Risk Aversion and the Response of the Macroeconomy to Uncertainty Shocks

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Abstract

Risk aversion (RA) affects the macroeconomic response to uncertainty shocks. Heightened level of RA during the 2008 crisis amplified the decline of output and investment by roughly 21% and 16%, respectively, at the recession bottom. Consistent with this evidence, degree of RA determines the impact of second moment shocks in DSGE models featuring stochastic volatility. Ceteris paribus, higher RA leads to stronger responses of macroeconomic variables to uncertainty shocks, potentially making uncertainty shocks as economically significant as level shocks. Conversely, elevated RA can amplify or dampen responses to level shocks depending on whether RA exaggerates or attenuates consumption growth expectations.

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1 Introduction

Risk matters. A growing strand of literature in economics is focused on documenting the effects of volatility shocks (uncertainty) on macroeconomic dynamics in equilibrium settings. For example, Bloom (2009) provides evidence of time-varying second moment to productivity growth causing significant distortions in output and employment. Fernández-Villaverde et al. (2011) estimate an open-economy model to demonstrate the impact of real interest rate volatility shocks on a number of macro variables. These papers find that time-varying risk (or uncertainty) can substantially alter the response of the macroeconomy to exogenous variations. If risk matters, then it is straightforward to conclude that the economic agent’s attitude towards risk (or risk aversion) should also matter.\footnote{This might appear to be a trivial point, but Tallarini (2000), in a model with Epstein and Zin (1989) – Weil (1990) recursive preferences and where the elasticity of intertemporal substitution is unity, numerically demonstrates the insignificance of the degree of risk aversion in generating macroeconomic fluctuations in an equilibrium model.} We document empirically that increased risk aversion exacerbated the fall in output and investment during the financial crisis of 2008. Consistently, in standard DSGE models, not only risk matters to equilibrium outcomes, but perhaps more importantly, the degree of risk aversion determines the magnitude of these outcomes. Higher risk aversion can amplify the effect of the uncertainty shock to be on par with that of the level, or first moment, shock. Our finding has significant ramification for general equilibrium modeling in monetary economics (to understand the Great Moderation, for example) and in asset pricing (to extend the Bansal and Yaron (2004) long-run risk mechanism from endowment to production economies).

The notion of time-varying risk aversion has gained traction in the macroeconomics and finance literature in recent decades. Grounded in theoretical models with habit (Abel (1990), Constantinides (1990), and Campbell and Cochrane (1999)) or heterogeneous agents (Dumas (1989)), aggregate risk aversion in the economy can exhibit countercyclical variation as evidenced by
the countercyclical risk premium in stock returns.\footnote{See \textcite{chan2002}.} Employing the Smoothed Local Projection (SLP) method of \textcite{barnichon2016}, we show that conditional on the fact that risk aversion (proxied by dividend-price, consumption-wealth, and leverage ratios) was elevated during the 2008 crisis, the fall in output and investment driven by uncertainty was deepened by 21%\footnote{Depending on the risk aversion proxy and the forecasting horizon, the amplifying effect on output can be as high as 50\% and as low as 20\%.} and 16%\footnote{Depending on the risk aversion proxy and the forecasting horizon, the amplifying effect on investment can be as high as 23\% and as low as 7\%.} respectively. This finding demonstrates that the interaction between risk aversion and uncertainty is a potential channel through which financial market conditions led to the deterioration of the macroeconomy during the crisis.

Theoretically, we demonstrate the interaction between risk aversion and uncertainty using two models with uncertainty. First, we build a standard New-Keynesian model with \textcite{campbell1999} habit to produce endogenous time-varying risk aversion, as approximated by the inverse of the surplus consumption ratio. The model features stochastic volatility in productivity. Second, we employ the small open economy model of \textcite{fernandez-villaverde2011} (FGRU henceforth) where the real interest rate process displays time-varying volatility. We replace the CRRA utility function with Epstein-Zin-Weil recursive utility to separate the effect of risk aversion from that of the elasticity of intertemporal substitution. We establish the following two main findings. First, risk aversion amplifies the magnitude of the response of macroeconomic quantities to uncertainty shocks. Precisely speaking, suppose there are two regime in the economy: low and high risk aversion. Our results show that uncertainty shocks have in general more impact when risk aversion is high. Second, the risk aversion implication on the economic effects stemming from level shocks is model-dependent: higher risk aversion can amplify or dampen macroeconomic responses through changing consumption growth expectations.
In both models, the endogenous macroeconomic response stemming from volatility shocks is amplified when agents display higher level of risk aversion. In FGRU (2011), however, the level shock to the real interest rate actually has a weaker impact on macroeconomic dynamics when risk aversion is high. This is because the level shock to the real interest rate increases consumption growth expectation by lowering the price of the internationally traded bond. When risk aversion is high, this downward pressure on bond price is less reflected in expected consumption growth, thus generating a dampened current macroeconomic reaction.

Our finding potentially has important applications in multiple areas of research. In monetary economics, for example, Fernández-Villaverde, Guerrón-Quintana and Rubio-Ramírez (2010) estimate a DSGE model with stochastic volatility and drifting parameters in the Taylor rule to examine the origins of the easing in business cycle fluctuations in the U.S. data between 1984 to 2007 (the Great Moderation). They find that modifications in monetary policy implementation contributed to the observed decline in macroeconomic volatility. However, the literature is inconclusive in the source of the Great Moderation as Cogley and Sargent (2005) and Sims and Zha (2006) have argued otherwise and show that fundamental changes in the volatility process is driving the decline. Fernández-Villaverde et al. (2010) employ log utility with simple habit in the estimation of conditional volatility processes. Given our documented joint determination of risk aversion and uncertainty shocks, the preference specification is non-trivial, and recursive utility with regime dependent risk aversion can be an essential ingredient in the structural model for the purpose of volatility estimation.

High-order perturbation techniques have become one of the standard method for solving DSGE models. It is also well know that risk premiums are unaffected by first-order terms and completely determined by those second- and higher-order terms. A widespread macro-finance separation paradigm, first

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proposed by [Tallarini (2000)], suggests that the moments of macroeconomic quantities are not very sensitive to the addition of second-order and higher-order terms. This result is important since it implies that by varying the risk aversion parameter while holding the other parameters of the model constant, one is able to fit the asset pricing facts without compromising the model's ability to fit the macroeconomic data.[6] Our paper suggests that risk aversion not only determines the level of asset returns, but it also matters in calibrating the model to match the standard deviations of macroeconomic variables to those in the data. The simultaneity of risk aversion and uncertainty in driving macroeconomic dynamics poses an additional challenge in our understanding of time-varying expected returns as risk cannot be filtered solely by observing macroeconomic volatilities. The macro-finance separation does not hold in DSGE models featuring stochastic volatility when the solution technique takes into account the non-linearity of the model.

Our paper is linked with different streams of literature in economics. The use of time-varying uncertainty has a long history in the financial economics literature. E.g. [Kandel and Stambaugh (1991)] study the implications for asset returns of time-varying first and second moments of consumption growth in a model with a representative Epstein-Zin investor. In a similar spirit, [Bansal and Yaron (2004)] incorporate time-varying first and second moments of consumption growth and recursive preferences in an endowment asset pricing model, and show that stochastic volatility not only generated time-variation in risk premiums but also significantly increased the mean equity risk premium. As already discussed, our result adds another dimension of complication in extending these types of macro-finance models that employ stochastic volatility from endowment economies to full general equilibrium as macroeconomic and asset pricing moments need to be calibrated simultaneously.

An increasing body of research has studied how uncertainty fluctuations influence business cycle dynamics. Within the framework of irreversible invest-
ment (see Bernanke (1983), Dixit and Pindyck (1994), Abel and Eberly (1996), Hassler (1996)), Bloom (2009) studies the propagation of firm-level uncertainty shocks. Following an increase in uncertainty about future profitability, firms will slow down activities that cannot be easily reversed, i.e. they “wait and see”. After the heightened uncertainty is resolved, pent-up demand for capital goods leads to an investment boom. Another growing literature stresses the interaction of risk and economic activity propagated through financial, rather than physical frictions. Using a model with financial frictions, Gilchrist, Sim and Zakrajsek (2014) argue that increases in firm risk lead to an increase in bond premia and the cost of capital which, in turn, triggers a decline in investment activity and measured aggregate productivity. Arellano, Bai and Keohoe (2016) show that firms downsize investment projects to avoid default when faced with higher risk. Finally, Christiano, Motto and Rostagno (2014) analyze the macroeconomic implications of volatility shocks in the context of a financial accelerator model adapted from Bernanke, Gertler and Gilchrist (1999). Our analysis shows that when risk aversion is elevated, uncertainty shocks have larger and more prolonged impact. Moreover, our study supports the literature that points to financial market frictions as an additional channel through which volatility fluctuations can affect macroeconomic outcomes: if risk aversion rises with tightening financial constraints, uncertainty may affect the economy via an increase in the risk premium.

More recently, the literature has also started investigating the impact of shocks to aggregate uncertainty. Justiniano and Primiceri (2008) and Fernández-Villaverde and Rubio-Ramírez (2007) estimate dynamic equilibrium models with heteroskedastic shocks and show that time-varying volatility helps to explain the Great Moderation between 1984 and 2007. Fernández-Villaverde, Guerrón-Quintana, Rubio-Ramírez and Uribe (2011) and Born and Pfeifer (2014) find that risk shocks are an important factor in explaining business cycles in emerging market economies. Fernández-Villaverde, Guerrón-Quintana, Kuester and Rubio-Ramírez (2015) document the important role of fiscal volatility for output fluctuations. Basu and Bundick (2017) study the inter-
action of aggregate risk shocks with precautionary saving in an environment with nominal rigidities. We contribute to this literature by investigating (theoretically and empirically) the interaction of aggregate uncertainty with risk aversion.

A number of papers have also investigated the possibility that spikes in uncertainty may be the result of adverse economic conditions rather than being a driving force of economic downturns, see e.g. [Van Nieuwerburgh and Veldkamp (2006); Fostel and Geanakoplos (2012); Bachmann, Elstner and Sims (2013)]. In this paper we study the amplification role of risk aversion for exogenous impulses to uncertainty, and we leave the analysis of the interaction between risk aversion with endogenous response of uncertainty as an interesting avenue for future research. In a related paper, [Alfaro, Bloom and Lin (2018)] show how financial friction can also amplify the impact of uncertainty shocks.

Gourio (2012) examines the joint implication of risk aversion and time-varying risk on macroeconomic dynamics, but we differ in the source of risk in our models. Rather than focusing on the time-varying probability of disaster risk as in Gourio (2012), we explore the interaction between stochastic volatility and risk aversion.

Finally, our paper is related to Gorodnichenko and Ng (2017) who use the insight from higher-order perturbation of policy functions to empirically separate the level from the volatility factors. Gorodnichenko and Ng (2017) conclude that “[T]he interaction between the first- and second-order dynamics is worthy of more theorizing in light of the evidence for non-trivial second-moment variations.” Our analysis of level and volatility shocks and their interaction with risk aversion is a first step in this direction.

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7 Gourio (2013) extends the application to credit spreads.
8 In Proposition 3, Gourio (2012) shows that uncertainty in the probability of disaster translates to a level shock to the time discount factor with constant volatility. Thus, our analysis of second moment shocks to productivity and interest rate is different.
2 Risk Aversion and Uncertainty: Empirical Evidence

In this section we estimate the dynamic responses of macroeconomic quantities to an uncertainty shock, conditional on risk aversion being low or high. The estimation of state-dependent impulse response functions have recently been the subject of expressed interest in macroeconomics, see e.g. Auerbach and Gorodnichenko (2012a), Auerbach and Gorodnichenko (2012b), and Ramey and Zubairy (2018) for investigations of the size of fiscal multipliers when the economy is in recession, or more broadly, during periods of economic slack. Tenreyro and Thwaites (2016) examine the response of the U.S. economy to monetary policy shocks predicated on the state of the business cycle. To the best of our knowledge, the role of risk aversion as a state variable concerning the macroeconomic response to uncertainty shocks is unexplored so far.

To estimate the state-dependent IRFs, we rely on the smoothed version of Jorda (2005) local projections developed by Barnichon and Brownlees (2016). The Smooth Local Projections (SLP) strikes a balance between the efficiency of Vector Autoregressions (VAR) and the robustness (to model misspecification) of the Local Projections (LP) approach. In practice, SLP consists in estimating LP under the assumption that the impulse response is a smooth function of the forecast horizon. Specifically, we estimate an $h$-step ahead predictive regressions,

$$y_{t+h} = \alpha_h + (\beta_{0,h} + \beta_{t,h} RA_t) UNCT_t + \sum_{i=0}^{p} \gamma_{i,h} w_{t-i} + u_{t+h} \quad (1)$$

where $h$ ranges from 0 to $H$ and $i$ is the number of lags used for the control variables, $w_t$. $y_{t+h}$ is the $h$ period ahead realization of the macroeconomic variable of interest. $RA_t$ is the state variable. $UNCT_t$ is our measure of uncertainty. To capture state dependence, the response of $y_{t+h}$ to uncertainty at

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9 We thank C. Brownless for clarifying various aspects about the SLP technique.
time $t$ is a linear function, $\beta_{0,h} + \beta_{1,h}RA_t$, of risk aversion. In what follows, the $\beta_{1,h}$ coefficient capturing the amplification/contraction effect due to risk aversion is called the state multiplier. We are interested in knowing whether uncertainty shock has a larger effect on, e.g., output during high risk aversion states.

For our empirical application, we follow Basu and Bundick (2017) and include gross domestic product (GDP), consumption, investment, hours worked, the GDP deflator, the M2 money stock, and a measure of the stance of monetary policy as control variables. We employ VXO as the uncertainty proxy because it is a well known and readily-observable measure of aggregate uncertainty. Since the VXO data start in 1986, we estimate our baseline empirical model using quarterly data over the 1986–2014 sample period. We choose $p = 2$, and let all other variables enter in log levels with the exception of the monetary policy measure. Finally, we use a recursive identification scheme with the VXO ordered first.\footnote{Appendix B.1 shows that the linear SLP methodology delivers responses that are almost identical to those obtained using a VAR of order four as in Basu and Bundick (2017).} \footnote{Appendix B.2 shows that our results are robust to using other proxies for uncertainty.}

To estimate the state dependent IRFs, we follow Barnichon and Brownlees (2016) and include the set of controls $w_t$ and their interaction with the state variable, $RA_t$. We report the estimation for three different proxies of risk aversion. The first two proxies are the dividend-price (see Figure 1) and the consumption-wealth (see Figure 2) ratios. These two proxies are motivated by the DSGE model with habit we build in Section 3. Second, motivated by the work of Santos and Veronesi (2016), we employ the financial intermediary
leverage as measured by He et al. (2017) (see Figure 3).13 Finally, we let $RA_t$ take value equal to 1 when the risk aversion proxies are above the 75\textsuperscript{th} percentile, $RA_t$ is equal to $-1$ when they are below the 25\textsuperscript{th} percentile, and $RA_t$ is set to zero otherwise. We then $RA_t$ standardize the variable. Very similar, yet slightly noisier, results are obtained when we use the continuous version of these proxies.

