The Term Structure and Time-Series Variation of the Pricing Kernel

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Abstract

We estimate the pricing kernel from portfolios of options on the S&P 500 index for different horizons and over time. This allows us to look both at the term structure and time-series variation of the pricing kernel. We document two striking new stylized facts. First, regarding the time-series variation, we find a U-shaped pricing kernel which is substantially more pronounced in good times, while in bad times the pricing kernel is almost flat. Second, we find that the pricing kernel is less U-shaped for longer horizons, indicating that investors are most sensitive towards short-term risks. We show that leading theoretical asset pricing models cannot explain these new stylized facts.

1 Introduction

One of the key building blocks of asset pricing is the pricing kernel, which represents the preferences of investors over risky outcomes. Both for practitioners and academics it is interesting to know how prices are set by investors, and estimating the pricing kernel is a way to obtain this information. Starting with the seminal work of Aït-Sahalia and Lo (2000) and Rosenberg and Engle (2002), many studies have used option pricing information to estimate pricing kernels for the U.S. equity market¹. The main finding in this literature is that the pricing kernel is a U-shaped function of the equity index return, a stylized fact often referred to as the “pricing kernel puzzle”. We

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¹See Cuesdeanu and Jackwerth (2017) for an overview of this literature.
contribute to this literature by presenting two striking new stylized facts for the pricing kernel, and by analyzing whether existing asset pricing models can explain these new stylized facts.

The first new stylized fact concerns the time-series variation of the pricing kernel. We find that the U-shape of the pricing kernel is substantially more pronounced in good times, while in bad times the pricing kernel is almost flat. The strong U-shaped pricing kernel in good times indicates that the investor is more sensitive towards risk and willing to pay more for her undesirable states, but also more sensitive to extreme positive returns, and thus willing to pay more for upside potential. In contrast, most asset pricing models predict that in bad times the investor is more sensitive towards risk.

The second stylized fact concerns the term structure of the pricing kernel. Whereas existing work focuses on the pricing kernel for a one-month horizon, we use options with maturities beyond one month to estimate the pricing kernel for longer horizons. Our results show that investors are most sensitive towards short-term risks. For a one-month horizon we find a strong U-shaped pricing kernel, which is line with existing work. However, for longer horizons the U-shape is less pronounced, and approaches a downward sloping pricing kernel. In order to present the information on the term structure of the pricing kernel in an accessible way, we define a forward kernel and estimate it empirically. The violation of monotonicity we find in the one month kernel disappears in the forward kernel with a forward horizon of twelve months. The finding that investors care most about short-term risks is in line with Dew-Becker et al. (2016), who find that hedging short-term realized volatility is more costly than hedging shocks to expected long-term volatility.

Our empirical procedure is as follows. We estimate the pricing kernel using portfolios of options on the S&P 500, because this index is a reasonable and commonly-used proxy for the market return and the options on the S&P 500 are quite liquid. The portfolios we construct are butterfly spreads. The prices of butterfly spreads capture the risk-neutral distribution. In order to estimate the pricing kernel we also need to model the actual returns of the underlying S&P 500 index. We assume a skewed $t$-distribution with time-varying volatility. The pricing kernel then follows from the risk-neutral and real world probability distributions. Conceptually, our method is similar, amongst others, to Aït-Sahalia and Lo (2000). Aït-Sahalia and Lo (2000) exploit the result of Breeden
and Litzenberger (1978) to estimate the risk-neutral distribution and model the underlying distribution non-parametrically. Our method uses a discrete-state analogue of the result of Breeden and Litzenberger (1978). It has the advantage that our proxy for the risk-neutral distribution is an investable portfolio which facilitates the economic interpretation. Given our methodology, we estimate the pricing kernel as a function of the market return. We do this for each day in our sample period, and for different horizons, ranging from 1 month to 12 months.

After establishing the time-series and term structure properties of the observed pricing kernel, we compare these newly established stylized facts to the implications of several leading theoretical asset pricing models. The models we incorporated so far are: habit formation from Campbell and Cochrane (1999), rare disasters from Wachter (2013), long run risks from Bansal and Yaron (2004), and time-varying recovery from Gabaix (2012). We obtain the pricing kernels for all these leading asset pricing models, which are designed to capture empirical facts of the equity index return distribution. We find the following puzzling results. All of the theoretical models produce a monotonic decreasing pricing kernel as a function of the market return. In each of the models, we can distinguish good and bad times according to the value of the respective state variables. Different from our empirical finding, we find that the curvature of the pricing kernel increases instead of decreases in the models for bad times. The forward kernel we defined in the paper can also be evaluated in the theoretical models. We find that, given that the state variables equal their averages, the one month forward kernel \( n \) periods ahead is equal to the current one month pricing kernel. Even if the state variables in the models are different from their averages, the term structure of the pricing kernel is essentially flat. These implications are at odds with our empirical findings, as we document significant variation both in the time-series and term structure.

2 Methodology

We estimate the pricing kernel using butterfly spreads, which is a portfolio of options. We use European options on the S&P500 as it is regarded as a reasonable proxy for the market portfolio and the option contracts are among the most traded options.
First, we explain how to construct butterfly spreads from options and later on we discuss the details of our estimation procedure.

2.1 Butterfly spreads

A Butterfly spread is a portfolio of options, to construct a butterfly we need three different call (or put) options. In Figure 1 we explain the construction of a butterfly spread with strike $K$ and spread $\Delta K$.

As seen in Figure 1(a) a call butterfly spread is constructed by a long position in call options with strikes $K - \Delta K$ (blue line) and $K + \Delta K$ (green line) and a short position in two calls with strike $K$ (black line). The profit of the butterfly spread is given by in the red line in the graph. In order to construct a put butterfly spread, one has to take the same positions in the put options as one would do to construct a call butterfly spread. When the spread of the butterfly is smaller, its payoff ranges over a smaller interval and therefore it identifies the pricing kernel better.

From Figures 1(a) and (b) it is clear that a butterfly, either from put or call options, should have the same price as the payoff is identical. As we will explain later on, we use out of the money options to construct the butterflies as these contracts are most...
liquid.

