Monetary Policy in Incomplete Market Models: Theory and Evidence*

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Abstract

The objective of this paper is to estimate a New Keynesian model with incomplete markets and to use the model to study theoretically and quantitatively the effects of monetary policy and the channels through which it operates. The key challenge, as well as the key quantitative insight in the paper, is that because of the failure of Ricardian equivalence monetary and fiscal policies are deeply intertwined and respond to each other, and to other shocks affecting the economy. We document these relationships in the data and exploit them to estimate the model. The analysis reveals that the macroeconomic impact of monetary policy is crucially affected by induced fiscal policy changes. Ignoring this policy interaction would lead to a severely biased assessment of monetary policy impacts and the mechanisms through which they operate.

Keywords: Monetary Policy, Fiscal Policy, Incomplete Markets, Estimation

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1 Introduction

The analysis of the effects of monetary and fiscal policies is the subject of extensive but largely separate literatures. The workhorse framework in public economics studying fiscal policy is the Bewley-Imrohoroglu-Huggett-Aiyagari incomplete markets model. It is considered an appropriate framework because it can match the joint distribution of earnings, consumption and wealth and generate a realistic distribution of marginal propensities to consume (MPCs), as well as empirically relevant consumption responses to permanent and transitory income changes. In monetary economics, on the other hand, the workhorse framework is the representative-agent New-Keynesian model. This framework features nominal rigidities, allowing output to be partially demand determined, it assigns a meaningful role for monetary policy, and can match various features of the aggregate data. The current research frontier attempts to combine the two models to allow for both demand determined output and rich distributional consequences of macroeconomic policies.[1]

Our objective in this paper is to facilitate quantitative empirical analysis using this rich new framework by estimating some of its key parameters. The key challenge as well as the key quantitative insight in the paper is that monetary and fiscal policies are deeply intertwined and respond to each other, and to other shocks affecting the economy. For example, shocks to technology or monetary policy that affect the aggregate economy also affect the government budget constraint, even if the fiscal authorities do not change taxes, spending, or transfer policies. As Ricardian equivalence does not hold in these models, we show that shocks to technology or monetary policy induce significant effects on prices and inflation through their effect on the level of government debt. Thus, the direct effect of shocks and monetary policy plus the indirect induced effect on fiscal policy together determine their total impact. Moreover, fiscal authorities can actively respond to shocks and monetary policy, further obscuring their direct impact. For example, we completely theoretically characterize a redistribution scheme through which the fiscal authority can fully offset any distributional consequences of monetary policy, eliminating the difference between complete and incomplete markets models.

[1]See Kaplan and Violante (2018) for a recent thorough review of this literature. Additional references include, among others, Oh and Reis (2012), Guerrieri and Lorenzoni (2015), Gornemann et al. (2012), Kaplan et al. (2016), Auclert (2016) and Lütticke (2015), McKay and Reis (2016), McKay et al. (2015), Bayer et al. (2015), Ravn and Sterk (2013),Den Haan et al. (2015), and Hagedorn et al. (2018a,b).

Thus, in the first part of the paper we document the response of the economy to shocks to neutral technology and monetary policy. We are particularly interested in the response of fiscal variables which has been largely ignored in the extensive related literature inspired by complete market New Keynesian models because changes in fiscal variables such as government debt or transfers matter little or not at all in those models. This is in contrast to models with incomplete markets where changes in these variables have the potential to significantly affect the distribution of income and aggregate outcomes.

In the second part of the paper we build on these results and estimate a New Keynesian incomplete markets model. This is important since one cannot simply import the parameters estimated using a complete markets model. The reason is not only abstract and theoretical but very concrete. As explained above, fiscal policy responds to a technology shock and this leads to a different price path in the model. For example, suppose that in response to a positive technology shock, government increases transfers to higher MPC individuals, implying that the total price response is a combination of the response to the technology shock and to the increase in transfers. The former leads to a price decrease whereas the latter leads to a price increase. Depending on the relative strength of the two responses, the total response might be a price increase, decrease or no response at all. This has obvious consequences for the estimation of the degree of price rigidities. For example, suppose that prices respond little to a positive technology shock. Viewed through the lens of a complete markets model, this would be interpreted as a very high degree of price rigidities. Viewed through the lens of an incomplete markets model where transfers are increased simultaneously, this can be consistent with very flexible prices.

Methodologically, our estimation approach follows Christiano et al. (2005). We calibrate a subset of the parameters and select the remaining ones to minimize the distance between the model and the empirical impulse response functions of various macroeconomic variables documented in the paper. Importantly, we impose on the model the same responses of fiscal and monetary policies to technology shocks and to each other that we document in the data. We find that our fairly parsimonious model is capable of generating the responses closely in line with the data.

We use the estimated model and theoretical analysis to understand the role of market incompleteness in shaping the impact of monetary policy. We do so by shutting down various components of fiscal policy response induced by monetary policy change. The key conclusion is that the macroeconomic impact of monetary policy is crucially affected by induced fiscal policy changes. Ignoring this policy interaction would lead to a severely biased assessment of monetary policy impacts and the mechanisms through which they operate.

2 Empirical Evidence

2.1 Data and Measurement

2.1.1 Fiscal Variables

Our measurement of nominal government spending, revenue, and transfers in the data aims to ensure that these variables are defined consistently with their meaning in the model developed below and that the government budget constraint holds. To study the aggregate economy, we consider a consolidated government combining federal, state, and local levels. At these level, the variables of interest can be constructed using the data from BEA NIPA Table 3.1. Specifically, we define them as follows (NIPA Table 3.1 line numbers reported in brackets):

Spending = Consumption expenditures [21] + Gross government investment [39]
+ Net purhases on nonproduced assets [41] – Consumption of fixed capital [42]

Revenue = Total receipts [34] – Current transfer receipts from the rest of the world [18]
– Subsidies [30] + Current surplus of government enterprises [19]

Transfers = Current transfer payments [22] + Capital transfer payments [40]
– Current transfer receipts from the rest of the world [18]

2.1.2 GDP

Data on nominal GDP are from BEA NIPA Table 1.1.5. The series is in billions of dollars and is seasonally adjusted.

2.1.3 Hours Worked and Wages

The Bureau of Labor Statistics provides data on total compensation (series PRS85006063) and total hours worked (series PRS85006033) in non-farm business sector. We define nominal hourly wages as the ratio of the two series.

2.1.4 Price Level and Deflator

We measure the (changes in) price level using BLS series on the Consumer Price Index for All Urban Consumers: All Items, 1982-1984=100, seasonally adjusted, quarterly average of monthly values. For consistency across variables, we use this series to deflate all nominal quantities.

2.1.5 Federal Funds Rate

Quarterly averages of monthly effective Federal Funds Rate. The series is downloaded from the FRED database of the Federal Reserve Bank of St. Louis (FEDFUNDS).

2.1.6 Neutral Technology Shocks

The time series for aggregate labor augmenting (Harrod-neutral) technology shocks comes from Bocola et al. (2018). The series is identified using implications of Uzawa (1961) theorem, the assumptions of which are satisfied by the theory developed below. The series is constructed using aggregate data extending back to 1947.

2.1.7 Monetary Policy Shocks

To measure shocks to monetary policy we use the series from Wieland and Yang (2017) who extended the series from Romer and Romer (2004). The series extends back to 1969 and represents residuals from a regression of the target federal funds rate on lagged values and the Federal Reserve's information set based on Greenbook forecasts. In unreported analysis we found that results remain qualitatively similar when we use monetary policy shocks measured using high frequency identification methodologies, but those series are much shorter.

2.1.8 Estimating IRFs

The objective of our empirical analysis is to document the impulse response of various outcome variables described above to identified shocks to technology or monetary policy. We measure the impulse response of an outcome variable X to identified shock ξ through the following regression:

$$100 * (\log(X_{t+k}) - \log(X_{t-1})) = \beta_k \, \log(\xi_t) + \epsilon_t.$$

The regressions are estimated on the data starting in 1983q1 through the end of available data for the identified technology shock (2014q4) and monetary policy shock (2007q4) series.

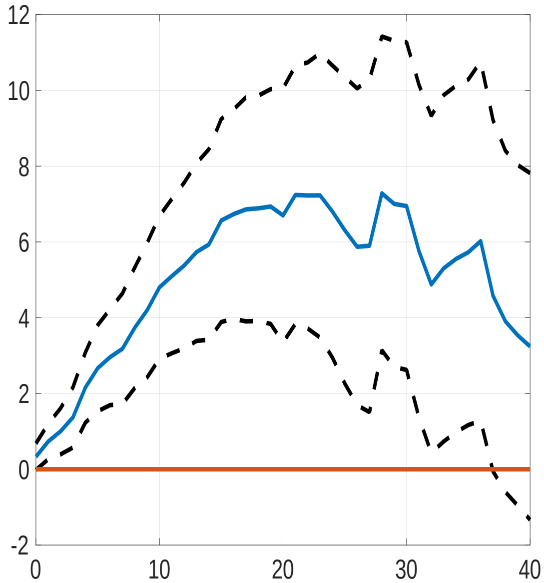
2.2 What Happens after a Technology Shock?

The impulse responses of various variables to a neutral technology shock are plotted in Figures 1 and 2 together with 95% confidence intervals. An improvement in neutral technology leads to a very persistent increase in output and the real wage and a more transitory increase in hours worked. Prices fall marginally in impact but build up steadily afterward.

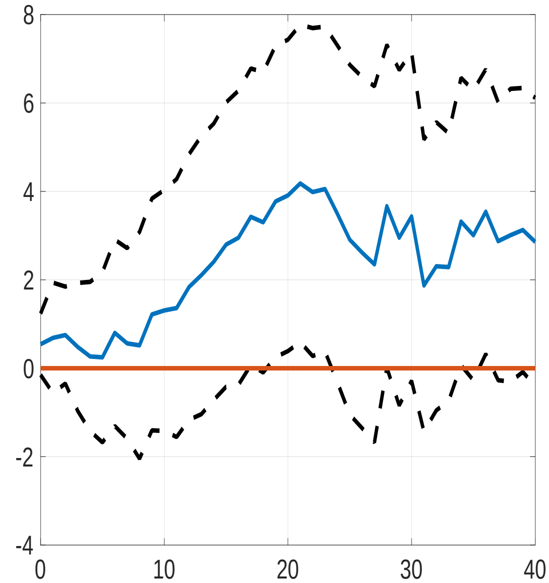
Notably, these expansionary consequences of neutral technology improvements are not solely due to technology shocks themselves. Instead, improvements in neutral technology are accompanied by significant changes in fiscal and monetary policies. Not only the government revenue increases substantially, the increase in fiscal spending is even more dramatic. The increase in transfers is more gradual but builds up significantly over time. These changes in fiscal policy are likely to be important contributors to the expansionary effects of technology improvements. Monetary policy also responds dramatically, first by raising interest rates significantly and then lowering them when fiscal stimulus induced by higher spending starts to stabilize and eventually decline.

It seems clear that one cannot study the effects of technology shocks, monetary policies or fiscal policies in isolation from each other. The economic responses to such shocks and policies are interdependent and are jointly determined.

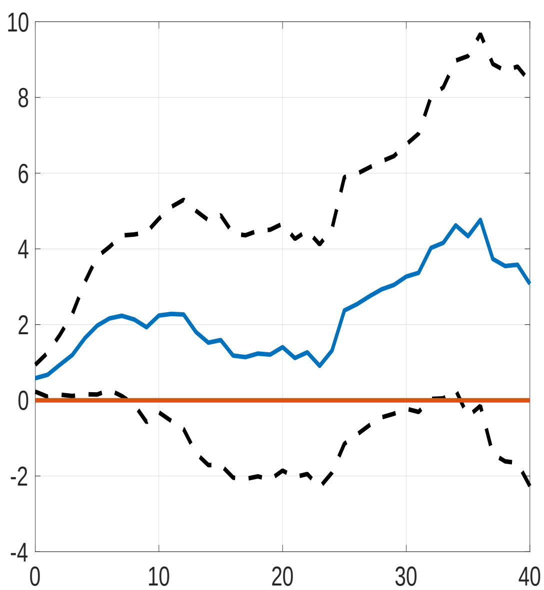
Figure 1: Response of Government Spending, Transfers, Revenue, and FFR to a neutral technology shock.