[Insert Figures 1, 2 and 3 about here.]

The left column in Figures 1 to 3 plot the responses of GDP, consumption, and investment to an uncertainty shock that is realized (i) in a high risk aversion state ($RA_t = 1$), (ii) in an average state ($RA_t = 0$), and (iii) in a low risk aversion state ($RA_t = -1$). The right column in Figures 1 to 3 plot the state multipliers obtained from SLP. Recall that the state multiplier, $\beta_{1,h}$, captures the extent to which the state variable, namely risk aversion, affects the IRF at each horizon. A negative value of the state multiplier implies that the IRF response to a positive uncertainty shock is more negative when the risk aversion is high in the economy ($RA_t = 1$).

Figures 1−3 deliver a clear message about the state-dependent nature of uncertainty shocks: the response of the macroeconomic aggregate to a VXO shock is substantially larger when risk aversion is heightened. The peak decline in output, consumption, and investment when $RA_t = 1$ is roughly twice as large as the decline obtained in a low risk aversion environment when $RA_t = -1$. In general, the impulse responses obtained in a low risk aversion state return to zero after about two years, whereas those in a high risk aversion state tend to stay low for longer.

To quantify the amplifying effect of increased risk aversion on the economic impact of uncertainty shocks, we use the estimates from Eq. (1) to generate the

\footnote{Our measure of leverage is based on market prices (market leverage). In the model of Santos and Veronesi (2016), the debt-to-wealth ratio is monotonically decreasing in the surplus consumption ratio (see their Corollary 13), which can be seen as the inverse of risk aversion.}
fitted values of output and investment with and without elevated risk aversion. In other words, we construct fitted values with the state multiplier, $\beta_{1,h}$, set to either the estimated value (high RA) or to zero (average RA). Specifically, we examine output and investment declines during the financial crisis using the post 2007Q4 sample and choose the forecast horizon, $h$, to be four quarters.$^{14}$

Figure 4 presents the time series plots of realized and fitted values of output, while Figure 5 presents the same plots for investment.$^{15}$ Focusing on output in Figure 4 we see the one-year ahead forecast of output (dashed line) from the SLP matches the realized path of output (solid line) in both subplots well. In particular, the forecasted maximal drop in output appears within two quarters of the actual minimal output during the financial crisis. Next, we set the state multiplier to zero and repeat the forecast while keeping all other coefficient estimates from the SLP. The resulting fitted output path (square-dashed line) is plotted along the original forecast for counter-factual analysis. Figure 4 subplot (a) shows that relative to the level of output at the onset of the crisis, the maximal decline in output due to uncertainty is exacerbated by roughly 50% conditional on high risk aversion (dashed vs. square-dashed lines), as proxied by leverage. In subplot (b), the amplifying effect is roughly 21% conditional on high risk aversion, as approximated by dividend-price ratio.

Quantitatively, heightened risk aversion during the financial crisis generates significantly larger decline in investment due to uncertainty. Figure 5 presents the realized (solid line), the actual one-year ahead forecast (dashed line), and the counter-factual forecast (square-dashed line) of investment. Similar to Figure 4, the SLP forecast of investment matches reasonably well with the realized path, especially in subplot (a) when leverage is used as the risk aversion proxy. Relative to the level of investment at the end of 2007, Figure 5

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$^{14}$Our results here are robust to using various other forecast horizons. Four quarters is chosen since it corresponds to the maximal impact on output, consumption, and investment in the uncertainty shock IRFs in Figures 1 and 3. Furthermore, given the short sample period, long horizon forecasts are less appropriate.

$^{15}$In the interest of space, we report only the case where $RA$ is proxied by either the intermediary leverage in subplot (a) or the dividend-price ratio in subplot (b). Please see Appendix B.3 for the corresponding figure using $\hat{cay}$ as the $RA$ proxy.
shows the maximal forecasted decline in investment is 23% and 16% greater, in
subplots (a) and (b) respectively, when we allow for risk-aversion-dependence
of uncertainty in the SLP.

Overall, the economic significance of risk aversion on macroeconomic dy-
namics cannot be overlooked. Our results from applying the SLP methodology
to examine the financial crisis can perhaps be viewed in one of two ways. First,
in the absence of state-dependence, the econometrician cannot decipher the
true impact of uncertainty shocks on economic aggregates. Second, conversely,
intensified risk aversion aggravated the depth of the recession by causing un-
certainty shocks to be more effective through general equilibrium mechanism.
In the next section, we explore the interaction between risk aversion and un-
certainty in relation to the macroeconomy in a structural setting employing
DSGE models.

3 The Effect of Uncertainty Shocks in the Pres-
ence of Time-Varying Risk Aversion

In this section, we first examine the interaction of risk aversion and un-
certainty in a structural setting. To study how uncertainty shocks and risk
aversion jointly determine business cycle moments we consider impulse re-
response functions (IRFs) and variance decomposition analysis. Appendix E
discusses the technical details behind the computation of IRFs and variance
decompositions in general equilibrium models featuring stochastic volatility.
Using a variant of the standard New-Keynesian model with endogenous time-
variation in risk aversion, we then demonstrate that the SLP results described
in Section 2 can be replicated using simulated data from our model.
### 3.1 A New-Keynesian Model with Recursive Preferences

We build a small-scale dynamic stochastic general equilibrium model with monopolistic competition and sticky prices and show that uncertainty about technology can generate a substantial fall in output, consumption, and investment. We adopt a recursive structure for intertemporal utility, where a representative household chooses sequences of consumption, $C_t$, and labor, $N_t$, to maximize

$$U_t = \left[ \left( C_t^\eta (1 - N_t)^{1-\eta} \right)^{\frac{1}{1-\eta}} + \beta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{\theta}{\gamma}} \right]^{\frac{1}{\theta-\gamma}},$$

(2)

where $\theta \equiv (1 - \gamma)/(1 - \psi)$, $\gamma$ determines the coefficient of relative risk aversion, $\psi$ is the inverse of intertemporal elasticity of substitution, $\beta$ is the subjective discount factor, and $\eta$ determines the Frisch elasticity of labor supply. Appendix C describes our baseline model economy which is a standard New-Keynesian model similar to Andreasen (2012) and Basu and Bundick (2017). Due to the nature of the model and Epstein and Zin (1989) preferences we refer to our baseline model with “NK-EZ”. The first column of Table 1 lists the parameters value used in our model (see Appendix C.6 for additional details).

We next investigate the interplay of risk aversion and stochastic volatility in the stationary technology shock process in our model. In particular, we consider intermediate goods-producing firms $i$ with the same constant returns to scale Cobb-Douglas production function, subject to a fixed cost of production $\Phi$ and their level of productivity $Z_t$: $Y_t(i) = (K_t(i) U_t(i))^{\alpha} (Z_t N_t(i))^{1-\alpha} - \Phi$, where $U_t(i)$ is the rate of utilization of their installed physical capital, $N_t(i)$ is labor, $Y_t(i)$ is the intermediate good. The technological process $z_t = \log(Z_t)$ evolves according to a first-order autoregressive process with stochastic volatil-
ity process

\[ z_{t+1} = (1 - \rho_z) z_{ss} + \rho_z z_t + e^{\sigma_{z,t}} \varepsilon_{z,t+1} \quad (3) \]

\[ \sigma_{z,t+1} = (1 - \rho_{\sigma}) \sigma_z + \rho_{\sigma} \sigma_{z,t} + \varepsilon_{\sigma,t+1} \quad (4) \]

with \( \varepsilon_{z,t} \sim \text{i.i.d. } N(0,1) \), \( z_{ss} = \log Z_{ss} \) where \( Z_{ss} \) is the steady state level of \( Z_t \), and \( \varepsilon_{\sigma,t} \sim \text{i.i.d. } N(0,\sigma_{\sigma}) \). The innovations \( \varepsilon_{z,t+1} \) and \( \varepsilon_{\sigma,t+1} \) are assumed to be mutually independent at all leads and lags. In words, two independent innovations affect the level of productivity. The first innovation, \( \varepsilon_{z,t+1} \), changes the level of productivity itself, while the second innovation, \( \varepsilon_{\sigma,t+1} \), determines the spread of values for the productivity level.

Figures 6(a) and 6(b) show the IRFs to a level and volatility shock to technology, respectively. The amplification effect of risk aversion is present only for impulse responses to a volatility shock in technology; responses to level shocks do not display any sensitivity to the risk aversion parameter, see Figure 6(a). Zooming in on the uncertainty channel, Figure 6(b) shows that a higher level of risk aversion generates a more pronounced decline in output, consumption, and hours in response to a uncertainty shock.

[Insert Figure 6 about here.]

In summary, the IRFs in Figure 6 are consistent with those in the data (c.f. Section 2).

Next, we compare the relative importance of level and volatility shocks in driving economic dynamics conditional on the magnitude of risk aversion within a model featuring stochastic volatility in productivity. Table 2 presents the model implied variance decomposition across low and high values of risk aversion, \( \gamma \). In Panel A, \( \gamma \) is set to be 80, while in Panel B, the calibrated value of \( \gamma \) is doubled to 160. We examine the sample standard deviation of output, consumption, investment and hours worked. Columns (3) and (4) in Table 2—Panel A show that level shocks to productivity are more important than
uncertainty shocks in driving macroeconomic volatility. However, in Panel B, when risk aversion is high, the contribution of uncertainty shocks to macroeconomic variation roughly doubles in column (4), while the contribution of the level shocks remains unchanged. This is consistent with the impulse response evidence in Figure 6 that demonstrates the amplifying effect of risk aversion on the macroeconomic response to uncertainty shocks but not necessarily to level shocks.

Our evidence here clearly shows that risk aversion has a role in influencing dynamics of the theoretical model. This is in line with our previously outlined empirical evidence that risk aversion is a crucial state variable that generates differential output and investment responses across business cycles. However, within the NK-EZ setup we are limited to address the empirical evidence by means of comparative statics, i.e. by exogenously changing the risk aversion parameter $\gamma$. To address this issue, in the next paragraph we extend our baseline New-Keynesian model and associated analysis to accommodate endogenous risk aversion through the nesting of Campbell and Cochrane (1999) habit within Epstein-Zin-Weil preferences.

### 3.2 A New-Keynesian Model with Recursive Preferences and Habit

To allow for endogenous time-varying risk aversion, we augment the NK-EZ model with Campbell and Cochrane (1999) external habit in the preference specification. To be precise, the Epstein-Zin utility function is defined over habit-adjusted consumption rather than consumption itself. In what follows, we refer to this model as “NK-EZ-Habit”. As the preference specification
changes, equation (2) becomes:

$$U_t = \left[ \left( C_t^{h(1 - N_t/1 - \eta)} \right)^{1 - \gamma} + \beta \left( E_t U_{t+1}^{1 - \gamma} \right)^{1 - \gamma} \right]^{1 - \gamma},$$

(5)

where $C_t^h$ is consumption with external habit such that $C_t^h = C_t \times S_t$. Consistent with [Campbell and Cochrane (1999)], we define $S_t$ as the surplus consumption ratio, which evolves according to the following process:

$$\log(S_t) = (1 - \rho_s) \bar{s} + \rho_s \log(S_{t-1}) + \lambda_t^h \log \left( \frac{C_t}{C_{t-1}} \right),$$

(6)

This is a generalized AR(1) process with mean $\bar{s}$ and autoregressive coefficient $\rho_s$. $\lambda_t^h$ is the sensitivity function of the surplus consumption ratio to consumption growth. It introduces non-linearity in the original [Campbell and Cochrane (1999)] model and is key for generating time-varying equity risk premium within their endowment framework. Denoting $\log(S_t)$ as $s_t$, we define $\lambda_t^h$ to be

$$\lambda_t^h = \sqrt{1 - \rho_s} \sqrt{1 - 2(s_t - \bar{s})} - 1^{16}$$

[Campbell and Cochrane (1999)] further show that the time-varying local risk aversion coefficient is equal to the inverse of the surplus consumption ratio. In other words, when $S_t$ is high, risk aversion of the representative agent is low and vice versa.

[Swanson (2018)] shows that, under regularity conditions, the coefficient of absolute wealth-gamble risk aversion under recursive preferences is given by

$$R_t^a = -\frac{\mathbb{E}_t[U_{t+1}^{1 - \gamma} U_{t+1}^n - \gamma U_{t+1}^{1 - \gamma - 1} U_{t+1}^{n - 2}]}{\mathbb{E}_t[U_{t+1}^{1 - \gamma} U_{t+1}^n]},$$

in which $U_{t+1}'$ and $U_{t+1}''$ denote the first and second derivatives of the utility function with respect to total wealth. In Appendix [D] we show that with our preference specification, similar to [Campbell and Cochrane (1999)], wealth-gamble risk aversion is a function of the inverse of the surplus consumption

\[^{16}\text{In the [Campbell and Cochrane (1999)] setting, this particular parameterization of } \lambda_t^h \text{ allows to achieve a constant risk free rate in the model. This is no longer the case in our DSGE model with Epstein-Zin preferences. Nonetheless, we rely on the same parameter values in our calibration as can be seen from the second column in Table 1.}\]
The NK-EZ-Habit setup allows us to study the endogenous response of the economy to productivity uncertainty shocks conditional on the level of risk aversion displayed by the representative agent. To do so, we use the model to simulate 200 economies spanning 116 periods\footnote{We use 116 periods to match the 116 quarters of data used in our empirical exercise in Section 2. Appendix B.4 verifies that our specification of (S)LP is able to recover the true, theoretical IRFs from the NK-EZ-Habit model.} We proxy for risk aversion with either the inverse of the surplus consumption ratio or the dividend-price ratio. We then use the simulated series of output, consumption, investment, dividend-price ratio, and the surplus consumption ratio and perform the smoothed local projection (SLP) as outlined in section 2.

