistence of the two regimes. The household can accumulate capital by investing:

$$K_t = (1 - \delta)K_{t-1} + \Phi_K \left( I_t/K_{t-1} \right) K_{t-1}$$

where capital adjustment costs are given by:

$$\Phi_K \left( I_t/K_{t-1} \right) = \frac{\phi_{K,1}}{1 - 1/\varphi_K} \left( I_t/K_{t-1} \right)^{1-1/\varphi_K} + \phi_{K,2} \tag{3}$$

Finally, the resource constraint for the economy is given by $Y_t = C_t + I_t$.

The first order optimality condition for the household is given by:

$$E_t[M_{t+1}R_{i,t+1}] = 1 \tag{4}$$

where the return on investment $R_{i,t+1}$ is defined as:

$$R_{i,t+1} \equiv \Phi' \left( I_t/K_{t-1} \right) \left[ \alpha \left( \frac{Z_{t+1}}{K_t} \right)^{1-\alpha} + \frac{(1 - \delta) + \Phi_K \left( I_{t+1}/K_t \right) - \frac{I_{t+1}}{K_t} \Phi_K' \left( I_{t+1}/K_t \right)}{\Phi_K' \left( I_{t+1}/K_t \right)} \right]$$

and the Stochastic Discount Factor (SDF) is given by:

$$M_{t+1} = \beta \left( C_{t+1}/C_t \right)^{-1/\psi} \left( V_{t+1}/E_t[V_{t+1}]^{1/(1-\gamma)} \right)^{1/\psi - \gamma}.$$ 

In order to make the model stationary, all variables are normalized with respect to the non-stationary TFP process $Z_t$. We then proceed with a log-linearization with risk adjustment. The idea behind the approximation method is that all expectational equations are approximated assuming that the variables are conditionally log-normal. The approximate solution indeed satisfies this condition, because it is linear in the state variables. Thus, (2) and (4) are approximated by:

$$\tilde{c}u_t = E_t[\tilde{v}_{t+1} + \tilde{x}_{t+1}] + .5 (1 - \gamma) Var_t[\tilde{v}_{t+1} + \tilde{x}_{t+1}]$$

$$0 = E_t[\tilde{m}_{t+1} + \tilde{r}_{i,t+1}] + .5 Var_t[\tilde{m}_{t+1} + \tilde{r}_{i,t+1}]$$

The rest of the system does not present any expectational term, so we obtain the standard log-linearized equations:

$$3 \phi_{K,1} = \left( \frac{\theta}{\kappa} \right)^{-\varphi_K}, \phi_{K,2} = -\frac{1}{\varphi_K - 1} \left( \frac{\theta}{\kappa} \right), \text{ where } \left( \frac{\theta}{\kappa} \right) = \frac{\hat{\mu}_t + \hat{\epsilon}_t}{\hat{\kappa}_t}.$$
1. Household value function

\[-\frac{1}{1-\beta_d} \tilde{v}_t + \tilde{c}_t + \frac{\beta^*}{1-\beta_d} \tilde{u}_t = 0\]

2. Production function

\[\tilde{y}_t = \alpha (\tilde{K}_{t-1} - \tilde{x}_t)\]

3. Capital accumulation equation

\[e^\mu (\tilde{K}_t + \tilde{x}_t) = (1 - \delta) \tilde{K}_{t-1} + \left( \frac{I}{K} \right) (\tilde{i}_t + \tilde{x}_t)\]

4. Resource constraint

\[\bar{Y}_{ss} \tilde{y}_t = \tilde{C}_{ss} \tilde{c}_t + \bar{I}_{ss} \tilde{i}_t\]

5. SDF

\[\bar{m}_{t+1} = -\psi^{-1} (\tilde{c}_{t+1} - \tilde{c}_t) + (\psi^{-1} - \gamma) (\tilde{v}_{t+1} - \tilde{u}_t) - \gamma \tilde{x}_{t+1}\]

6. Return on investment

\[M_{ss}^{-1} \tilde{r}_{t+1} = -\frac{1}{M_{ss} \varphi_K} \left( \tilde{i}_t - \tilde{K}_{t-1} + \tilde{x}_t \right) + (1 - \alpha) (M_{ss}^{-1} - 1 + \delta) (\tilde{x}_{t+1} - \tilde{K}_t) + \frac{1 - \delta + \left( I/K \right)}{\varphi_K} \left( \tilde{i}_{t+1} - \tilde{K}_t + \tilde{x}_{t+1} \right)\]

where \(M_{ss} = \beta e^{-\mu/\psi}\), \(\left( I/K \right) = \left( \bar{I}_{ss}/\bar{K}_{ss} \right) e^\mu\), and \(\beta^* = \beta e^{\mu (1 - 1/\psi)}\).