2.2 Estimation

The estimation method we use to estimate the pricing kernel exploits the discrete-state analogue of the result by Breeden and Litzenberger (1978). In the paper they prove that the price of an Arrow-Debreu security is given by the following equation:

\[
P(K, T) = \lim_{\Delta K \to 0} \frac{c(K + \Delta K, T) + c(K - \Delta K, T) - 2c(K, T)}{(\Delta K)^2} = \frac{\partial^2 c(K, T)}{\partial K^2},
\]

where \(P(K, T)\) represents the price of an Arrow-Debreu security paying one if \(S_T = K\). \(c(K, T)\) is the price of a call option, therefore the numerator of equation (1) corresponds to the price of a butterfly spread with strike \(K\), spread \(\Delta K\) and maturity \(T\). Aët-Sahalia and Lo (2000) utilize this result to estimate the risk-neutral probability density semiparametrically. They assume the price of a call option to equal the Black-Scholes equation with volatility modeled nonparametrically, and differentiating twice gives them the risk-neutral distribution. Together with a non-parametric model of the S&P 500 return distribution, it allows them to tease out the pricing kernel. Conceptually our methodology is similar to Aët-Sahalia and Lo (2000), but we model the S&P 500 parameterically. First, we use butterfly spreads to estimate the discrete-state risk-neutral distribution. Second, we assume a parametric model for the return distribution of the S&P 500 we calculate expected payoffs of the butterfly spreads to extract the pricing kernel of the discrete states. The advantage of our methodology is that we use economically meaningful instruments to estimate the risk-neutral distribution because it is possible to invest in these butterfly spreads. Given that we can invest in these portfolios, we can check if our estimates of the kernel make sense as high estimates for the stochastic discount factor corresponds to low returns and vice versa.

The pricing kernel follows from the classical asset pricing equation: \(p = \mathbb{E}(mX)\). We know that if the market is complete, the pricing kernel is unique. In order to solve for the discrete state pricing kernel, we write the price of a butterfly spread as a linear combination of expected payoff in each state times the corresponding discount factor. To do so, we discretize the outcome space and assume that stochastic discount factor is
constant within each state.

Let us now give an example of how the discretization works. Consider an index with current price $S_0 = 1000$ and a butterfly $i$ with strike $K_i = 1000$ and spread $\Delta K = 10$. Assume for now that the state space is discretized with distance between states equal to $1\% \cdot S_0 = 10$. Therefore, the state where the return of horizon $T$ is $0\%$ has the interval $[995; 1005]$. The example of butterfly $i$ is graphically represented Figure 2.

Figure 2: In the graph the example is graphically illustrated, a butterfly which has a payoff on the range $[990; 1010]$ is represented. Three states are important for the valuation of the instrument, namely $x_{i,j-1}$, $x_{i,j}$ and $x_{i,j+1}$.

Three expected payoffs over the states are distinguished in the example which are important for the valuation of the butterfly spread. Expected payoff $x_{i,j-1}$ calculated over the range $[985; 995]$, expected payoff $x_{i,j}$ calculated over range $[995; 1005]$ and expected payoff $x_{i,j+1}$ calculated over range $[1005; 1015]$. The expected payoff over the middle state is given by:

$$x_{i,j} = \int_{995}^{1005} X_i(S_T) \cdot f(S_T) dS_T.$$  

Where $X_i(S_T)$ is the payoff function of a butterfly with strike $K_i = 1000$. To calculate expected payoffs $x_{i,j-1}$ and $x_{i,j+1}$ the intervals have to be adjusted accordingly. In this way the grid is discretized, the butterfly spread is valued as if there are only three states possible where the instrument has a positive payoff.

On a certain trading day we observe several butterfly spreads which have positive payoffs on different regions of the outcome space. Assume that, similar to the example given above, the butterflies we observe all have three discrete state in which the payoff of the butterfly is positive. For each butterfly spread observed on a trading day, we write
the price as a linear combination of the expected payoff in that state times the stochastic
discount factor of the particular state. In matrix notation, it is represented as follows,
where \( b_j, b_{j+1} \) represent the bounds of the state:

\[
\begin{pmatrix}
p_1 \\
p_2 \\
p_3 \\
p_n
\end{pmatrix} =
\begin{pmatrix}
x_{1,1} & x_{1,2} & x_{1,3} & \cdots & 0 \\
0 & x_{2,2} & x_{2,3} & \cdots & 0 \\
0 & \cdots & x_{n-1,n-2} & x_{n-1,n-1} & 0 \\
0 & \cdots & x_{n,n-2} & x_{n,n-1} & x_{n,n}
\end{pmatrix}
\begin{pmatrix}
m_1 \\
m_2 \\
m_3 \\
m_n
\end{pmatrix}
\]

The price of butterfly \( i \) observed on a certain trading day is given by \( p_i \). Furthermore,
the stochastic discount factor corresponding to state \( j \) is given by \( m_j \). In general the
expected payoff of butterfly \( i \) in state \( j \), with interval \( [b_j, b_{j+1}] \) is given by:

\[
x_{i,j} = \int_{b_j}^{b_{j+1}} X_i(S_T) \cdot f(S_T) dS_T,
\]

where \( X_i(S_T) \) is the payoff function of butterfly \( i \). When the number of states \( (j) \) equals
the number of butterflies \( (i) \), the expected payoff matrix is square, and invertible, making
pricing kernel \( (m) \) unique.

To construct the expected payoff matrix, the return distribution of the S&P 500
is modeled. We assume the return distribution to follow a Skewed-\( t \) distribution with the
volatility a polynomial function of the volatility index (VIX). The details of the model
are discussed in the next section.

2.3 Return distribution of the S&P 500

In order to estimate the pricing kernel, a return distribution for the underlying
asset, in this case the S&P 500, is assumed. The distributions are estimated on S&P 500
returns from 1990 to 2015.
First, we introduce the skewed $t$-distribution and then we will report the MLE results. The distribution is introduced in Bauwens and Laurent (2002). Different from this paper, we consider the distribution with time-varying volatility. The return distribution depends on a standardized skewed $t$-distribution. The returns of the S&P 500 follow the process:

\[
\begin{align*}
    r_t &= \mu + \epsilon_t, \\
    \sigma_t &= \alpha + \beta VIX_t + \gamma VIX_t^2, \\
    \epsilon_t &= \sigma_t \varsigma_t, 
\end{align*}
\]

where $r_t$ are returns and the standard deviation is a quadratic function of the $VIX$ at time $t$. The random variable $\varsigma_t$ is $SKST(0, 1, \xi, v)$, i.e. it follows a standardized skewed $t$-distribution with parameters $v > 2$ (degrees of freedom) and $\xi > 0$ (skewness parameter).

