Asset Pricing when Intermediaries have Market Power

Financial Oligopoly and Oligopsony

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Motivation
Asset Pricing with Financial Intermediaries (FIs)

Crisis → more attention to FIs

Standard setup: intermediation frictions+FIs perfectly competitively
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This paper
FIs are “big” and compete strategically in a bilateral market

- FIs have market power on the sell (oligopoly) and the buy side (oligopsony)
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**Questions**

1. Do intermediary returns reflect risk premia or rents?
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**Questions**

1. Do intermediary returns reflect risk premia or rents?
2. Do intermediary fees reflect $\alpha$ or $\beta$?
3. Is $\alpha$ market power or skill?
We develop a dynamic general equilibrium asset pricing model with strategic intermediaries with bilateral market power

- develop a computational algorithm to determine the dynamic Cournot equilibrium with strategic intermediaries (GEE)
- provide a decomposition of returns and fees into risk premia and rents
- take a step towards the quantitative modeling of the industrial organization of the intermediary sector
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**Why?**
1. Mutual fund fee puzzle: Wide dispersion in index fund fees in US
2. PE funds and Hedge funds document high $\alpha$, fees, and margins
3. Banks retain substantial market power in deposit, loan, and syndication markets
Model

- 3 types of agents:
  1. Household: representative
  2. Financial intermediary: discrete number $I$
  3. Entrepreneurs: discrete number $\Theta$
Model

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- Each intermediary competes simultaneously in 2 markets:
  1. Selling $i$-fund shares $S(i)$ at price $p(i)$ to households
  2. Buying $\theta$-equity shares $\tilde{S}_\theta(i)$ at a price $\tilde{p}_\theta(i)$
Overview

Model

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- Financial differentiation:
  1. $i$-fund provides a differentiated stream of dividends to HH
  2. $\theta$-equity is financed through differentiated financing
Model

○ 3 types of agents:
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○ Financial differentiation:
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  2. \( \theta \)-equity is financed through differentiated financing

○ Role of strategic interactions in a GE setting
Model
Two periods set-up: period 0

**Inflow**
- $e_0$
- $p_0(i)S_0(i)$
- $\tilde{p}_\theta, t\tilde{S}_\theta, 0$

**Consumption**
- $C_0$
- $\Pi_0(i)$
- $\mathcal{D}_\theta, 0$

**Outflow**
- $\sum_i p_0(i)S_0(i)$
- $\sum_\Theta \tilde{p}_\theta, 0\tilde{S}_\theta, 0$
- 0
Two periods set-up: period 1

\[ \mathcal{D}_1 \]

**Inflow**
\[
\sum_i S_0(i)d_1(i) \quad \sum_\theta \tilde{S}_{\theta,0}(i)\tilde{d}_{\theta,1} \quad \tilde{d}_{\theta,1}
\]

**Consumption**
\[
\mathcal{D}_1 \quad \Pi_1(i) \quad C_{\theta,1}
\]

**Outflow**
\[
0 \quad S_0(i)d_1(i) \quad \sum_i \tilde{S}_{\theta,0}(i)\tilde{d}_{\theta,1}
\]
Household: problem

The representative HH solves\(^1\):

\[ \mathcal{V}(S, d) = \max_{C, S'} \mathcal{U}(C) + \beta \mathbb{E} [\mathcal{V}(S', d') | S, d] \]

Where:

\[ C = e + \mathbf{p}^T (S - S') + \mathcal{D} \]

\[ \mathcal{D} = \left( \sum_{i=1}^{I} \left( d(i) S(i) \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \]

\(^1\mathbf{p}, S, d \in \mathbb{R}^I\)
Household: optimality

Optimality:

$$\forall i : p(i) = \mathbb{E} \left[ \beta \frac{\mathcal{U}'(C')}{\mathcal{U}'(C)} \left( p'(i) + \left( \frac{D'(i)}{D'} \right)^{-\frac{1}{\sigma}} d'(i) \right) \right]$$  \hspace{1cm} (\star)

Novel terms$^2$:

$$\left( \frac{D'(i)}{D'} \right)^{-\frac{1}{\sigma}} : \text{relative price of dividend } i$$

$$d(i) = \tilde{S}(i)^T \tilde{d}(i): \text{portfolio technology}$$

$^2\tilde{S}(i), \tilde{d}(i) \in \mathbb{R}^\Theta$
Entrepreneurs: problem

An entrepreneur $\theta$ solves$^3$:

$$\mathcal{V}_{\theta}(\tilde{S}_\theta, \tilde{d}_\theta) = \max_{C_\theta, \tilde{S}'_\theta} \mathcal{U}_\theta(C_\theta) + \beta \mathbb{E} \left[ \mathcal{V}_\theta(\tilde{S}'_\theta, \tilde{d}'_\theta) | \tilde{S}_\theta, \tilde{d}_\theta \right]$$