The key step to implement our method consists of realizing that in a model in which stochastic volatility is modelled as a Markov-switching process, uncertainty at time \(t\) only depends on the regime in place at time \(t\), \(\xi_t\). Thus, the system of equations presented above can be written by using matrix notation (Sims (2002)) as:

\[\Gamma_0 S_t = \Gamma_1 S_{t-1} + \Gamma_2 \sigma_{\xi \varepsilon} \varepsilon_t + \Gamma_3 \eta_t + \Gamma_4 \xi_t\] (5)

where the DSGE state vector \(S_t\) contains all variables of the model known at time \(t\), \(\eta_t\) is a vector containing the expectation errors (for a variable \(x_t\), \(\eta_t^x = x_t - E_{t-1}[x_t]\)), and the Markov-switching constant \(\Gamma_{\xi_t}\) captures the effects of uncertainty:

\[\Gamma_{c,\xi_t} = \begin{pmatrix} \frac{1-\gamma}{2} Var_t [c_1 S_{t+1} + c_2 \tilde{x}_{t+1}] \\ \frac{1}{2} Var_t [d_1 S_{t+1} + d_2 \tilde{x}_{t+1}] \\ \vdots \end{pmatrix} \]
where we have used the fact uncertainty at time $t$ only depends on the regime in place at time $t$, $\xi_t$.

However, we cannot compute the volatility terms in $\Gamma_{c\xi_t}$ without knowing the solution for $S_t$. This is because we need to know how the economy reacts to the exogenous shock $\varepsilon_{x,t}$. Therefore, we employ the following iterative procedure. First, note that given some $\Gamma_{c\xi_t} = \tilde{\Gamma}_{c\xi_t}$, the solution to (5) is given by

$$S_t = TS_{t-1} + R\sigma_{\xi_t} \varepsilon_{x,t} + C_{\xi_t}$$

Taking (6) as given, we can now compute the implied level of uncertainty, i.e. the implied $\tilde{\Gamma}_{c\xi_t}$. In particular:

$$\text{Var}_t \left[ c_1 S_{t+1} + c_2 \tilde{\varepsilon}_{t+1} \right] = (c'_1 R + c_2)^2 E_t \left[ \sigma^2_{\xi_{t+1}} \right] + c'_1 \text{Var}_t \left[ C_{\xi_{t+1}} \right] c_1$$

Note that the changes in constant induced by the risk adjustment are themselves a source of uncertainty. Given the new value for $\tilde{\Gamma}_{c\xi_t}$, we repeat the iteration: (1) compute a new solution to (5); (2) update $\tilde{\Gamma}_{\xi_t}$. This iterative procedure continues until the desired level of accuracy is reached. It is worth emphasizing that only $C_{c\xi_t}$ depends on $\Gamma_{c\xi_t}$, while the matrices $T$ and $R$ do not depend on it. So, we only need to iterate on $C_{c\xi_t}$. Furthermore, the stationarity of the solution only depends on $T$, so we know that a finite level of uncertainty exists as long as the shocks are stationary.

In the solution (6), the matrices $T$ and $R$ are equivalent to a standard log-linear solution, so conditional on the volatility regime, the dynamics of the model are the same as in a standard log-linear solution. Volatility matters in two ways. First, like in loglinearized models, it affects the size of the innovations. This term is captured by $\sigma_{\xi_t}$. Second, it affects uncertainty and so it affects the risk adjustment term $C_{\xi_t}$. This term is not present in a standard log-linear approximation. It adjusts the levels of the variables, determines the dynamics in response to a volatility regime change, and it contributes to generate uncertainty.

The approach proposed above can be generalized and applied to richer models with more shocks. In a general setting, the solution approach delivers the following canonical form:

$$\Gamma_0 S_t = \Gamma_1 S_{t-1} + \Gamma_\sigma Q_{\xi_t} \varepsilon_t + \Gamma_\eta \eta_t + \Gamma_{c\xi_t}$$

where the DSGE state vector $S_t$ contains all variables of the model known at time $t$, $Q_{\xi_t}$ is a regime-dependent diagonal matrix with all standard deviation of the shocks on the main diagonal, $\varepsilon_t$ is a vector with all structural shocks, and the Markov-switching constant $\Gamma_{\xi_t}$ captures the effects of uncertainty. If a solution exists, it can be characterized as a MS-VAR,
Table 1: Comparison of global solution with approximate solution with risk adjustment. The table compares macroeconomic and asset pricing moments based on two alternative solution approaches. The left column computes the moments based on the approximate solution with risk adjustment, the right column reports the same moments based on the global solution.

of the kind studied by Hamilton (1989), Chib (1996), and Sims and Zha (2006):

$$S_t = C (\xi_t, \theta^p, \theta^v, H) + T (\theta^p) S_{t-1} + R (\theta^p) Q (\xi_t, \theta^v) \varepsilon_t$$  \hspace{1cm} (8)

where $\theta^p$ and $\theta^v$ are vectors that contain the structural parameters and the stochastic volatilities, respectively. It is important to emphasize that the constant $C (\cdot)$ depends on the structural parameters because for a given volatility of the exogenous disturbances, different structural parameters determine different levels of uncertainty. In a standard log-linearization this term would always be zero. As shown below, this hybrid approach allows us to capture salient asset pricing features despite having approximated a model with a conditionally linear solution. Furthermore, it is worth emphasizing that given that agents are aware of the possibility of regime changes, uncertainty also depends on the possibility of moving across regimes. This is why the constant term depends on the transition matrix $H$. Finally, given that in the model regime changes enter additively, the conditions for existence and uniqueness of a solution are not affected by the presence of regime changes. The model can then be solved by using solution algorithms developed for fixed coefficient general equilibrium models such as Blanchard and Kahn (1980) and Sims (2002). Of course, the model can also be solved by using the solution algorithms explicitly developed for MS-DSGE models such as Farmer et al. (2009, 2011), Cho (2016), and Foerster et al. (2016).