The density is given by:

\[
f(\varsigma_t|\xi, v) = \begin{cases} 
    \frac{2}{\xi + \frac{1}{\xi}} \cdot s \cdot g \left[ \frac{\xi (s \varsigma_t + m)}{v} \right] & \text{if } \varsigma_t < -\frac{m}{s} \\
    \frac{2}{\xi + \frac{1}{\xi}} \cdot s \cdot g \left[ \frac{1}{\xi} (s \varsigma_t + m) \right] & \text{if } \varsigma_t \geq -\frac{m}{s},
\end{cases}
\]

where $g(\cdot|v)$ is a symmetric (zero mean and unit variance) Student $t$-distribution with $v$ degrees of freedom, defined by:

\[
g(x|v) = \frac{\Gamma \left( \frac{v+1}{2} \right)}{\sqrt{\pi (v-2)} \Gamma \left( \frac{v}{2} \right)} \left[ 1 + \frac{x^2}{v-2} \right]^{-\frac{(v+1)}{2}},
\]

where $\Gamma(\cdot)$ is Euler’s gamma function. The constants $m(\xi, v)$ and $s^2(\xi, v)$ are the mean and standard deviation of the non-standardized skewed $t$-distribution, $SKST(m, s^2, \xi, v)$, and given by:

\[
\begin{align*}
    m(\xi, v) &= \frac{\Gamma \left( \frac{v+1}{2} \right) \sqrt{v-2}}{\sqrt{\pi \Gamma \left( \frac{v}{2} \right)}} \left( \xi - \frac{1}{\xi} \right), \\
    s^2(\xi, v) &= \left( \xi^2 + \frac{1}{\xi^2} - 1 \right) - m^2.
\end{align*}
\]
The standard deviation of returns is not constant over time and the changes of this variable have a significant impact on prices of butterfly spreads. We assume that the standard deviation of the S&P 500 return distribution is a polynomial function of the model-free risk-neutral volatility introduced by Britten-Jones and Neuberger (2000). The CBOE introduced the VIX index where it uses the result of Britten-Jones and Neuberger (2000) to estimate risk-neutral volatility from monthly options. In the estimation of our model for the S&P 500, we scale the yearly VIX (252 trading days) to the corresponding maturity.

A skewed $t$-distribution supports some important stylized facts of index returns. Namely, left-skew and fat tailed distributions. The $\xi$ is a skewness related parameter and has an economic interpretation. The probability mass right from the mode divided by the probability mass left from the mode equals $\frac{1}{\xi^2}$. Therefore, when $\xi < 1$, the distribution is left skewed and when $\xi > 1$ the distribution is right skewed. Fat tails are related to the parameter of the degrees of freedom $v$ corresponding to the $t$-distribution, the lower the parameter the fatter the tails and when $v \to \infty$ the distribution converges to a skewed normal distribution.

The parameters of equations (2)-(8) we estimate using Maximum Likelihood on $n$-month simple overlapping returns of the S&P 500 from 1990 to 2015. Our methodology allows us to estimate the pricing kernel for different maturities if we adjust the maturity of the options and horizon of return distribution accordingly. As we are interested in the term structure of the pricing kernel, we estimate the pricing kernel for different horizons $n$, where $n$ ranges from one up to twelve months. The estimates for the one month S&P 500 return distribution are represented in Table 1.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\xi$</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0058</td>
<td>-0.0066</td>
<td>0.8488</td>
<td>-</td>
<td>0.7022</td>
<td>14.0910</td>
</tr>
<tr>
<td>0.0058</td>
<td>-0.0148</td>
<td>1.1629</td>
<td>-2.6792</td>
<td>0.7028</td>
<td>14.6734</td>
</tr>
</tbody>
</table>

We also do the estimation where volatility is linear in VIX to observe the impact of the squared term.
The $\mu$ of Table 1 is equal to the sample average of the return sample. Given the estimates of $\alpha$ and $\beta$ of the first row, we conclude that the overall level of the VIX is larger than the realized volatility of the S&P 500 index. This phenomenon is known as the variance premium, which Baele et al. (2016) discuss as well. The estimate of $\xi$ is smaller than 1 so it indicates that the return distribution is left-skewed and this left-skew is also economically sizable. Furthermore, some evidence of fat tails is found as the estimate of the degrees of freedom $v$ is not too large.

To see how well the underlying distribution captures the S&P 500 returns, we compare expected with realized payoffs of butterflies in our sample from 1996-2016. We construct butterflies using raw option data from OptionMetrics with $\Delta K = 10$. To construct a butterfly we need three options with consecutive distance of 10 index points in strike. Given that we use the raw option data we have one trading day each month where we observe different butterfly spreads. The moneyness of the butterfly is calculated dividing the strike of the butterfly by the current index value ($S_0$). For the butterflies we observe in the sample we can calculate the expected payoff using our return distribution of the S&P 500. Given that the likelihood of a butterfly with fixed spread ending up in the money is decreasing $S_0$, we take out this effect dividing the realized payoff and expected payoff by $S_0$. In order to assess the overall ability of our return distribution to capture the expected payoff of S&P 500 butterfly spreads, we calculate for each moneyness level the time-series average of realized payoff divided by $S_0$ and the time-series average of the expected payoff divided by $S_0$. The results are given in the following figure.
Two things are important from Figure 3, first the expected payoff matches the realized payoff quite well. Secondly, the expected payoff of the butterflies is more or less the same when we add a squared term to the volatility equation. The squared term in the volatility equation makes expected volatility lower for extreme values of VIX. For this reason, as the squared term only does something for crises levels of the VIX, we choose to use the equation with the squared term to model volatility.

### 2.4 Forward pricing kernel

We are interested in pricing kernels with different horizons, and we want to observe its differences. To compare a one period kernel with a two period kernel, we construct the one period forward kernel. Otherwise, it is not straightforward to compare, for instance, a one-month pricing kernel with a two-month pricing kernel. Given that,
the one-month and two-month return distribution are quite different, it is not possible to compare the one-month and two-month kernel directly. Estimating the forward pricing kernel allows us to compare a one-month pricing kernel dependent on one-month returns with a forward kernel dependent on one-month forward returns. As the one-month return distribution and the one-month forward distribution are arguably similar, it allows us to compare the sensitivity of the investor towards one-month risks and one-month forward risks. First, we illustrate the definition of the forward pricing kernel in an example and in a later stage, we discuss what theoretical models predict for the forward kernel.