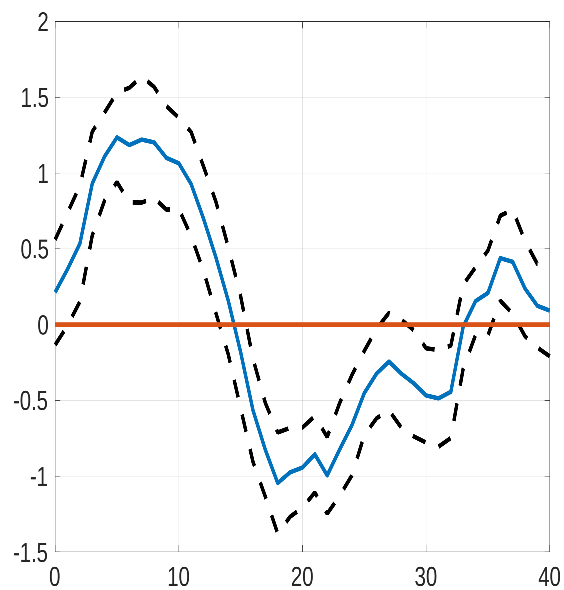
(a) Spending



(b) Transfers

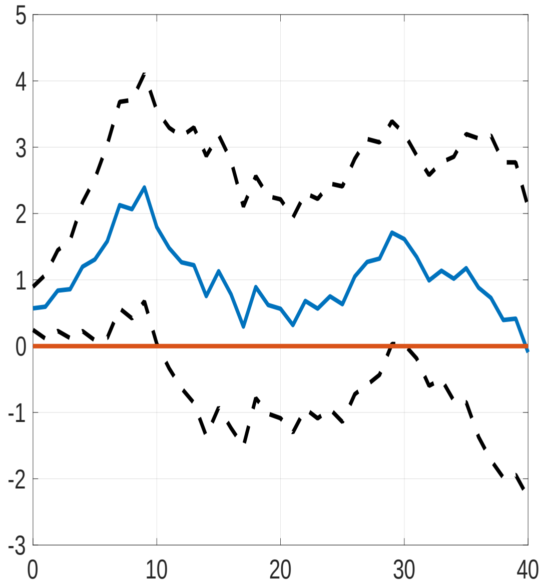


(c) Revenue

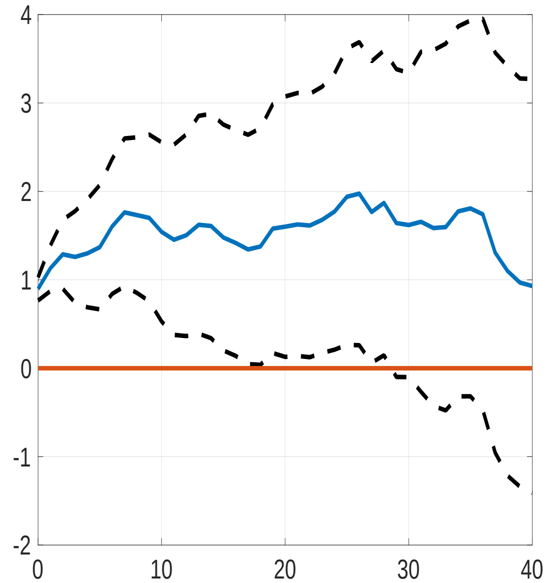


(d) FFR

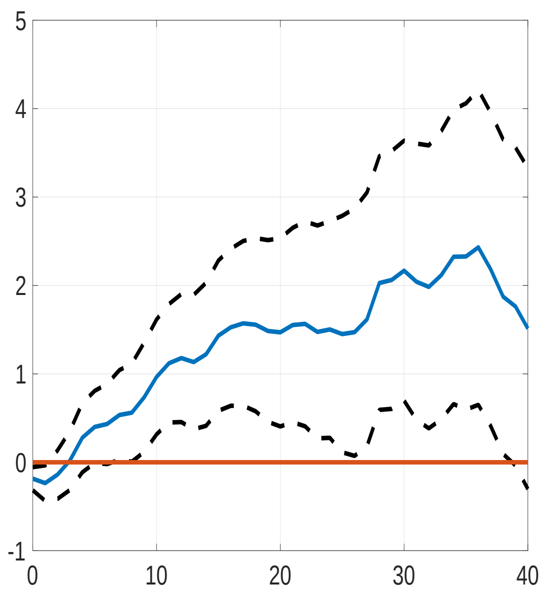
Figure 2: Response of Hours Worked, Output, Price Level, and Real Wage to a neutral technology shock.



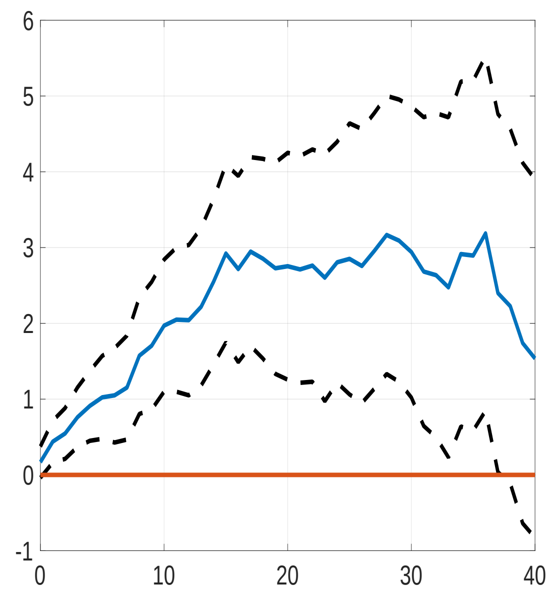
(a) Hours



(b) Output



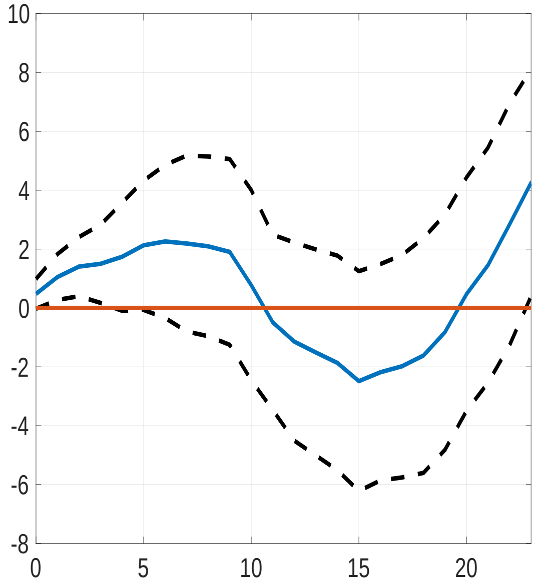
(c) Price Level



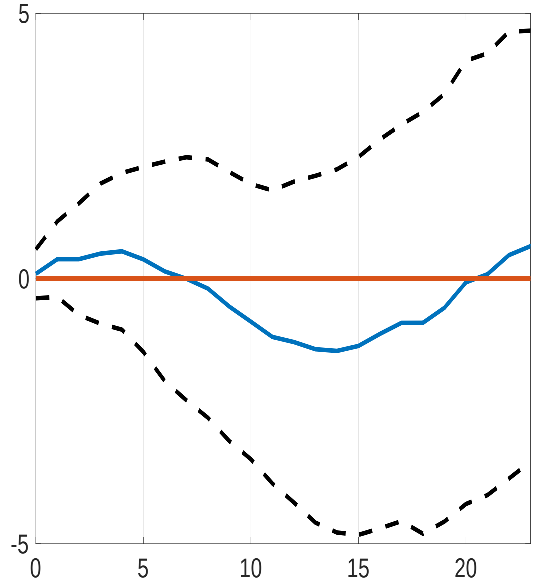
(d) Wages

2.3 What Happens after a Monetary Policy Shock?

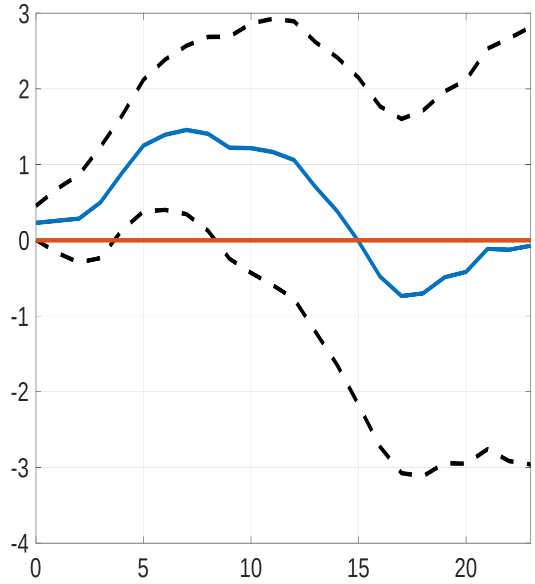
Figure 3: Response of Hours Worked, Output, Price Level, and Real Wage to a monetary policy shock.



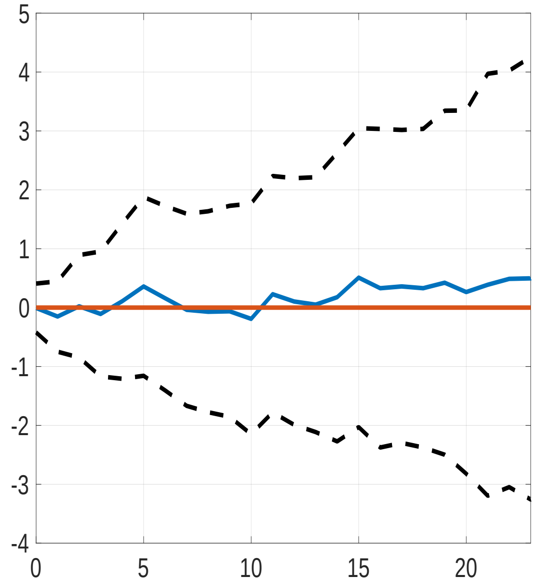
(a) Hours



(b) Output

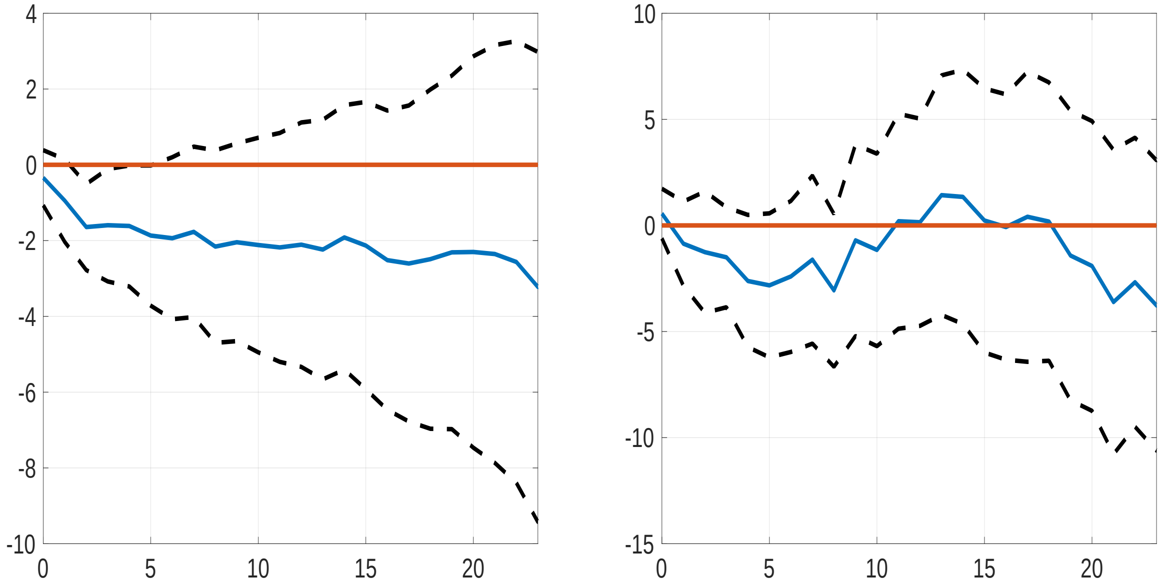


(c) Price Level



(d) Real Wage

Figure 4: Response of Government Spending and Transfers to a monetary policy shock.



(a) Spending

(b) Transfers

Figures 3 and 4 plot the impulse responses of various variables to a monetary policy shock. Interestingly, the data suggest that unexpected increases in federal funds rate are mildly expansionary in the short run. Their effects are likely significantly influenced, however, by a substantial and persistent reduction in fiscal spending.

3 Model

The model is a standard New Keynesian model with one important modification: Markets are incomplete as in Aiyagari (1994, 1995). Price setting faces some constraints as price adjustments are costly as in Rotemberg (1982) leading to price rigidities. As is standard in the New Keynesian literature, final output is produced in several intermediate steps. Final good producers combine the intermediate goods to produce and sell their output in a competitive goods market. Intermediate goods producers are monopolistically competitive. They set a price they charge to the final good producer to maximize profits taking into account the price adjustment costs they face. The intermediate goods producer rent inputs, capital and a composite of differentiated labor, in competitive factor markets. We also allow for sticky wages and assume that differentiated labor is monopolistically supplied as well.

3.1 Households

The economy consists of a continuum of agents normalized to measure 1 who are ex-ante heterogenous with respect to their subjective discount factors and who have CRRA preferences over consumption and additively separable preferences for leisure:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t),$$

where

$$u(c,h) = \begin{cases} \frac{c^{1-\sigma}-1}{1-\sigma} - g(h) & \text{if } \sigma \neq 1 \\ \log(c) - g(h) & \text{if } \sigma = 1, \end{cases}$$

$\beta \in (0,1)$ is the household-specific subjective discount factor, and $g(h)$ is the disutility of labor. Agents' labor productivity $\{s_t\}_{t=0}^{\infty}$ is stochastic and is characterized by an N-state Markov chain that can take on values $s_t \in \mathcal{S} = \{s_1, \cdots, s_N\}$ with transition probability characterized by $p(s_{t+1}|s_t)$ and $\int s = 1$. Agents rent their labor services, hs, to firms for a real wage w_t and their nominal assets a to the capital market for a nominal rent i^a and a real return $(1 + r_t^a) = \frac{1+i_t^a}{1+\pi_t}$, where $1 + \pi_t = \frac{P_t}{P_{t+1}}$ is the inflation rate (the equilibrium final good price P_t is derived below). There are two classes of assets, bonds and capital with potentially different returns, but households can invest in one asset A, which the mutual fund (described below) collects and allocates to bonds and capital.

To allow for sticky wages we follow the literature and assume that each household j provides differentiated labor services, h_{jt}. These differentiated labor services are transformed by a representative, competitive labor packer firm into an aggregate effective labor input, H_t, using the following technology:

$$H_t = \left(\int_0^1 s_{jt} (h_{jt})^{\frac{\epsilon_w - 1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{1}$$

where ϵ_w is the elasticity of substitution across labor services.