To demonstrate the quality of the approximation we compare the log-linear solution with risk adjustment (6) to a global solution obtained by using a value function iteration. We use the same quarterly calibration as in Kaltenbrunner and Lochstoer (2010): $\alpha = 0.36$, $\delta = 0.021$, $\gamma = 5$, $\mu = 0.4\%$, $\psi = 1.5$, $\beta = 0.998$, $\varphi_K = 18$, $\sigma = 4.1\%$. The volatility
Figure 2: Impulse response functions to a change in volatility regime from low volatility to high volatility. \(c\) - log consumption, \(i\) - log investment, \(y\) - log output. Macro variables are expressed in percentage deviations from steady state. \(r_f\) - log risk-free rate, \(r_i\) - log return on investment. Blue solid line - global solution, obtained by value function iteration. Red dotted line - log-linear approximation with risk-adjustment.

<table>
<thead>
<tr>
<th>Approximate solution</th>
<th>Below 5%</th>
<th>Above 95%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>5.54%</td>
<td>4.98%</td>
</tr>
</tbody>
</table>

Table 2: This table reports the proportion of realized Den Haan (1994) test statistics below 5. We simulate 5000 economies for 3500 periods and discard the first 500 observations.

regimes are chosen to be \(\sigma_1^2 = 0.9\sigma^2\) and \(\sigma_2^2 = 1.1\sigma^2\). We make the high volatility regime more persistent than the low volatility regime by setting \(p_{11} = .9\) and \(p_{22} = .95\) in \(H\).

Table 1 reports selected macroeconomic and asset pricing moments. Overall the log-linear approximation produces values which are close to a full numerical solution. Figure 2 shows the impulse response functions to an increase in uncertainty (change from the low volatility to the high volatility regime). The log-linear solution with risk adjustment approximates those responses well.

Finally, to assess the accuracy of the log-linear solution with risk adjustment in a more formal way we also conduct a Den Haan and Marcet (1994) test. We simulate 5000 economies for 3500 periods and drop first 500 observations. Then we use the conditionally linear policy functions for consumption, value function, capital and investment to compute the time path of the corresponding variables. We then plug these series into the non-linear equation \(err_{t+1} = M_{t+1}R_{i,t+1} - 1\). Then we compute Den Haan-Marcet statistic:

\[
DM = \left[ T \left( \frac{\sum_{s=1}^{T} (err_s)}{T} \right)^2 \right] / \left[ \frac{\sum_{s=1}^{T} (err_s^2)}{T} \right].
\]

Under the null hypothesis that the approximation is exact this statistic should have a chi-squared distribution because under the null hypothesis the Euler equation implies \(E_t(err_{t+1}) =\)
Thus, we obtain 5,000 statistics, one for each simulated economy. We check how many of them are above the 95% and below the 5% chi-squared critical values. Table 2 shows that the percentages of realized test statistics below 5% and above 95% critical values of a $\chi^2$ distribution are very close to the theoretical ones.

3 The Model

In this section, we present the model that we use to study the link between the macroeconomy and the term structure of interest rates taking into account the effects of uncertainty. As it will become clear below, uncertainty plays a key role in generating an upward sloping yield curve.

3.1 Household

Assume that the representative household has recursive utility over streams of consumption $C_t$ and labor $L_t$:

$$V_t = \left(1 - \left(1 + \beta e^{\tilde{b}_t}\right)^{-1}\right) u(C_t, L_t)^{1-1/\psi} + \left(1 + \beta e^{\tilde{b}_t}\right)^{-1} \left(E_t \left[V_{t+1}^{1-\gamma}\right]\right)^{1-1/\psi}^{1-1/\psi}$$

$$u(C_t, L_t) = (C_t - h C_t e^{-\tau t_{t-1}})^{1-1+\tau t_{t-1}}$$

where $\gamma$ is the coefficient of risk aversion, $\psi$ is the elasticity of intertemporal substitution, $\theta \equiv 1 - \frac{1-\gamma}{1-1/\psi}$ is a parameter defined for convenience. Also define $\beta_t = \left(1 + \beta e^{\tilde{b}_t}\right)^{-1}$, where $\tilde{b}_t$ is a preference shock

$$\tilde{b}_{t+1} = \rho \tilde{b}_t + \sigma \xi_{t+1} \varepsilon_{t+1}$$

The variable $\xi_t$ follows a Markov-switching process with transition matrix $H$ and determines the volatility regime in place at time $t$. In the limit, when $\psi \rightarrow 1$, preferences collapse to

$$V_t = u(C_t, L_t)(1-\beta_t) \left(E_t \left[V_{t+1}^{1-\gamma}\right]\right)^{\beta_t}$$

The time $t$ budget constraint of the household is

$$P_t C_t + P_t (e^{\xi_t \chi_t} Y_t)^{-1} I_t + B_{t+1}/R_t = D_t + P_t W_t L_t + B_t + P_t K_{t-1} r_t U_t - P_t I_t$$

where $P_t$ is the nominal price of the final good, $B_{t+1}$ are nominal one-period bonds, $R_t$ is the gross nominal interest rate set at time $t$ by the monetary authority, $D_t$ is nominal
dividend income received from the intermediate firms, $W_t$ is the nominal wage rate, $L_t$ is labor supplied by the household, and $T_t$ denotes lump-sum taxes. Following Justiniano et al. (2011), we allow for a decline in the price of investment. The gross rate of the decline is controlled by $\gamma^{-1}$. We also allow for shocks to the price of investment modelled as an AR(1) process:

$$\zeta_{\gamma,t+1} = \rho_{\gamma}\zeta_{\gamma,t} + \sigma_{\gamma}\zeta_{\gamma,t+1}\varepsilon_{\gamma,t+1}$$