Figure 7 plots the SLP results from simulated data where we employ the surplus consumption ratio as a proxy for risk aversion. Similar to Figure 1−3, the left column shows the conditional IRFs of output, consumption, and investment following a positive one standard deviation shock to productivity volatility, and the right column shows the state multipliers from estimating equation \( \beta_{1,h} \), over horizons \( h \in \{1 \ldots 22 \} \) for the same three variables. To ease the comparison to the empirical data counterpart, we assign values of \([-0.25, 0, +0.25]\) to the state variable \( RA_t \) according to the relative time-series magnitude of the surplus consumption ratio in the simulated sample. For example, for a given period \( t \), if the value of the surplus consumption ratio is in the top quartile of the sample, then risk aversion is low, thus \( RA_t = -0.25 \). Conversely, if the value of the surplus consumption ratio is below the 25\textsuperscript{th} percentile, then \( RA_t \) is assigned to be \(+0.25\). There are two main takeaways in Figure 7. First, conditional on high risk aversion (surplus consumption ratio is low and \( RA_t \) is positive), output, consumption, and investment react more negatively to a positive uncertainty shock relative to the scenario where risk aversion is neutral (\( RA_t = 0 \)). This is consistent with the corresponding empirical IRFs shown in Figures 1 and 3. Second, the estimated state multipliers are significantly negative for output, consumption, and investment in
the simulated data causing high risk aversion ($RA_t = +0.25$ and $\beta_{1,h}RA_t < 0$) to generate stronger declines in those variables following an increase in uncertainty.

[Insert Figures 7 and 8 about here.]

Figure 8 shows the results when we use the dividend-price ratio as a proxy for risk aversion. In this case, $RA_t$ takes on the value of $\{-1, 0, +1\}$ depending on whether the dividend-price ratio is below the 25th percentile, between the 25th and the 75th percentile, or above the 75th percentile, respectively. In line with Figure 7, the IRFs in Figure 8 suggest that conditional on a high dividend-price ratio relative to its average (risk aversion is high), output, consumption, and investment drop more after a positive uncertainty shock. Conversely, when dividend-price ratio is relatively low, the economic responses to the same shock are significantly milder. The similarities between Figures 7 and 8 provide some assurance that the dividend-price ratio is indeed a credible instrument to proxy for the level of risk aversion in the data. Furthermore, our model is doing a reasonable job in capturing the amplifying effect of risk aversion on the impact of uncertainty shocks.

To align the SLP estimations on empirical and simulated data as closely as possible, we also report the responses analogous to Figures 7 and 8 but where productivity uncertainty is replaced by the model-implied VXO index (i.e. the expected conditional volatility of the return on firm equity). Figures 9 and 10 shows that our conclusion continues to hold also in this case.

[Insert Figures 9 and 10 about here.]

Finally, Panel C of Table 2 presents the model implied variance decomposition for the NK-EZ-Habit model. Overall, the second column reports that the volatilities of output and consumption are comparable in terms of magnitude with the results from NK-EZ. Conversely, the variation of investment
and hours worked is larger in the NK-EZ-Habit model. Another similarity to the NK-EZ model is the fact that level shocks are the main driver of variation in macro variables. However, due to the endogenous counter-cyclical variation of risk aversion, uncertainty shocks explain a larger part of the variation in macro variables compared to the NK-EZ specification with the same value for \( \gamma \), i.e. Panel A.

4 Impact of Risk Aversion on Uncertainty Shocks in an Open Economy Model

To complement the analysis of the New-Keynesian model with uncertainty in productivity from section 3, we study the open economy model from Fernández-Villaverde et al. (2011) (FGRU) in this section. We show that the effects of volatility shocks on the real economy are intertwined with the magnitude of risk aversion in a model with no rigidities and in which conditional volatility affects the real interest rate. To study how uncertainty shocks and risk aversion jointly determine business cycle moments, we consider impulse response functions (IRFs) and variance decompositions in the analysis. We refer interested readers to Appendix E for the technical details behind the computation.

4.1 The FGRU Model

The FGRU model is a standard small open economy business cycle model calibrated to match data from four emerging economies: Argentina, Brazil, Ecuador, and Venezuela. The small open economy is populated by a representative household.¹⁸ In contrast to Fernández-Villaverde et al. (2011), we model the preferences of the household with a recursive utility function similar to equation 2 (see Epstein and Zin (1989) and Weil (1990)). We do so because

¹⁸For the interested reader, a detailed derivation of the model equations, and steady states is available in Fernández-Villaverde et al. (2011), and hence not repeated here.
we want to separate the effect of risk aversion from that of intertemporal substitution. Trivially, when risk aversion equals the inverse of the elasticity of substitution we obtain exactly the same results as Fernández-Villaverde et al. (2011) (see Table F.1 in Appendix F.1). The household can invest in two types of assets: the stock of physical capital, $K_t$, and an internationally traded bond, $D_t$. Firms maximize profits by equating wages and the rental rate of capital to marginal productivities. Thus,

$$Y_t - C_t - I_t = D_t - \frac{D_{t+1}}{1 + r_t} + \frac{\Phi_D}{2} (D_{t+1} - D_t)^2$$

where $\Phi_D > 0$ is a parameter that controls the costs of holding a net foreign asset position.

The model is calibrated to monthly frequency. Following the original approach, we construct quarterly simulated data, and we report results on a quarterly basis. We refer the interested reader to the online Appendix F.2 for details on the model aggregation. Finally FGRU takes the real interest rate, $r_t$, as an exogenously defined process. We now turn to describe these dynamics.

### 4.1.1 Stochastic Volatility in Real Interest Rate

The real interest rate, $r_t$, a country faces on loans denominated in US dollars is decomposed as the international risk-free real rate plus a country–specific spread:

$$r_t = r + \varepsilon_{rb,t} + \varepsilon_{r,t}$$

where $r$ is the mean of the international risk-free real rate plus the mean of the country spread; the term $\varepsilon_{rb,t}$ equals the international risk-free real rate subtracted from its mean, and $\varepsilon_{r,t}$ equals the country spread subtracted from
its mean. Both $\varepsilon_{tb,t}$ and $\varepsilon_{r,t}$ follow AR(1) processes described by

$$
\varepsilon_{tb,t} = \rho_{tb}\varepsilon_{tb,t-1} + \sigma_{tb,t} u_{tb,t} \\
\varepsilon_{r,t} = \rho_{r}\varepsilon_{r,t-1} + \sigma_{r,t} u_{r,t},
$$

where both $u_{r,t}$ and $u_{tb,t}$ are normally distributed random variables with mean zero and unit variance. Importantly, the process for interest rates displays stochastic volatility. In particular, the standard deviations $\sigma_{tb,t}$ and $\sigma_{r,t}$ follow an AR(1) process:

$$
\begin{align*}
\sigma_{tb,t} &= (1 - \rho_{\sigma_{tb}}) \sigma_{tb} + \rho_{\sigma_{tb}} \sigma_{tb,t-1} + \eta_{tb} u_{\sigma_{tb},t} \\
\sigma_{r,t} &= (1 - \rho_{\sigma_{r}}) \sigma_{r} + \rho_{\sigma_{r}} \sigma_{r,t-1} + \eta_{r} u_{\sigma_{r},t},
\end{align*}
$$

where both $u_{\sigma_{tb},t}$ and $u_{\sigma_{r},t}$ are normally distributed random variables with mean zero and unit variance.

Each of the components of the real interest rate is affected by two innovations. For instance, $\varepsilon_{tb,t}$ is hit by $u_{tb,t}$ and $u_{\sigma_{tb},t}$. The first innovation, $u_{tb,t}$, changes the rate, while the second innovation, $u_{\sigma_{tb},t}$, affects the standard deviation of $u_{tb,t}$. The innovations $u_{r,t}$ and $u_{\sigma_{r},t}$ have a similar reading. Section F.3 highlights why it is key to have two separate innovations, one to the level of the interest rate and one to the volatility of the level.

In comparison with the country spread, the international risk-free real rate has both lower average standard deviation of its innovation ($\sigma_{tb}$ is smaller than $\sigma_{r}$ for all four countries) and less stochastic volatility ($\eta_{tb,t}$ is smaller than $\eta_{r,t}$ for all four countries). These relative sizes justify why in our analysis we concentrate only on the innovation to the volatility of the country spread, $u_{\sigma_{tb},t}$, and forget about shocks to the international risk-free real rate. For simplicity, we refer to the innovation $u_{\sigma_{tb},t}$ as the stochastic volatility shock.

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19This specification has been adopted by Justiniano and Primiceri (2008) among others.
4.1.2 Volatility Shocks, Risk Aversion and Macro Dynamics

We examine the impulse response functions (IRFs) of the model to shocks in the productivity, country spread, and country spread volatility. We report the results only for the model calibrated to Argentina. We consider both the original calibration of Fernández-Villaverde et al. (2011) and the re-calibrated model of Born and Pfeifer (2014). The IRFs for a positive one standard deviation shock are reported in Figure 11. We plot the IRFs of output (first row of panels), consumption (second row), investment (third row) to the three shocks (columns).

![Insert Figure 11 about here.]

The amplifying effect of risk aversion on the macroeconomic dynamics is apparent only following uncertainty shocks. The third column plots the IRFs to a one-standard-deviation shock to the volatility of the Argentinean country spread, $u_{\sigma_{tb},t}$. This column shows that there is a large effect of risk aversion on macro dynamics. The first two columns of Figure 11 show that IRFs to shocks in the level are hardly affected. In response to a positive volatility shock, output, consumption, and investment fall more in the case of high risk aversion than in the case of low risk aversion. For example, after a shock to volatility, consumption drops 0.5% upon impact when risk aversion equals to 5; the contraction is larger (1.1% percent at impact) when risk aversion equals to 15. Similarly, we observe a slow fall in output (after 10 quarters, it falls 0.16 percent) when risk aversion is low. However, for high risk aversion, the fall is deeper and more persistent (after 11 quarters years, it falls 0.32 percent). The implication is the same for investment. Columns (2) – (4) in Table 3 display the drop in macroeconomic variables, and the length of the recovery phase, for alternative values of risk aversion within FGRU.

![Insert Table 3 about here.]

We use the same parameters as in Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014); these are reported in Table F.2 for the reader’s convenience.
Table 4 shows the variance decomposition of output, consumption, and investment. Each column corresponds to a specific simulation: (1) the benchmark case with all three shocks (productivity, the country spreads and its volatility); (2) when we have a shock only to productivity; (3) when we have a shock to productivity and to the interest rate (with volatility fixed at its unconditional value); (4) when we have shocks to interest rate and to volatility; (5) when we have a shock only to the interest rate level; and (6) when we have shocks only to interest rate volatility.

The last column shows that volatility alone makes a relatively important contribution to the fluctuations of consumption (the standard deviation is 0.75) and investment (standard deviation of 3.08). Again, increasing the risk aversion almost doubles these contributions.

We next consider an alternative calibration of the FGRU model. In particular, Born and Pfeifer (2014) note an error in the time aggregation of flow variables, and they show that the model of Fernández-Villaverde et al. (2011) must be recalibrated. Figure 12 compares the IRFs for the recalibrated model with the original IRFs in Fernández-Villaverde et al. (2011). The figure shows that a one standard deviation positive volatility shock now leads to a larger drop in macro quantities than originally reported in Fernández-Villaverde et al. (2011). The difference between the two calibrations is further magnified under high risk aversion in the right column of the figure. Table 3 columns (5) – (7) display the drop in macroeconomic variables, and the length of the recovery phase, for varying degrees of risk aversion.

Appendix F.2 provides additional details on how to obtain the variance decomposition for the FGRU economy.
Table 5 shows the variance decomposition for the alternative calibration proposed by Born and Pfeifer (2014). First, we find that in the re-calibrated model that corrects for the time-aggregation, the contribution of volatility shocks to business cycle volatility increase, and more so the higher the risk aversion. Second, by comparing Table 5 with Table 4, an important insight emerges: risk aversion amplifies not only the simulation with volatility shocks only in column (6), but also the simulation where both level and volatility shocks are active in column (4). For example, in the Born and Pfeifer (2014) re-calibrated economy, investment raises by about 28% (18.11/14.19) when risk aversion raises from 5 to 15. On the other hand, the original Fernández-Villaverde et al. (2011) calibration does not show any sensitivity of investment to risk aversion in column (4) of Table 4. This makes us conclude that: (1) volatility shocks are amplified by the magnitude of risk aversion; (2) the amplification effect of risk aversion in a simulation where both level and volatility shocks are active depends on the specific calibration of the model. The next section digs deeper into this issue and highlights the key role played by the cost of debt parameter $\Phi_D$ (which is higher in Fernández-Villaverde et al. (2011) and lower in Born and Pfeifer (2014), see Table F.2) in determining the interaction between risk aversion and the level shock to interest rates.

[Insert Table 5 about here.]

4.1.3 Level Shock to the Country Spread

Examining the variance decomposition in Table 4, it is striking that as risk aversion increases from Panel A to Panel B, the unconditional volatilities of macroeconomic aggregates actually drop in columns (1), (3), (4) and (5). This is driven by the level shock to country spread in column (5), as columns (2)
and (6) show that the level shock to TFP and the volatility shock to country spread generate higher economic volatilities with increasing risk aversion. This implies that risk aversion can dampen the macroeconomic response to some shocks while strengthening the response to others.

To understand the mechanism causing elevated risk aversion to attenuate output, consumption and investment volatilities following level shocks to country spread, we focus on the Euler equation specific to the open economy model of FGRU (2011):

\[
\frac{1}{1 + r_t} = \Phi_D (D_{t+1} - D) + \beta \mathbb{E}_t \left[ \left( \frac{C_{t+1}}{C_t} \right)^{-\nu} \right].
\]

Here, like the original model, we assume CRRA utilities for the ease of exposition. To start with, assume the debt adjustment parameter, \( \Phi_D \), is zero. A positive level shock to \( r_t \) increases the country spread and lowers the price \( \frac{1}{1 + r_t} \) of the internationally traded bond. Under the low risk aversion calibration, \( \nu = 5 \) for example, lower bond price today translates into higher expected consumption growth between today and tomorrow in the Euler equation. As a result, the representative agent optimally decides to borrow more today and invest less in capital. As risk aversion increases, to \( \nu = 15 \) for example, a level shock of the same magnitude to country spread does not raise consumption growth expectation as significantly. To see this, rewrite the Euler equation in logs while keeping \( \Phi_D = 0 \):

\[
e^{-r_t} = \beta \mathbb{E}_t \left[ e^{-\nu(c_{t+1}-c_t)} \right].
\]

Holding the increase in \( r_t \) constant, larger \( \nu \) means smaller \( (c_{t+1} - c_t) \). As consumption growth expectation is tempered due to high risk aversion, the representative agent does not adjust borrowing and investment after the level shock is realized as dramatically relative to the case when risk aversion is low. Taken together, high risk aversion attenuates the dynamic response of macroeconomic variables with respect to level shocks through the consumption
growth expectations.

In the benchmark case, however, $\Phi_D$ is not zero, and the debt adjustment cost term enters the Euler equation. Under this scenario, a positive level shock to country spread lowers the price of the internationally traded bond and causes debt level to decline (since $\Phi_D > 0$). Furthermore, because the debt adjustment term partially absorbs the price drop, consumption growth expectation does not alter between high and low risk aversion calibrations as much compared to the scenario when $\Phi_D$ is zero. Therefore, the representative agent assuages the disinvestment to similar degrees regardless of high or low risk aversion. In other words, debt adjustment cost renders the impact of risk aversion on the debt and investment responses to level shocks to country spread ineffective.