Where:

$$C_\theta = \tilde{d}_\theta - \tilde{S}_\theta^T(\tilde{p}'_\theta + \tilde{d}'_\theta 1_{1,1}) + \mathcal{D}_\theta$$

$$\mathcal{D}_\theta = \left[ \sum_{i=1}^{I} \left( \tilde{p}_\theta(i) \tilde{S}_\theta(i) \right) \right]^{\frac{v-1}{v}} \left[ \right]^{\frac{v}{v-1}}$$

$^3\tilde{S}_\theta, \tilde{p}_\theta \in \mathbb{R}^I$
Optimality:

$$\forall \theta, i : \tilde{p}_\theta(i) \left( \frac{D_\theta(i)}{D_\theta} \right)^{-\frac{1}{\nu}} = \mathbb{E} \left[ \beta \frac{\mathcal{U}'(C'_\theta)}{\mathcal{U}'(C_\theta)} \left( \tilde{p}'_\theta(i) + \tilde{d}'_\theta \right) \right]$$

Where:

$$\left( \frac{D_\theta(i)}{D_\theta} \right)^{-\frac{1}{\nu}} : \text{relative price of the financing resource}$$
An intermediary $i$ solves:

$$V(S_i, \tilde{S}_i, S_{-i}, \tilde{S}_{-i}, \tilde{d}) = \max_{S'_i, \tilde{S}'_i} \Pi + \beta \mathbb{E} \left[ V(S'_i, \tilde{S}'_i, S'_{-i}, \tilde{S}'_{-i}, \tilde{d}') \right]$$

Subject to:

1. $\Pi + \tilde{p}^T \tilde{S}' + (p + d)S \leq (\tilde{p}^T + \tilde{d}^T)\tilde{S} + pS'$
2. Portfolio tech.: $d = \tilde{S}_i^T \tilde{d}$
3. HH Euler ★
4. Ent. Euler ★
The GEEs on the **HH side** are mark-up rules (∀i):

\[
p = \frac{1}{1 + \frac{\partial p}{\partial S'} \frac{S' - S}{p}} \beta \mathbb{E} \left[ p' + d' \right]
\]

\(\epsilon^{-1}\): oligopoly power ↑ fund’s price today but lead to a loss because the old fund’s stock need to be re-payed at a higher price.
The GEEs on the Ent.s side are mark-up rules ($\forall \theta, i$):

\[
\tilde{p}_\theta = \frac{1}{1 + \frac{\partial \tilde{p}_\theta}{\partial \tilde{S}_\theta} \frac{\tilde{S}'_\theta - \tilde{S}_\theta}{\tilde{p}_\theta} - \frac{\partial p}{\partial \tilde{S}_\theta} \frac{S' - S}{\tilde{p}_\theta}} 
\times \left[ \beta^{E} \left[ \tilde{p}'_\theta + (1 - S')\tilde{d}'_\theta \right] \right] \quad (★★★)
\]

- $E^{-1}$: oligopsony power ↓ equity’s price today but lead to a loss from yesterday investment making the old equity cheaper
- $E^{*-1}$: cross inverse elasticity
A Symmetric Cournot-Nash Equilibrium is:

- A sequence of vectors \( \{S_{t}^{HH}, S_{t}^{I}, p_{t} \in \mathbb{R}^{I}\}_{t=0}^{\infty} \)
- A sequence of matrices \( \{\tilde{S}_{t}^{E}, \tilde{S}_{t}^{I}, \tilde{p}_{t} \in \mathbb{R}^{I \times \Theta}\}_{t=0}^{\infty} \)

Such that at every \( t \):

1. The \( 2I(\Theta + 1) \) Eqs. (★), (★), (★☆), (★☆) are solved
2. Markets are clear: \( S_{t}^{HH} = S_{t}^{I} = S_{t} \) and \( \tilde{S}_{t}^{E} = \tilde{S}_{t}^{I} = \tilde{S}_{t} \)
3. Strategies are symmetric: \( S_{t} = S_{t}^{*}1_{I} \) and \( \forall i : \tilde{S}_{t}(i) = \tilde{S}_{t}^{*} \)

