**Paper:** about effect of uncertainty on investment and productivity with relational contracts

- **issue:** how to reconcile evidence that effect is adverse with traditional theory that, with risk-neutral agents, would not be

**Recent literature:** focussed on option value of not making irreversible investments (Dixit-Pindyck, 1994)

- gives rise to adjustment, not long-run equilibrium, effect
  - calibration in Bloom (*Ecta*, 2009) has most of adjustment in 3 years
  - now that > 10 years after 2007 crisis, know not what happened
  - Bloom *et al* (*Ecta*, 2018) find need (unappealing) negative mean productivity shock to capture, not just increase in uncertainty

**With relational contracts:** greater uncertainty affects long-run equilibrium investment

- calibration with parameters of Bloom *et al* (*Ecta*, 2018) can generate:
  - decrease in general investment in data with just greater uncertainty
  - increase in specific investment: of interest because Bloom (*JEP*, 2014) comments that some investments increase with greater uncertainty
With each recession (shaded area), investment drops, then starts to grow but not back to previous path (as would with irreversible investment):
The model: key elements

- **MacLeod & Malcomson (Econometrica, 1989)** plus:
  - **uncertainty:** productivity of “effort” has iid shock each period;
  - **investment in capital:** can enhance productivity of relationship.

- **Principal’s payoff** in period $t$ conditional on being matched:
  \[ y(e_t, K, \theta_t) - W_t, \]  
  where $W_t$ is payment to agent and:
  - $y(e_t, K, \theta_t)$: (non-contractible) output at $t$;
  - $\theta_t \in [\theta, \bar{\theta}]$: iid random variable distributed $F(\theta, \sigma)$, with $dF(\theta, \sigma) > 0$ for all $\theta \in [\theta, \bar{\theta}]$, parameterized by $\sigma$ and observed by both parties at start of period $t$;
  - $e_t \in [0, \bar{e}]$: agent’s non-contractible effort at $t$, chosen after $\theta_t$ known;
  - $K \in [0, \bar{K}]$: capital investment at start of relationship at cost $C(K)$.

- **Agent’s payoff** in period $t$ conditional on being matched:
  \[ W_t - c(e_t), \]  
  where $c(e_t)$ is increasing and convex cost of effort.

- **Payoffs if unmatched:** principal $v(K) \geq 0$, agent $u(K) \geq 0$, with $s(K) := u(K) + v(K) > 0$, for all $K \in [0, \bar{K}]$.

- **Discount factor** for both parties $\delta$. 
Key result

Effort cannot be enforced contractually in court because output and effort non-contractible

- so limited to what is in current interest of both parties

Proposition

An effort schedule $e(K, \theta)$ that generates expected joint payoff $S(K, \sigma)$ each period with capital stock $K$ can be implemented by a stationary contract if and only if

$$
\frac{\delta}{1 - \delta} [S(K, \sigma) - s(K, \sigma)] \geq c(e(K, \theta)), \quad \text{for all } \theta \in [\theta, \bar{\theta}] .
$$

- $S(K, \sigma)$: joint (principal + agent) payoff from one period before shock $\theta$ realized given $K$ and $\sigma$:
  - irrelevant how divided between principal and agent.
- (1) requires joint payoff gain from future exceeds cost of effort now.
Implications of key result

Key equation is

$$\frac{\delta}{1 - \delta} [S(K, \sigma) - s(K, \sigma)] \geq c(e(K, \theta)), \quad \text{for all } \theta \in [\theta, \bar{\theta}]. \quad (2)$$

- With iid shocks, left-hand side is independent of current $\theta$
  - first-best effort $e^*(K, \theta)$ is increasing in $\theta$
  - if constraint not binding for $\theta$, implement first-best effort
  - so: if constraint binding for $\theta = \tilde{\theta}$, it is certainly binding for all higher $\theta$.