Figure 13 demonstrates the FGRU model implied impulse response functions for output, consumption, investment, hours, $q$ and debt following a positive level shock to country spread under low and high risk aversion calibrations. Panel A presents the IRFs when the debt adjustment cost parameter is set to close to zero ($\Phi_D = 0.0001$), while Panel B contains the IRFs when the same parameter is set to 0.1. When the adjustment term is small in Panel A, higher $r_t$ causes output, investment, hours and $q$ to decline. At the same time, consumption and borrowing increase due to a rise in consumption growth expectation, consistent with the mechanism described above. Furthermore, as risk aversion is elevated from 5 to 15 in Panel A, the drops in output and investment and the increase in borrowing are less exaggerated.

In Panel B of Figure 13, we set the debt adjustment term to be large. Three takeaways are immediate. First, in comparison to Panel A, the positive level shock to country spread leads to moderate declines in consumption and borrowing as the adjustment cost absorbs most of the “good news” generated by lower price of debt. Second, as consumption growth expectation is mitigated
by the cost of changing the debt level, investment only dips mildly in contrast to when \( \Phi_D = 0 \). Finally, the differential impact of the country spread shock under low and high risk aversion is completely nullified in Panel B in the presence of debt adjustment cost. These implications are in line with our priors formed by examining the Euler equation of the FGRU model and provide us with insights on the interaction between risk aversion and first moment shocks to the real interest rate.

5 Conclusion

Our study shows that not only risk matters to equilibrium outcomes, but more importantly, the degree of risk aversion determines the magnitude of these outcomes.

The effects we document here are quantitatively important: after a positive shock to volatility, the higher the risk aversion, the larger and more prolonged the decline in economic activity. Empirically, we show that conditional on the fact that risk aversion was elevated during the 2008 crisis, the fall in output and investment driven by uncertainty was deepened by 21% and 16%, respectively.

From a theoretical perspective, a New-Keynesian model with endogenous time-variation in risk aversion is able to reproduce our empirical evidence. Moreover, the risk-aversion-dependence of macroeconomic quantitative response to volatility shocks continue to hold in an open economy model with no rigidities and in which conditional volatility affects the real interest rate.

In contrast to volatility shocks, the responses of macroeconomic quantities to level shocks are in general more robust to varying degrees of risk aversion.

Our results could be relevant for policymakers to consider stochastic volatility, and its interplay with financial quantities via risk-aversion, when implementing fiscal and monetary policy.
References


## Tables

### Table 1: Model Parameters

This table reports the parameters used for the model with Epstein-Zin (EZ) preferences and the model incorporating Epstein-Zin preferences over habit-adjusted consumption (EZ-Habit) rather than consumption itself.

<table>
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<td><strong>Preferences:</strong></td>
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### Table 2: Variance Decomposition - The Effect of Structural Shocks

This table reports the variance decomposition for the different structural shocks. The first column shows data moments. The second column reports the model moments simulations with all shocks, the third column reports model moments with level shocks to productivity only and column four shows model moments with volatility shocks to productivity only. Panel A (B) shows simulation results for the model with Epstein-Zin (EZ) preferences and a \( \gamma \) parameter of 80 (160). Panel C, accordingly, reports simulation results for the model incorporating Epstein-Zin preferences over habit-adjusted consumption (EZ-Habit) rather than consumption itself. All moments are calculated based on 200 simulations of the pruned state-space system.

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<th>(3) Level only</th>
<th>(4) Volatility only</th>
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<tr>
<td>( \sigma_Y )</td>
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Table 3: IRF Analysis - The Effect of a Volatility Shocks in FGRU

This table reports the drops in macroeconomic variables, and the length of the recovery phase, for alternative values of risk aversion. Results are shown for both the model by Fernández-Villaverde et al. (2011) and the recalibrated corrected model by Born and Pfeifer (2014). “Recovery time” is defined as time (closest quarter) it takes for a variable to revert back to its unconditional mean. An example: With a risk aversion of 15, it takes 4 quarters for consumption to revert back to its mean level after a one standard deviation shock in volatility. See also Figure 12.

<table>
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<td>11</td>
<td>26</td>
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<tr>
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<td>12</td>
<td>27</td>
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<td>11</td>
<td>26</td>
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Panel A: Output

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<td>-2.757</td>
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</tr>
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<td>-4.390</td>
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Panel B: Consumption

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<tr>
<td>15</td>
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<td>15</td>
<td>-9.778</td>
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<tr>
<td>20</td>
<td>-5.774</td>
<td>5</td>
<td>15</td>
<td>-15.353</td>
<td>4</td>
<td>14</td>
</tr>
<tr>
<td>25</td>
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<td>16</td>
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<td>14</td>
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Panel C: Investment
TABLE 4: Variance Decomposition FGRU - The Effect of Structural Shocks

This table reports the variance decomposition for the different structural shocks in the model of Fernández-Villaverde et al. (2011) with stochastic volatility. First column: 200 simulations of the model; second column: TFP shocks only; third column: without volatility shocks to spread and T-bill rate; fourth column: without TFP shocks; fifth column: only level shocks to the spread and the T-bill rate; sixth column: only shocks to the volatility of spread and the T-bill rate.

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Shocks</td>
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<td>5.02</td>
<td>5.09</td>
<td>1.10</td>
<td>0.64</td>
<td>0.16</td>
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<tr>
<td>TFP only</td>
<td>7.60</td>
<td>2.63</td>
<td>4.70</td>
<td>7.11</td>
<td>4.00</td>
<td>0.75</td>
</tr>
<tr>
<td>w/o volatility</td>
<td>20.63</td>
<td>5.00</td>
<td>12.71</td>
<td>19.90</td>
<td>11.63</td>
<td>3.08</td>
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</table>

Panel A: γ = 5, Pruning, 200 Replications

<table>
<thead>
<tr>
<th>Interest Rate</th>
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<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<td>1.07</td>
<td>0.57</td>
<td>0.31</td>
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<td>6.86</td>
<td>3.53</td>
<td>1.48</td>
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<tr>
<td>w/o volatility</td>
<td>20.42</td>
<td>5.42</td>
<td>11.86</td>
<td>19.57</td>
<td>10.54</td>
<td>5.89</td>
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</tbody>
</table>

Panel B: γ = 15, Pruning, 200 Replications

TABLE 5: Variance Decomposition FGRU - The Effect of Structural Shocks

This table reports the variance decomposition for the different structural shocks relying on the recalibration of Born and Pfeifer (2014) with stochastic volatility. First column: 200 simulations of the model; second column: TFP shocks only; third column: without volatility shocks to spread and T-bill rate; fourth column: without TFP shocks; fifth column: only level shocks to the spread and the T-bill rate; sixth column: only shocks to the volatility of spread and the T-bill rate.

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>4.49</td>
<td>0.75</td>
<td>0.40</td>
<td>0.27</td>
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<tr>
<td>TFP only</td>
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<td>3.81</td>
<td>1.76</td>
<td>1.40</td>
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<tr>
<td>w/o volatility</td>
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<td>9.78</td>
<td>14.19</td>
<td>7.80</td>
<td>5.46</td>
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</table>

Panel A: γ = 5, Pruning, 200 Replications

<table>
<thead>
<tr>
<th>Interest Rate</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Shocks</td>
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<td>4.47</td>
<td>0.93</td>
<td>0.38</td>
<td>0.66</td>
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<tr>
<td>TFP only</td>
<td>5.24</td>
<td>2.40</td>
<td>2.65</td>
<td>4.63</td>
<td>1.20</td>
<td>3.38</td>
</tr>
<tr>
<td>w/o volatility</td>
<td>19.54</td>
<td>6.68</td>
<td>9.85</td>
<td>18.11</td>
<td>7.11</td>
<td>13.10</td>
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Panel B: γ = 15, Pruning, 200 Replications

35
Figures

Figure 1: **State-dependent (dividend-price) IR to an uncertainty shock**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the VXO. Our proxy for risk aversion is the log dividend-price ratio. The state variable $RA_t$ takes values -1 (low risk aversion; blue line with squares), 0 (black line), and +1 (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.
Figure 2: **State-dependent (consumption-wealth ratio) IR to an uncertainty shock**: This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the VXO. Our proxy for risk aversion is the CAY variable (Lettau and Ludvigson (2001)). The state variable $RA_t$ takes values -1 (low risk aversion; blue line with squares), 0 (black line), and +1 (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.
Figure 3: **State-dependent (leverage) IR to an uncertainty shock:** This figure plots the empirical impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the VXO. Our proxy for risk aversion is intermediary leverage (He et al. (2017)). The state variable $RA_t$ takes values -1 (low risk aversion; blue line with squares), 0 (black line), and +1 (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.
(a) Output and Leverage.

(b) Output and Dividend-price ratio.

**Figure 4:** The Role of the interaction between Uncertainty and Risk aversion in Output. The solid line displays (log) per capita, real GDP for our sample. The dashed line is the four periods ahead forecasts from direct regressions that allow for an interaction between risk aversion and uncertainty. The line with squares is the four periods ahead forecasts from direct regression with no interaction between risk aversion and uncertainty. Shaded areas indicate NBER recession dates. We measure uncertainty using the VXO, a well-known and readily-observable measure of aggregate uncertainty. To proxy for risk aversion we use either the intermediary (equity-based) leverage measure by He et al. (2017) (Panel A), or the dividend-price ratio (Panel B).
Figure 5: The Role of the interaction between Uncertainty and Risk aversion in Investment: The solid line displays (log) per capita, real investment for our sample. The dashed line is the four periods ahead forecasts from direct regressions that allow for an interaction between risk aversion and uncertainty. The line with squares is the four periods ahead forecasts from direct regressions with no interaction between risk aversion and uncertainty. Shaded areas indicate NBER recession dates. We measure uncertainty using the VXO, a well-known and readily-observable measure of aggregate uncertainty. To proxy for risk aversion we use either the intermediary (equity-based) leverage measure by He et al. (2017) (Panel A), or the dividend-price ratio (Panel B).
Figure 6: Impulse Response Function to Technology Shock – NK-EZ Model

This figure plots the impulse responses for a one standard deviation shock to the (i) level, and (ii) volatility of technology. Impulse responses are for a one standard deviation shock when the model is approximated to the third order. To construct these responses, we set the exogenous shocks in the model to zero and iterate our third-order solution forward. After a sufficient number of periods, the endogenous variables of the model converge to a fixed point, which we denote the stochastic steady state. We then hit the economy with a one standard deviation uncertainty shock but assume the economy is hit by no further shocks. We compute the impulse response as the percent deviation between the equilibrium responses and the pre-shock stochastic steady state.
Figure 7: Model implied state-dependent (RA) IR to a productivity uncertainty shock: This figure plots the model-implied impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the productivity volatility time series from model simulations. Risk aversion is the inverse of the consumption surplus ratio. The state variable $RA_t$ takes values $-.25$ (low risk aversion; blue line with squares), $0$ (black line), and $+.25$ (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.
**Figure 8: Model implied state-dependent (dividend-price) IR to a productivity uncertainty shock.** This figure plots the model-implied impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the productivity volatility time series from model simulations. Our proxy for risk aversion is the model-implied log dividend-price ratio. The state variable $RA_t$ takes values -1 (low risk aversion; blue line with squares), 0 (black line), and +1 (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.
Figure 9: Model implied state-dependent (RA) IR to a stock market uncertainty shock:
This figure plots the model-implied impulse responses (estimated using SLP) to an uncertainty shock for
different levels of risk aversion. We measure uncertainty using the model-implied VXO index which equals
the expected conditional volatility of the return on firm equity. The state variable $RA_t$ takes values -0.25 (low
risk aversion; blue line with squares), 0 (black line), and 0.25 (high risk aversion; red line with circles) units
of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment
(bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IR of
GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The shaded areas denote 90%
confidence intervals.
Figure 10: Model implied state-dependent (dividend-price) IR to a stock market uncertainty shock: This figure plots the model-implied impulse responses (estimated using SLP) to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the model-implied VXO index which equals the expected conditional volatility of the return on firm equity. Our proxy for risk aversion is the model-implied log dividend-price ratio. The state variable $RA_t$ takes values -1 (low risk aversion; blue line with squares), 0 (black line), and +1 (high risk aversion; red line with circles) units of $\sigma(RA_t)$. The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column displays the state multiplier of the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The shaded areas denote 90% confidence intervals.
Figure 11: Impulse Response Functions – FGRU: This figure plots the impulse responses for a one standard deviation shock to the (i) technology level (ii) interest rate (iii) conditional volatility in interest rate. Impulse responses are for a one standard deviation shock when the model is approximated up to third order. The cost of debt is set to $\Phi_D = 0.001$. To construct these responses, we set the exogenous shocks in the model to zero and iterate our third-order solution forward. After a sufficient number of periods, the endogenous variables of the model converge to a fixed point, which we denote the stochastic steady state. We then hit the economy with a one standard deviation to e.g. country spread shock but assume the economy is hit by no further shocks. The IRFs must be interpreted as the percentage deviations from the ergodic mean in absence of shocks, EMAS. See Appendix F.1 for a comparison between steady state, ergodic mean, and EMAS in the Fernández-Villaverde et al. (2011) model, and Appendix E for additional details.
Low Risk Aversion: $\gamma = 5$

High Risk Aversion: $\gamma = 15$

**Figure 12: Impulse Response Function to a Volatility Shock Interest Rates – Fernández-Villaverde et al. (2011) vs Born and Pfeifer (2014) Calibrations**

Impulse responses are for a one standard deviation shock to the conditional volatility in interest rate when the model is approximated up to third order. The IRFs must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016).
Figure 13: Impulse Response Function to Country Spread Level Shock for different values of the holding cost of debt: This figure plots the impulse responses for a one standard deviation shock to the country spread level shock when the (i) cost of debt $\Phi_D = 0.0001$, and (ii) cost of debt $\Phi_D = 0.1$. Impulse responses are for a one standard deviation shock when the model is approximated up to third order. To construct these responses, we set the exogenous shocks in the model to zero and iterate our third-order solution forward. After a sufficient number of periods, the endogenous variables of the model converge to a fixed point, which we denote the stochastic steady state. We then hit the economy with a one standard deviation country spread shock but assume the economy is hit by no further shocks.
A Illustrative Example

In this section, we write down a simple dynamic model with Epstein and Zin (1989) and Weil (1990) preferences and stochastic volatility to demonstrate the steps in solving such a model using higher order perturbation techniques. The exercise is for illustration purposes only since solving the model in closed form is too burdensome and numerical solutions are well known. The reader familiar with perturbation theory can safely skip this material, and hop to Section 4.