Given:

- A sequence of vector of dividends \( \{\tilde{d}_{t} \in \mathbb{R}^{\Theta}\}_{t=0}^{\infty} \)
- Initial stocks \( S_{-1}^{HH} \in \mathbb{R}^{I}, \tilde{S}_{-1}^{E} \in \mathbb{R}^{I \times \Theta} \)
### 2 periods solution

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
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<td>$e_0$</td>
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<tr>
<td>$\sigma$</td>
<td>Elasticity</td>
<td>${10,1000}$</td>
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### Intermediary

<table>
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<td>$I$</td>
<td>Numbers</td>
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### Entrepreneurs

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<tr>
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<th>Description</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>$\eta$</td>
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<tr>
<td>$\nu$</td>
<td>Elasticity</td>
<td>${10,1000}$</td>
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<tr>
<td>$\Theta$</td>
<td>Number</td>
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<tr>
<td>$\mathbb{E}_0\tilde{d}$</td>
<td>Dividends</td>
<td>$[9,2]$</td>
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<tr>
<td>$\Sigma_{\tilde{d}}$</td>
<td>Var-Covar</td>
<td>$\begin{bmatrix} 2 &amp; 0.15 \ 0.15 &amp; 0.1 \end{bmatrix}$</td>
</tr>
<tr>
<td>pdf</td>
<td>Distribution</td>
<td>Multivariate Gaussian</td>
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</table>
Profit decomposition
1. **Oligopoly markup** \((1/(1 + \epsilon^{-1}))\) is given by:

\[
\epsilon^{-1} = \mathcal{K} - \frac{\gamma S_0 \beta \mathbf{E} d_1}{e_0 - p_0 S_0 I} \quad \& \quad \mathcal{K} = \left(\frac{1}{\sigma} - \gamma\right) \frac{1}{I} - \frac{1}{\sigma}
\]

Without oligopoly, \(i\)-MC \((p_0)\) is equals to its expected MR \((\beta \mathbf{E} d_1)\).

2. **Oligopsony inverse markup** \((1/(1 + \mathbf{E}^{-1} - \mathbf{E}^{*-1}))\) is given by:

\[
\mathbf{E}^{-1} = \frac{\mathbf{K} + \eta \frac{\tilde{S}_{\theta,0}}{1 - \tilde{S}_{\theta,0} I}}{1 - \mathbf{K}} \quad \& \quad \mathbf{K} = \left(\eta - \nu\right) \frac{1}{I} + \frac{1}{\nu}
\]

Without oligopsony, \(\theta\)-MC \((\tilde{p})\) is higher than its expected MR:

\[
\tilde{p}_{\theta,0} = \frac{1 - S_0}{1 - \mathbf{E}^{*-1}} \beta \mathbf{E} \tilde{d}_{\theta,0} \quad \& \quad \mathbf{E}^{*-1} = \frac{\partial p_0}{\partial \tilde{S}_{\theta,0}} \tilde{p}_{\theta,0} > 0
\]
All market power comes from risk aversions $\gamma$ and $\eta$! With $I = 5$, Oligopoly and full market power reduce HH $\mathbb{E}C_1$ of $\sim 50\%$ and Oligopsony undercuts Ent. financial resources by a factor $\sim 2X$. 
HH equity premium increases as $I$ increases:

$$\begin{align*}
\mathbb{E} \left( \frac{1}{\sigma-1} d_1(\tilde{S}) \right) - R^f_1 &= -R^f_1 \text{Cov} \left[ R^{HH}_1(\tilde{S}), \beta \left( \frac{D_1(\tilde{S})}{C_0} \right)^{-\gamma} \right] \\
&\quad \text{with } \frac{\partial R^f_1}{\partial I} > 0 \quad \text{and} \quad \frac{\partial \text{Cov}(\cdot, \cdot)}{\partial I} < 0
\end{align*}$$

As $I$ increases, the fund becomes cheaper faster than the speed at which $\sigma(d_1)$ decreases, $R^{HH}_1$ is increasingly riskier in $I$:

$$\begin{align*}
\sqrt{\text{Var}(R^{HH}_1)} &= I^{\frac{1}{\sigma-1}} \frac{1}{p_0} \sigma(\tilde{S}^T \tilde{d}_1) \\
&\quad \text{with } \frac{\partial p_0}{\partial I} < 0 \quad \text{and} \quad \frac{\partial \sigma(\cdot)}{\partial I} < 0
\end{align*}$$