We start with the general asset pricing equation to price an asset which does not pay dividends over the following period.

\[ P_t = E_t (m_{t+1}P_{t+1}) = E_t (m_{t+1}m_{t+2}P_{t+2}) . \]

The second equality is obtained by iterating the equation one period ahead. Given our framework, we are able to estimate a pricing kernel using options with one month horizon but also for two months horizon. We assume that the pricing kernels are a function of the return over the next period. Using that assumption, we can write a two-period pricing kernel in the following way:

\[ m_{t,t+2}(r_{t+1,t+2}) = m_{t,t+1}(r_{t+1,t+2}) \cdot m_{t+1,t+2}(r_{t+1,t+2}) . \]

The one-period forward pricing kernel is represented by the last term in the equation. In order to compare the one-period kernel with the one-period forward kernel, we calculate an expected forward pricing kernel. Which is done taking the expectation over the first period return conditional on the one-period forward return.

\[ E_t(m_{t+1,t+2}(r_{t+1,t+2})|r_{t+1,t+2} = r) = \int_0^\infty \frac{m_{t,t+2}(r_{t+1,t+2}) \cdot f_t(r_{t+1})}{m_{t,t+1}(r_{t+1})} dr_{t,t+1} , \]

where \( r_{t,t+k} = \frac{P_{t+k}}{P_t} \) and \( f_t(r_{t+1}) \) the one-period return distribution conditional on time \( t \) and second period return \( r_{t+1,t+2} \).
3 Data

We use European options on the S&P 500 available on OptionMetrics to construct the volatility surface from 1996-2016. One of the reasons to make use of the surface is to address noise in the data and make our methodology less vulnerable to outliers. We estimate the pricing kernel from a system of equations using prices of butterfly spreads, which are portfolios of three different call (put) options. In the volatility surface of OptionMetrics options with a delta of 5% to 95% for call and -5% to -95% for put options are represented. One of our goals is to explore the behavior of the pricing kernel in the tails of the return distribution, therefore we use the same kernel smoothing algorithm to obtain estimates up to a delta (-)1 basis point. For the construction of the butterfly spreads we make use of out of the money options as these are the most liquid.

Given that for most of the sample European options on the S&P 500 have only one exercise date every month, we have only one observation each month if we used the raw data. On all trading days options are observed, not necessarily having a one-month maturity, as these options do contain relevant information we construct the volatility surface for the desired maturity. In this way we have approximately 20 observations each month, i.e. the amount of trading days each month.

It is possible to construct the volatility surface for a given maturity and delta on each trading day. We use the algorithm to construct one up to twelve month options with a (minimum) maximum delta (-)1 basis point. Note, we only use out of the money options to construct butterflies. We denote the spread of the butterfly by $\Delta K$, which captures the distance between the strike $K$ and the point where the payoff of the butterfly is zero. We make $\Delta K$ increasing in $S_0$, $T$ and implied volatility by choosing $\Delta K$ equal to the difference in strike of call options with deltas 0.35 and 0.50 on a certain trading day. Approximately, we keep the expected payoff of a butterfly given a moneyness level constant, as the expected value would decrease in any of the aforementioned variables. To make sure that the prices of butterflies with moneyness more towards the tails of the distribution are significantly larger than zero, we scale $\Delta K$ in the tails of the return distribution by two. By doing so, we can still exploit the differences in price for butterflies with extreme moneyness levels.

As we are discretizing the state-space, we have to choose the distance between
two consecutive states. We choose the amount of states on which the pricing kernel is estimated to be stable at 20 estimates per day. The spread of the butterfly is twice as large in the tails of the return distribution, similarly we choose the distance between consecutive states to be twice as large in the tails. Calculating the implied strike from the volatility surface of options with delta -1 and 1 basis points gives us the interval on which we can estimate the pricing kernel. The amount of states are distributed over the interval as follows, relatively twice as many states lay in the interval of one-standard deviation around the mode of skewed-$t$ distribution adjusted for the skew. Calculated in the following way:

$$\left[ m_t - \frac{c \cdot \sigma_t}{1 + \xi^2}, m_t + \frac{c \cdot \sigma_t}{1 + \frac{1}{v}} \right]$$

where $m_t$ is the mode of the distribution and $c = 2 \cdot t^{-1}(0.84, v)$ comes from a $t$-distribution with $v$ degrees of freedom. The described interval is the skewed-$t$ equivalent of the interval $[\mu - \sigma; \mu + \sigma]$ of a normal distribution. Outside the interval with the largest probability mass, the relative amount of states goes down by a factor two or equivalently, the step-size between intermediate states goes up by a factor of two. This makes sense, given that we scale the spread of the butterfly in the tails of the return distribution by two.

4 Results

We start present the results of the estimation of the one-month kernel first. Later on in the section we discuss the results when we estimate the kernel for a longer horizon. We focus on maturities of one, six and twelve months, for each of these maturities we estimate the forward pricing kernel and discuss the term-structure implications. In the last part of the results section we will discuss the time-series variation of the pricing kernel for each of the maturities.
4.1 One-month Pricing Kernel

To estimate the one-month pricing kernel, we need a model for the underlying distribution as explained in 2.3.1, we carry-out the MLE estimation on an overlapping return sample of the corresponding maturity. We start by estimating the one-month kernel for which we use the results of the first rule of table 2. The second and third rule are used to estimate the kernel with six and twelve month maturity, respectively.

Table 2: Maximum Likelihood Estimates of the parameters of the skewed $t$-distribution with volatility a second order polynomial of the VIX. These parameters are estimated using monthly overlapping simple returns of the S&P 500 from 1990-2015.

<table>
<thead>
<tr>
<th>T</th>
<th>$\mu$</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$\gamma$</th>
<th>$\xi$</th>
<th>$v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0058</td>
<td>-0.0148</td>
<td>1.1629</td>
<td>-2.6792</td>
<td>0.7028</td>
<td>14.6734</td>
</tr>
<tr>
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<td>-0.0050</td>
<td>1.0667</td>
<td>-1.0351</td>
<td>0.8179</td>
<td>7.3702</td>
</tr>
</tbody>
</table>

The average return estimate increases with the maturity as expected. Even though the VIX equals the model-free one-month risk neutral volatility, it does capture time-variation of the six and twelve month distribution. We see that the skewness related parameter goes up when the maturity is longer, indicating that the left-skew of the distribution becomes less pronounced. Lastly, for each of the maturities we find evidence of fat tails given that the degrees of freedom are rather small.