A.1 Economic Environment

We augment a standard real business cycle model with Epstein-Zin-Weil preferences. For simplicity, we abstract away from labor supply. A representative firm takes capital and produces a single consumption good. Capital accumulation is frictionless, meaning there is no adjustment cost other than a constant depreciation each period. We further assume the production technology has decreasing return to scale.

With Epstein-Zin-Weil recursive utilities, the representative agent maximizes lifetime utility by solving the following:

\[
\max_v V_t(C_t, V_{t+1}) = \left[ (1 - \beta) \frac{C_t^{1-\psi}}{1-\psi} + \beta \left\{ E_t \left[ V_{t+1}^{1-\gamma} \right] \right\}^{\frac{1-\psi}{1-\gamma}} \right]^{\frac{1}{1-\gamma}},
\]

subject to

\[
C_t + K_{t+1} = e^{z_t} K_t^\alpha + (1 - \delta) K_t,
\]

where \(\psi\) is the inverse of elasticity of intertemporal substitution and \(\gamma\) is the coefficient of relative risk aversion. The budget constraint is a combination of the aggregate resource constraint, the production function, and the capital accumulation equation after substituting out investment. The production function is \(Y_t = e^{z_t} K_t^\alpha\), and \(z_t\) is a transitory TFP shock subject to stochastic volatility. Similar to Caldara et al. (2012), we specify \(z_t\) to follow the exogenous process:

\[
\begin{align*}
\dot{z}_t &= \rho z_{t-1} + \chi e^{\sigma_t} \epsilon_t \\
\sigma_t &= (1 - \phi) \tilde{\sigma} + \phi \sigma_{t-1} + \chi \nu \omega_t,
\end{align*}
\]

where \(\epsilon_t\) and \(\omega_t\) are uncorrelated normal shocks. \(\chi\) is the perturbation parameter that allows us to turn the stochastic model into a deterministic model, and vice versa.
A.2 Equilibrium Conditions

Optimal decision on consumption and investment result in two standard equilibrium conditions: the stochastic discount factor and the \(q\)-investment equation. Respectively,

\[
\frac{\lambda_{t+1}}{\lambda_t} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\psi} \left[ \frac{V_{t+1}}{\mathbb{E}_t \left[ V_{t+1}^{1-\gamma} \right]^{1-\gamma}} \right]^{\psi-\gamma}
\]

and

\[
1 = \mathbb{E}_t \left[ \frac{\lambda_{t+1}}{\lambda_t} \left\{ \alpha e^{z_{t+1}} K_{t+1}^{\alpha-1} - (1 - \delta) \right\} \right],
\]

in which \(\lambda_t\) is the Lagrangian multiplier of the resource constraint and \(\alpha e^{z_t} K_t^{\alpha-1}\) is the marginal productivity of capital. The equilibrium conditions can be combined to get the following rational expectation condition:

\[
\mathbb{E}_t \left[ \beta \left\{ f(K_{t+1}, z_{t+1}) - K_{t+2} \right\}^{-\psi} V_t^{\psi-\gamma} \left\{ \alpha e^{z_{t+1}} K_{t+1}^{\alpha-1} - (1 - \delta) \right\} - C_t^{-\psi} \mathbb{E}_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{\psi-\gamma}{1-\psi}} \right] = 0. \quad (A.1)
\]

Denoting \(e^{z_t} K_t^{\alpha} + (1 - \delta) K_t\) as \(f(K_t, z_t)\), the budget constraint implies \(C_t = f(K_t, z_t) - K_{t+1}\), which we can plug into Equation (A.1) to arrive at the following:

\[
\mathbb{E}_t \left[ \beta \left\{ f(K_{t+1}, z_{t+1}) - K_{t+2} \right\}^{-\psi} V_t^{\psi-\gamma} \left\{ \alpha e^{z_{t+1}} K_{t+1}^{\alpha-1} - (1 - \delta) \right\} - \left\{ f(K_t, z_t) - K_{t+1} \right\}^{-\psi} \mathbb{E}_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{\psi-\gamma}{1-\psi}} \right] = 0. \quad (A.2)
\]

Before we can solve the model using perturbation, we need to include a second equation to completely identify the system of equations. Following Caldara, Fernández-Villaverde, Rubio-Ramírez and Yao (2012), we employ equilibrium condition perturbation (ECP) and include the Epstein Zin value function evaluated at its arguments of the maxima such that

\[
V_t - \left[ \frac{1 - \beta}{1 - \psi} \left\{ f(K_t, z_t) - K_{t+1} \right\}^{1-\psi} + \beta \mathbb{E}_t \left[ V_{t+1}^{1-\gamma} \right]^{\frac{1-\gamma}{1-\psi}} \right]^{\frac{1}{1-\psi}} = 0, \quad (A.3)
\]

after substituting out consumption. Together, Equations (A.2) and (A.3) and the shock processes for \(z_t\) and \(\sigma_t\) constitute the system of rational expectation equations to be solved by perturbation.

A.3 Solution Strategy

Equations (A.2) and (A.3) can be seen as error functions, \(F(s_t)\). They have to hold under any values of \(K_t\) and \(V_t\). By the implicit function theorem, the derivatives of the equilibrium conditions also have to hold to be zeros, which allows us to solve for higher order terms in the policy functions of the endogenous variables. As a first step, we rewrite
the error functions in logs such that

\[
\begin{align*}
F^1(s_t) &= \mathbb{E}_t \left[ \beta \left\{ f(e^{k_{t+1}}, z_{t+1}) - e^{k_{t+2}} \right\}^{-\psi} e^{(\psi-\gamma)\nu_{t+1}} \left\{ \alpha e^{z_{t+1} + (\alpha - 1)k_{t+1}} - (1 - \delta) \right\} \\
- \left\{ f(e^{k_t}, z_t) - e^{k_{t+1}} \right\}^{-\psi} \mathbb{E}_t \left[ e^{(1-\gamma)\nu_{t+1}} \right]^{\frac{1}{1-\psi}} \right] = 0 \\
F^2(s_t) &= e^{\nu_t} - \left[ \frac{1 - \beta}{1 - \psi} \left\{ f(e^{k_t}, z_t) - e^{k_{t+1}} \right\}^{1-\psi} + \beta \left\{ \mathbb{E}_t \left[ e^{(1-\gamma)\nu_{t+1}} \right] \right\}^{\frac{1}{1-\psi}} \right] = 0,
\end{align*}
\]

where \( s_t \) contains the state variables \((k_t, z_{t-1}, \epsilon_t, \sigma_{t-1}, \omega_t, \chi)\). Policy functions of the endogenous variables can be expressed in terms of \( s_t \) following Taylor series expansions around the non-stochastic steady state, \((k_{ss}, 0, 0, \bar{\sigma}, 0, 0)\). Under the second order approximation, policy functions \(k_{t+1} = g^1(s_t)\) and \(v_t = g^2(s_t)\) can be written as:

\[
k_{t+1} = k_{ss} + g^1_k(k_t - k_{ss}) + g^1_z z_{t-1} + g^1_\epsilon \epsilon_t + g^1_\sigma (\sigma_{t-1} - \bar{\sigma}) + g^1_\omega \omega_t + g^1_\chi \chi \\
+ \frac{1}{2} \left[ g^2_{kk}(k_t - k_{ss})^2 + g^2_{zz} z_{t-1}^2 + g^2_{k\epsilon} \epsilon_t^2 + g^2_{k\sigma} (\sigma_{t-1} - \bar{\sigma})^2 + g^2_{k\omega} \omega_t^2 + g^2_{k\chi} \chi^2 \right] \\
+ g^2_{kz}(k_t - k_{ss})z_{t-1} + g^2_{k\epsilon} (k_t - k_{ss})\epsilon_t + g^2_{k\sigma} (k_t - k_{ss}) (\sigma_{t-1} - \bar{\sigma}) + g^2_{k\omega} (k_t - k_{ss}) \omega_t \\
+ g^2_{k\chi}(k_t - k_{ss}) \chi + g^1_z z_{t-1} \epsilon_t + g^1_\epsilon \epsilon_t \chi + g^1_\sigma (\sigma_{t-1} - \bar{\sigma}) \omega_t + g^1_\sigma (\sigma_{t-1} - \bar{\sigma}) \chi \\
+ g^1_\omega \omega_t \chi, \quad (A.4)
\]

and

\[
v_t = v_{ss} + g^2_v(k_t - k_{ss}) + g^2_z z_{t-1} + g^2_\epsilon \epsilon_t + g^2_\sigma (\sigma_{t-1} - \bar{\sigma}) + g^2_\omega \omega_t + g^2_\chi \chi \\
+ \frac{1}{2} \left[ g^2_{vv}(k_t - k_{ss})^2 + g^2_{zz} z_{t-1}^2 + g^2_{v\epsilon} \epsilon_t^2 + g^2_{v\sigma} (\sigma_{t-1} - \bar{\sigma})^2 + g^2_{v\omega} \omega_t^2 + g^2_{v\chi} \chi^2 \right] \\
+ g^2_{vz}(k_t - k_{ss})z_{t-1} + g^2_{v\epsilon} (k_t - k_{ss})\epsilon_t + g^2_{v\sigma} (k_t - k_{ss}) (\sigma_{t-1} - \bar{\sigma}) + g^2_{v\omega} (k_t - k_{ss}) \omega_t \\
+ g^2_{v\chi}(k_t - k_{ss}) \chi + g^2_{vz} z_{t-1} \epsilon_t + g^2_{v\epsilon} \epsilon_t \chi + g^2_{v\sigma} (\sigma_{t-1} - \bar{\sigma}) \omega_t + g^2_{v\sigma} (\sigma_{t-1} - \bar{\sigma}) \chi \\
+ g^2_\omega \omega_t \chi. \quad (A.5)
\]

Steady state values \( k_{ss} \) and \( v_{ss} \) can be found by solving the system of equations \( F(k_{ss}, 0, 0, \bar{\sigma}, 0, 0) \).

### A.4 First Order Approximation

Define \( z_t = h^1(z_{t-1}, \epsilon_t, \sigma_{t-1}, \omega_t, \chi) = h^1(s_t') \) and \( \sigma_t = h^2(\sigma_{t-1}, \omega_t, \chi) = h^2(s_t') \) for exposition purposes. To demonstrate the solution technique, we focus on the error function \( F^1 \). Under a first order approximation, only the
first order terms in the Taylor series expansion appear (no cross-derivatives). After plugging in \( g^1(s_t), g^2(s_t), \) and \( h(s'_t) \)

\[
P^1(s_t) = E_t \left[ \beta \left\{ f(e^{g^1(s_t)}, h^1(s'_t)) - e^{g^1(s_{t+1})} \right\}^{(\psi-\gamma)} e^{(1-\gamma)g^2(s_{t+1})} \left\{ \alpha e^{h^1(s'_{t+1})+(\alpha-1)g^1(s_t)} - (1 - \delta) \right\} \right.

- \left. \left\{ f(e^{k_t}, h^1(s'_t) - e^{g^1(s_t)}) \right\}^{(\psi-\gamma)} E_t \left[ e^{(1-\gamma)g^2(s_{t+1})} \right]^{\frac{(\psi-\gamma)}{\psi}} \right]

= \beta \left\{ f(e^{g^1(s_t)}, h^1(s'_t)) - e^{g^1(s_{t+1})} \right\}^{(\psi-\gamma)} e^{(1-\gamma)g^2(s_{t+1})} \left\{ \alpha e^{h^1(s'_{t+1})+(\alpha-1)g^1(s_t)} - (1 - \delta) \right\} - \left\{ f(e^{k_t}, h^1(s'_t) - e^{g^1(s_t)}) \right\}^{(\psi-\gamma)} E_t \left[ e^{(1-\gamma)g^2(g^1(s_t),h^1(s_t))} \right]^{\frac{(\psi-\gamma)}{\psi}}

= 0,

and take derivative with respect to \( k_t \):

\[
P^1(s_t)|^{k_t} = E_t \left[ -\beta \psi \left\{ f(e^{g^1(s_t)}, h^1(s'_t)) - e^{g^1(s_{t+1})} \right\}^{(\psi-\gamma)} \left\{ f_k(e^{g^1(s_t)}, h^1(s'_t))g_k - e^{g^1(s_{t+1})} \right\} + \beta \left\{ f(e^{g^1(s_t)}, h^1(s'_t)) - e^{g^1(s_{t+1})} \right\}^{(\psi-\gamma)} (\psi-\gamma)e^{(1-\gamma)g^2(g^1(s_t),h^1(s'_t))} + \beta \left\{ f(e^{g^1(s_t)}, h^1(s'_t)) - e^{g^1(s_{t+1})} \right\}^{(\psi-\gamma)} \left\{ f_k(e^{g^1(s_t)}, h^1(s'_t))g_k - e^{g^1(s_{t+1})} \right\} \right.

+ \left. \left\{ f(e^{k_t}, h^1(s'_t) - e^{g^1(s_t)}) \right\}^{(\psi-\gamma)} \left\{ f_k(e^{k_t}, h^1(s'_t)) - e^{g^1(s_t)} \right\}^{(\psi-\gamma)} g_k \right]

= 0.

Evaluating at the steady state, \( s_t = (k_{ss}, 0, 0, \bar{\sigma}, 0, 0) \) (i.e. by setting shocks to zeros), the expectations drop out from the above express while \( g^1(s_t) = k_{ss} \),
\[ g^2(s_t) = u_{ss}, \text{ and } h(s_t') = 0: \]

\[
\mathbf{F}^1(s_t)|_{s_{ss}} = -\beta \psi \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(s_t)} \right\}^{-(\psi - 1)} \left\{ f_k(e^{g^1(s_t)}, h^1(h^1(s_t')))g^1_k - e^{g^1(g^1(s_t))}g^2_k \right\} \\
e^{(\psi - \gamma)g^2(g^1(s_t),h^1(s_t'))} \left\{ \alpha e^{h^1(h^1(s_t')) + (\alpha - 1)g^1(s_t)} - (1 - \delta) \right\} + \beta \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{-(\psi - \gamma)}g^2(g^1(s_t),h^1(s_t')) \\
(\psi - \gamma)e^{(\psi - \gamma)g^2(g^1(s_t),h^1(s_t'))}g^1_kg^2_k \left\{ \alpha e^{h^1(h^1(s_t')) + (\alpha - 1)g^1(s_t)} - (1 - \delta) \right\} \\
+ \beta \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{-(\psi - \gamma)}e^{(\psi - \gamma)g^2(g^1(s_t),h^1(s_t'))} \alpha(\alpha - 1)e^{h^1(h^1(s_t')) + (\alpha - 1)g^1(s_t)}g^1_k \\
+ \psi \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{-(\psi - \gamma)}f_k(e^{k_t}, h^1(s_t')) - e^{g^1(g^1(s_t))}g^1_k \\
- \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{-(\psi - \gamma)}(\psi - \gamma)e^{(\psi - \gamma)g^2(g^1(s_t),h^1(s_t'))}g^1_kg^2_k \\
= -\beta \psi \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{-(\psi - 1)} \left\{ f_k(e^{g^1(s_t)}, h^1(h^1(s_t')))g^1_k - e^{g^1(g^1(s_t))}g^2_k \right\} \\
\left\{ \alpha e^{h^1(h^1(s_t')) + (\alpha - 1)g^1(s_t)} - (1 - \delta) \right\} + \beta \left\{ f(e^{g^1(s_t)}, h^1(h^1(s_t'))) - e^{g^1(g^1(s_t))} \right\}^{-(\psi - 1)} \alpha(\alpha - 1)e^{h^1(h^1(s_t')) + (\alpha - 1)g^1(s_t)}g^1_k \\
+ \psi \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{-(\psi - 1)}f_k(e^{k_t}, h^1(s_t')) - e^{g^1(g^1(s_t))}g^1_k \\
- \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{-(\psi - 1)}(\psi - \gamma)g^1_kg^2_k \\
= 0,
\]

which is a non-linear equation of two unknowns, \( g^1_k \) and \( g^2_k \).