At the equilibrium the sharpe ratio of the HH’s portfolio does not depend on the market structure!
Household: premium and risk $\sigma = \nu = 10$

Full market power reduces the HH’premium and risk!
Intermediary: premium and risk

Intermediaries equity premium decreases as $I$ increases:

$$\mathbb{E}R^I_1 - R^f_1 = (1 - S_0) \mathbb{E} \left[ \frac{d_1}{\tilde{p}_0^T \tilde{S}_0} \right] - \beta^{-1}$$

\[ \frac{\partial}{\partial I} \frac{1 - S_0}{0} < 0 \quad \frac{\partial}{\partial I} < 0 \]

As $I$ increases, intermediaries provide better financing deals to the Ent.s, this reduces their equities expected return and risk. As $I \uparrow$, intermediaries pass through a higher portion of the risky assets:

$$\sigma(R^I_1) = (1 - S_0) \frac{\sqrt{\tilde{S}^T \Sigma \tilde{S}}}{\tilde{p}^T \tilde{S}}$$
Intermediary: premium and risk $\sigma = \nu = \infty$

As a result more market power translates into more risky assets retained by the intermediaries...
Intermediary: premium and risk $\sigma = \nu = 10$

...this difference is even more significant with finite elasticities (market power is even higher)!
INFINITE HORIZON
1. Strategic interactions $\rightarrow$ **more state variables** (others intermediaries strategies are considered in order to calculate best responses)

2. Euler's as constraints of the intermediary's problem $\rightarrow$ equilibrium involves **derivatives of the policies** (GEEs)

3. Polynomials provide a poor approximation in this case $\rightarrow$ a **non-parametric approach** is needed (e.g. piecewise splines)
Global solution: why is it hard? Focus GEEs

**GEE-HH:** it does not only contain the future price’s policy but also the derivative of the today’s price’s policy

\[ p(S, \tilde{S}, \tilde{d}) = \frac{1}{1 + \frac{\partial p}{\partial S'} \frac{s' - S}{p}} \beta \mathbb{E} [p'(S', \tilde{S}', \tilde{d}') + \tilde{S}'^T \tilde{d}'] \]

**GEE-End:** Harder to solve because it contains policy functions for the current financing price, its derivative, the future one and also the fund’s price policy and its derivative

**Note:** The implicit function theorem and a lot of patience yield an analytical expression for the policies derivatives
Computational approach

- Start with a non parametric guess for the functions $S'(S, \tilde{S}, \tilde{d})$, $\tilde{S}'(S, \tilde{S}, \tilde{d})$, $p(S, \tilde{S}, \tilde{d})$, $\tilde{p}_\theta(S, \tilde{S}, \tilde{d})$.
- For every $S, \tilde{S}$ and $\tilde{d}$:
  - Rewrite the system of all GEEs and Eulers in functions of prices, shares and exogenous variables, in the form: $[S' \quad \tilde{S}' \quad p \quad \tilde{p}_\theta] = \mathbb{E}f(S, \tilde{S}, p, \tilde{p}, S', \tilde{S}', p', \tilde{p}', \tilde{d}, \tilde{d}')$.
  - Use guesses and multivariate Gaussian quadrature to evaluate $\mathbb{E}f$ and get (analytically) an updated vector $[S' \quad \tilde{S}' \quad p \quad \tilde{p}_\theta]$.
- Update the guesses and start again. Stop when the guesses solve the system of Eulers and GEEs.

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4E.g. pure grids method, piecewise spline, machine learning
### Infinite horizon solution

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<td>$E_0\tilde{d}$</td>
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<td>pdf</td>
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</table>
Solution: slice of $S_t$ for a low $L$ shock

The shares of fund don’t depend on the portfolio composition and they are roughly stable over time.
Solution: slice of $S_{\theta,t}$ for a low $L$ shock

Under symmetry, with 3 intermediaries the maximum amount of equity each intermediary can buy is $1/3$, in proximity of this limit the policy presents a kink.
Solution: slice of $p_t$ for a low $L$ shock

The more risky asset in the portfolio the pricier the fund becomes. The price decreases as the amount of shares increases.
The price of the risky asset is decreasing when the number of shares bought in the current period is small («1/3). When this limit is approached the price shows a first kink and starts to be increasing till when, in the limits the entire amount of risky assets is bought and the price presents a singularity point.
Thank you!