- Implication: binding constraint restricts how much can adjust to $\theta$
  - if without constraint joint payoff is linear in $\theta$ (risk neutrality)
  - with constraint, joint payoff is strictly concave
    - so: make general investment choice as if risk averse
  - specific investment relaxes constraint (2) because increases joint payoff from future so greater uncertainty may increase return to it.
Illustration of effort constraint

- Thin line: first-best effort for given capital stock
- Dotted line: highest effort sustainable given total future payoff gain
- Thick line: optimal effort with relational contract
- Use term *cutoff shock* for shock above which effort with relational contract constrained
  - equals 1 in case illustrated in figure.
Functional forms for calibration

- As literature, Cobb-Douglas production and iso-elastic cost functions:
  \[ y(e, K, \theta) = \theta^\gamma K^\alpha e^\beta, \quad \alpha, \beta, \gamma > 0, \quad \alpha + \beta \leq 1; \]
  \[ c(e) = ce^n, \quad c > 0, n \geq 1; \]
  \[ C(K) = CK^k, \quad C > 0, k \geq 1. \]

- Joint payoff if no relational contract constraint (first-best effort)
  \[ s(e^* (K, \theta), K, \theta) = \left(1 - \frac{\beta}{n}\right) \left(\frac{\beta}{nc}\right)^{\frac{\beta/n}{1-\beta/n}} \theta^{\frac{\gamma}{1-\beta/n}} K^{\frac{\alpha}{1-\beta/n}}. \]

- Risk neutrality requires \( \gamma = 1 - \beta / n \):
  - so expected joint payoff affected by \( \theta \) only through its mean.

- Distribution of \( \theta \) log-normal:
  - stationary counterpart to autoregressive process with normally distributed logs of innovations;
  - implies always *interior cutoff shock* for first-best effort:
    - marginal productivity of effort \( \to \infty \) as \( \theta \to \infty \); to 0 as \( \theta \to 0 \).
Implications of functional forms for relational contract

- Joint payoff with relational contract and $\tilde{\theta}$ cutoff shock

$$s \left( e^* (K, \tilde{\theta}) , K, \theta \right) = \left( \frac{\beta}{nc} \right)^{\frac{\beta}{n}} \tilde{\theta}^{1-\frac{\gamma}{n}} K^{\frac{\alpha}{1-\frac{\gamma}{n}}} \left[ \left( \frac{\theta}{\tilde{\theta}} \right)^{\gamma} - \frac{\beta}{n} \right]. \quad (3)$$

Note that strictly concave in $\theta$ for given $\tilde{\theta}$ when $\gamma < 1$.

- For notational convenience, define

$$\hat{S} (\sigma) = \frac{\delta}{1-\delta} [S (K, \sigma) - s (K, \sigma)]. \quad (4)$$

- For different $\sigma_L$ and $\sigma_H$, use first-order conditions for general capital to express cutoff shock for $\sigma_H$, $\hat{G}^G (\sigma_H)$, in terms of cutoff shock for $\sigma_L$, $\hat{G}^G (\sigma_L)$.

- Optimal general investment $\hat{K}^G (\sigma)$ for different $\sigma_L$ and $\sigma_H$ satisfies

$$\frac{\hat{K}^G (\sigma_L)}{\hat{K}^G (\sigma_H)} = \left[ \frac{\hat{S} (\sigma_L) \hat{G}^G (\sigma_H)}{\hat{S} (\sigma_H) \hat{G}^G (\sigma_L)} \right]^{\frac{1-\frac{\beta}{n}}{\alpha}}. \quad (5)$$
Parameters for calibration

Based on Bloom et al (Ecta, 2018), combining aggregate $\sigma^A$ and firm $\sigma^Z$ shocks (higher $\sigma$ corresponds to higher variance):

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>0.25</td>
<td>Factor share with isoelastic demand, 33% markup</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.5</td>
<td>As $\alpha$ with labour share 2/3, capital share 1/3</td>
</tr>
<tr>
<td>$n$</td>
<td>1</td>
<td>Implied by Bloom et al (2018) model</td>
</tr>
<tr>
<td>$k$</td>
<td>1</td>
<td>Implied by Bloom et al (2018) model</td>
</tr>
<tr>
<td>$\sigma^A_L$</td>
<td>0.67</td>
<td>Bloom et al (2018) estimate, %</td>
</tr>
<tr>
<td>$\sigma^A_H / \sigma^A_L$</td>
<td>1.6</td>
<td>Bloom et al (2018) estimate</td>
</tr>
<tr>
<td>$\sigma^Z_L$</td>
<td>5.1</td>
<td>Bloom et al (2018) estimate, %</td>
</tr>
<tr>
<td>$\sigma^Z_H / \sigma^Z_L$</td>
<td>4.1</td>
<td>Bloom et al (2018) estimate</td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>0.10</td>
<td>Calculated combined $\sigma^A_L$ and $\sigma^Z_L$ for $\theta$</td>
</tr>
<tr>
<td>$\sigma_H / \sigma_L$</td>
<td>4.07</td>
<td>Calculated from combined $\sigma^A_H$ and $\sigma^Z_H$ for $\theta$</td>
</tr>
</tbody>
</table>