Capital can be accumulated by the household subject to an adjustment cost:

$$K_t = K_{t-1}(1 - \delta(U_t)) + e^{\zeta_{i,t}}\Phi_K\left(\frac{I_t}{K_{t-1}}\right)K_{t-1}$$

$$\Phi_K\left(\frac{I_t}{K_{t-1}}\right) = \frac{\phi_{K,1}}{1 - 1/\phi_{K}}\left(\frac{I_t}{K_{t-1}}\right)^{1-1/\phi_{K}} + \phi_{K,2}$$

Households rent out capital services $K_t = U_tK_{t-1}$ to a competitive capital markets at the rate $P_{t_k}$. The depreciation rate of capital depends on capital utilization:

$$\delta(U_t) = \delta_0 + \delta_1(U_t - U_{ss}) + \frac{\delta_2}{2}(U_t - U_{ss})^2.$$  

Finally, we also allow for a shock to the marginal efficiency of investment as a parsimonious way to capture frictions in financial markets:

$$\zeta_{i,t+1} = \rho_{i}\zeta_{i,t} + \sigma_{i}\zeta_{i,t+1}\varepsilon_{i,t+1}.$$  

### 3.2 Final Goods

A representative firm produces the final (consumption) good in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods $X_{i,t}$ as input in the CES production technology

$$Y_t = \left(\int_0^1 X_{i,t}^{1+\lambda_p} di\right)^{1+\lambda_p}$$

where $\lambda_p$ determines elasticity of substitution between intermediate goods. The profit maximization problem of the firm yields the following isoelastic demand schedule with price elasticity $\nu = \frac{1+\lambda_p}{\lambda_p}$

$$X_{i,t} = Y_t\left(\frac{P_{i,t}}{P_t}\right)^{\frac{1+\lambda_p}{\lambda_p}}$$
where $\mathcal{P}_t$ is the nominal price of the final good and $\mathcal{P}_{i,t}$ is the nominal price of intermediate good $i$. The inverse demand schedule is

$$\mathcal{P}_{i,t} = \mathcal{P}_t Y_t^{\lambda_p} X_{i,t}^{-\lambda_p}$$

### 3.3 Intermediate Goods

The intermediate goods sector is characterized by a continuum of monopolistic competitive firms. Each intermediate goods firm produces $X_{it}$ using labor $L_{it}$ and capital $K_{i,t}$ with constant returns to scale technology:

$$X_{it} = K_{it}^\alpha (Z_t L_{it})^{1-\alpha}$$

where $Z_t$ is an aggregate technology shock that contains transitory and permanent components:

$$Z_t = e^{\alpha t + n_t}$$
$$a_t = \rho_a a_{t-1} + \sigma_a \xi_a \varepsilon_{a,t}$$
$$\Delta n_t = \mu + x_t$$
$$x_t = \rho_x x_{t-1} + \sigma_x \xi_x \varepsilon_{x,t}$$

$\mu$ is the unconditional mean of productivity growth, and $\rho_x$ is the persistence parameter of the autoregressive process $x_t$.

The intermediate firms face a cost of adjusting the nominal price a la Rotemberg (1982), measured in terms of the final good as

$$G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}, Y_t) = \frac{\phi_R}{2} \left( \frac{\mathcal{P}_{i,t}}{(\Pi^*)^{\kappa_2} \Pi_t^{1-\kappa_2} \mathcal{P}_{i,t-1}} - 1 \right)^2 Y_t$$

where $\Pi^*$ is the central bank inflation target and $\phi_R$ is the magnitude of the adjustment costs. The source of funds constraint is

$$\mathcal{P}_t D_{i,t} = \mathcal{P}_{i,t} X_{i,t} - \mathcal{P}_t W_t L_{i,t} - \mathcal{P}_t k_{i,t} K_{i,t} - \mathcal{P}_t G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}, Y_t)$$

where $D_{i,t}$ is the real dividend paid by the firm. The objective of the firm is to maximize shareholder’s value $V_t^{(i)} = V^{(i)}(\cdot)$ taking the pricing kernel $M_t$, the competitive nominal wage
\[ V^{(i)}(P_{i,t-1}; \Psi_t) = \max_{P_{i,t}, Y_{i,t}} \left\{ D_{i,t} + E_t \left[ M_{t+1} V^{(i)}(P_{i,t}; \Psi_{t+1}) \right] \right\} , \]

subject to

\[ D_{i,t} = \frac{P_{i,t}}{P_t} X_{i,t} - W_t L_{i,t} - r^k K_{i,t} - G(P_{i,t}, P_{i,t-1}, Y_t) \]

\[ X_{i,t} = Y_t \left( \frac{P_{i,t}}{P_t} \right)^{-1 + \lambda_{p,t}} \]

\[ X_{it} = K_{it}^\alpha (Z_t L_{it})^{1-\alpha} \]