In the next figure, the time-series average (median) of the one-month pricing kernel is represented. The range of the figure corresponds to a 99% confidence interval based on the return distribution being normally distributed. In order to construct the confidence interval we use the average return of Table 2 and the volatility is equal to the sample average of the fitted values in our volatility model. We choose a 99% confidence according to a normal distribution for the ease of interpretation. Using this metric it seems that we go quite far into the tails, but the empirical stock return distribution is heavy-tailed and left-skewed. When we increase the maturity of the pricing kernel, we adjust the confidence interval according to the estimates of our underlying model.
Figure 4: In the figure the time-series average (solid) and median (dashed) of the one month pricing kernel are shown. The dotted line represents the pricing kernel of the risk neutral investor. Note, for each moneyness level 1% highest and lowest estimates are discarded.

As one can see in Figure 10, investors are willing to pay more for instruments which pay in bad (or very good) states of the economy. Bad states are characterized by monthly returns being negative. Economically, an investor is willing to pay about 1.5 times (median) the expected value for an Arrow-Debreu security paying when the stock market looses 10% (0.90 in the figure), which is equivalent to accepting an expected return of -33%. In the extremely good states of the economy, monthly stock returns greater than 5%, the estimates of the pricing kernel are sloping upward again. The U-shape, or violation of monotonicity, of the pricing kernel for one month maturity has been documented before. This result can be characterized as a preference of investors for betting on (extremely) good states of the economy and willing to accept a negative expected return. The difference between the mean and median increases more towards the tails of the return distribution, indicating that the time-series distribution of the pricing kernel for a given return level is right-skewed. It appears that there are times when we estimate large values of the pricing kernel in the tails and times where we estimate lower ones. Later in the paper we take a look at the differences in the time-series of the kernel.
4.2 Term Structure

We start with exploring the term structure implication looking at the six and twelve month pricing kernel. To do so, we carry out the same exercise using butterflies with a maturity of six and twelve months and using the underlying model of rule two and three of Table 2. The results for the six month pricing kernel are represented in the next figure, where we adjusted the scale accordingly.

Figure 5: In the figure the time-series average (solid) and median (dashed) pricing kernel are shown for a maturity of six months. The dotted line represents the Pricing Kernel of the risk neutral investor. Note, for each moneyness level 1% highest and lowest estimates are discarded.

The first observation from Figure 5 is that even for an horizon of six months, although being less extreme, the pricing kernel slopes upward at some point. A natural question is whether this upward slope is due to the accrual of pricing kernels with short horizon or investors also have this preference for extreme good states for instruments with longer horizon. The forward pricing kernel helps us addressing this question and we will explore the implications later in this section. Interestingly, the difference between the
mean and median is smaller for the six month kernel than for the one month kernel. This is a first indication that the six month kernel varies less in the time-series. Furthermore, the value of the median for the worst return level in the figure is around 3, indicating that the investor is willing to accept an huge negative return for instruments paying in these states. The median of the six-month pricing kernel for the worst state went up substantially compared to the median one-month pricing kernel.

The pricing kernel with maturity of twelve months is the third maturity for which we calculate the average pricing kernel over our sample. As before, we plot the mean and median where we scale the return interval accordingly.

The patterns emerging from the comparison of the one with six month kernel continue to hold for the twelve month kernel. First, the upward slope becomes again less extreme, only in the extremely good tail the level is above one. Second, the difference between the mean and median has become smaller indicating a more symmetric distribution. Third, the median level of the pricing kernel for the worst state in the figure has gone up even more.

Figure 6: In the figure the time-series average (solid) and median (dashed) pricing kernel are shown for a maturity of twelve months. The dotted line represents the Pricing Kernel of the risk neutral investor. Note, for each moneyness level 1% highest and lowest estimates are discarded.
4.2.1 Forward Pricing Kernel

Our next step is to disentangle the short and long-term effects. In order to present the information on the term structure of the pricing kernel in an accessible way, we defined the forward kernel in section 2.4. As in the previous section we are going to focus on the one, six and twelve month kernel. We calculate the expected forward kernel with a forward horizon of six months in the following way:

\[
E_t(m_{t+5,t+6}|r_{t+5,t+6}) = \int_0^\infty \frac{m_{t,t+5}(r_{t,t+5})}{m_{t+5,t+5}(r_{t,t+5})} f_t(r_{t+5}) dr_{t+5}.
\]

The expected one-month forward kernel with a forward horizon of five months is dependent on the pricing kernel with six months horizon and pricing kernel with five months horizon. Therefore, we first estimate the five-month pricing kernel similar to we did before. Estimating the five-month kernel involves estimating the five-month return distribution, which we are going to use to calculate the expectation in the above expression. Additionally, we assume that the return in the sixth month is independent of the total return in the first five months. We refer to the expected one-month forward kernel with forward horizon of five month as the six months forward kernel. In order to estimate the expected one-month forward kernel with forward horizon of eleven months, we have to adjust the maturities accordingly.

The first thing we compare are the means and medians of the one-month kernel, the six months forward kernel and twelve months forward kernel.

Figure 7: The figure on the left (right) represents the time-series mean (median) of the one-month (forward) pricing kernel. The solid, dashed and dot-dashed line represent the one, six and twelve month forward pricing kernel, respectively. Forward pricing kernels are estimated using the methodology described in section 2.4.
Interestingly, the curvature of the one-month kernel is largest, indicating that the investor is most sensitive towards short-term risks. The U-shape is most pronounced for the one-month kernel, but also for the kernel with forward horizon of six months the pricing kernel slopes upwards when forward return on the market is large. For the pricing kernel with horizon twelve months forward, the monotonic decreasing relation holds between the pricing kernel and the market return. Our results are quantified in the next table.

Table 3: The mean differences between seven points of the (forward) pricing kernel are tested. \( m_1(r) \) corresponds to the one-month pricing kernel, \( m_6(r) \) corresponds to six months forward kernel and \( m_{12}(r) \) corresponds to the twelve months forward kernel. In brackets the hac \( t \)-statistics are given, with appropriate amount of lags according to the ACF of the residuals.