**Table:** Parameter values for calibration
Parameters for calibration (cont)

- To fully calibrate relational contract model, also need specifications for \( \delta \) and \( s(K, \sigma) \).
  - \( s(K, \sigma) \) is joint payoff if separate from relationship
    - no counterpart in Bloom et al (Ecta, 2018)
    - no obvious way to derive from data.
    - any brilliant suggestions more than welcome!

- Alternative used here: combination of \( \delta \) and \( s(K, \sigma) \) implies cutoff shock for given \( \sigma \):
  - illustrate effect of change in uncertainty by calculating for different possible values of cutoff shock for \( \sigma_L \);
  - to be interpreted as values on domain of log-normal distribution with mean 1.

- Two kinds of shock:
  - systemics: affects values of both continuing and ending relationship
    - for systemic shocks, \( \hat{S}(\sigma) \) independent of \( \sigma \).
  - idiosyncratic: affects value only of continuing relationship.
Table: Effect of increase in systemic risk with general capital for given cutoff shock for $\sigma_L$

- Effects for *given* cutoff shock for $\sigma_L$ (log-normal distribution, mean 1);
  - for cutoff shock for $\sigma_L = 1$, capital falls by 6%, productivity by 11%;
  - effect smaller as cutoff shock for $\sigma_L$ moves away from mean;
    - still substantial even for cutoff shock for $\sigma_L$ 50% above or below mean;
    - recall: effects would be zero in absence of relational contract.

- To fit their model to data, Bloom *et al* (*Ecta*, 2018)
  - add -2% *first* moment aggregate shock to increased second moment
  - without relational contract, here gives c. 4% fall in general capital
    - within range in table from higher second moment alone.
Specific capital with idiosyncratic shocks

Specific capital more complicated as relaxes relational contract constraint
- consider only idiosyncratic shocks here and all capital specific
- presumably only a small share of capital so calibrated $\alpha$ far too high
  - any suggestions for appropriate value welcomed!
- only limited range of cutoff shock for $\sigma_L$ can be optimal values for any $c, C > 0$:
  - between 3.24 and 77.4 for calibrated parameters, well above mean
- so: use calibration only to illustrate theoretical effect that greater uncertainty can increase specific capital:

<table>
<thead>
<tr>
<th>Cutoff shock for $\sigma_L$</th>
<th>75.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cutoff shock for $\sigma_H$</td>
<td>14.23</td>
</tr>
<tr>
<td>Capital change (%)</td>
<td>2434.2</td>
</tr>
</tbody>
</table>

**Table:** Effect of increase in idiosyncratic risk with specific capital for given cutoff shock for $\sigma_L$
Comments on calibration results

- Compared *long-run equilibria* for $\sigma_L$ and $\sigma_H$, no dynamics
  - even that non-trivial with relational contract constraint!
  - I conjecture theory with probabilistic switching between $\sigma_L$ and $\sigma_H$
    - regimes doable
      - but needs *better computational skills than mine* to do calibration
- Effects in tables are for *capital* not *gross investment*
  - in comparison of long-run equilibria, related by
    - replacement for depreciation in continuing firms
    - investment by replacement firms for those going out of business
  - if these in constant proportions, percentage changes same
- *Capital rigidity*: model allows no adjustment of capital, upwards as well as downwards, in response to shocks
  - one-sided irreversibility tricky to handle with relational contracts.
Conclusion

Theoretical effects of greater uncertainty

- **general investment**: risk-neutral parties choose as if risk-averse when rely on relational contract
  - so: greater uncertainty reduces general investment for same mean

- **specific investment**: relaxes relational contract constraint
  - greater uncertainty for same mean may increase specific investment because relaxing constraint becomes more valuable

Calibrated effects of greater uncertainty
with functions and parameters based on Bloom *et al* (*Ecta*, 2018):

- **general investment**: no need for (unappealing) negative first-moment shock to capture fall of magnitude in data

- **specific investment**: increases with greater uncertainty
  - of interest because Bloom (*JEP*, 2014) comments that some investments increase with greater uncertainty.