### 3.4 Central Bank

The central bank follows a modified Taylor rule that depends on output and inflation deviations from their corresponding targets:

\[
\ln \left( \frac{R_t}{R^*_t} \right) = \rho_r \ln \left( \frac{R_{t-1}}{R^*_t} \right) + \\
+ (1 - \rho_r) \left[ \rho_y \ln \left( \frac{\Pi_t}{\Pi^*} \right) + \rho_{ss} \ln \left( \frac{\hat{Y}_t}{Y^*_t} \right) \right] + \sigma_m \xi_m \varepsilon_{m,t}
\]

where \( R_t \) is the gross nominal short rate, \( \hat{Y}_t \equiv \frac{Y_t}{Z_t} \) is detrended output, and \( \varepsilon_{m,t} \sim N(0,1) \) is an i.i.d. monetary policy shock. \( R^*_t \) is a nominal interest rate target, which is determined by \( \Pi^* \) and steady state value of real interest rate \( R^*_t \): \( R^* = R^*_t \Pi^* \). We assume that the fiscal authority moves lump sum taxes to guarantee debt stability.

### 3.5 Symmetric Equilibrium

In equilibrium, all firms make identical decisions. The aggregate resource constraint is

\[
Y_t = C_t + (e^{\xi_t} Y_t^{\nu})^{-1} I_t + \phi_R \left( \frac{\Pi_t}{(\Pi^* \nu_2 (\Pi_{t-1})^{1-\nu_2} - 1)^2} Y_t + G_t \right)
\]

where \( \Pi_t \equiv P_t/P_{t-1} \) is the gross inflation rate.
4 Empirical Analysis

We estimate the model by using Bayesian methods. We consider the sample 1984:Q2-2007:Q4. We choose this sample because it is commonly believed that monetary policy was substantially stable over this period of time. The priors for the model parameters are combined with the likelihood to obtain the posterior distribution. Details about the priors can be found in the Appendix. We use eleven observables: GDP per-capita growth, Inflation, FFR, consumption growth, investment growth, price of investment growth, one-year yield, two-year yield, three-year yield, four-year yield, and five-year yield. Given that we have more observables than shocks, we allow for observation errors on all yields.

4.1 Parameter estimates

Table 3 reports the posterior mode values for the structural parameters. A few comments are in order. First, we fix the elasticity of intertemporal substitution to 1. Our estimated risk aversion parameter is around 6.50, which is a very moderate value in the asset pricing literature. Therefore, the results that are presented below do not rely on a high value of risk aversion. Second, the parameters controlling the magnitude of the price adjustment cost (\( \phi_R \)) and the average markup (\( \nu \)) cannot be separately identified. Thus, we fixed \( \nu \) to 6, a value that implies an average net markup of 20\% and that is considered in the ballpark (see Gali (1999)). The resulting estimated value for \( \phi_R \) implies a level of price stickiness in line with the existing new-Keynesian literature. Third, in line with previous results in the literature, we find a strong response of the FFR to inflation. The fact that the response is well above 1 guarantees that the Taylor principle is satisfied.

Table 4 reports the estimates for the volatilities of the shocks and the persistence for the two regimes. Figure 3 reports the probability of Regime 1. Regime 2, is characterized by higher volatility of the preference shock and the non-stationary productivity shock, while Regime 1 presents higher volatility for the MEI shock. Regime 2 is also less persistent than Regime 1. As we will show below, while the volatilities of the shocks do not move all in the same direction, Regime 2 unequivocally leads to an increase in uncertainty with respect to Regime 1.

4.2 Yield Curve

As a first basic check on the fit of the model, Figure 4 presents the yields as implied by our model and as they are in the data. Recall that we have observation error on all yields: The model implied yields are obtained by removing such observation errors. Clearly, observation
Table 3: Posterior mode parameter values. The model is estimated with Bayesian methods, details on the prior and the methodology can be found in the Appendix.

errors do not play a key role in matching the observed path for yields. Table 5 reports the nominal and real yield curve as implied by our estimated model. Despite using a very parsimonious specification with only two volatility regimes, our model is able to generate a sizeable slope in the term structure using moderate risk aversion, both for nominal and real yields.

Our model generates an upward-sloping real and nominal yield curve primarily through the persistent shocks to time discount rates coupled with recursive preferences. To understand the role of preference shocks, Figure 5 presents the impulse responses to such a shock for some key variables. A negative time preference shock (less patience) induces the agent to consume more and save less, which decreases the wealth-to-consumption ratio. A drop in the wealth-to-consumption ratio implies a decline in the return on a claim to aggregate consumption. When the agent prefers an early resolution of uncertainty (i.e., $\psi > 1/\gamma$), a decrease in the return on the consumption claim increases marginal utility. When the shock