A middleman firm (e.g. a union) sells households labor services to the labor packer, which given aggregate labor demand H_t by the intermediate goods sector, minimizes costs

$$\int_0^1 W_{jt} s_{jt} h_{jt} dj, \tag{2}$$

implying a demand for the labor services of household j:

$$h_{jt} = h(W_{jt}; W_t, H_t) = \left(\frac{W_{jt}}{W_t} \right)^{-\epsilon_w} H_t, \tag{3}$$

where W_t is the (equilibrium) nominal wage which can be expressed as

$$W_t = \left(\int_0^1 s_{jt} W_{jt}^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}}.$$

The middleman sets a nominal wage $\hat{W}_t$ for an effective unit of labor (so that $W_{jt} = \hat{W}_t$) to maximize profits subject to wage adjustment costs modeled similarly to the price adjustment costs in Rotemberg (1982). These adjustment costs are proportional to idiosyncratic productivity s, are measured in units of aggregate output, and are given by a quadratic function of the change in wages above and beyond steady state wage inflation $\overline{\Pi}^w$,

$$\Theta\left(s_{jt}, W_{jt} = \hat{W}_t, W_{jt-1} = \hat{W}_{t-1}; H_t\right) = s_{jt} \frac{\theta_w}{2} \left(\frac{W_{jt}}{W_{jt-1}} - \overline{\Pi}^w \right)^2 H_t = s_{jt} \frac{\theta_w}{2} \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w \right)^2 H_t.$$

The middleman's wage setting problem is to maximize[2]

$$
\begin{aligned}
V_t^w\left(\hat{W}_{t-1}\right) \equiv & \max_{\hat{W}_t} \int \Big(\frac{s_{jt}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - \frac{g(h(\hat{W}_t; W_t, H_t))}{u'(C_t)}\Big) dj \\
- & \int s_{jt} \frac{\theta_w}{2} \left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t dj + \frac{1}{1+r_t} V_{t+1}^w \left(\hat{W}_t\right),
\end{aligned} \tag{4}
$$

where C_t is aggregate consumption.

Some algebra (see the appendix) yields, using $h_{jt} = H_t$ and $\hat{W}_t = W_t$ and defining the real wage $w_t = \frac{W_t}{P_t}$, the wage inflation equation

$$
\theta_w \left(\pi_t^w - \overline{\Pi}^w\right) \pi_t^w = (1-\tau_t)(1-\epsilon_w) w_t + \epsilon_w \frac{g'(h(\hat{W}_t; W_t, H_t))}{u'(C_t)} + \frac{1}{1+r_t} \theta_w \left(\pi_{t+1}^w - \overline{\Pi}^w\right) \pi_{t+1}^w \frac{H_{t+1}}{H_t}. \tag{5}
$$

The wage adjustment process does not involve actual costs but is as-if those costs were actually present. We make this assumption to avoid significant movements of these adjustment costs in response to e.g. a fiscal stimulus or in a liquidity trap. Such swings would matter in our incomplete markets model and might yield quite different implications from price setting à la Calvo.

Thus, at time t an agent faces the following budget constraint:

$$
P_t c_t + a_{t+1} = (1 + i_t^a) a_t + (1 - \tau_t) P_t w_t h_t s_t + T_t,
$$

where τ_t is a proportional labor tax and T_t is a nominal lump sum transfer. Agents are price takers. In addition, households take wages and hours from the middleman's wage setting problem as given. Thus, we can rewrite the agent's problem recursively:

$$
\begin{aligned}
V(a, s, \beta; \Omega) = & \max_{c \geq 0, a' \geq 0} u(c, h) + \beta \sum_{s \in \mathcal{S}} p(s'|s) V(a', s', \beta; \Omega') \qquad (6) \\
& \text{subj. to} \quad Pc + a' = (1 + i^a) a + P(1-\tau) whs + T \\
& \qquad\qquad\quad \Omega' = \Gamma(\Omega),
\end{aligned}
$$

where $\Omega(a, s, \beta)$ is the distribution on the space $X = A \times S \times \mathcal{B}$, agents asset holdings $a \in A$,

[2] Equivalently one can think of a continuum of middlemen each setting the wage for a representative part of the population with $\int s = 1$ at all times.

labor productivity $s \in S$ and discount factor $\beta \in \mathcal{B}$, across the population, which will together with the policy variables determine the equilibrium prices. Let $\mathbb{B}(X) = \mathbb{A} \times \mathcal{P}(S) \times \mathcal{P}(\mathcal{B})$ be the σ-algebra over X, defined as the cartesian product over the Borel σ-algebra on A and the power sets of S and $\mathcal{B}$. Define our space $M = (X, \mathbb{B}(X))$, and let $\mathcal{M}$ be the set of probability measures over M. Γ is an equilibrium object that specifies the evolution of the distribution Ω.

3.2 Production

3.2.1 Final Good Producer

A competitive representative final goods producer aggregates a continuum of intermediate goods indexed by $l \in [0,1]$ and with prices p_l:

$$Y_t = \left(\int_0^1 y_{lt}^{\frac{\epsilon-1}{\epsilon}} dl \right)^{\frac{\epsilon}{\epsilon-1}},$$

where ϵ is the elasticity of substitution across goods. Given a level of aggregate demand Y, cost minimization for the final goods producer implies that the demand for the intermediate good l is given by

$$y_{lt} = y(p_{lt}; P_t, Y_t) = \left(\frac{p_{lt}}{P_t} \right)^{-\epsilon} Y_t, \tag{7}$$

where P is the (equilibrium) price of the final good and can be expressed as

$$P_t = \left(\int_0^1 p_{lt}^{1-\epsilon} dl \right)^{\frac{1}{1-\epsilon}}.$$

3.2.2 Intermediate-Goods Firms

A monopolist produces intermediate good $l \in [0,1]$ using the following technology:

$$Y_{lt} = \begin{cases} Z_t K_{lt}^{\alpha} H_{lt}^{1-\alpha} - Z_t F & \text{if} \geq 0 \\ 0 & \text{otherwise} \end{cases}, \tag{8}$$

where $0 < \alpha < 1$, K_{lt} is capital services rented, H_{lt} is labor services rented and the fixed cost of production are denoted $F > 0$.

Intermediate-goods firms rent capital and labor in perfectly competitive factor markets. Profits are fully taxed by the government. A firm's real marginal cost is $mc_{lt} = \partial S_t(Y_{lt})/\partial Y_{lt}$, where

$$S_t(Y_{lt}) = \min_{K_{lt},H_{lt}} r_t^k K_{lt} + w_t H_{lt}, \text{ and } Y_{lt} \text{ is given by (8).} \tag{9}$$

Given our functional forms, we have

$$mc_{lt} = \left(\frac{1}{\alpha}\right)^{\alpha} \left(\frac{1}{1-\alpha}\right)^{1-\alpha} \frac{(r_t^k)^{\alpha}(w_t)^{1-\alpha}}{Z_t} \tag{10}$$

and

$$\frac{K_{lt}}{H_{lt}} = \frac{\alpha w_t}{(1-\alpha) r_t^k}. \tag{11}$$

Prices are sticky as intermediate-goods firms face Rotemberg (1982) price adjustment costs. Given last period's individual price p_{jt-1} and the aggregate state $(P_t, Y_t, Z_t, w_t, r_t)$, the firm chooses this period's price p_{jt} to maximize the present discounted value of future profits, satisfying all demand. The firm's pricing problem is

$$V_t\left(p_{lt-1}\right) \equiv \max_{p_{lt}} \frac{p_{lt}}{P_t} y\left(p_{lt}; P_t, Y_t\right) - S(y(p_{lt}; P_t, Y_t)) - \frac{\theta}{2}\left(\frac{p_{lt}}{p_{lt-1}} - \overline{\Pi}\right)^2 Y_t - Z_t F + \frac{1}{1+r_t} V_{t+1}\left(p_{lt}\right),$$

where Φ are fixed operating costs.

Some algebra (in the appendix) yields the New Keynesian Phillips Curve

$$(1-\epsilon) + \epsilon mc_t - \theta\left(\pi_t - \overline{\Pi}\right)\pi_t + \frac{1}{1+r_t}\theta\left(\pi_{t+1} - \overline{\Pi}\right)\pi_{t+1}\frac{Y_{t+1}}{Y_t} = 0.$$

The equilibrium real profit of each intermediate goods firm is then

$$d_t = Y_t - Z_t F - S(Y_t).$$

3.2.3 Mutual Fund

The mutual fund collects households savings A_{t+1}/P_{t+1} and pays a real return $\tilde{r}_t^a$, invests them in real bonds B_{t+1}/P_{t+1} and capital K_{t+1}, and pays a capital income tax $\tau_k(r_t^k - \delta)K_t$. It maximizes

$$\Phi(K_{t+2}, K_{t+1}) + (1 + (1-\tau_k)(r_{t+1}^k - \delta))K_{t+1} + (1 + r_{t+1})B_{t+1}/P_{t+1} - (1 + \tilde{r}_{t+1}^a)(A_{t+1}/P_{t+1}),$$

such that $A_{t+1}/P_{t+1} = K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t)$ and for adjustment costs $\Phi(K_{t+1}, K_t)$, taking K_t and K_{t+2} as given. In equilibrium,

$$\begin{aligned}
&r_{t+1} = \tilde{r}^a_{t+1} \\
&1 + (1-\tau_k)(r^k_{t+1} - \delta) = (1 + \tilde{r}^a_{t+1})(1 + \Phi_1(K_{t+1}, K_t)) - \Phi_2(K_{t+2}, K_{t+1}) \\
&A_{t+1}/P_{t+1} = K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t).
\end{aligned}$$

The same first order conditions would arise in an intertemporal optimization problem where profits are discounted at rate $\tilde{r}^a$. The objective function above shows all parts of the full intertemporal objective function where $t+1$ terms appear, evaluated in period $t+1$.

The total profits of the fund are

$$D^{MF}_{t+1} = (1-\tau_k)(1 + r^k_{t+1} - \delta)K_{t+1} + (1 + r_{t+1})B_{t+1}/P_{t+1} - (1 + \tilde{r}^a_{t+1})(A_{t+1}/P_{t+1}),$$

and per unit of investment they are $d^{MF}_{t+1} = D^{MF}_{t+1}/(A_{t+1}/P_{t+1})$. Households therefore receive (or have to pay) $d_{t+1}A_{t+1}/P_{t+1}$ in period $t+1$ per unit invested such that households' return equals

$$(1 + r^a_{t+1}) = (1 + \tilde{r}^a_{t+1} + d^{MF}_{t+1}).$$

3.3 Government

The government obtains revenue from taxing labor income, capital income, and profits as well as issuing bonds. Household labor income wsl is taxed progressively with a nominal lump-sum transfer T_t and a proportional tax τ:

$$\tilde{T}(wsh) = -T + \tau P wsh.$$

The government issues nominal bonds denoted by B^g, with negative values denoting government asset holdings. It fully taxes profits away, obtaining nominal revenue Pd. It also taxes capital income at the rate τ_k. The government uses the revenue to finance exogenous nominal government expenditures, G_t, interest payments on bonds and transfers to households.

The government budget constraint is therefore given by

$$B^g_{t+1} = (1 + i_t)B^g_t + G_t - P_t d_t - \tau_k(r^k_t - \delta)K_t - \int \tilde{T}_t(w_t s_t h_t)d\Omega_t. \tag{12}$$

3.4 Equilibrium

Market clearing requires that the labor demanded by the firm is equal to the aggregate labor supplied by households, that the demand for bonds issued by the government and capital equal their supplies and that the amount of assets provided by households equals their demand by the mutual fund:

$$K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t) = A_{t+1}/P_{t+1} = \int_{a_t} \sum_{s_t \in \mathcal{S}} \sum_{b_t \in \mathcal{B}} a_{t+1}(a_t, s_t, \beta_t) d\Omega_t(a_t, s_t, \beta_t) \tag{13}$$

$$B_t = B^g_t \tag{14}$$

$$K_t = \int K_{lt} dl \tag{15}$$

$$H_t = \int H_{lt} dl = H_{lt} = \int h_{jt} dj = h_{jt} \tag{16}$$

where we have abused notation slightly here, $a_{t+1}(a_t, s_t, \beta_t)$ is the asset choice of an agent with asset level a_t and productivity s_t and discount factor β_t.

Definition: A monetary competitive equilibrium is a sequence of prices P_t, tax rates τ_t and τ_k, nominal transfers T_t, nominal government spending G_t, bonds B^g_t, a value functions $v_t : M \to \mathcal{R}$ with policy functions $a_t : M \to \mathcal{R}_+$ and $c_t : M \to \mathcal{R}_+$, hours choices $H_t, H_{lt}, h_{jt} : M \to \mathcal{R}_+$, capital decisions $K_t, K_{lt} : M \to \mathcal{R}_+$, pricing functions $r_t, r^k_t, r^a_t, \tilde{r}^a_t : M \to \mathcal{R}$ and $w_t : M \to \mathcal{R}_+$, and a law of motion $\Gamma : \mathcal{M} \to \mathcal{M}$, such that:

1. v_t satisfies the Bellman equation with corresponding policy functions a_t and c_t given price sequences $r^a_t(), w_t()$ and hours h_t.