Next, we move on to the second error function. After replacing \( k_{t+1} = g^1(s_t), v_t = g^2(s_t), \text{ and } z_t = h^1(s_t): \)

\[
\mathbf{F}^2(s_t) = e^{g^2(s_t)} - \left[ \frac{1 - \beta}{1 - \psi} \left\{ f(e^{k_t}, h^1(s_t')) - e^{g^1(s_t)} \right\}^{1-\psi} + \beta \left\{ E_{k_t} \left[ e^{(1-\gamma)g^2(g^1(s_t),h^1(s_t'))} \right] \right\}^{1-\psi} \right]^{1-\psi} \\
= 0.
\]
Similarly, the derivative of $F^2(s_t)$ with respect to $k_t$ can be written as

$$
F^2(s_t)|_{k_t} = e^{g^2(s_t)}g_k^2 - \left(\frac{1}{1-\psi}\right) \left[\frac{1-\beta}{1-\psi} \left\{ f(e^{k_t}, h^1(s'_t)) - e^{g^1(s_t)} \right\} \right]^{1-\psi} \\
+ \beta \left\{ \mathbb{E}_t \left[ e^{(1-\gamma)g^2(g^1(s_t), h^1(s'_t))} \right] \right\}^{\frac{1-\psi}{1-\gamma}} \left[ (1-\beta) \left\{ f(e^{k_t}, h^1(s'_t)) - e^{g^1(s_t)} \right\} \right]^{-\psi} \\
\left\{ f_k(e^{k_t}, h^1(s'_t)) - e^{g^1(s_t)}g_k^1 \right\} + \beta \left( \frac{1-\psi}{1-\gamma} \right) \left\{ \mathbb{E}_t \left[ e^{(1-\gamma)g^2(g^1(s_t), h^1(s'_t))} \right] \right\}^{\frac{\gamma-\psi}{\gamma}} \\
(1-\gamma)e^{(1-\gamma)g^2(g^1(s_t), h^1(s'_t))}g_k^1g_k^2 \\
= 0.
$$

When evaluated at the steady state, $g^1(s_t) = k_{ss}$, $g^2(s_t) = v_{ss}$, and $h(s'_t) = 0$, and the above expression becomes

$$
F^2(s_t)|_{k_t}^{ss} = e^{g^2(s_t)}g_k^2 - \left(\frac{1}{1-\psi}\right) \left[\frac{1-\beta}{1-\psi} \left\{ f(e^{k_t}, h^1(s'_t)) - e^{g^1(s_t)} \right\} \right]^{1-\psi} \\
+ \beta e^{(1-\psi)g^2(g^1(s_t), h^1(s'_t))} \left[ (1-\beta) \left\{ f(e^{k_t}, h^1(s'_t)) - e^{g^1(s_t)} \right\} \right]^{-\psi} \\
\left\{ f_k(e^{k_t}, h^1(s'_t)) - e^{g^1(s_t)}g_k^1 \right\} + \beta e^{(1-\psi)g^2(g^1(s_t), h^1(s'_t))}g_k^1g_k^2 \\
= 0, \quad (A.7)
$$

which is again a non-linear equation in $g_k^1$ and $g_k^2$. The process is then repeated for each of the remaining state variables ($z_{t-1}, \epsilon_t, \sigma_{t-1}, \omega_t, \chi$) to generate corresponding error functions evaluated at the steady state, $F(s_t)|_{ss}^{(z_{t-1}, \epsilon_t, \sigma_{t-1}, \omega_t, \chi)}$. Combining Equations (A.6) and (A.7) with the rest of the error functions, we can solve for all the unknown coefficient loadings in the policy function, $g_j^{(k, z, \epsilon, \sigma, \omega, \chi)}$ for $j \in \{1, 2\}$. It is widely known in the literature that $g_{\sigma} = g_{\omega} = 0$ in general, and the coefficient of risk aversion $\gamma$ does not affect $g_k, g_z, g_\epsilon$, under first order perturbation due to certainty equivalence, consistent with the Tallarini (2000) result.
A.5 Higher Order Approximation

The perturbation solution technique is increasingly complex in several dimensions: the size of the system, the number of state variables, and the order of approximation. Higher order perturbation takes the same approach as that in Section A.4 of the online appendix, but the error functions are 2nd, 3rd, and so on, derivatives of the original system of equations. It is cumbersome to solve for the policy functions analytically even for a two-equation model under first order approximation, as demonstrated previously. For that reason, numerical programs (AIM, Dynare, etc.) are often employed for the purposes of taking derivatives.

In general, the online appendix of Andreasen (2012) shows that the second order terms in the policy function are zeros when they are related to the cross-derivatives of the state variables and the perturbation parameter (χ in the illustrative example here). Numerical simulation of the two-equation model with stochastic volatility used here produces policy functions in line with that observation:

\[ k_{t+1} = k_{ss} + g_k^1 (k_t - k_{ss}) + g_z^1 z_{t-1} + g_\epsilon^1 \epsilon_t + g_\sigma^0 (\sigma_{t-1} - \bar{\sigma}) + g_\omega^0 \omega_t + g_\chi^0 \chi \]

\[ + \frac{1}{2} \left[ g_{kk}^1 (k_t - k_{ss})^2 + g_{zz}^2 z_{t-1}^2 + g_{\epsilon\epsilon}^2 \epsilon_t^2 + g_{\sigma\sigma}^0 (\sigma_{t-1} - \bar{\sigma})^2 + g_{\omega\omega}^0 \omega_t^2 + g_{\chi\chi}^0 \chi^2 \right] \]

\[ + g_{kz}^0 (k_t - k_{ss}) z_{t-1} + g_{ke}^0 (k_t - k_{ss}) \epsilon_t + g_{k\sigma}^0 (k_t - k_{ss}) (\sigma_{t-1} - \bar{\sigma}) + g_{k\omega}^0 (k_t - k_{ss}) \omega_t \]

\[ + g_{k\chi}^0 (k_t - k_{ss}) \chi + g_{z\epsilon}^1 z_{t-1} \epsilon_t + g_{z\sigma}^1 z_{t-1} (\sigma_{t-1} - \bar{\sigma}) + g_{z\omega}^1 z_{t-1} \omega_t + g_{z\chi}^1 z_{t-1} \chi \]

\[ + g_{\epsilon\sigma}^0 (\sigma_{t-1} - \bar{\sigma}) + g_{\epsilon\omega}^0 \epsilon_t \omega_t + g_{\epsilon\chi}^0 \epsilon_t \chi + g_{\sigma\omega}^0 (\sigma_{t-1} - \bar{\sigma}) \omega_t + g_{\sigma\chi}^0 (\sigma_{t-1} - \bar{\sigma}) \chi \]

\[ + g_{\omega\chi}^0 \omega_t \chi, \]  

(A.8)
and

\[ v_t = v_{ss} + g_k^2(k_t - k_{ss}) + g_k^2\varepsilon_t + \frac{1}{2} \left[ g_{kk}^2 (k_t - k_{ss})^2 + g_{zz}^2\varepsilon_t^2 + g_{\sigma\sigma}^2 (\sigma_t - \bar{\sigma})^2 + g_{\omega\omega}^2 \omega_t^2 + g_{\chi\chi} \chi^2 \right] \]

\[ + g_{kz}^2 (k_t - k_{ss})z_t \varepsilon_t + g_{k\varepsilon}^2 \varepsilon_t^2 + g_{k\omega}^2 \omega_t^2 + g_{k\chi} \chi \]

where the surviving cross-derivatives \((g_{kz}, g_{k\varepsilon}, g_{k\omega}, g_{k\chi})\) are small relative to the first order terms. This renders stochastic volatility ineffective in driving the dynamics of the model.

Furthermore, Binsbergen et al. (2012) and Caldara et al. (2012) show that in models with Epstein-Zin-Weil preferences, parameter of relative risk aversion \((\gamma)\) only enters the solution in \(g_{\chi\chi}\), generating a constant in the policy rule that can be interpreted as the precautionary behavior toward risk. However, a cursory inspection of the policy functions shows there is no direct interaction between \(g_{\chi\chi}\), which contains \(\gamma\), and stochastic volatility \((\sigma_t - 1)\) and the shock to stochastic volatility \((\omega_t)\). This implies that a second order perturbation is not enough to explain how increasing risk aversion can amplify the response of the macroeconomic variables to uncertainty shocks. To do so, at least a third order approximation is required because risk aversion enters into all non-zero coefficient loadings in the policy function in front of the square of the perturbation parameter \((\chi^2)\), such as \(g_{\sigma\chi\chi}\) and \(g_{\omega\chi\chi}\). This insight allows us to break the Tallarini (2000) irrelevance result making risk matter in DSGE models.
B Local Projections: Additional Results

B.1 Local Projections Vs. VAR

In this section we check whether the linear (i.e. $\beta_{1,h} = 0$, in Eq. (1) in the text) LP (and SLP) methodology delivers similar results to the original specification in BB (2017), which is instead based on a VAR. To this end, Figure B.1 displays the original VAR-based response of BB (2017) overlaid with the responses from our LP and SLP estimation.

Figure B.1: IR to a uncertainty shock: This figure plots the empirical impulse responses to uncertainty shock. We measure uncertainty using the VXO. The IR of GDP (top), consumption, investment, and inflation (bottom) to a volatility shock are estimated using LP (blue line) and SLP (red line). The black dashed line is the original response in Basu–Bundick (2017). The dashed lines denote the 68% confidence interval.

The figure shows that our methodology replicates in a nonparametric setting the findings of BB (2017) such that higher uncertainty causes declines in output, consumption, investment, and inflation.
B.2 Local Projections: Alternative Measure of Uncertainty

This section shows that replacing the VXO for the aggregate uncertainty index proposed by Jurado et al. (2015) does not affect our results. To this end, we run the same specification in Eq. (1) (c.f. Section 2), but we proxy for $UNC_t$ using the financial uncertainty series. Comparing (the right panels in) Figure B.2 to Figure 3 we observe that the state multiplier is hardly affected by the measure of uncertainty. Similar results are obtained when we proxy risk aversion with the dividend-price ratio.

![Figure B.2: State-dependent (leverage) IR to an uncertainty shock](image)

**Figure B.2: State-dependent (leverage) IR to an uncertainty shock:** This figure plots the empirical impulse responses to an uncertainty shock for different levels of risk aversion. We measure uncertainty using the aggregate financial uncertainty index (Ludvigson et al. (2017)). Our proxy for risk aversion is intermediary leverage (He et al. (2017)). The left column shows the state-dependent IR of GDP (top), consumption (mid), and investment (bottom) to a volatility shock. The right column reports the state multiplier of the state-dependent IRs. The shaded areas denote 90% confidence intervals.
B.3 Local Projections: Alternative Measure of Risk Aversion

Figure B.3 presents the time series plots of realized and fitted values of output (Panel A) and investment (Panel B) when RA is proxied by the consumption-wealth ratio.

Figure B.3: The Role of the interaction between Uncertainty and Risk aversion in Output and Investment: The solid line in Panel A displays (log) per capita, real GDP for our sample. The solid line in Panel B displays (log) per capita, real investment for our sample. The dashed line is the four periods ahead forecasts from direct regressions that allow for an interaction between risk aversion and uncertainty. The line with squares is the four periods ahead forecasts from direct regression with no interaction between risk aversion and uncertainty. Shaded areas indicate NBER recession dates. We measure uncertainty using the VXO, a well-known and readily-observable measure of aggregate uncertainty. To proxy for risk aversion we use the CAY variable by Lettau and Ludvigson (2001).
B.4 Local Projections Vs. Generalized IRFs

In this section we investigate whether the SLP method is able to recover the unconditional (Generalized) IRFs to technology uncertainty shocks from the NK-EZ-Habit model. We compare the unconditional responses with the SLP-estimated IRFs recovered from 200 economies that each span 116 periods. Further, the technology uncertainty shock in the SLP exercise is re-scaled such that the SLP and unconditional responses of output coincide on impact. As can be seen from figure B.4 below, the two sets of IRFs are indeed very similar on impact but also at longer horizons.

![Graphs showing output, consumption, and investment responses to a technology uncertainty shock.](image)

**Figure B.4: Unconditional Model IR vs SLP IR to a uncertainty shock.** This figure plots the unconditional impulse responses of output, consumption, and investment to an uncertainty shock in the NK-EZ-Habit model (blue line) against the median impulse responses recovered from SLP on model simulated data with risk aversion being neutral, i.e. $RA_t = 0$ (black dashed line).

---

23We use 116 periods to match the 116 quarters of data used in our empirical exercise.
C  New-Keynesian Model Economy

This section describes the building blocks of the New-Keynesian model employed in Section 3.

C.1 Preferences

As discussed in section 3, the difference between the models NK-EZ and NK-EZ-Habit lies in the preference structure. The NK-EZ model employs standard Epstein and Zin (1989) preferences where lifetime utility at time \( t \) and the corresponding stochastic discount factor are defined as follows:

\[
U_t = \left[ (C^\eta_t(1 - N_t)^{1-\eta} \right)^{1-\gamma} + \beta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{C.1}
\]

and

\[
M_{t,t+1} = \beta \left( \frac{C_{t+1}^\eta(1 - N_{t+1})^{1-\eta}}{C_t^\eta(1 - N_t)^{1-\eta}} \right)^{(1-\gamma)/\theta} \left( \frac{C_{t+1}}{C_t} \right)^{-1} \left( \frac{U_{t+1}^{1-\gamma}}{E_t U_{t+1}^{1-\gamma}} \right)^{1-1/\theta}, \tag{C.2}
\]

respectively.