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Preferences</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>Mean of subjective discount factor</td>
<td>0.9922</td>
</tr>
<tr>
<td>$\rho_d$</td>
<td>Persistence of subjective discount factor</td>
<td>0.9809</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Elasticity of intertemporal substitution (fixed)</td>
<td>1.0000</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Risk aversion</td>
<td>6.5088</td>
</tr>
<tr>
<td>$\tau$</td>
<td>Inverse of Frisch elasticity of labor supply</td>
<td>0.5496</td>
</tr>
<tr>
<td>B. Productivity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>Persistence of transitory productivity shocks</td>
<td>0.4199</td>
</tr>
<tr>
<td>$\mu^*$</td>
<td>Average economic growth</td>
<td>0.0045</td>
</tr>
<tr>
<td>$\log(\bar{T})$</td>
<td>Average decline in price of investment</td>
<td>0.0051</td>
</tr>
<tr>
<td>$\rho_x$</td>
<td>Persistence of permanent productivity shocks</td>
<td>0.2468</td>
</tr>
<tr>
<td>C. Technology</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_R$</td>
<td>Magnitude of price adjustment costs</td>
<td>17.5258</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Price elasticity for intermediate goods (fixed)</td>
<td>6.0000</td>
</tr>
<tr>
<td>$\bar{\lambda}_p$</td>
<td>Average net markup: $1/(\nu - 1)$ (fixed)</td>
<td>0.2000</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital share in production</td>
<td>0.1921</td>
</tr>
<tr>
<td>$\delta_0$</td>
<td>Average capital depreciation</td>
<td>0.0287</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>Coefficients in capital depreciation function</td>
<td>7.4853</td>
</tr>
<tr>
<td>$\phi_K$</td>
<td>Capital adj. cost parameter</td>
<td>0.5531</td>
</tr>
<tr>
<td>$\rho_T$</td>
<td>Persistence of shocks to price of investment</td>
<td>0.9372</td>
</tr>
<tr>
<td>$\rho_l$</td>
<td>Persistence of shocks to marginal efficiency of investment</td>
<td>0.9477</td>
</tr>
<tr>
<td>D. Policy</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>Degree of monetary policy inertia</td>
<td>0.8444</td>
</tr>
<tr>
<td>$\rho_n$</td>
<td>Sens. of interest rate to inflation</td>
<td>2.3324</td>
</tr>
<tr>
<td>$\rho_y$</td>
<td>Sensitivity of interest rate to output</td>
<td>0.0119</td>
</tr>
<tr>
<td>$\pi_{ss}$</td>
<td>Inflation in deterministic steady state</td>
<td>0.0120</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>Degree of indexation to past inflation</td>
<td>0.9704</td>
</tr>
<tr>
<td>$\eta_G$</td>
<td>St.st. share of gov.spending in GDP</td>
<td>0.1022</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>Persistence of gov. spending shocks (fixed)</td>
<td>0.9800</td>
</tr>
</tbody>
</table>
Table 4: Posterior mode parameter values. The model is estimated with Bayesian methods, details on the prior and the methodology can be found in the Appendix.

<table>
<thead>
<tr>
<th>Shock</th>
<th>Regime 1</th>
<th>Regime 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Regime persistence</td>
<td>0.97</td>
<td>0.93</td>
</tr>
<tr>
<td>Prob. of regime</td>
<td>0.68</td>
<td>0.32</td>
</tr>
<tr>
<td>Preference, $\sigma_{b,\xi}$</td>
<td>0.0257</td>
<td>0.0679</td>
</tr>
<tr>
<td>Stationary product., $\sigma_{a,\xi}$</td>
<td>0.0126</td>
<td>0.0121</td>
</tr>
<tr>
<td>Non-Stationary product., $\sigma_{x,\xi}$</td>
<td>0.0185</td>
<td>0.0190</td>
</tr>
<tr>
<td>Monetary, $\sigma_{m,\xi}$</td>
<td>0.0013</td>
<td>0.0016</td>
</tr>
<tr>
<td>IST, $\sigma_{\tau,\xi}$</td>
<td>0.0035</td>
<td>0.0039</td>
</tr>
<tr>
<td>MEI, $\sigma_{I,\xi}$</td>
<td>0.0623</td>
<td>0.0382</td>
</tr>
<tr>
<td>Gov. spending, $\sigma_{g,\xi}$</td>
<td>0.0566</td>
<td>0.0682</td>
</tr>
</tbody>
</table>

is persistent, this leads to a sharp increase in marginal utility.

A persistent negative time preference shock also increases the real rate persistently, which erodes the payoffs of long real bonds more than short ones. Given that a negative time preference shock is associated with high marginal utility, long real bonds provide less insurance against bad states of the world relative to short real bonds. In equilibrium, this is reflected in an upward-sloping real yield curve and positive real term premia.

The time preference shock endogenously generates a negative relation between marginal utility and inflation, which translates to positive inflation risk premia increasing with maturity. A persistent negative time preference shock increases aggregate demand, which raises inflation persistently. The negative time preference shock is also associated with high marginal utility as discussed above. Persistently higher inflation erodes the value of long nominal bonds more than short nominal bonds during high marginal utility states. Consequently, the nominal yield curve is upward-sloping and nominal term premia is positive.

Our model is also able to reproduce the predictive regressions using the Cochrane-Piazzesi factor. Table 6 reports forecasts of one year excess bond returns using the Cochrane-Piazzesi factor for bonds with maturities of 2, 3, 4 and 5 years. First, the factor is obtained by running the regression $\frac{1}{4}\sum_{n=2}^{5} r_{x_{t+1}}^{(n)} = \gamma' f_t + \epsilon_{t+1}$, where $\gamma' f_t = \gamma_0 + \gamma_1 y_t^{(1)} + \gamma_2 f_t^{(2)} + \ldots + \gamma_5 f_t^{(5)}$, $r_{x_{t+1}}^{(n)}$ is the one year excess bond return of a bond with maturity of n-years over the one-year yield, $f_t^{(n)}$ is a forward rate between $t + n - 1$ and $t + n$, $y_t^{(1)}$ is the yield on a 1-year bond. Second we use this factor to forecast bond excess returns $r_{x_{t+1}}^{(n)} = \beta^{(n)} \gamma' f_t + \epsilon_{t+1}$. HAC t-stats are
Figure 4: The figure compares the yields as implied by our model (blue solid line) with the yields observed in the data (black dashed line). The model implied yields are obtained by removing the estimated observation errors.