2. Firms maximize profits taking prices P_t, r^k_t, w_t as given.

3. Wages are set optimally by middlemen.

4. The mutual fund maximizes profits taking prices as given.

5. For all $\Omega \in \mathcal{M}$:

$$\begin{aligned}
K_{t+1} + B_{t+1}/P_{t+1} + \Phi(K_{t+1}, K_t) &= \int \frac{a_{t+1}(a_t, s_t, \beta_t)}{P_{t+1}} d\Omega_t, \\
B_t &= B_t^g \\
K_t &= \int K_{lt} dl \\
H_t &= \int H_{lt} dl = H_{lt} = \int h_{jt} dj = h_{jt} \\
Y_t = Z_t K_t^\alpha H_t^{1-\alpha} &= \int c_t(a_t, s_t, \beta_t) d\Omega_t + \frac{G_t}{P_t} + Z_t F + K_{t+1} \\
&\quad -(1-\delta)K_t + \Phi(K_{t+1}, K_t).
\end{aligned}$$

6. Aggregate law of motion Γ generated by a' and p.

4 Monetary Policy in Complete and Incomplete Markets

The complete markets model arises as a special case without idiosyncratic uncertainty, $s \equiv 1$. In this section we compare the response of the economy to a monetary policy shock in a model with complete and incomplete markets.[3]

To this aim, let $i_0 = i^*, i_1, i_2, \ldots, i_t, \ldots$ be a sequence of nominal interest rates describing a monetary policy innovation in the complete market model in period 0, where i^* is the steady state nominal interest rate and $\lim_{t\to\infty} i_t = i^*$. The nominal interest rate in the zero-inflation IM steady state is i_{ss}^{IM} and monetary policy in the IM economy is a sequence $1 + i_0^{IM} = 1 + i_{ss}^{IM}, 1 + i_1^{IM} = (1 + i_{ss}^{IM})\frac{1+i_1}{1+i^*}, 1 + i_2 = (1 + i_{ss}^{IM})\frac{1+i_2}{1+i^*}, \ldots, 1 + i_t^{IM} = (1 + i_{ss}^{IM})\frac{1+i_t}{1+i^*}, \ldots$.

This is a well-defined experiment in incomplete market models in which government bonds are nominal as shown in Hagedorn (2016, 2018). The price level and the inflation rate are both determinate for arbitrary sequences of nominal interest rates. In particular, determinacy does not require to assume that the central bank follows a Taylor rule with a sufficiently strong response to inflation as is the case in complete market models.

Determinacy implies that we obtain a unique sequence of prices to the monetary policy

[3] In this theoretical section, we consider a model without capital and linear production in hours.

innovation in the incomplete markets model, $\mathcal{P} = P_0^{IM}, \ldots, P_t^{IM}, \ldots$, and we use this same sequence of prices in the complete markets model. Prices and nominal interest rates allow us to construct real interest rates in the complete markets model which is sufficient to compute all remaining variables.

The variables in the complete markets model are denoted with a superscript CM: Inflation rate π_t^{CM}, real interest rate $(1+r_t^{CM}) = \frac{1+i_t}{1+\pi_t^{CM}}$, output Y_t^{CM}, total hours H_t^{CM}, consumption C_t^{CM} and wages w_t^{CM}. Real government spending is fixed at g. Transfers and bonds are indeterminate by Ricardian equivalence in the complete markets/representative agent economy. The proportional tax rate is unchanged at its steady state level τ_{ss}. Real government expenditure is allowed to respond and now equals g_t and is equal to g_{ss} in a steady state and we define $\gamma_t^g = \frac{g_t}{g_{ss}}$ as the change in government expenditure.

These variables satisfy

$$\begin{aligned}
Y_t^{CM} = Z_t H_t^{CM} &= C_t^{CM} + g_t + F + \frac{\theta}{2}\left(\pi_t^{CM} - \overline{\Pi}\right)^2 Y_t^{CM} && (17)\\
w_t^{CM}(1-\tau_t)(C_t^{CM})^{-\sigma} &= D(H_t^{CM})^{\phi} && (18)\\
u_c(C_t^{CM}) = (C_t^{CM})^{-\sigma} &= \beta \frac{1+i_{t+1}}{1+\pi_{t+1}^{CM}} u_c(C_{t+1}^{CM}) = \beta(1+r_{t+1}^{CM})(C_{t+1}^{CM})^{-\sigma} && (19)\\
(1-\epsilon) + \frac{\epsilon}{1-\alpha}\frac{w_t^{CM}}{Z_t} &= \theta\left(\pi_t^{CM} - \overline{\Pi}\right)\pi_t^{CM} - \frac{1}{1+r_t^{CM}}\theta\left(\pi_{t+1}^{CM} - \overline{\Pi}\right)\pi_{t+1}\frac{Y_{t+1}^{CM}}{Y_t^{CM}}. && (20)
\end{aligned}$$

Note that output is linear in hours, $Y = ZH$, and that the function describing the disutility of labor is $g(h) = \frac{Dh^{1+\phi}}{1+\phi}$.

The question we are asking is whether or when we get the same sequence of aggregate consumption, hours and output in the incomplete markets case for the same monetary policy impulse. Define therefore the percentage deviations from steady state of the impulse responses of consumption, hours, output and wages:

$$\begin{aligned}
\gamma_t^C &= \frac{C_t^{CM}}{C_{ss}^{CM}}, && (21)\\
\gamma_t^H &= \frac{H_t^{CM}}{H_{ss}^{CM}}, && (22)\\
\gamma_t^Y &= \frac{Y_t^{CM}}{Y_{ss}^{CM}}, && (23)\\
\gamma_t^w &= \frac{w_t^{CM}}{w_{ss}^{CM}}, && (24)
\end{aligned}$$

where C_{ss}^{CM}, H_{ss}^{CM}, Y_{ss}^{CM} and w_{ss}^{CM} are steady-state aggregate consumption, output, hours and wages respectively.

Define now the corresponding variables for the incomplete markets model. Steady state consumption, hours worked and savings of a household with a assets and productivity s are denoted $c_{ss}^{IM}(a, s)$, $h_{ss}^{IM}(a, s)$, and $a_{ss}^{IM}(a, s)$.[4] Each household receives transfers T_{ss}^{IM}. The steady state price level is P_{ss}^{IM}. Our aim is to compare the consumption decision of household i at time t in the stationary allocation without any aggregate shocks and in the allocation when an aggregate shock occurred at time 0. We start with the stationary allocation where a household i has a_0 asset at time 0 and experiences shocks $s_0, s_1, \ldots s_t$. Household i then starts period t with $a_t(a_0, s_0, s_1, \ldots, s_{t-1})$ assets. The behavior of this individual in period t is therefore fully characterized by its assets $a_t(a_0, s_0, s_1, \ldots, s_{t-1})$ and the productivity s_t. To simplify notation, we therefore identify this individual by the pair (a, s).

We also identify an individual i at time t in the "shock" allocation through its pair (a, s) in the stationary no-shock allocation. Concretely, the consumption of this individual i who holds a assets and faces shock s in period t in the stationary allocation and is thus identified with the pair (a, s), is denoted $c_t^{IM}(a, s)$ in the shock world. Note that although this agent has the same productivity s in period t both in the transition and in the stationary allocation (since s follows an exogenous process), the endogenous asset level might not be the same. Due to this notational simplification, the difference in consumption is then simply $c_t^{IM}(a, s) - c_{ss}^{IM}(a, s)$ for this individual at time t. Similarly hours worked are denoted $h_t^{IM}(a, s)$, so that aggregate consumption and hours at time t equal

$$H_t^{IM} = \int s h_t^{IM}(a, s) d\Omega_{ss} \tag{25}$$

$$C_t^{IM} = \int c_t^{IM}(a, s) d\Omega_{ss} \tag{26}$$

For their steady state counterparts

$$H_{ss}^{IM} = \int s h_{ss}^{IM}(a, s) d\Omega_{ss} \tag{27}$$

$$C_{ss}^{IM} = \int c_{ss}^{IM}(a, s) d\Omega_{ss} \tag{28}$$

[4] We suppress the dependence on β for expositional convenience.

The price level is P_t^{IM}, transfers are T_t^{IM}, bonds are B_t^{IM} and the wage is w_t^{IM}.

In addition to labor and asset income households also receive real dividend income and real transfers from the government. The aggregate transfers are denoted Γ_t^{IM} and Γ_{ss}^{IM} respectively which satisfy

$$\Gamma_t^{IM} = d_t^{IM} + \tau w_t^{IM} H_t^{IM} + \frac{B_{t+1}^{IM} - B_t^{IM}(1+i_t^{IM})}{P_t^{IM}} - g_t^{IM}, \tag{29}$$

$$\Gamma_{ss}^{IM} = d_{ss}^{IM} + \tau w_{ss}^{IM} H_{ss}^{IM} + \frac{B_{ss} - B_{ss}(1+i_{ss}^{IM})}{P_{ss}^{IM}} - g_{ss}. \tag{30}$$

Each household with current productivity s receives a share $\lambda(s)$ of the transfer, such that $\int \lambda(s)ds = 1$. We denote $\gamma_t^{\Gamma} = \Gamma_t^{IM}/\Gamma_{ss}^{IM}$.

We now define a transfer Δ which also depend on the (a,s) in the stationary economy. We will show that paying those individual specific transfers makes the impulse response of the complete and the incomplete markets economy identical in terms of aggregate consumption, output and hours worked.

$$\begin{aligned} & \Delta_t(a,s) \\ = \; & (\gamma_t^C - 1)c_{ss}(a,s) - \left(\gamma_t^H \gamma_t^w - 1\right) w_{ss}^{IM}(1-\tau_{ss}) s h_{ss}(a,s) \\ - \; & \lambda(s)(\gamma_t^{\Gamma} - 1)\Gamma_{ss}^{IM} + a\left(\frac{1+i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}^{IM}}{P_{ss}} \frac{1+i_t^{IM}}{P_t^{IM}}\right) \end{aligned} \tag{31}$$

These payments do not depend on households decisions in the new equilibrium but are conditional on households decisions in the steady state. In particular, the household cannot affect those transfer by adjusting the saving behavior. Transfers also depend on productivity but this process is exogenous and thus also beyond the household's control.

As a comparison define the representative agent counterpart of Δ as

$$\begin{aligned} & \overline{\Delta}_t \\ = \; & (\gamma_t^C - 1)C_{ss} - \left(\gamma_t^H \gamma_t^w - 1\right) w_{ss}^{IM}(1-\tau_{ss}) H_{ss} \\ - \; & (\gamma_t^{\Gamma} - 1)\Gamma_{ss}^{IM} + A\left(\frac{1+i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}^{IM}}{P_{ss}} \frac{1+i_t^{IM}}{P_t^{IM}}\right), \end{aligned} \tag{32}$$

so that the difference makes the various redistributions clear:

$$
\begin{aligned}
& \Delta_t(a,s) - \overline{\Delta}_t \\
= & \; (\gamma_t^C - 1)(c_{ss}(a,s) - C_{ss}) \\
- & \; \left(\gamma_t^H \gamma_t^w - 1\right) w_{ss}^{IM}(1-\tau_{ss})(sh_{ss}(a,s) - H_{ss}) \\
- & \; (\lambda(s) - 1)(\gamma_t^\Gamma - 1)\Gamma_{ss}^{IM} \\
+ & \; (a - A)\left(\frac{1+i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}^{IM}}{P_{ss}} \frac{1+i_t^{IM}}{P_t^{IM}}\right)
\end{aligned} \tag{33}
$$

The individual specific transfer Δ has four components. The first component is the difference between the change in individual consumption and the change in aggregate consumption. Although the percentage change in consumption is the same for everyone, the absolute change in consumption is larger for high consumption households than for low consumption households. Thus, this transfer redistributes towards high consumption households if $\gamma_t^C > 1$. The second component is the difference between the change in individual labor income and the change in aggregate labor income. Note that $\gamma_t^H \gamma_t^w w_{ss} H_{ss}^{IM} = w_t^{IM} H_t^{IM}$ is aggregate labor income at time t during the transition. Although the percentage change in labor income is the same for everyone, $\gamma_t^H \gamma_t^w - 1$, the absolute change in labor income is larger for high labor income households than for low labor income households. Thus, this transfer redistributes towards high labor income households if $\gamma_t^H \gamma_t^w - 1 > 0$. The third component is redistribution through dividend and transfer receipts. Although the shares $\lambda(s)$ do not change, aggregate dividends and transfers do. The fourth component redistributes between asset holders. Note that an asset holder looses in the new equilibrium without transfers if the real interest rate $\frac{(1+i_t^{IM})P_{t-1}^{IM}}{P_t^{IM}}$ is lower than in the steady state, $1 + i_{ss}^{IM}$ (note that steady state inflation is zero). In this case the transfer redistributes towards households with assets above the average, $a - B_{ss} > 0$. Note, however, that in the first period when the nominal interest rate has not changed yet, the change in the price level alone determines who benefits and who looses. If, for example, the initial price level increases then asset rich households loose.

The next theorem shows that this transfer which for an expansionary monetary policy redistributes towards high-consumption, high labor income and high asset households renders the complete and the incomplete market economies identical in terms of aggregate variables.

Theorem 4.1 *Consider the complete market economy with aggregate consumption C_t^{CM}, aggregate hours H_t^{CM}, wages w_t^{CM}, inflation rates π_t^{CM} and nominal interest rates $1+i_t$ satisfying equations (17) - (20). The incomplete market economy with transfers $\Delta(a,s)$ as in (31) and the same nominal interest rate changes, $1+i_t = (1+i_{ss}^{IM})\frac{1+i_t}{1+i^*}$ has the same aggregate consumption, hours, wages and inflation rates as the complete markets case. Furthermore, individual consumption, hours, and savings satisfy*

$$
\begin{aligned}
c_t^{IM}(a,s) &= \gamma_t^C c_{ss}(a,s) &(34)\\
h_t^{IM}(a,s) &= \gamma_t^H h_{ss}(a,s) &(35)\\
a_{t+1}^{IM}(a,s) &= \frac{P_t}{P_{ss}} a_{ss}^{IM}(a,s), &(36)
\end{aligned}
$$

for a price sequence P_t. Real bonds are unchanged, $B_t = \frac{P_t}{P_{ss}} B_{ss}$ and transfers are adjusted to balance the government period-budget constraint.