<table>
<thead>
<tr>
<th>( r )</th>
<th>0.90</th>
<th>0.95</th>
<th>0.98</th>
<th>1.00</th>
<th>1.02</th>
<th>1.05</th>
<th>1.10</th>
</tr>
</thead>
<tbody>
<tr>
<td>( m_1(r) - m_6(r) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>3.54</td>
<td>-0.25</td>
<td>-0.12</td>
<td>-0.08</td>
<td>-0.07</td>
<td>0.01</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td>(2.15)</td>
<td>(-6.12)</td>
<td>(-6.68)</td>
<td>(-8.99)</td>
<td>(-2.69)</td>
<td>(0.17)</td>
<td>(2.03)</td>
</tr>
<tr>
<td>( m_1(r) - m_{12}(r) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>3.69</td>
<td>-0.21</td>
<td>-0.09</td>
<td>-0.07</td>
<td>-0.03</td>
<td>0.12</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>(2.18)</td>
<td>(-9.07)</td>
<td>(-10.01)</td>
<td>(-10.93)</td>
<td>(-0.99)</td>
<td>(1.81)</td>
<td>(2.19)</td>
</tr>
<tr>
<td>( m_6(r) - m_{12}(r) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean</td>
<td>0.17</td>
<td>0.04</td>
<td>0.02</td>
<td>0.01</td>
<td>0.04</td>
<td>0.12</td>
<td>0.31</td>
</tr>
<tr>
<td></td>
<td>(4.26)</td>
<td>(3.77)</td>
<td>(3.48)</td>
<td>(1.77)</td>
<td>(5.99)</td>
<td>(11.19)</td>
<td>(4.49)</td>
</tr>
</tbody>
</table>

The results from the table are the same as seen in the figures. First, the difference between the one-month kernel and the forward kernels is large, especially in the tails, indicating that the one-month kernels is more curved than the forward kernels. Furthermore, the difference between the forward kernel of with forward horizon six and twelve months is much smaller than its respective difference with the one-month kernel. The mean of the forward pricing kernel with forward horizon six months is always larger than the forward pricing kernel with forward horizon twelve months. However, it does not indicate that the six month forward kernel is always larger than the twelve month forward kernel.
4.3 Time-series variation

In this subsection we check how the pricing kernel varies over time. The first natural thing to check is if there are differences for subsamples, where we are going to split according to economic environment. We split the sample in two subsamples according to the level of the VIX, which captures volatility of the stock market relatively well. Periods where the VIX is lower than the median VIX level from 1996-2016, we will refer to as good times and for bad times vice versa. As a first indication, we calculate the average (median) pricing kernel for each of the maturities in good and bad times. As before, we plot the average and median of the pricing kernel over the range corresponding to a 99% confidence interval if the return distribution would have been normally distributed. Only in this case, we calculate the average volatility according to our return distribution in good and bad times separately. The volatility in good times is lower than in bad times, therefore the average and median pricing kernel in good times is plotted on a smaller range.

Figure 8: The blue (black) lines represent the time-series mean and median of the one month pricing kernel conditional on being in good (bad) times. The solid and dashed line represent the mean and median, respectively.

Clearly, the U-shape increases in good times compared to bad times. The will-
ingness of the investor to pay for extreme events of the economy increases in good times, indicated by the pricing kernel exceeding one for events beyond 5% movement. Theoretically, many consumption based asset pricing models will have difficulty explaining this result, even when just looking at the pricing kernel for low returns. For instance, the habit model predicts that in bad times when consumption is falling risk-aversion goes up which makes the pricing kernel more curved rather than less. Similarly, we compute the average and median six month kernel in the same subsamples as we used before.

Figure 9: The blue (black) lines represent the time-series mean and median of the six month pricing kernel conditional on being in good (bad) times. The solid and dashed line represent the mean and median, respectively.

The results for the six month kernel are similar to the results from the analysis for the one-month kernel. The curvature of the pricing kernel increases in bad times compared to good times. This increased curvature indicates that the investor is more sensitive towards risk in good versus bad times. As the six-month kernel accruals the shorter term kernels, it is interesting whether short or long-term sensitivity towards risk is driving the results. When we analyze the forward kernel, we can answer this question. Furthermore, the difference between the mean and median is larger in good than in bad times, indicating that the time-series distribution of the pricing kernel for a given return is less skewed in bad times. The next graphs represent the same analysis for the
The pattern for the curvature is the same for the twelve as for the six-month pricing kernel. In bad times the kernel is flatter and the time-series distribution less skewed. Also, the pricing kernel is less U-shaped in both good and bad times.

In order to test the significance of the change of curvature we do a regression of the slope of the pricing kernel in a certain region on a dummy variable equaling one in good times. We are going to take a look at four different slopes, defined in the following way:

\[
\begin{align*}
    s_1(T) &= m_T(1 - 0.10 \cdot \sqrt{T}) - m_T(1.00), \\
    s_2(T) &= m_T(1 - 0.05 \cdot \sqrt{T}) - m_T(1.00), \\
    s_3(T) &= m_T(1.00) - m_T(1 + 0.05 \cdot \sqrt{T}), \\
    s_4(T) &= m_T(1.00) - m_T(1 + 0.10 \cdot \sqrt{T}).
\end{align*}
\]

The slope is dependent on the horizon of the pricing kernel \(T\). As for the figures, we distinguish a horizon of one, six or twelve months and we scale the shocks accordingly.
In Table 4 the first column of each T-month slope corresponds to the total sample average. The second column adds a good times dummy equaling one when the VIX is larger than the median VIX over our sample period. Similarly, in the third column we check robustness of our results of the second column by redefining the good times dummy using data on the price-dividend ratio from Shiller’s website. To account for the trend in the ratio over our sample period we take first differences and define the dummy to equal one when in the previous month the increase in price-dividend ratio is larger than the median over our sample period. The results are a bit weaker if we define the dummy in this way, but this is caused by the fact that the price-dividend data is monthly therefore it can only vary on the monthly frequency, whereas the VIX dummy possibly on the daily level.