Figure 5: Impulse responses to a negative preference shock (less patience).
Table 5: Average nominal and real yield curves as implied by the model with parameters set at the posterior model.

<table>
<thead>
<tr>
<th>Maturity</th>
<th>1-year</th>
<th>2-year</th>
<th>3-year</th>
<th>4-year</th>
<th>5-year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>5.82</td>
<td>5.94</td>
<td>6.03</td>
<td>6.10</td>
<td>6.15</td>
</tr>
<tr>
<td>Real</td>
<td>2.57</td>
<td>2.65</td>
<td>2.73</td>
<td>2.79</td>
<td>2.84</td>
</tr>
</tbody>
</table>

Table 6: Cochrane-Piazzesi regressions.

<table>
<thead>
<tr>
<th>$\beta^{(n)}$</th>
<th>2Y</th>
<th>3Y</th>
<th>4Y</th>
<th>5Y</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.21)</td>
<td>0.43</td>
<td>0.84</td>
<td>1.20</td>
<td>1.53</td>
</tr>
<tr>
<td>(3.36)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.41)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3.42)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5.61)</td>
<td>0.45</td>
<td>0.84</td>
<td>1.23</td>
<td>1.48</td>
</tr>
<tr>
<td>(5.79)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6.21)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6.13)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

| $R^2$ | 0.19 | 0.21 | 0.22 | 0.22 | 0.37 | 0.37 | 0.40 | 0.38 |
| N obs | 95   | 95   | 95   | 95   | 95   | 95   | 95   | 95   |

Figure 6 reports the results. The horizontal axis reports the time horizon $s$ used to compute uncertainty. The figure clearly shows that Regime 2 is characterized by higher uncertainty, despite the fact that the volatility of the MEI shock is larger under Regime 1. For all variables and for all horizons uncertainty is higher under this regime. This is for two reasons. First, the larger volatility of the preference shock implies substantial fluctuations both in the term structure and the real economy. Second, Regime 2 is less persistent than Regime 1. As a result, when agents are under this regime they expect the possibility of large

4.3 Uncertainty shocks, Real Activity, and the Term Structure

We are interested in characterizing the level of uncertainty across the two regimes. Uncertainty is computed taking into account the possibility of regime changes, following the methods developed in Bianchi (2016). For each variable $z_t$, we measure uncertainty by computing the conditional standard deviation $sd_t(z_{t+s}) = \sqrt{V_t(z_{t+s})} = \sqrt{E_t[z_{t+s} - E_t(z_{t+s})]^2}$, where $E_t(\cdot) \equiv E(\cdot|I_t)$ and $I_t$ denotes the information available at time $t$. We assume that $I_t$ includes knowledge of the regime in place at time $t$, the data up to time $t$, and the model parameters for each regime. These assumptions are consistent with the information set available to agents in our model and so our measure of uncertainty reflects uncertainty supposedly faced by the agent in the model across the two regimes. Uncertainty is computed taking into account that future regimes are unknown.

Figure 6 reports the results. The horizontal axis reports the time horizon $s$ used to compute uncertainty. The figure clearly shows that Regime 2 is characterized by higher uncertainty, despite the fact that the volatility of the MEI shock is larger under Regime 1. For all variables and for all horizons uncertainty is higher under this regime. This is for two reasons. First, the larger volatility of the preference shock implies substantial fluctuations both in the term structure and the real economy. Second, Regime 2 is less persistent than Regime 1. As a result, when agents are under this regime they expect the possibility of large
swings in real activity and the term structure. Both these channels will be studied below.

It is also interesting to notice that uncertainty is in some cases hump-shaped. This is particularly visible for investment, but also consumption and GDP present this feature. About investment growth is hump-shaped. In other words, uncertainty is not monotonically increasing with the time horizon. In other words, agents are more uncertain about the short run than the long run. This is because two competing forces are at play. On the one hand, events that are further into the future are naturally harder to predict, as the possibility of shocks and regime changes increase. On the other hand, in the long run the probability of still being in the high uncertainty regime declines.

Figure 7 presents a simulation to understand the effects of changes in uncertainty on the macroeconomy and the term structure of interest rates. We take the most likely regime sequence, as presented in Figure 3 and we simulate the economy based on the parameters at the posterior mode. The top panel reports the behavior of GDP and the slope of the yield curve. An increase in uncertainty determines both a drop in real activity and an increase in the slope. This generates a negative co-movement between the slope of the yield curve and real activity, as in the data. The increase in the slope is relatively larger than the drop in GDP. The lower panel of the figure compares the movements in the slope induced by the increase in uncertainty with the actual series. It is clear that the changes are quite large. For GDP the fluctuations are smaller with respect to the overall GDP volatility. An increase in uncertainty leads to a decline in (annualized) real activity of around 6%.