The appendix shows that the average transfer $\int \Delta(a,s)d\Omega = \overline{\Delta}_t = 0$, that is Δ is just redistributing.

A simpler case obtains if we make some additional assumptions. We assume no government expenditures, $g = 0$, no fixed costs, $F = 0$, and the cost of price adjustments are as-if, so that consumption equals output, $C = Y$. We also assume that the labor tax is zero, $\tau = 0$, and that dividends are distributed proportional to wages, implying that $w_t s h_t + \lambda(s)\Gamma_t = Z_t s h_t$.

Then the transfer Δ simplifies to

$$
\begin{aligned}
& \Delta_t(a,s) & (37)\\
= \ & (\gamma_t^Y - 1)(c_{ss}(a,s) - Z_t s h_{ss}(a,s)) \\
+ \ & a\left(\frac{1+i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}^{IM}}{P_{ss}}\frac{1+i_t^{IM}}{P_t^{IM}}\right)
\end{aligned}
$$

Result 1 **[Special Case]** *If $g = 0$, $F = 0$, the cost of price adjustments are as-if, $\tau = 0$ and $w_t s h_t + \lambda(s)\Gamma_t = Z_t s h_t$, then the incomplete market economy with transfers*

$$
\begin{aligned}
& \Delta_t(a,s) & (38)\\
= \ & (\gamma_t^Y - 1)(c_{ss}(a,s) - Z_t s h_{ss}(a,s)) \\
+ \ & a\left(\frac{1+i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}^{IM}}{P_{ss}}\frac{1+i_t^{IM}}{P_t^{IM}}\right)
\end{aligned}
$$

has the same aggregate consumption, hours, wages and inflation rates as the complete markets *case. Furthermore, individual consumption, hours, and savings satisfy*

$$c_t^{IM}(a,s) = \gamma_t^C c_{ss}(a,s) \tag{39}$$

$$h_t^{IM}(a,s) = \gamma_t^H h_{ss}(a,s) \tag{40}$$

$$a_{t+1}^{IM}(a,s) = \frac{P_t}{P_{ss}} a_{ss}^{IM}(a,s). \tag{41}$$

A further special case - the autarky case, where the incomplete market allocation coincides with the complete market one - arises when households consumption always equals their current period labor income since in addition there is no government, i.e. no bond supply, no government spending and no taxes and transfers. This follows directly from Result 1 since in this case the transfers are zero

$$\Delta^{Autarky}(a,s) \equiv 0, \tag{42}$$

as $c_{ss}(a,s) = Z_t s h_{ss}(a,s)$ and $a = 0$.

Proposition 1 (Autarky-Equivalence) *Consider an autarky economy, $a = B = G = \tau_{ss} = 0$. In addition assume that $F = 0$ and the cost of price adjustments are as-if, then*

$$\Delta^{Autarky}(a,s) \equiv 0. \tag{43}$$

Aggregate consumption, hours, output and inflation in the complete and the incomplete market economy are identical without any additional transfers.

Werning (2015) obtains a similar result. He considers a zero liquidity economy, that is $a = B = 0$, implying that consumption equals income for every household. In addition he assumes that the ratio of individual to aggregate income to be constant for every household, which implies here that

$$\begin{aligned} & \left(\gamma_t^H \gamma_t^w - 1\right) w_{ss}^{IM}(1-\tau_{ss}) s h_{ss}(a,s) - \lambda(s)(\gamma_t^\Gamma - 1)\Gamma_{ss}^{IM} \\ = \quad & (\gamma_t^Y - 1) w_{ss}^{IM} s h_{ss}(a,s), \end{aligned} \tag{44}$$

so that $\Delta(a,s) = 0$ and Theorem 4.1 implies the equivalence of complete and incomplete markets, just as in Werning (2015).

In the results Section we will decompose the difference between the incomplete and complete markets case by shutting down the various components of individual specific transfers.

5 Quantitative Analysis

To quantitatively assess the effects of shocks as well as monetary and fiscal policies we now estimate the model. We calibrate a subset of the parameters and estimate the slopes of New Keynesian price and wage Philips curves to minimize the distance between the model and the empirical impulse response functions. The results of Boppart et al. (2018) imply that matching perfect foresight impulse resposes are an alternative method of linearization for impulse response matching.

5.1 Calibration

Preferences Households have separable preferences over labor and constant relative risk aversion preferences for consumption. We set the risk-aversion parameter, σ, equal to 1. Following Krueger et al. (2016), we assume permanent discount factor heterogeneity across agents. We allow for two values of the discount factor, which we choose along with the relative proportions to match the Gini of net worth net of home equity and the ratio of median and 30th percentile of networth net of home equity in the 2013 SCF, and aggregate savings to quarterly GDP of 11.46.[5] We assume the functional form for g:

$$g(h) = \psi \frac{h^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}. \tag{45}$$

We set the Frisch elasticity, $\varphi = 0.5$, following micro estimates. We choose $\psi = 0.6$ such that in steady state $h = 1/3$.

Productivity Process We follow Krueger et al. (2016) who use data from the Panel Study of Income Dynamics to estimate a stochastic process for labor productivity. They estimate that log income consists of a persistent and transitory component. They estimate that the persistent shock has an annual persistence of 0.9695 and variance of innovations of 0.0384.

[5] We calibrate to a capital to quarterly output ratio of 10.26, and government debt to quarterly GDP ratio of 1.2.

The transitory shock is estimated to have variance 0.0522. We follow Krueger et al. (2016) in converting these annual estimates into a quarterly process. We discretize the persistent shock into a seven state Markov chain using the Rouwenhorst method and integrate over the transitory shock using Gauss-Hertmite quadrature with three nodes.

Production Technology We set the capital share $\alpha = 0.36$. We choose the quarterly depreciation rate $\delta = 0.032$ to generate a real return on capital net of depreciation of 0 BP when the capital output ratio is 10.26. We asume the function form for Φ:

$$\Phi(K', K) = \frac{\phi_k}{2} \left(\frac{K' - K}{K} \right)^2 K, \tag{46}$$

and set $\phi_k = 17$ to match estimates of the elasticity of investment to Tobin's q from Eberly, Rebelo, and Vincent (2008). We choose the elasticity of substitution between intermediate goods, $\epsilon = 10$, to match an average markup of 10%. We set the firm operating cost Φ equal to the steady state markup such that steady state profits equal 0 (Basu and Fernald, 1997). These profits are fully taxed and are distributed to households as lump-sum transfers.

Fiscal and Monetary Policies We set the proportional labor income tax, τ equal to 25%. We set nominal government spending, G in steady state equal to 6% of output (Brinca et al., 2016). The value of of lump-sum transfers T is set to 8.55% of output such that roughly 40% of households receive a net transfer from the government (Kaplan et al., 2016). The monetary authority operates a constant interest rate peg of $i = 0$.

5.2 Estimation

We estimate the slopes of New Keynesian price and wage Philips curves by matching the impulse response functions documented in Section 2. Starting from the steady state, we feed in the shocks and the dynamic response of monetary and fiscal policies as measured in the data in Figure 1 and estimate the degree of price an wage rigidities, θ_p and θ_w, to best match the dynamic responses of output, hours worked, prices and wages described in Figure 2. The estimated parameter values are summarized in Table II.

Table I: Calibrated Parameters

Parameter	Interpretation	Internally Calibrated	Value
σ	Risk-aversion	N	1
β_i	Discount Factors	Y	(0.9994,0.9929)
φ	Frisch Elasticity	N	0.5
ψ	Labor disutility	Y	0.6
ϵ	Elas. substitution intermediates	N	10
ϵ_w	Elas. substitution labor	N	10
F	Firm Fixed Cost/GDP	Y	0.1
τ	Labor tax	N	25%
T	Transfer/GDP	Y	8.55%

Table II: Estimated Parameters

Parameter	Interpretation	Value
θ_p	Price adjustment	1000
θ_w	Wage adjustment	1000

5.3 Steady State Model Fit

Before describing the dynamic properties of the model in response to shock and policy changes, we note that in the steady state the model features reasonable distributional properties. Specifically, 3% of agents have 0 wealth, and 10% of agents less than $1000. The annual MPC out of transitory income equals 0.46, which is in the middle range of empirical estimates 0.2-0.6 (e.g., Johnson et al., 2006).[6]

5.4 Results, Technology Shock

Taking the impulse responses of monetary and fiscal policies as given, the estimated model delivers a fairly good fit to the dynamic responses of output, hours worked, prices and wages as described in Figure 5, which superimposes impulse responses to a neutral technology shock in the model on those measured above in the data. Of course, these responses were targeted in estimation. Nevertheless, it is quite remarkable that a parsimonious AiyaGalí model proposed in this paper can match the data to such an extent.

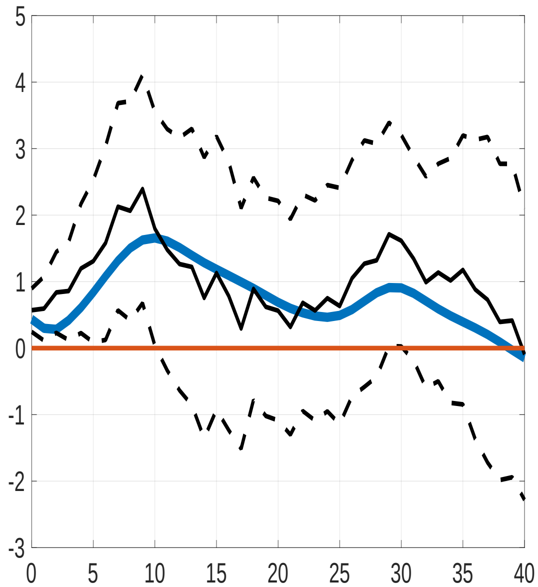
[6] We compute the annual MPC using the quarterly MPC via the formula: $MPC_a = 1 - (1 - MPC_q)^4$. The quarterly MPC in the model is 0.144.

We now use the estimated model to disentangle the effects of fiscal and monetary policy responses by counterfactually shutting them down. In Figure 6 we shut down both fiscal and monetary policy responses to an improvement in neutral technology. The resulting impulse responses in the model are far from those observed in the data. Hours worked now fall in the short run and output increases very gradually over time. Price level exhibits a very persistent decline.

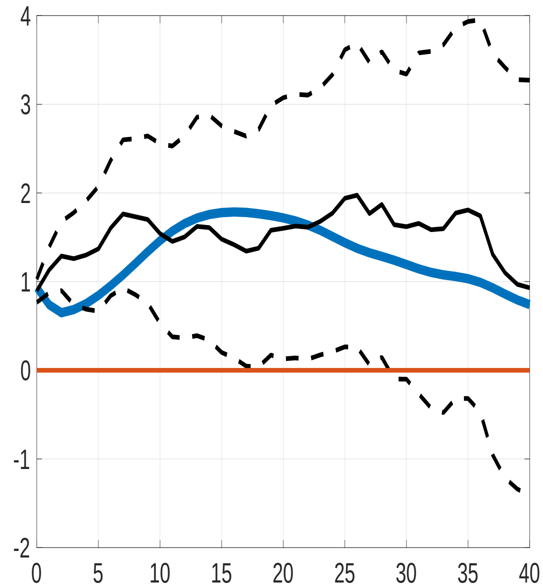
In Figure 7 we switch on the response of monetary policy described in Panel d of Figure 1. As monetary policy responds to a technology improvement by raising nominal interest rates in the short to medium run, the combined response to the shock and monetary policy response is quite contractionary, with a large decline in output and hours in the short run. A drop in interest rates that begins approximately 10 quarters after the technology shock leads to a faster growth in output relative to the experiment in which all policy responses were eliminated.

In Figure 8 we once again eliminate the responce of the monetary policy but switch on the response of fiscal policy as described in Panels a, b, and c of Figure 1. This experiment highlights the powerful effects of fiscal policy response in reinforcing the effects of a technological improvement. The increase in spending and transfers are both highly expansionary. It is the response of these variables that is largely responsible for a substantial increase in hours worked and in output following the improvement in technology. Although, as we have seen above, the monetary policy response partially counteracts these effects in the short run, the increase in the nominal interest rates is too small to offset them completely. Similarly, the long run increase in the price level is entirely due to the fiscal policy response.