The results in Table 4 show the statistical significance of the change in slope over time. For the one-month kernel, we can see in the table that the slope for negative market returns (slope 1 and 2) is driven by the highly negative slope in good times. Furthermore,
the U-shape, or positive slope for positive market returns (slope 3 and 4), is also driven by the large slope in good times. The pattern for the slope of negative market returns of the one-month kernel is similar for the pricing kernels with longer horizons. However, the U-shape disappears for long horizons, as we only find a positive slope for the six-month kernel in slope 4. For the twelve-month kernel, the U-shape disappeared.

5 Theoretical Models

In this section we discuss what leading asset pricing models predict with respect to the average kernel, time-series variation and the term structure. We incorporate the following models: habit model by Campbell and Cochrane (1999), rare disaster model by Wachter (2013), Long run risk model by Bansal and Yaron (2004) and time-varying recovery by Gabaix (2012). All simulation we carry out for the models are based on a monthly calibration.

5.1 Habit model

Consider the Habit model of Campbell and Cochrane (1999), the $k$-period pricing kernel at time $t$ is given by the following equation:

$$M_t^k = e^{-k\delta} \left( \frac{C_{t+k}}{C_t} \right)^{-\gamma} \left( \frac{S_{t+k}}{S_t} \right)^{-\gamma}, \quad (9)$$

where $C_t$ and $S_t := (C_t - X_t)/C_t$ are the consumption and surplus ration at time $t$, respectively. Log consumption growth is assumed to be i.i.d. normally distributed and the surplus ratio evolved according to:

$$\log \left( \frac{C_{t+1}}{C_t} \right) = g + v_{t+1},$$

$$s_{t+1} = (1 - \phi) s_t + \phi s_t + \lambda(s_t) v_{t+1}.$$
Where $v_{t+1}$ follows a $N(0, \sigma_v^2)$, $s_t = \log(S_t)$ and $\lambda(s_t)$ is given by:

$$
\lambda(s_t) = \frac{1}{\bar{S}} \sqrt{1 - 2(s_t - \bar{s})} - 1,
$$

$$
\bar{S} = \sigma_v \sqrt{\frac{\gamma}{1 - \phi - b/\gamma}}.
$$

Note, $\lambda(s_t)$ is set to zero when $s_t > s_{\text{max}}$.

$$
S_{\text{max}} = \bar{s} + \frac{1}{2}(1 - S^2).
$$

We solve the following identity given the above parameters, where $P_t$ denotes the ex-dividend price of the claim to the future consumption stream:

$$
E_t[M_t^1 \left( \frac{P_{t+1}}{C_{t+1}} + 1 \right) \frac{C_{t+1}}{C_t}] = \frac{P_t}{C_t}.
$$

If we are to assume that $C_t$ is the dividend paid by the aggregate market, $P_t/C_t$ is the price-dividend ratio. Note, Campbell and Cochrane (1999) also have a version of the model where the dividend process is not perfectly correlated to the consumption process.

We obtain the price dividend ratio as a function of the state variable, which is $s_t$ for the habit model. Returns are obtain in the model via the following equation:

$$
R_{m,t,t+1}^m = \frac{P_{t+1}/C_{t+1} + 1}{P_t/C_t} \cdot \frac{C_{t+1}}{C_t}.
$$

In later models where we model dividends separately, we replace consumption with dividends.

Using the calibration from Campbell and Cochrane (1999), we do a simulation and obtain an equity premium of 6.5% with volatility 14.7% in line with their result. From this simulation we obtain the following average pricing kernel as a function of the market return. We take averages of the realizations of the simulation, each shock to consumption growth corresponds to a certain market return and pricing kernel.
As expected, the average pricing kernel is monotonically decreasing in the market return. If one starts at the long run average of the surplus ratio, we know that on average the \( n \)-period forward kernel is approximately equal to the current pricing kernel. This result follows from the fact that the expected future level of the habit also be equal to the long term average. Given the persistence of the habit in the model, the term structure is quite flat even if one would not start from the long term average. If the surplus ratio is larger than the long term average, current consumption exceeds the habit level more, implied risk aversion is lower which decreases the curvature. If the surplus ratio is larger than the average, it is an indication of good times. Together it contradicts our empirical findings.

### 5.2 Rare disasters model

In this section we will do the same analysis for the model of Wachter (2013) as we did for the Habit model. We follow the notation and calibration of Dew-Becker et al.
Log consumption growth in a discrete time version of this model is given by:

\[ \Delta c_t = \mu_c + \sigma_c \epsilon_{c,t} + J_t, \]
\[ \Delta d_t = \eta \Delta c_t, \]

where \( J_t \) is the jump process, which we assume for simplicity to be a poisson mixture of normal distributions. It is given by:

\[ J_t = \sum_{i=1}^{N_t} \xi_{i,t}, \]
\[ \xi_{i,t} \sim N(\mu_d, \sigma_d), \]
\[ N_t \sim Poisson(\lambda_t). \]

The intensity of the jump process follows:

\[ \lambda_{t+1} = \phi \lambda_t + (1 - \phi) \mu + \sigma \lambda \sqrt{\lambda_t} \epsilon_{\lambda,t}. \]

The utility function and pricing kernel are given in this framework by:

\[ v_t = (1 - \beta) c_t + \frac{\beta}{1 - \alpha} \log E_t \exp \left( v_{t+1} (1 - \alpha) \right), \]
\[ M_{t+1} = \beta \exp \left( - \Delta c_{t+1} \right) \frac{\exp \left( (1 - \alpha) v_{t+1} \right)}{E_t \exp \left( (1 - \alpha) v_{t+1} \right)}, \]

which is a recursive utility function with EIS equal to one. Note, \( \epsilon_c \) and \( \epsilon_\lambda \) are standard normal distributed. The returns on the market in this model are given by:

\[ R_{t+1}^m = \frac{P_{t+1}/D_{t+1} + 1}{P_t/D_t} \cdot \frac{D_{t+1}}{D_t}, \]

as the dividends are a leveraged claim on consumption in this model. Note, both the returns and pricing kernel are driven by shocks to consumption and shocks to the disaster probability. The monthly calibration is in line with Wachter (2013), where only the disaster distribution differs. In the table the parameters are represented:
Table 5: Calibration of the Wachter (2013) model.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_c$</td>
<td>0.0252/12</td>
<td>$\sigma_c$</td>
<td>0.02/√12</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>$-0.15$</td>
<td>$\sigma_d$</td>
<td>0.10</td>
</tr>
<tr>
<td>$\mu_\lambda$</td>
<td>0.0355/12</td>
<td>$\sigma_\lambda$</td>
<td>0.067/12</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$\exp(-0.08/12)$</td>
<td>$\beta$</td>
<td>$\exp(-0.012/12)$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>2.6</td>
<td>$\gamma$</td>
<td>$4.9 = 1 - \alpha$</td>
</tr>
</tbody>
</table>

We solve for the price dividend ratio as a function of the state variable, the disaster probability in this case, using the global method. The next step is to do a simulation, different from before is that we now have to shocks to consumption and the disaster probability. Doing so, we find an equity premium of 4.7% and volatility of 17.2%. Still, we are interested in the average pricing kernel from the simulation as a function of the market return. The result is given in the following graph.