Conversely, in the NK-EZ-Habit model the Epstein-Zin utility function is defined over habit-adjusted consumption rather than consumption itself. Hence, lifetime utility at time \( t \) and the corresponding stochastic discount factor are defined as follows:

\[
U_t = \left[ (C^{h\eta}_t(1 - N_t)^{1-\eta} \right)^{1-\gamma} + \beta \left( E_t U_{t+1}^{1-\gamma} \right)^{\frac{1}{\theta}} \right]^{\frac{\theta}{1-\gamma}}, \tag{C.3}
\]

and

\[
M_{t,t+1} = \beta \left( \frac{C_{t+1}^{h\eta}(1 - N_{t+1})^{1-\eta}}{C_t^{h\eta}(1 - N_t)^{1-\eta}} \right)^{(1-\gamma)/\theta} \left( \frac{C_{t+1}^h}{C_t^h} \right)^{-1} \left( \frac{U_{t+1}^{1-\gamma}}{E_t U_{t+1}^{1-\gamma}} \right)^{1-1/\theta}, \tag{C.4}
\]

respectively.
C.2 Intermediate Goods Producers

There is a continuum of intermediate goods producers that rent labor from households. The intermediate goods market is monopolistically competitive and producers face each period quadratic costs \( \phi_P \) when changing their nominal price \( P_t(i) \). The firms own their capital stocks \( K_t(i) \) and face convex costs \( \phi_K \) of changing the quantity of capital installed. In addition to prices, firms choose the rate of utilization of their installed physical capital \( U_t(i) \) which affects its depreciation rate. Firm \( i \) chooses labor input \( N_t(i) \), investment \( I_t(i) \), and prices \( P_t(i) \) to maximize cash flows \( D_t(i)/P_t(i) \) given aggregate demand \( Y_t \) and the aggregate goods price index \( P_t \). Further, the intermediate goods firms all have the same constant-returns-to-scale Cobb-Douglas production function, subject to a fixed production cost \( \Phi \).

Each firm maximizes the discounted cash flows:

\[
\max\mathbb{E}_t \sum_{s=0}^{\infty} M_{t,t+s} \frac{D_{t+n}(i)}{P_{t+s}} \tag{C.5}
\]

subject to the production function,

\[
\left[ \frac{P_t(i)}{P_t} \right]^{-\theta} Y_t \leq [K_t(i)U_t(i)]^\alpha [Z_tN_z(i)]^{1-\alpha} - \Phi, \tag{C.6}
\]

and subject to the capital accumulation equation,

\[
K_{t+1}(i) = \left( 1 - \delta (U_t(i)) - \frac{\phi_K}{2} \left( \frac{I_t(i)}{K_t(i)} - \delta \right)^2 \right) K_t(i) + I_t(i) \tag{C.7}
\]

The cash flows are defined as follows:

\[
\frac{D_t(i)}{P_t} = \left[ \frac{P_t(i)}{P_t} \right]^{1-\theta} Y_t - \frac{W_t}{P_t} N_t(i) - I_t(i) - \frac{\phi_P}{2} \left[ \frac{P_t(i)}{P_{t-1}(i)} - 1 \right]^2 Y_t, \tag{C.8}
\]

and capital depreciation is non-linearly determined by utilization:

\[
\delta(U_t(i)) = \delta + \delta_1 (U_t(i) - U) + \left( \frac{\delta_2}{2} \right) (U_t(i) - U)^2 \tag{C.9}
\]
Finally, each intermediate goods firm finances a fraction $\nu$ of its capital stock each period with one-period risk-less bonds which pay the one-period real risk-free interest rate. Therefore, total firm cash flows are split into payments to bond and equity holders:

$$\frac{D^E_t(i)}{P_t} = \frac{D_t(i)}{P_t} - \nu \left( K_t(i) - \frac{1}{R^R_{t+1}} K_{t+1}(i) \right)$$  \hspace{1cm} (C.10)

### C.3 Final Goods Producers

The representative final goods uses $Y_t(i)$ units of each intermediate good, where $i \in [0,1]$ to assemble the final good. The market for final goods is perfectly competitive which results in zero profits in equilibrium. The aggregate goods price index $P_t$ is defined as:

$$P_t = \left[ \int_0^1 P_t(i)^{1-\theta} \nu di \right]^{\frac{1}{1-\theta}}$$  \hspace{1cm} (C.11)

### C.4 Monetary Policy

The monetary authority sets the nominal interest rate, $r_t$, to stabilize inflation and output growth. Doing so, the monetary policy is in accordance with the Taylor rule:

$$r_t = r + \rho_\pi (\pi_t - \pi) + \rho_y \Delta y_t$$  \hspace{1cm} (C.12)

where $r_t = \ln(R_t)$, $\pi_t = \ln(\Pi_t)$, and $\Delta_t = \ln(Y_t/Y_{t-1})$. The gross nominal interest rate $R_t$ is further pinned down by the standard Euler equation:

$$1 = R_t E_t \left[ M_{t+1} \frac{1}{\Pi_{t+1}} \right]$$  \hspace{1cm} (C.13)

### C.5 Equilibrium

In the symmetric equilibrium, all intermediate goods firms choose the same price, employ the same amount of labor, the same amount of capital and the same utilization rate. As a result, all intermediate goods firms have the same cash flows that are financed with the exogenously determined mix of
bonds and equity. Intuitively, one can interpret the continuum of firms as one representative intermediate goods producer.

C.6 Calibration

In the calibration of the model parameters we rely to a large extent on parameter values used in the literature. As such, our parametrization of the NK-EZ model follows closely the one from Basu and Bundick (2017). We deviate from their benchmark calibration by increasing the capital adjustment cost parameter $\phi_K$ to 10 and lowering the leverage ratio to 0.3. In our setup, leverage does not affect firm value or optimal firm decisions. Leverage simply makes the price of equity more volatile. A leverage ratio of 0.3 implies price-dividend dynamics that are roughly in line with the data. Further, the parameters governing the exogenous productivity process are comparable with the parameters from Andreasen (2012). Finally, for the NK-EZ-Habit model we calibrate the habit related parameters according to Campbell and Cochrane (1999).
C.7 Proxies for Risk Aversion: Simulations

Figure C.1 shows one simulation path for model variables that proxy for risk aversion: surplus consumption ratio, price/dividend ratio, and wealth/consumption.

![Figure C.1: Different Proxies for Risk Aversion within the Model](image-url)

**Figure C.1: Different Proxies for Risk Aversion within the Model**: This figure plots one simulation paths (1000 periods with a burn-in period of 5000) for the various proxies of risk aversion within our model environment, i.e. surplus consumption ratio, price/dividend ratio, and wealth/consumption ratio. The correlation between consumption surplus and price/dividend ratios is 0.927; the correlation between consumption surplus and wealth/consumption ratios 0.854; and the correlation between the price/dividend and wealth/consumption ratios is 0.767. Note that the wealth/consumption ratio is defined as follows: \( W/C = (E_t [U_{t+1}/C_t])^{(1-1/\psi)} \).
D Consumption Surplus and Risk Aversion

From Swanson (2018), wealth-gamble risk aversion with recursive preferences can be written as:
\[
R^a(a_t; \theta_t) = \frac{-\mathbb{E}_t[U(a_{t+1}^*; \theta_{t+1})^{\gamma}U_{11}(a_{t+1}^*; \theta_{t+1}) - \gamma U(a_{t+1}^*; \theta_{t+1})^{-\gamma}U_1(a_{t+1}^*; \theta_{t+1})^2]}{\mathbb{E}_t[U(a_{t+1}^*; \theta_{t+1})^{-\gamma}U_1(a_{t+1}^*; \theta_{t+1})]},
\]
where \(a_{t+1}^*\) is optimal wealth through the budget constraint \(a_{t+1}^* \equiv (1 + R_t) a_t + W_t N_t^* + D_t - C_t^*\). \(\theta_t\) represents the set of exogenous shocks driving the dynamics of the economy. Subscripts 1 and 11 denote the first and second derivatives, respectively, with respect to future wealth.

Assuming within-period power utility over habit adjusted consumption only, the Epstein-Zin value function has the following form:
\[
U(a_{t+1}^*; \theta_{t+1}) = \left[ \frac{C_{t+1}^h}{1 - \psi} + \beta \mathbb{E}_{t+1} \left[ U(a_{t+2}^*; \theta_{t+2})^{1-\gamma} \right] \right]^{\frac{1}{1-\psi}}.
\]
Recall that \(C_{t+1}^h = C_{t+1}^* S_{t+1}\).

We can apply the chain rule to calculate the derivative of \(U(a_{t+1}^*; \theta_{t+1})\) with respect to \(a_{t+1}^*\) such that
\[
\frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial a_{t+1}^*} = \frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} \frac{\partial C_{t+1}^h}{\partial a_{t+1}^*} + \frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} \frac{\partial C_{t+1}^h}{\partial a_{t+1}^*} = \frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} S_{t+1}(1 + R_{t+1}),
\]
and
\[
\frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} = \frac{1}{1 - \psi} \left[ U(a_{t+1}^*; \theta_{t+1})^{1-\gamma} \right]^{\frac{1}{1-\psi} - 1} C_{t+1}^h^{-\psi} = \frac{1}{1 - \psi} U(a_{t+1}^*; \theta_{t+1})^{\psi} C_{t+1}^h^{-\psi}.
\]

The second derivative of \(U(a_{t+1}^*; \theta_{t+1})\) with respect to \(a_{t+1}^*\) can be found by repeated application of the product rule to the expression \(\frac{\partial U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h} S_{t+1}(1 + R_{t+1})\) and notice that \(\frac{\partial S_{t+1}}{\partial a_{t+1}^*} = 0\) and \(\frac{\partial (1 + R_{t+1})}{\partial a_{t+1}^*} = 0\). Thus we have:
\[
\frac{\partial^2 U(a_{t+1}^*; \theta_{t+1})}{\partial a_{t+1}^* \partial a_{t+1}^*} = \frac{\partial^2 U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h \partial C_{t+1}^h} S_{t+1}^2(1 + R_{t+1})^2,
\]
\[
\frac{\partial^2 U(a_{t+1}^*; \theta_{t+1})}{\partial C_{t+1}^h \partial C_{t+1}^h} = \frac{1}{1 - \psi} \left[ -\psi U(a_{t+1}^*; \theta_{t+1}) \psi C_{t+1}^h \psi \right]
\]

Then, we can calculate the terms in \( R^a \):

\[
U(a_{t+1}^*; \theta_{t+1})^{-\gamma} U_1(a_{t+1}^*; \theta_{t+1}) = \frac{1}{1 - \psi} U(a_{t+1}^*; \theta_{t+1}) \psi - \gamma C_{t+1}^h \psi S_{t+1} (1 + R_{t+1}),
\]

\[
U(a_{t+1}^*; \theta_{t+1})^{-\gamma} U_{11}(a_{t+1}^*; \theta_{t+1}) = \frac{\psi}{\psi - 1} U(a_{t+1}^*; \theta_{t+1}) \psi - \gamma C_{t+1}^h \psi \left[ C_{t+1}^h \psi \right] S_{t+1}^2 (1 + R_{t+1})^2,
\]

\[
U(a_{t+1}^*; \theta_{t+1})^{-\gamma - 1} U_1(a_{t+1}^*; \theta_{t+1})^2 = \left( \frac{1}{1 - \psi} \right)^2 U(a_{t+1}^*; \theta_{t+1})^2 \psi - \gamma - 1 C_{t+1}^h \psi S_{t+1}^2 (1 + R_{t+1})^2,
\]
such that

\[
R^a(a_t; \theta_t) = \mathbb{E}_t \left[ \frac{\psi}{\psi - 1} U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} \left( C_{t+1}^{\psi} \left[ C_{t+1}^{\psi - 1} - \frac{1}{\psi - 1} U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} C_{t+1}^{\psi - 1} \right] S_{t+1}^{2} (1 + R_{t+1})^2 \right] \right. \\
+ \left. \mathbb{E}_t \left[ \frac{1}{\psi - 1} U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} C_{t+1}^{\psi - 1} S_{t+1} (1 + R_{t+1}) \right] \right]
\]

\[
\approx \psi \left[ C_{t+1}^{\psi - 1} - \frac{1}{1 - \psi} U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} C_{t+1}^{\psi - 1} \right] S_{t+1} (1 + R_{t+1}) \\
+ \gamma \left( \frac{1}{1 - \psi} \right) U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} C_{t+1}^{\psi - 1} S_{t+1} (1 + R_{t+1}) \\
= \psi \left[ C_{t+1}^{\psi - 1} + \frac{1}{\psi - 1} U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} C_{t+1}^{\psi - 1} S_{t+1}^{1 - \psi} \right] (1 + R_{t+1}) \\
+ \gamma \left( \frac{1}{1 - \psi} \right) U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} C_{t+1}^{\psi - 1} S_{t+1}^{1 - \psi} (1 + R_{t+1}),
\]

where the approximation relies on the fact that the risk involved in the wealth gamble is minuscule.

Recall that \( \psi \) is the inverse of the elasticity of intertemporal substitution. Under standard calibration, \( \psi > 1 \), and \( R^a(a_t; \theta_t) \) can be written as:

\[
R^a(a_t; \theta_t) \approx \mathbb{E}_t \left[ \psi \left\{ \frac{1}{C_{t+1}^{\psi - 1}} + \frac{1}{\psi - 1} U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} \frac{1}{C_{t+1}^{\psi - 1}} \frac{1}{S_{t+1}^{\psi - 1}} \right\} (1 + R_{t+1}) \right. \\
+ \left. \gamma \left( \frac{1}{1 - \psi} \right) U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} \frac{1}{C_{t+1}^{\psi - 1}} \frac{1}{S_{t+1}^{\psi - 1}} (1 + R_{t+1}) \right]
\]

\[
= \mathbb{E}_t \left[ \psi \left\{ \frac{1}{C_{t+1}^{\psi - 1}} + \frac{\psi - \gamma}{\psi - 1} U(a_{t+1}^*; \theta_{t+1})^{\psi - 1} \frac{1}{C_{t+1}^{\psi - 1}} \frac{1}{S_{t+1}^{\psi - 1}} \right\} (1 + R_{t+1}) \right]
\]

which is an inverse function of \( S_{t+1} \). In other words, when the consumption surplus ratio is high, wealth-gamble risk aversion is low and vice versa. The derivation with leisure preference is similar but slightly more involved.
E Perturbation Methods and Generalized Impulse Response Function

This appendix includes a more detailed discussion of the solution of the model and the explanation of how we compute the IRFs and the variance decomposition of the model. We refer the interest reader to the Born and Pfeifer (2014) Appendix for an exhaustive discussion of the use of perturbation and pruning techniques, and their implications for simulation and IRFs.

To judge the importance of volatility shocks for business cycle moments, and their interaction with risk aversion, our analysis relies on perturbation methods. Perturbation methods were first extensively applied to dynamic stochastic models by Judd (1998).