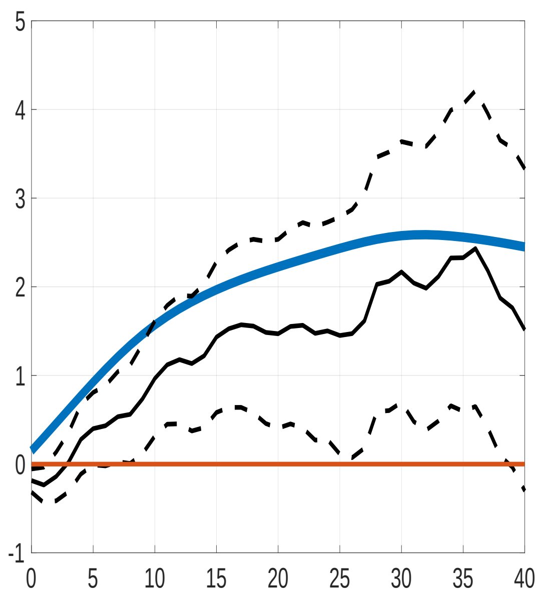
Figure 5: Results, Technology Shock + FP and MP Response



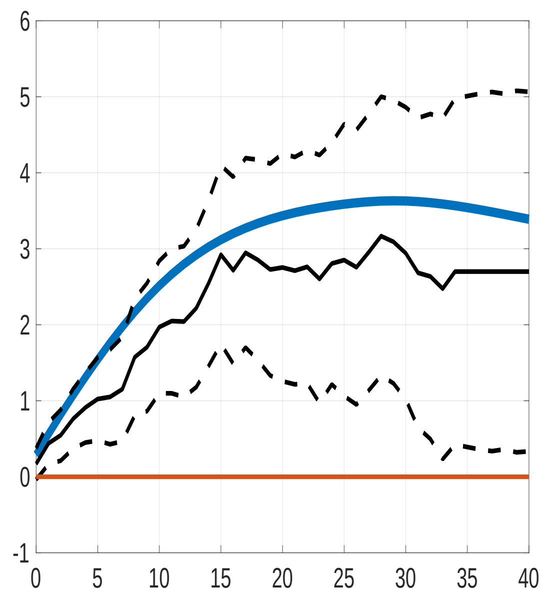
(a) Hours



(b) Output

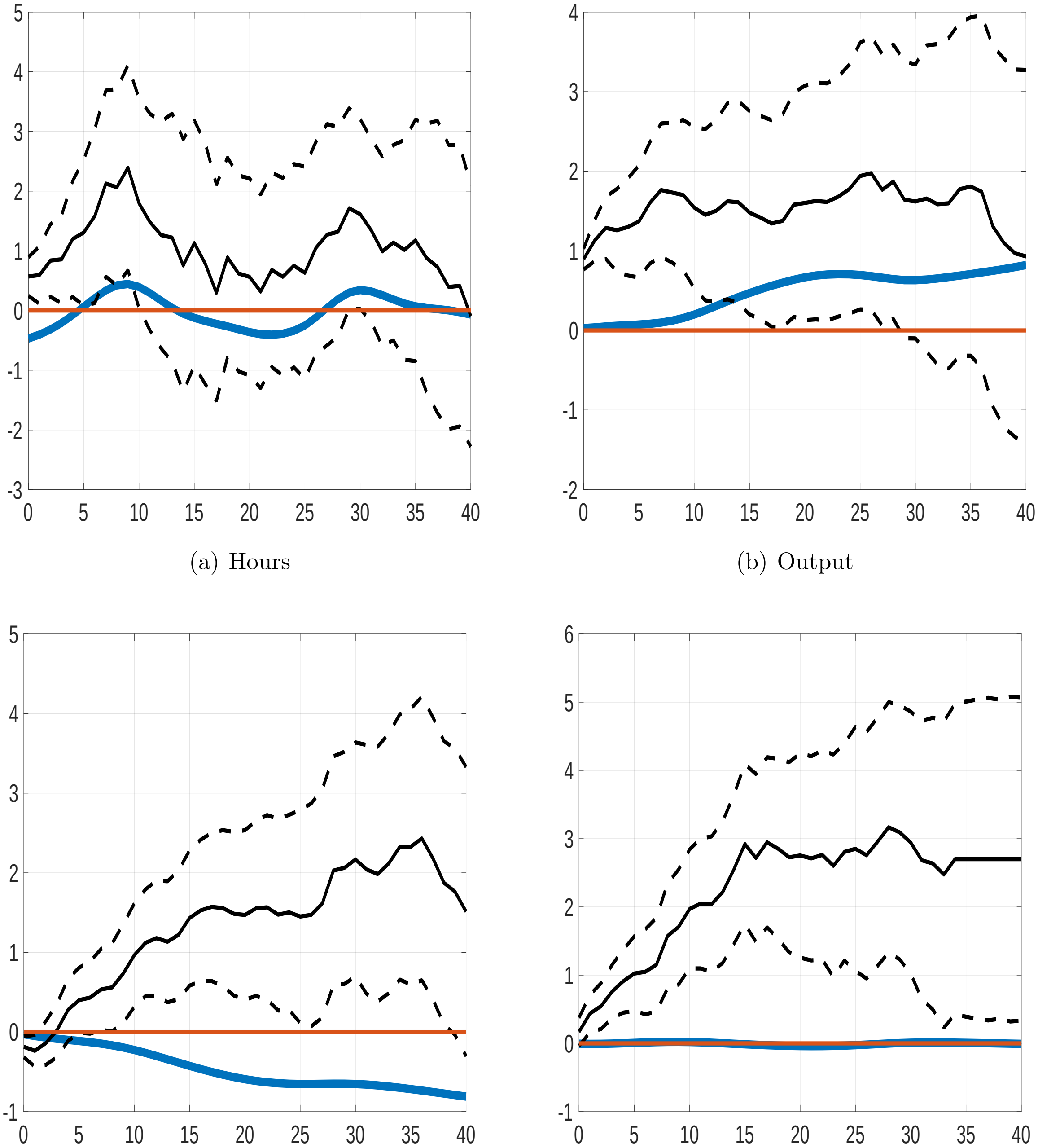


(c) Price Level



(d) Wages

Figure 6: Results, Technology Shock + No Policy Response



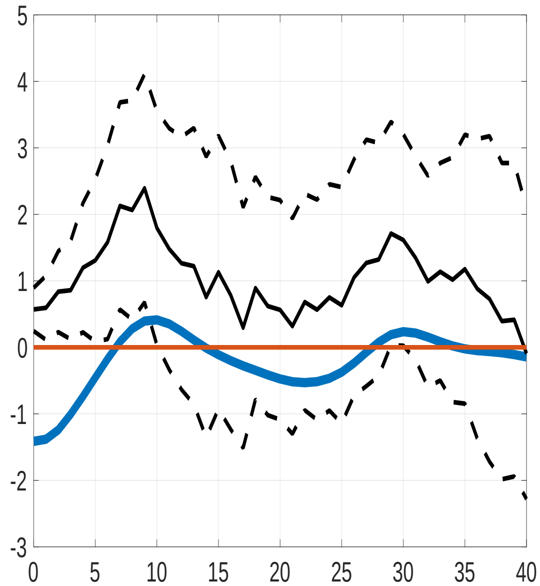
(a) Hours

(b) Output

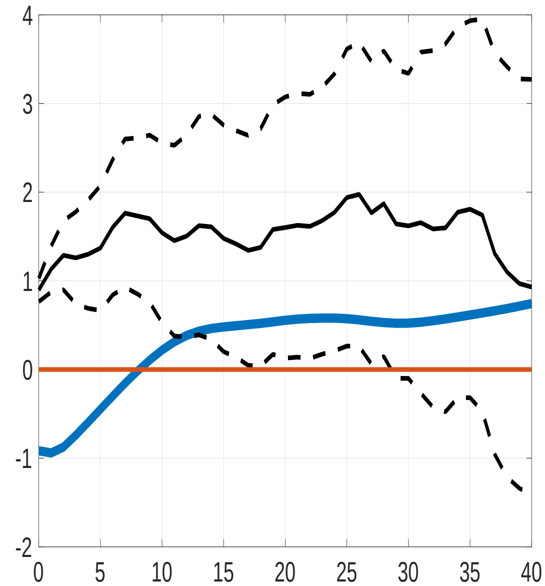
(c) Price Level

(d) Wages

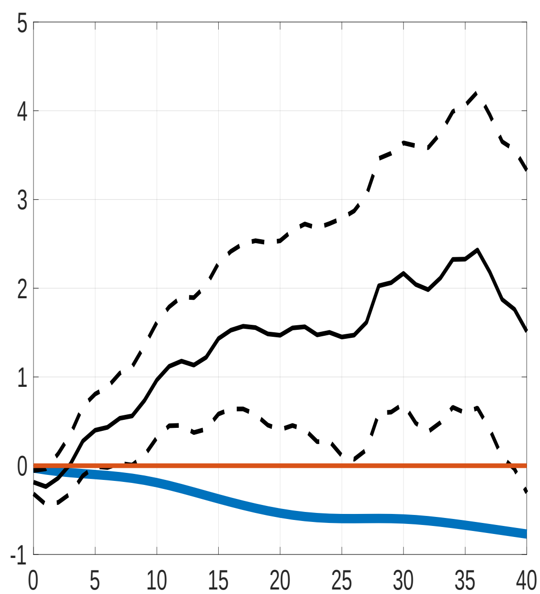
Figure 7: Results, Technology Shock + Only MP response



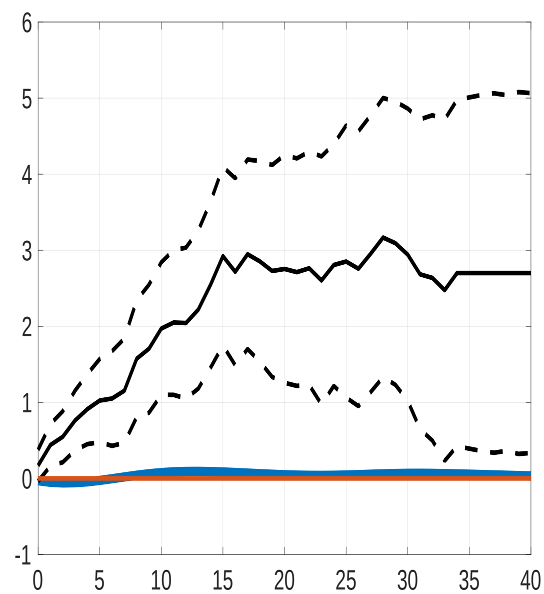
(a) Hours



(b) Output

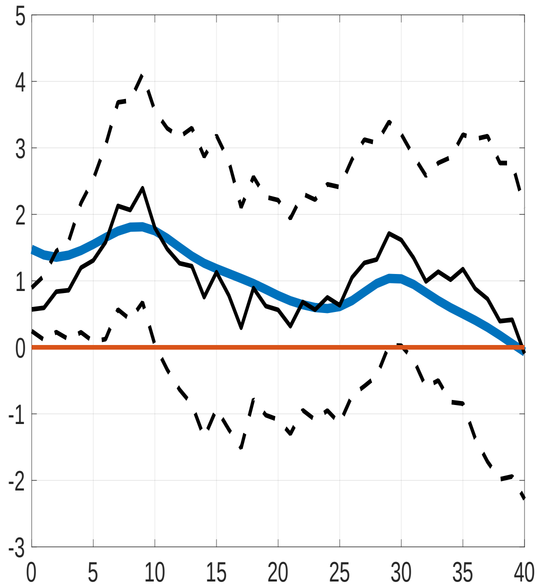


(c) Price Level

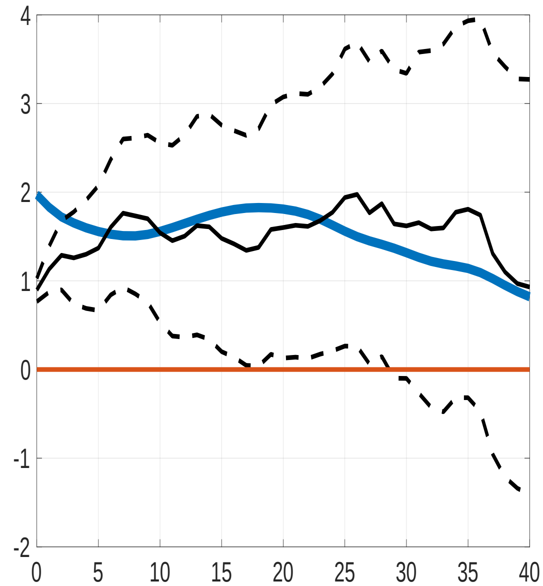


(d) Wages

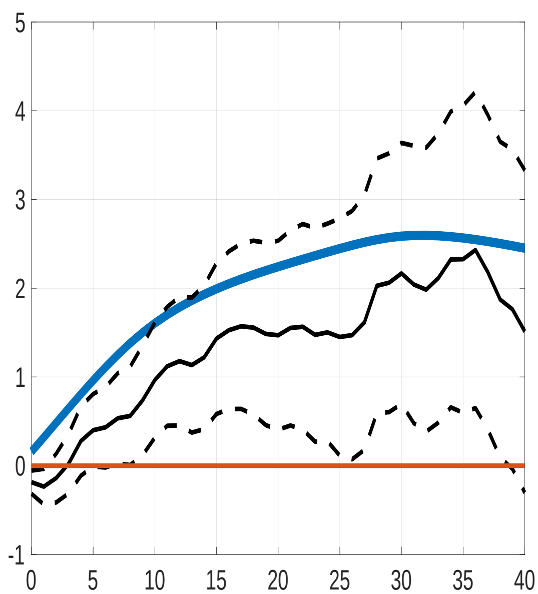
Figure 8: Results, Technology Shock + Only FP response



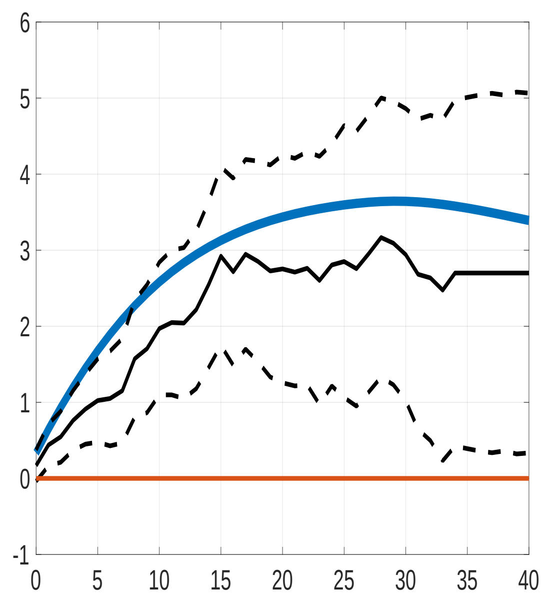
(a) Hours



(b) Output



(c) Price Level



(d) Wages

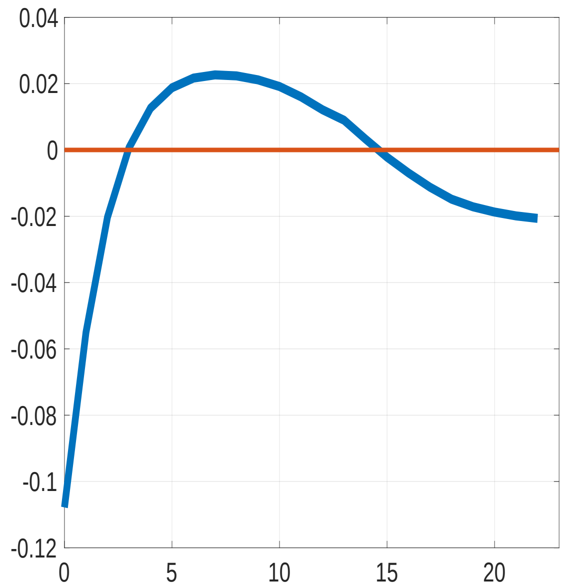
5.5 Results, Monetary Policy Shock

We now consider the effects of a monetary policy shock in the estimated model. We model this shock as a .25pp nominal interest rate increase (with quarterly persistence of .6).