The fluctuations in uncertainty also lead to significant breaks in the term premium. Term premium is defined as the difference between the yield on an 20-period bond and the

Figure 6: Uncertainty. The figure reports the level of uncertainty at different horizons. Uncertainty is computed taking into account the possibility of regime changes.
expected average short-term yield (1 quarter) over the same 20 periods (following Rudebusch et al. (2006)). The expected value is computed taking into account the possibility of regime changes using the methods developed in Bianchi (2016). In the low uncertainty regime, we obtain an annualized theoretical term premium of 0.29%, while in the high uncertainty regime the annualized term premium is 0.67%. These values are computed under the assumption that the economy starts from the unconditional steady state and then moves to one of the two regimes. However, we can also ask how these premia would fluctuate if we consider the estimated regime sequence. The last panel of Figure 3 shows that periods of high uncertainty are associated with an increase in term premia.

In order to explore the importance of uncertainty shocks with respect to other disturbances, we compute a variance decomposition. We consider the unconditional variance as implied of the model when only one shock is active and compare it to the overall variance. The results are reported in Table 7. Uncertainty shocks play an important role to explain fluctuations in the slope of the term structure. In fact, the contribution of changes in the

![Graphs showing GDP, slope, and term premium over time.](image)

**Figure 7:** Simulation with estimated regime sequence. Top panel: Simulated dynamics of GDP and slope of the yield curve. GDP is expressed in percentage deviations from steady state. Second panel: Slope of yield curve in the data and also from simulation of the model. Bottom panel: Term premium. All variables are annualized.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Preference</th>
<th>TFP growth</th>
<th>MEI</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta GDP)</td>
<td>0.42</td>
<td>69.44</td>
<td>4.65</td>
<td>0.10</td>
</tr>
<tr>
<td>Inflation</td>
<td>49.46</td>
<td>2.00</td>
<td>4.95</td>
<td>4.82</td>
</tr>
<tr>
<td>FFR</td>
<td>87.59</td>
<td>1.04</td>
<td>3.42</td>
<td>3.48</td>
</tr>
<tr>
<td>(\Delta Inv)</td>
<td>17.00</td>
<td>34.57</td>
<td>45.85</td>
<td>0.04</td>
</tr>
<tr>
<td>(\Delta Cons)</td>
<td>1.48</td>
<td>90.08</td>
<td>1.74</td>
<td>0.34</td>
</tr>
<tr>
<td>Slope</td>
<td>27.80</td>
<td>15.55</td>
<td>16.64</td>
<td>28.62</td>
</tr>
</tbody>
</table>

**Table 7:** Variance decomposition. This table presents the contribution of the different shocks to the unconditional variance of the macroeconomic variables and the slope of the yield curve.
volatility regime is larger than any other shock. However, other shocks also play an important role: Preference, TFP growth, and the shock to the marginal efficiency of investment. This is not surprising given that as illustrated by the rest of the table, these shocks generate sizeable fluctuations in macroeconomic variables and inflation. In line with previous results in the literature, we find that uncertainty has an impact on macro variables, but this is limited with respect to the contribution of the other shocks. With respect to this, it is important to remind the reader that our model and estimation focuses on the effects of aggregate macro uncertainty.

4.4 Response to shocks

In this section, we analyze in detail the role of other shocks in generating macro and asset pricing fluctuations.

4.4.1 Long run growth uncertainty

We have discussed above that the preference shock plays an important role in generating the upward slope in the term structure. However, other uncertainty shocks also play an important role. For example, shocks to TFP volatility generate a positive commovement in the macro variables, while generating a shift in the opposite direction for the slope. This effect is illustrated in 8. The magnitude of this effect is small, but so is the change in the volatility of the shocks to TFP growth: from 1.85% to 1.90%. This suggests that this channel can be potentially important in situations of particular stress about the future value of TFP growth.
4.4.2 Preference and MEI shocks

So far, we have explored the role of changes in uncertainty in determining fluctuations in the term structure and real activity. Here, we focus on Gaussian shocks. Figure 9 reports the impulse response to a shock to the marginal efficiency of investment. Note that such shock generates positive commovement in the macro variables, while at the same time generating a drop in the slope of the term structure. This is the pattern that we documented in the introduction.

Figure 10 reports the impulse response to a preference shock. As explained above, this shock plays a key role in obtaining a positive slope. Note that on impact the shock generates an increase in consumption, as agents become more impatient. However, the large drop in investment determines an overall decline in GDP. As a result, consumption declines over time.
because of smaller capital accumulation. The shock determines a large increase in inflation to which the central bank responds aggressively. This determines a prolonged period of above-target inflation and above-target real interest rates.

5 Conclusions

In this paper, we have presented and estimated a general equilibrium model that allows for a joint analysis of the term structure of interest rates and the level of real activity. The model features endogenous term premia, and allows us to quantify the effects of uncertainty both on the real economy and the term structure of interest rates. The model is able to generate a sizable and positive term premia using moderate risk aversion. Shocks to time discount rates generate positive co-movement between interest rates and marginal utility, which in turn makes long maturity bonds riskier than short maturity ones. The model is solved using a novel iterative algorithm that captures the affect of uncertainty in a conditionally linear framework. We find that uncertainty shocks produce negative co-movement between GDP and the slope of the term structure, as in the data. The effects of uncertainty on the term structure are large and contribute to explaining the fluctuations in the slope observed in the data and variation in term premia.
References


Roger E.A. Farmer, Daniel F. Waggoner, and Tao Zha. Minimal State Variable Solutions