Our investigation faces a number of computational challenges. First, we are interested in the implications of a volatility increase while keeping the level of the variable constant. We thus have to consider a third-order Taylor expansion of the solution of the model, see e.g., Schmitt-Grohe and Uribe (2004), Fernández-Villaverde et al. (2011) and Fernández-Villaverde et al. (2015). Indeed, in a first-order approximation, stochastic volatility would not even play a role, since the policy rules of the representative agent follow a certainty equivalence principle. In the second-order approximation, only the product of the two innovations appears in the policy function. Only in the third-order approximation do the innovations to volatility play a role by themselves.

Second, higher order perturbation solutions tend to explode due to the accumulation of terms of increasing order. For example, in a second order approximated solution, the quadratic term at time $t$ will be raised to the power of two in the quadratic term at $t+1$, thus resulting in a quartic term, which will become a term of order 8 at $t+2$ and so on. As a solution, we adopt the pruning scheme described in Andreasen et al. (2016). This pruning scheme augments the state space to keep track of first to third order terms and uses the Kronecker product of the first and second order terms to compute the third order term. In contrast, Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014) use a IRF-pruning scheme were all higher order terms are based

\footnote{Recently, de Groot (2016) shows that to risk-correct the constant term for the standard deviation of stochastic volatility innovations (a.k.a. vol of vol) a fourth (or sixth, depending on the functional form of the volatility process) order expansion is further needed. de Groot (2016) shows that this risk-correction has important consequences for the bond and equity risk premia as well as for understanding the welfare cost of business cycle fluctuations.}
on the first-order terms. Also, whereas in Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014) the IRF-pruning scheme differs from the scheme used for simulations, we use the same pruning for both IRFs and simulations.

Third, computing IRFs in a nonlinear environment is somewhat involved, since the IRFs are not invariant to rescaling and to the previous history of shocks. To circumvent this problem, we consider the generalized impulse response function (GIRF) proposed by Koop et al. (1996). In particular, we follow Fernández-Villaverde et al. (2011), Born and Pfeifer (2014) and Basu and Bundick (2017), and we start the IRFs at the ergodic mean in the absence of shocks (EMAS).

Fourth, to judge the importance of risk shocks for business cycle moments, it is instructive to consider a variance decomposition. However, computing a variance decomposition is complicated because, with a third-order approximation to the policy function and its associated nonlinear terms, we cannot neatly divide total variance among the shocks as we would do in the linear case. Thus, to gauge the relative importance of shocks we follow Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014), and we simulate the model with only a subset of the shocks. In particular, we set the realizations of one or two of the shocks to zero and measure the volatility of the economy with the remaining shocks. The agents in the model still think that the shocks are distributed by the law of motion that we specified: it just happens that their realizations are zero in the simulation.
F The Fernández-Villaverde et al. (2011) Model: Additional Details

F.1 Steady State, EMAS, and Ergodic Mean

This appendix compares the deterministic steady state, the ergodic mean in the absence of shocks (EMAS), and the ergodic mean for the Fernández-Villaverde et al. (2011) model. It is in fact well known that time-varying volatility moves the ergodic distribution of the endogenous variables of the model away from their deterministic steady state. The theoretical mean are based on the third-order pruned state space of Andreasen et al. (2016). We use the term EMAS for Fernández-Villaverde et al. (2011)’s concept of “[s]tarting from the ergodic mean and in the absence of shocks” (p. 10 in their technical appendix). The EMAS is the fixed point of the third order approximated policy functions in the absence of shocks. Sometimes, it is referred to as the “stochastic steady state” (e.g. Juillard and Kamenik 2005), because it is the point of the state space where, in absence of shocks in that period, agents would choose to remain although they are taking future volatility into account.

Table F.1 compares steady state, ergodic mean, and EMAS in the original model of Fernández-Villaverde et al. (2011). Results for the Born and Pfeifer (2014) re-calibrated model after correcting for time aggregation are available upon request.

Table F.1: Ergodic Mean FGRU (2011)

This table reports the steady-state values, the analytical ergodic means, and the simulated ergodic means in the absence of shocks for the FGRU (2011) model. We consider also the model with Epstein-Zin preferences when risk aversion equals the inverse of the elasticity of substitution.

<table>
<thead>
<tr>
<th></th>
<th>Analytical Ergodic Mean</th>
<th>Simulated EMAS</th>
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<tbody>
<tr>
<td></td>
<td>Steady State FGRU</td>
<td>FGRU EZ</td>
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<tr>
<td></td>
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<tr>
<td></td>
<td>$D_t$</td>
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<td></td>
<td>$K_t$</td>
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<td>$X_t$</td>
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</tr>
<tr>
<td></td>
<td>$C_t$</td>
<td>0.00</td>
</tr>
</tbody>
</table>

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Table F.2: Parameters for FGRU (2011) model economy

This table reports the parameters used for the Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014) models. These are the same values as in their original papers, and reported here for readers convenience. The high risk aversion scenario refers to a change in $\gamma$ from 5 to 15, while leaving all other parameters unchanged.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<td>$\psi$</td>
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<td>5.0000</td>
</tr>
<tr>
<td>$\eta$</td>
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<td>5.0000</td>
</tr>
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<td>$\beta$</td>
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<td>0.9804</td>
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<tr>
<td>$\delta$</td>
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<tr>
<td>$\alpha$</td>
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<td>0.3200</td>
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<tr>
<td>$\Phi_D$</td>
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<td>0.0006</td>
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<td>$D$</td>
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<tr>
<td>$\varphi$</td>
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<td>$r$</td>
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<td>0.02</td>
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<tr>
<td>$\rho_x$</td>
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<td>0.9500</td>
</tr>
<tr>
<td>$\sigma_x$</td>
<td>-3.2168</td>
<td>-4.1997</td>
</tr>
<tr>
<td>$\rho_r$</td>
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<td>0.9700</td>
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<tr>
<td>$\sigma_r$</td>
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<td>-5.7100</td>
</tr>
<tr>
<td>$\rho_{\sigma_r}$</td>
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<td>0.9400</td>
</tr>
<tr>
<td>$\eta_r$</td>
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<td>0.4600</td>
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<tr>
<td>$\rho_{tb}$</td>
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<td>$\rho_{\sigma_{tb}}$</td>
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<tr>
<td>$\sigma_{tb}$</td>
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<td>0.9400</td>
</tr>
<tr>
<td>$\eta_{tb}$</td>
<td>0.1300</td>
<td>0.1300</td>
</tr>
</tbody>
</table>
F.2 Time Aggregation: Moments, IRFs and Variance Decomposition

Fernández-Villaverde et al. (2011) set up their model in monthly terms, but report results at quarterly frequency. We follow their approach, and we aggregate monthly output, consumption, investment to quarterly frequency by summing up monthly percentage deviations. The only exceptions are Figure 12 and Table 5 where we follow Born and Pfeifer (2014) and we aggregate by averaging percentage deviations of monthly flow variables.

For the moment computations, the percentage deviations are from the deterministic steady state. Following Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014), the quarterly variables are HP-filtered before using them to compute the moments.

The variance decomposition in Tables 4 and 5 are obtained as follows. We simulate the model, starting from the ergodic mean, for 96 periods. We hit the equilibrium system with a subset of the shocks. As we mentioned in the main text as well as in the next section, since the data come in quarterly frequency, we build quarters of data from the model-simulated variables, and we H-P filter them. The simulations are always restarted at this point after 96 periods and there is no burn-in. We repeat this exercise 200 times to obtain the mean of the moments over the 200 simulations. Table F.3 check the stability of our simulations.

For the impulse response functions in Figure F.1 the percentage deviations are from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016). In particular, we compute GIRFs at the true ergodic mean using the methods proposed in Andreasen et al. (2016). However, the analytical expression for the ergodic mean is available only for the third-order (or lower) pruned state space described in Andreasen et al. (2016). Thus in the second and third columns in Figure F.1 we compute the IRFs at the EMAS. In particular, we first simulate the model, starting from the ergodic mean (obtained analytically using a third-order pruned state space), for 2,096 periods. We disregard the first 2,000 periods as a burn-in and use the last 96 periods to compute the IRFs. In period 2,001 we set the realization of one of the shocks (productivity, the country spreads and its volatility) to one. We repeat this exercise 200 times to obtain the mean of the IRFs over the 200 simulations. As already mentioned, since the data come in quarterly frequency, we build quarters of data from the model-simulated monthly IRFs.
F.3 Robustness

In the interest of space we run a battery of robustness tests for the FGRU (2011) economy. Analogous results for the BB (2017) economy are available upon request.

Simulation Table F.3 checks the stability of our simulations. In particular it shows that our results are robust to an increase in the number of replications, and to removing pruning from the simulations.

Table F.3: Variance Decomposition FGRU (2011) - Robustness Tests

This table reports the variance decomposition for the different structural shocks in the model of FGRU (2011) with stochastic volatility. First column: 200 simulations of the model; second column: TFP shocks only; third column: without volatility shocks to spread and T-bill rate; fourth column: without TFP shocks; fifth column: only level shocks to the spread and the T-bill rate; sixth column: only shocks to the volatility of spread and the T-bill rate.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
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<tr>
<td>All Shocks</td>
<td>γ = 15, Pruning, 1000 Replications</td>
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<tr>
<td>$\sigma_Y$</td>
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<td>5.14</td>
<td>5.18</td>
<td>1.07</td>
<td>0.58</td>
<td>0.31</td>
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<tr>
<td>$\sigma_C$</td>
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<td>2.81</td>
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<td>7.00</td>
<td>3.63</td>
<td>1.52</td>
</tr>
<tr>
<td>$\sigma_I$</td>
<td>20.81</td>
<td>5.55</td>
<td>12.13</td>
<td>20.01</td>
<td>10.89</td>
<td>6.01</td>
</tr>
<tr>
<td></td>
<td>Panel B: γ = 15, No Pruning, 200 Replications</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>5.22</td>
<td>5.01</td>
<td>5.07</td>
<td>1.01</td>
<td>0.58</td>
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</tr>
<tr>
<td>$\sigma_C$</td>
<td>7.18</td>
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<tr>
<td>$\sigma_I$</td>
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<td>11.97</td>
<td>18.11</td>
<td>10.63</td>
<td>5.74</td>
</tr>
</tbody>
</table>

Stochastic Volatility and Perturbation Solution To compute impulse response functions we have so far relied on a pruned state-space system for non-linear DSGE models when the model is approximated up to third order, see Andreasen et al. (2016). Moreover, we have followed Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014) so that the IRFs we reported so far must be interpreted as deviations from the ergodic mean in absence of shocks, EMAS.
In Figure F.1 we investigate the effects of pruning and the order of approximation on our results.

**Figure F.1:** Impulse Responses to a Volatility Shock in Interest Rates – FGRU (2011): Impulse responses are for a one standard deviation shock when the model is approximated up to third order (first and second columns) and to the fifth order (last column). The IRFs in the first column must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016). The IRFs in the second to third column must be interpreted as percentage deviations from the ergodic mean in the absence of shocks (EMAS).

The first column shows the IRFs obtained when we use the analytical expressions for the generalized impulse response function (GIRF) derived in Andreasen et al. (2016) for a model that is pruned and approximated up to...
third order. These IRFs must be interpreted as the percentage deviations from the theoretical mean based on the third-order pruned state space of Andreasen et al. (2016). The second column in Figure F.1 shows GIRF when the system has not been pruned. Comparing these impulse responses with those in Figure 11 it is clear that our results are not affected by pruning, nor by choosing EMAS or ergodic mean as the initial condition.

The third column in Figure F.1 investigates how our results are affected by adopting a fifth-order (rather than a third-order) solution for the decision rules. We rely on the approach developed by Fernández-Villaverde and Levintal (2016) to overcome the computational challenges associated with higher than third-order approximation. The figure shows that fourth- and fifth-order terms are not important for the Fernández-Villaverde et al. (2011) calibration. Clearly, there might exist parameter values for which these orders are relevant.

Stochastic Volatility: Alternative Functional Forms Figure F.2 shows how our results change depending on the functional form used for the stochastic volatility process. This analysis is important because the finance and macro literature have largely specified stochastic volatility processes differently. One can re-write Eq. (7) as follows:

\[
\begin{align*}
\varepsilon_{r,t} &= \rho_r \varepsilon_{r,t-1} + m(x_t)u_{r,t} \\
x_{t+1} &= (1 - \rho_x) x + \rho_x x_t + \varepsilon_{x,t+1}
\end{align*}
\]

where now the innovations are scaled by \( m(x_t) \), but we leave unspecified the functional form of \( m(\cdot) \). The previous analysis focused on the functional form \( m(\cdot) \equiv \exp(\cdot) \) and \( x_t = \sigma_{r,t} \). This specification is commonly used in macroeconomics. In contrast, finance papers like to use \( m(\cdot) \equiv \sqrt(\cdot) \) and \( x_t = \sigma^2_{r,t} \) \(^{25}\) Figure F.2 shows that our results are not affected by either functional choice.

The ARCH model (Engle, 1982), and its various generalizations, provides another candidate to model time-varying volatility. In a GARCH model, the

\(^{25}\)The benefit of the \( m(\cdot) \equiv \sqrt(\cdot) \) specification is that the stochastic process is still conditionally normal and can be exploited to generate a conditionally log-normal linear approximation that accounts for risk as in Campbell and Shiller (1988). The drawback of this functional form is that it is possible to get a negative standard deviation. The functional form \( m(\cdot) \equiv \exp(\cdot) \) ensures the standard deviation remains strictly positive but, as pointed out by Andreasen (2010), has the drawback that the level of the process does not have any moments.
**Figure F.2: Impulse Responses to a Volatility Shock Interest Rates – FGRU (2011):** IRFs are for a one standard deviation shock to the conditional volatility in interest rate when the model is approximated up to third order. The IRFs must be interpreted as percentage deviations from the theoretical mean based on the third-order pruned state space of [Andreasen et al.] (2016). The first column focuses on the specification that is commonly used in macroeconomics: the functional form is \( m(x_t) \equiv \exp(x_t) \) with \( x_t = \sigma_{r,t} \). The second column focuses the typical specification used in finance papers: the functional form is \( m(x_t) \equiv \sqrt{x_t} \) and \( x_t = \sigma_{r,t}^2 \). In all cases, \( x_t \) follows an exogenous AR(1) process.

Conditional volatility is a function of lagged volatility and lagged squared residuals of the level process. Thus, a GARCH process is not driven by separate innovations relative to the level process. On the contrary, the specifications we have analyzed so far admitted two innovations, one to the country-spread and one to the volatility of the country spread, respectively. In unreported results, we analyze the IRFs to a real rate shock when stochastic volatility is modeled with GARCH, and we show that the risk aversion does not affect macro dynamics when the time-varying volatility has no separate innovations relative to the level process.