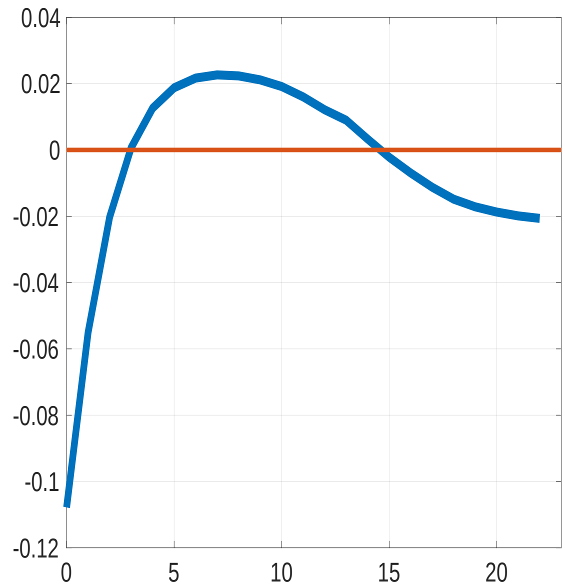
We have seen that increases in interest rates are mildly expansionary in the data. Figure 9 indicates that the same holds in the estimated model albeit with a lag of several quarters after the shock. While perhaps puzzling at first sight, the economics behind this effect is simple. Even if fiscal policy is entirely passive, changes in monetary policy necessarily impact the government budget constraint and induce changes in fiscal variables. For example, a contractionary monetary policy lowers government revenues (holding tax rates fixed). To finance the same level of government spending and transfers, the government must issue new debt. As discussed above, in incomplete market models with nominal government bonds the price level and the inflation rate are both determinate for any given sequences of nominal interest rates. The increase in nominal bonds must then be accompanied by an increase in the price level. This lowers the real interest rate and stimulates private spending. In other words, the increase of government debt is expansionary. This counteracts the effects of a contractionary monetary policy and turns out to more than offset it a few quarters after a monetary policy shock.

This logic is further illustrated in Figure 10, where we report the response to the same monetary policy shock while holding the effects of fiscal policy fixed. To do so, we do not allow nominal government debt to increase following a contractionary monetary policy by reducing transfers to keep the government budget balanced. As expected, with this fiscal environment in place, raising interest rates is indeed contractionary at all horizons. Price level, output, and hours all fall significantly more relative to the benchmark scenarion where nominal government debt (implicitly) was allowed to increase.

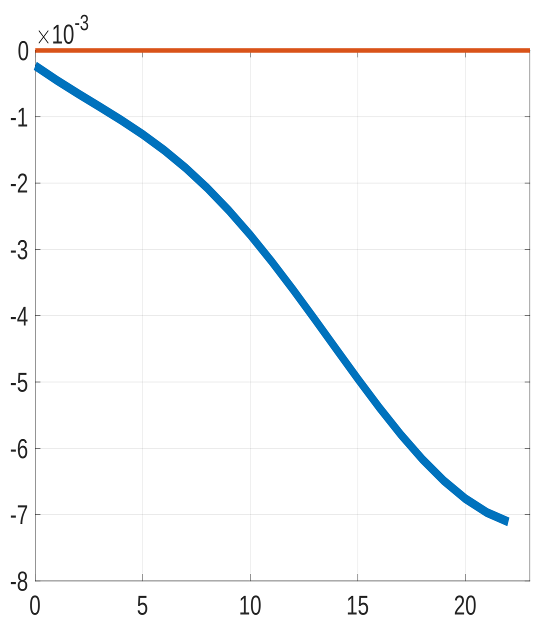
Figure 9: Results, Monetary Policy Shock



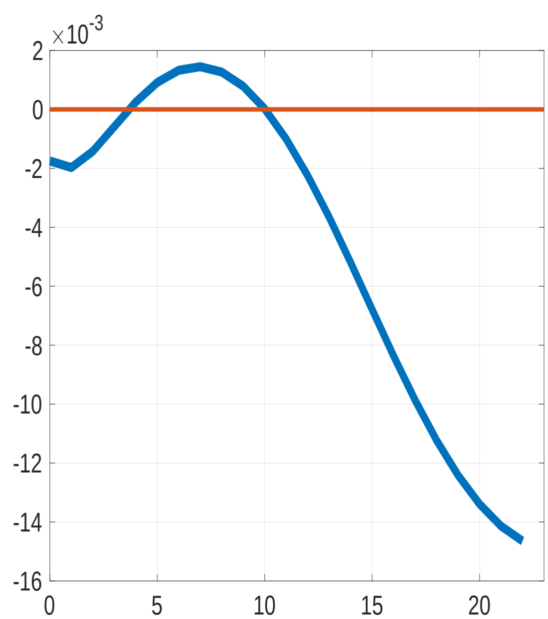
(a) Hours



(b) Output

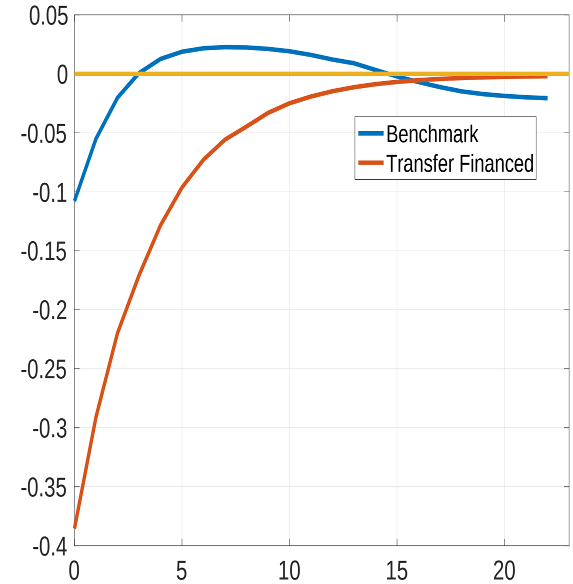


(c) Price Level

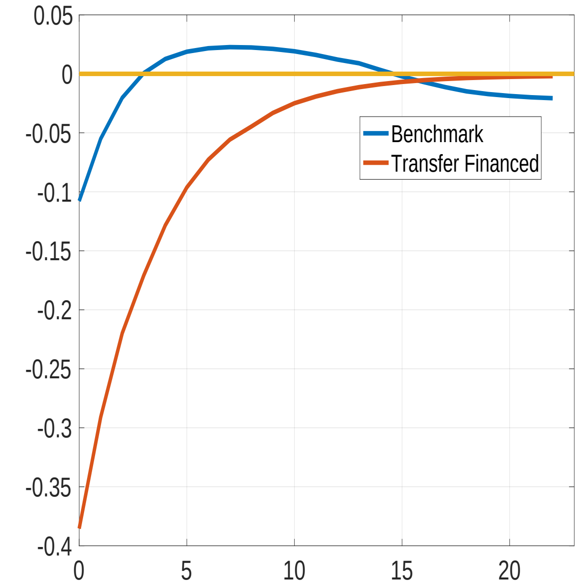


(d) Wages

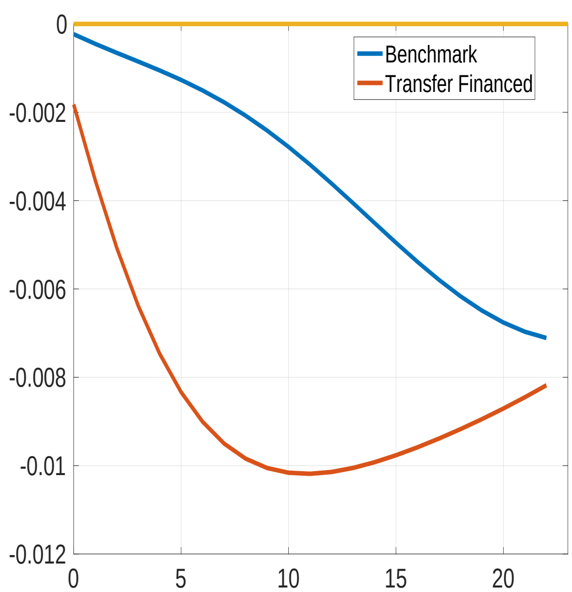
Figure 10: Results, MP Shock + No FP Response



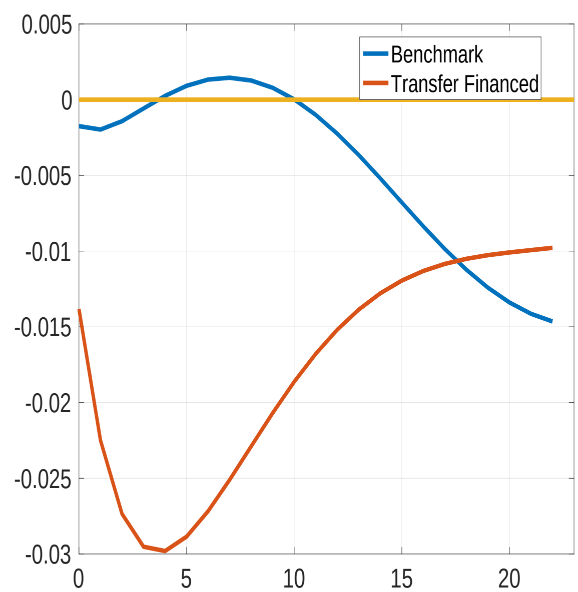
(a) Hours



(b) Output



(c) Price Level



(d) Wages

6 Conclusion

A novel framework that has holds significant promise for understanding the effects of monetary policy combines New Keynesian nominal rigidities with incomplete markets. We study theoretically the role of incomplete markets and of fiscal policy in shaping the effects of monetary policy. We then estimate the model to enable a quantitaive analysis of the effects of monetary policy and the channels through which it operates. The key challenge, as well as the key quantitative insight in the paper, is that because of the failure of Ricardian equivalence monetary and fiscal policies are deeply intertwined and respond to each other, and to other shocks affecting the economy. We document these relationships in the data and exploit them to estimate the model. We find that our fairly parsimonious model fits the dynamic responses of the economy to shocks quite well. The analysis reveals that the macroeconomic impact of monetary policy is crucially affected by induced fiscal policy changes. Ignoring this policy interaction would lead to a severely biased assessment of monetary policy impacts and the mechanisms through which they operate.

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APPENDICES

I Derivations and Proofs

I.1 Derivation Pricing Equation

The firm's pricing problem is

$$V_t\left(p_{lt-1}\right) \equiv \max_{p_{lt}} \frac{p_{lt}}{P_t} y\left(p_{lt}; P_t, Y_t\right) - S_t(Y_{lt}) - \frac{\theta}{2}\left(\frac{p_{lt}}{p_{lt-1}} - \overline{\Pi}\right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}\left(p_{lt}\right),$$

subject to the constraints $y_{lt} = Z_t K_{lt}^{\alpha} H_{lt}^{1-\alpha}$ and $y(p_{lt}; P_t, Y_t) = \left(\frac{p_{lt}}{P_t}\right)^{-\epsilon} Y_t$. Equivalently,

$$V_t\left(p_{lt-1}\right) \equiv \max_{p_{lt}} \frac{p_{lt}}{P_t}\left(\frac{p_{lt}}{P_t}\right)^{-\epsilon} Y_t - S_t(Y_{lt}) - \frac{\theta}{2}\left(\frac{p_{lt}}{p_{lt-1}} - \overline{\Pi}\right)^2 Y_t + \frac{1}{1+r_t} V_{t+1}\left(p_{lt}\right).$$

The FOC w.r.t p_{jt} is

$$(1-\epsilon)\left(\frac{p_{lt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon m c_{lt} - \theta\left(\frac{p_{lt}}{p_{lt-1}} - \overline{\Pi}\right) \frac{Y_t}{p_{lt-1}} + \frac{1}{1+r_t} V_{t+1}'(p_{lt}) = 0 \tag{A1}$$

and the envelope condition is

$$V_{t+1}' = \theta\left(\frac{p_{lt+1}}{p_{lt}} - \overline{\Pi}\right) \frac{p_{lt+1}}{p_{lt}} \frac{Y_{t+1}}{p_{lt}}. \tag{A2}$$

Combining the FOC and and the envelope condition,

$$\begin{aligned} & (1-\epsilon)\left(\frac{p_{lt}}{P_t}\right)^{-\epsilon} \frac{Y_t}{P_t} + \epsilon m c_{lt} \\ - \quad & \theta\left(\frac{p_{lt}}{p_{lt-1}} - \overline{\Pi}\right) \frac{Y_t}{p_{lt-1}} + \frac{1}{1+r_t} \theta\left(\frac{p_{lt+1}}{p_{lt}} - \overline{\Pi}\right) \frac{p_{lt+1}}{p_{lt}} \frac{Y_{t+1}}{p_{lt}} = 0. \end{aligned} \tag{A3}$$

Using that all firms choose the same price in equilibrium,

$$\begin{aligned} & (1-\epsilon) + \epsilon m c_t \\ - \quad & \theta\left(\pi_t - \overline{\Pi}\right) \pi_t + \frac{1}{1+r_t} \theta\left(\pi_{t+1} - \overline{\Pi}\right) \pi_{t+1} \frac{Y_{t+1}}{Y_t} = 0. \end{aligned} \tag{A4}$$

I.2 Derivation Wage Equation

$$\Theta\left(s_{jt}, W_{jt}, W_{jt-1}; Y_t\right) = s_{jt}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t.$$

The middleman's wage setting problem is to maximize

$$\begin{aligned}
& V_t^w\left(\hat{W}_{t-1}\right) \\
\equiv\ & \max_{\hat{W}_t} \int \Big(\frac{s_{jt}(1-\tau_t)\hat{W}_t}{P_t} h(\hat{W}_t; W_t, H_t) - s_{jt} g(h(\hat{W}_t; W_t, H_t)) dj - \int s_{jt}\frac{\theta_w}{2}\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi}^w\right)^2 H_t dj \\
+\ & \frac{1}{1+r_t} V_{t+1}^w\left(\hat{W}_t\right),
\end{aligned} \tag{A5}$$

where $h_{jt} = h(W_{jt}; W_t, H_t) = \left(\frac{W_{jt}}{W_t}\right)^{-\epsilon_w} H_t$.