Figure 12: Average pricing kernel from the disaster model of Wachter (2013) with the calibration from Table 5.

![Average Kernel](image)

Again, we find that the pricing kernel is monotonically decreasing in market
returns. In the model the pricing kernel is marginally affected by an increase in disaster probability, yielding a flat term structure. If the probability is larger than the long term average, indicating bad times, the curvature increases marginally.

5.3 Long run risk model

The next theoretical model that we consider is the long run risk model from Bansal and Yaron (2004) and we use the specification of Bansal et al. (2012) which is given by:

\[
\begin{align*}
\Delta c_{t+1} &= \mu_c + x_t + \sigma_t \eta_{t+1}, \\
x_{t+1} &= \rho x_t + \varphi_e \sigma_t e_{t+1}, \\
\sigma^2_{t+1} &= \bar{\sigma}^2 + \nu (\sigma^2_t - \bar{\sigma}^2) + \sigma_w w_{t+1}, \\
\Delta d_{t+1} &= \mu_d + \phi x_t + \pi \sigma_t \eta_{t+1} + \varphi_d \sigma_t u_{t+1}.
\end{align*}
\]

As in the original specification, the representative agent is assumed to have Epstein and Zin (1989) recursive preferences and maximizes lifetime utility:

\[
V_t = \left[ (1 - \delta) C_t^{\frac{1-\gamma}{\bar{\gamma}}} + \delta E_t [V_{t+1}^{1-\gamma}] \right]^{\frac{1}{1-\gamma}}.
\]

The calibration we use is from Bansal and Yaron (2004) and represented in the following table. Note, \( \theta = \frac{1-\gamma}{1-\bar{\gamma}}. \)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_c )</td>
<td>0.0015</td>
<td>( \mu_d )</td>
<td>0.0015</td>
</tr>
<tr>
<td>( \bar{\sigma} )</td>
<td>0.0078</td>
<td>( \sigma_w )</td>
<td>0.0000023</td>
</tr>
<tr>
<td>( \rho )</td>
<td>0.979</td>
<td>( \varphi_e )</td>
<td>0.044</td>
</tr>
<tr>
<td>( \phi )</td>
<td>3</td>
<td>( \pi )</td>
<td>0</td>
</tr>
<tr>
<td>( \varphi )</td>
<td>4.5</td>
<td>( \delta )</td>
<td>0.998</td>
</tr>
<tr>
<td>( \gamma )</td>
<td>10</td>
<td>( \psi )</td>
<td>1.5</td>
</tr>
<tr>
<td>( \nu )</td>
<td>0.987</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
For a similar simulation as we did for the previous models, we find an equity premium of 4.1% and volatility of 16.4%. We estimate the average pricing kernel as a function of the market return from the simulation and obtain the following:

Figure 13: Average pricing kernel from the long run risk model of Bansal and Yaron (2004) with the calibration from Table 6.

Pricing kernel monotonically decreasing in market returns. The curvature of the kernel is driven by the level stochastic volatility, if volatility is high indicate bad times and vice versa. If the stochastic volatility is above the long term average, curvature increases. Furthermore, given the persistence of the state variables the term structure is quite flat.

5.4 Time-varying Recovery

The next theoretical model we consider is the time-varying recovery model of Gabaix (2012), we use the same specification as in Dew-Becker et al. (2016) and is
specified as follows:

\[
\Delta c_t = \mu_c + \sigma_c \epsilon_{c,t} + J_{c,t},
\]

\[
L_t = (1 - \rho_L) \bar{L} + \rho_L L_{t-1} + \sigma_L \epsilon_{L,t},
\]

\[
\Delta d_t = \lambda \sigma_c \epsilon_{c,t} - L_t \cdot 1_{J_{c,t} \neq 0}.
\]

Table 7: Calibration of Dew-Becker et al. (2016).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_c )</td>
<td>0.01/12</td>
<td>( \sigma_c )</td>
<td>0.02/( \sqrt{12} )</td>
</tr>
<tr>
<td>( \mu_d )</td>
<td>-0.3</td>
<td>( \sigma_d )</td>
<td>0.15</td>
</tr>
<tr>
<td>( \bar{L} )</td>
<td>- log(0.5)</td>
<td>( \sigma_L )</td>
<td>0.04</td>
</tr>
<tr>
<td>( \rho_L )</td>
<td>0.87( \sqrt{12} )</td>
<td>( \lambda )</td>
<td>5</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.96( \sqrt{12} )</td>
<td>( \gamma )</td>
<td>7</td>
</tr>
</tbody>
</table>

The simulation given these parameters yields an equity premium of 6.8% and volatility of 13.7%.

Dew-Becker et al. (2016) assume the representative agent to have a power utility function. Corresponding to a pricing kernel with the following specification.

\[
M_{t+1} = \beta \cdot \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma}.
\]

As one can see from the formulas, the pricing kernel is only driven by shocks to consumption, \( \epsilon_c \) or disaster \( J_c \). Whereas returns are driven by shocks to consumption \( \epsilon_c \) and shocks to the severity of the impact for consumption disasters to dividends \( \epsilon_L \). The results of the average kernel in the simulation are given in the next figure.
The mechanisms yielding the time-variation and term structure are in line with the previous models.

6 Conclusion

We document two new stylized facts of the pricing kernel with respect to the time-variation and term structure. First, the U-shape in the pricing kernel is more pronounced in good than in bad times. Second, the U-shape disappears for longer horizons, making investors more sensitive towards short-term risks compared to long-term risks. Both results are puzzling given that leading asset pricing models cannot explain them.
7 Bibliography