The FOC w.r.t $\hat{W}_t$

$$(1-\tau_t)(1-\epsilon_w)\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w}\frac{H_t}{P_t} + \epsilon_w g'(h(\hat{W}_t; W_t, H_t))\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w - 1}\frac{H_t}{W_t} \tag{A6}$$

$$-\theta_w\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi^w}\right)\frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1+r_t}V'_{t+1}(\hat{W}_t) = 0 \tag{A7}$$

and the envelope condition

$$V'_{t+1} = \theta_w\left(\frac{\hat{W}_{t+1}}{\hat{W}_t} - \overline{\Pi^w}\right)\frac{\hat{W}_{t+1}}{\hat{W}_t}\frac{H_{t+1}}{\hat{W}_t}, \tag{A8}$$

where we have used that $\int s = 1$.

Combining the FOC and and the envelope condition

$$\begin{aligned}
& (1-\tau_t)(1-\epsilon_w)\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w}\frac{H_t}{P_t} + \epsilon_w g'(h(\hat{W}_t; W_t, H_t))\left(\frac{\hat{W}_t}{W_t}\right)^{-\epsilon_w - 1}\frac{H_t}{W_t} \\
- \ & \theta_w\left(\frac{\hat{W}_t}{\hat{W}_{t-1}} - \overline{\Pi^w}\right)\frac{H_t}{\hat{W}_{t-1}} + \frac{1}{1+r_t}\theta_w\left(\frac{\hat{W}_{t+1}}{\hat{W}_t} - \overline{\Pi^w}\right)\frac{\hat{W}_{t+1}}{\hat{W}_t}\frac{H_{t+1}}{\hat{W}_t} = 0
\end{aligned} \tag{A9}$$

Using that $\hat{W}_t = W_t$, $\pi_t^w = \frac{W_t}{W_{t-1}} = \frac{\hat{W}_t}{\hat{W}_{t-1}}$ and $h_{jt} = H_t$:

$$\begin{aligned} & (1-\tau_t)(1-\epsilon_w)\frac{W_t}{P_t} + \epsilon_w g'(h(\hat{W}_t; W_t, H_t)) \\ - & \theta_w \left(\pi_t^w - \overline{\Pi^w}\right)\pi_t^w + \frac{1}{1+r_t}\theta_w\left(\pi_{t+1}^w - \overline{\Pi^w}\right)\pi_{t+1}^w \frac{H_{t+1}}{H_t} = 0 \end{aligned} \tag{A10}$$

Proof of Theorem 4.1 To prove the theorem we have to show that the conjectures allocation satisfies all equilibrium restrictions, that is the consumption Euler equation, household labor supply, households' and government's budget constraints.

Average Transfer Δ

First we show that the average transfer is indeed zero:

$$\int\limits_{a,s} \Delta_t(a,s) d\Omega_t(a,s) \tag{A11}$$

$$= (\gamma_t^C - 1)\int\limits_{a,s} c_{ss}(a,s) d\Omega_t(a,s) \tag{A12}$$

$$- \left(\gamma_t^H \gamma_t^w - 1\right) w_{ss}^{IM}(1-\tau_{ss}) \int\limits_{a,s} s h_{ss}^{IM}(a,s) d\Omega_t(a,s) \tag{A13}$$

$$- \underbrace{\int \lambda(s) d\Omega_t(a,s)}_{=1} (\gamma_t^\Gamma - 1)\Gamma_{ss}^{IM} \tag{A14}$$

$$+ \left(\frac{1+i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}}{P_{ss}}\frac{1+i_t^{IM}}{P_t})\right) \int\limits_{a,s} d\Omega_t(a,s) \tag{A15}$$

$$= (\gamma_t^C - 1)C_{ss} - \left(\gamma_t^H \gamma_t^w - 1\right) w_{ss}^{IM}(1-\tau_{ss}) H_{ss}^{IM} \tag{A16}$$

$$- \left(d_t^{IM} + \tau w_t^{IM} H_t^{IM} + \frac{B_{t+1}^{IM} - B_t^{IM}(1+i_t^{IM})}{P_t^{IM}} - g_t^{IM}\right), \tag{A17}$$

$$+ \left(d_{ss}^{IM} + \tau w_{ss}^{IM} H_{ss}^{IM} + \frac{B_{ss} - B_{ss}(1+i_{ss}^{IM})}{P_{ss}^{IM}} - g_{ss}\right) \tag{A18}$$

$$+ \left(\frac{1+i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}}{P_{ss}}\frac{1+i_t^{IM}}{P_t})\right) B_{ss} \tag{A19}$$

$$= (\gamma_t^C - 1)C_{ss} - \left(\gamma_t^H \gamma_t^w - 1\right) w_{ss}^{IM} H_{ss}^{IM} + d_t^{IM} - d_{ss}^{IM} + (g_t^{IM} - g_{ss}) \tag{A20}$$

$$= (C_t^{IM} - C_{ss}^{IM}) - (Y_t^{IM} - Y_{ss}^{IM}) + (g_t^{IM} - g_{ss}) \tag{A21}$$

$$= 0. \tag{A22}$$

Government Budget Constraint and Transfers

The amount of transfers are set such that the government budget constraint holds during the transition and in the steady state:

$$T_t^{IM} = \tau P_t^{IM} w_t^{IM} H_t^{IM} + B_{t+1}^{IM} - B_t^{IM}(1+i_t^{IM}) - G_t^{IM} \tag{A23}$$
$$T_{ss}^{IM} = \tau P_{ss}^{IM} w_{ss}^{IM} H_{ss}^{IM} + B_{ss} - B_{ss}(1+i_{ss}^{IM}) - G_{ss}. \tag{A24}$$

<u>Household Budget Constraints</u>

The conjectured allocation of consumption and hours also satisfy the households budget constraints without changing the amount of individual real savings, that is $a_{t+1}^{IM}(a,s) = \frac{P_t^{IM}}{P_{ss}^{IM}} a_{ss}^{IM}(a,s)$. In a steady state, the budget constraint for a household with a assets and productivity e is

$$c_{ss}^{IM}(a,s) + \frac{a_{ss}^{IM}(a,s)}{P_{ss}^{IM}} = \frac{(1+i_{ss})a}{P_{ss}^{IM}} + (1-\tau_{ss})w_{ss}^{IM} h_{ss}^{IM} s + \lambda(s)\Gamma_{ss}^{IM} \tag{A25}$$

and the transfer is

$$\begin{aligned}\Delta_t(a,s) &= (\gamma_t^C - 1)c_{ss}(a,s) - \left(\gamma_t^H \gamma_t^w - 1\right) w_{ss}^{IM}(1-\tau_{ss}) s h_{ss}(a,s) \\ &- \lambda(s)(\gamma_t^\Gamma - 1)\Gamma_{ss}^{IM} + a\left(\frac{1+i_{ss}^{IM}}{P_{ss}} - \frac{P_{t-1}^{IM}}{P_{ss}} \frac{1+i_t^{IM}}{P_t^{IM}}\right).\end{aligned} \tag{A26}$$

For the same household, the budget constraint is now

$$\begin{aligned}
& P_t^{IM} c_t^{IM}(a,s) + a_{t+1}^{IM}(a,s) && \text{(A27)}\\
=\ & P_t \gamma_t^C c_{ss}^{IM}(a,s) + \frac{P_t^{IM}}{P_{ss}^{IM}} a_{ss}(a,s) && \text{(A28)}\\
=\ & P_t^{IM}\left\{(\gamma_t^C - 1)c_{ss}^{IM}(a,s) + c_{ss}^{IM}(a,s) + \frac{a_{ss}^{IM}(a,s)}{P_{ss}^{IM}}\right\} && \text{(A29)}\\
=\ & P_t^{IM}\{(\gamma_t^C - 1)c_{ss}^{IM}(a,s) + \frac{(1+i_{ss})a}{P_{ss}^{IM}} + (1-\tau_{ss})w_{ss}^{IM} h_{ss}^{IM} s + \lambda(s)\Gamma_{ss}^{IM}.\} && \text{(A30)}\\
=\ & P_t^{IM}\{\Delta(a,s) + \gamma_t^H \gamma_t^w (1-\tau_{ss}) w_{ss}^{IM} s h_{ss}^{IM}(a,s) + \frac{P_{t-1}^{IM}}{P_{ss}^{IM}} \frac{1+i_t^{IM}}{P_t^{IM}} a + \lambda(s)\Gamma_t^{IM}\} && \text{(A31)}\\
=\ & P_t^{IM}\Delta(a,s) + P_t^{IM} w_t^{IM} s(1-\tau_{ss}) h_t^{IM}(a,s) + (1+i_t^{IM}) a_t^{IM}(a,s) + \lambda(s)\Gamma_t^{IM}, && \text{(A32)}
\end{aligned}$$

proving that the flow budget constraint is satisfied. Note that since the real value of transfers is kept constant, the nominal transfer has to change by $\frac{P_t}{P_{ss}}$. The third equality uses that the budget constraint is satisfied in a steady state, the fourth equality uses the definition of Δ, and the last equality uses the definition of h_t^{IM}, c_t^{IM} and a_t^{IM}.

<u>Consumption Euler Equation</u>

The conjectured allocation also satisfies the consumption Euler equation since the CM economy implies that

$$(\frac{\gamma_{t+1}^C}{\gamma_t^C})^{-\sigma} = \frac{(\frac{C_{t+1}^{CM}}{C_{ss}^{CM}})^{-\sigma}}{(\frac{C_t^{CM}}{C_{ss}^{CM}})^{-\sigma}} = \frac{1+r_{ss}^{CM}}{1+r_{t+1}^{CM}} = \frac{1+r_{ss}^{IM}}{1+r_{t+1}^{IM}}. \tag{A33}$$

For households (a,s) who are credit constrained in the steady state at period t are still constrained so that their consumption Euler equation holds with inequality. For non-constrained households

$$c_t^{IM}(a,s)^{-\sigma} = (\gamma_t^C c_{ss}^{IM}(a,s))^{-\sigma} = (\gamma_t^C)^{-\sigma}\beta(1+r_{ss}^{IM})E_t(c_{ss}^{IM}(a',s))^{-\sigma} \tag{A34}$$

$$= (\gamma_{t+1}^C)^{-\sigma}\beta(1+r_{t+1}^{IM})E_t(c_{ss}^{IM}(a',s))^{-\sigma} = \beta(1+r_{t+1}^{IM})E_t(c_{t+1}^{IM}(a',s))^{-\sigma}, \tag{A35}$$

where we have used that the derivation for CM implies that $(1+r_{ss}^{IM})(\gamma_t^C)^{-\sigma} = (1+r_{t+1}^{IM})(\gamma_{t+1}^C)^{-\sigma}$

Labor Supply

Households are on their labor supply curve in the CM both during the transition and in the steady state

$$\gamma_t^w = \frac{w_t^{CM}}{w_{ss}^{CM}} = (\frac{H_t^{CM}}{H_{ss}^{CM}})^{\phi}(\frac{C_t^{CM}}{C_{ss}^{CM}})^{\sigma} = (\gamma_t^H)^{\phi}(\gamma_t^C)^{\sigma}, \tag{A36}$$

implying that households are also on the labor supply curve during the transition in the IM economy:

$$s w_t^{IM}(1-\tau_{ss})(c_t^{IM}(a,s))^{-\sigma} = s\gamma_t^w w_{ss}^{IM}(1-\tau_{ss})(\gamma_t^C c_{ss}^{IM}(a,s))^{-\sigma} \tag{A37}$$

$$= \gamma_t^w(\gamma_t^C)^{-\sigma}B(h_{ss}^{IM}(a,s))^{\phi} = \gamma_t^w(\gamma_t^C)^{-\sigma}(\gamma_t^H)^{-\phi}B(h_t^{IM}(a,s))^{\phi} = B(h_t^{IM}(a,s))^{\phi} \tag{A38}$$

since $\gamma_t^w(\gamma_t^C)^{-\sigma}(\gamma_t^H)^{-\phi} = 1$.

Investment and Capital Stock

The investment sector is the same in the complete and incomplete markets model and therefore the allocations necessarily coincide.

This completes the proof.