Valuation Risk Revalued

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ABSTRACT

The asset pricing literature assigns a key role to valuation risk, which is modelled as a time-preference shock within an Epstein-Zin utility function. In a stylized model where logic would imply no equity premium, we analytically show the current valuation risk specification paradoxically predicts one. Empirically, our baseline model with the current specification fits the data well and resolves many asset pricing puzzles. However, the results stem from an undesirable asymptote that permeates the determination of asset prices in the model because the preferences fail to satisfy an important restriction on the weights in the Epstein-Zin time-aggregator. When we revise the preferences to satisfy the restriction, the model does not fit the data as well and the puzzles resurface. In an extended model with Bansal-Yaron long-run risk, we estimate that valuation risk with the revised specification plays only a minor role in matching the mean equity premium and risk-free rate but is crucial for matching the volatility of the risk-free rate.

Keywords: Epstein-Zin Utility; Valuation Risk; Equity Premium Puzzle; Risk-Free Rate Puzzle

JEL Classifications: D81; G12

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1 INTRODUCTION

In standard asset pricing models, uncertainty enters through the supply side of the economy, either through endowment shocks in a Lucas (1978) tree model or productivity shocks in a production economy model. Recently, several papers introduced demand side uncertainty or “valuation risk” as a potential explanation of key asset pricing puzzles (Albuquerque et al. (2016, 2015); Creal and Wu (2017); Maurer (2012); Nakata and Tanaka (2016); Schorfheide et al. (2018)). In macroeconomic parlance, valuation risk is usually referred to as a discount factor or time preference shock.¹

The literature contends valuation risk is an important determinant of key asset pricing moments when it is embedded in Epstein and Zin (1991) recursive utility preferences. We show the success of valuation risk rests on an undesirable asymptote that permeates the determination of asset prices. The influence of the asymptote is easily identified in a stylized model where logic would imply no equity premium. In that model, the valuation risk specification used in the current literature (henceforth, the “current specification”) paradoxically predicts a positive equity premium. de Groot et al. (2018) show that with Epstein-Zin preferences, time-varying weights in a CES time-aggregator must sum to 1 to prevent an undesirable asymptote from determining equilibrium outcomes. The current specification fails to impose this important restriction. de Groot et al. (2018) propose an alternative specification (henceforth, the “revised specification”) that eliminates the asymptote.²

This paper uses the revised specification to re-evaluate the role of valuation risk in explaining key asset pricing moments. While the change to the model will appear minor, it profoundly alters the equilibrium predictions of the model. Key comparative statics, such as the response of the equity premium and the risk-free rate to a rise in the intertemporal elasticity of substitution (IES) parameter, switch sign. This means that once we re-estimate the model, the parameters that best fit the data change dramatically. For example, our baseline model with the revised specification requires a coefficient of relative risk aversion (RA) well above the accepted range in the literature.

For intuition, consider the log-stochastic discount factor (SDF) under Epstein-Zin preferences

\[ \hat{m}_{t+1} = \theta \log \beta + \theta (\omega \hat{a}_{t+1} - \hat{a}_t) - (\theta / \psi) \Delta \hat{c}_{t+1} + (\theta - 1) \hat{r}_{y,t+1}, \]  

where the first, third, and fourth terms—subjective discount factor \( (\beta) \), log-consumption growth \( (\Delta \hat{c}_{t+1}) \), and log-return on the endowment \( (\hat{r}_{y,t+1}) \)—are standard in this class of asset pricing models. The second term captures valuation risk, where \( \hat{a}_t \) is a time preference shock. In the current asset pricing literature, \( \omega = 1 \). Once we revise the preferences and re-derive the log-SDF, we find \( \omega = \beta \). Since \( \beta \) is the subjective discount factor at a monthly frequency, it is very close to 1.

¹Discount factor shocks have become common in the business cycle literature since the 2007 financial crisis because they are an effective reduced-form mechanism for getting to the zero lower bound on the nominal interest rate.

At first sight, this innovation appears innocuous. However, when we apply this single, seemingly minor, alteration to the model, the asset pricing predictions are starkly different. In particular, it becomes difficult to resolve the equity premium (Mehra and Prescott (1985)), risk-free rate (Weil (1989)) and correlation puzzles (Campbell and Cochrane (1999)) with the revised specification.

The asymptote in the original valuation risk specification is related to the preference parameter \( \theta \equiv (1 - \gamma)/(1 - 1/\psi) \) that enters the log-SDF, where \( \gamma \) is RA and \( \psi \) is the IES. Under constant relative risk aversion (CRRA) preferences, \( \gamma = 1/\psi \). In this case, \( \theta = 1 \) and the log-SDF becomes

\[
\hat{m}_{t+1} = \log \beta + (\omega \hat{a}_{t+1} - \hat{a}_t) - \Delta \hat{c}_{t+1}/\psi.
\] (2)

The return on the endowment drops out of (1), so the log-SDF is simply composed of the subjective discount factor and consumption growth terms. The advantage of Epstein-Zin preferences is that they decouple \( \gamma \) and \( \psi \), so it is possible to simultaneously have high RA and a high IES. However, there is a nonlinear relationship between \( \theta \) and \( \psi \), as shown in figure 1. A vertical asymptote occurs at \( \psi = 1 \): \( \theta \) tends to infinity as \( \psi \) approaches 1 from below while the opposite occurs as \( \psi \) approaches 1 from above. When the IES equals 1, \( \theta \) is undefined. In addition to the vertical asymptote in \( \theta \), there is also a horizontal asymptote at \( 1 - \gamma \) as the IES becomes perfectly elastic.

Figure 1: Preference parameter \( \theta \) in the stochastic discount factor from a model with Epstein-Zin preferences.

Under the Epstein and Zin (1989) preferences and the generalization in de Groot et al. (2018) to include valuation risk, the asymptote in figure 1 does not affect asset prices. There is a well-defined equilibrium when the IES equals 1 and asset pricing predictions are robust to small variations in the IES around 1. Continuity is preserved because the weights in the time-aggregator always sum to unity. An alternative interpretation is that the time-aggregator maintains the well-known property that a CES aggregator tends to a Cobb-Douglas aggregator as the elasticity approaches 1. The current specification violates the restriction on the weights so the limiting properties of the CES aggregator break down. As a result, the asymptote in figure 1 permeates key asset pricing moments.
Taken at face value, the asymptote that occurs with the current valuation risk specification resolves several long-standing asset-pricing puzzles in the literature. Furthermore, when we estimate a model that includes valuation risk and long-run risk following Bansal and Yaron (2004), counterfactual exercises demonstrate that the asset pricing moments are almost completely explained by valuation risk, rather than long-run risk. The reason is straightforward. The asymptote stemming from $\theta$ allows the model to deliver an arbitrarily large equity premium and an arbitrarily low risk-free rate as the IES tends to 1 from above with modest risk aversion. With the revised preferences, valuation risk behaves more like conventional types of risk, which causes the puzzles to re-emerge.

We summarize our main results as follows: (1) The current valuation risk specification always fits the data well due to an undesirable asymptote; (2) In our baseline model, the revised specification does not perform as well at matching the data, even with more extreme parameter values; (3) In an extended model with Bansal-Yaron long-run risk, valuation risk with the revised preferences is estimated to play a small role in determining most asset pricing moments but is important for matching the volatility of the risk-free rate; (4) Allowing valuation risk shocks to directly affect cash flow growth helps fit the data better than introducing stochastic volatility on cash flow growth.

The paper proceeds as follows. Section 2 describes the baseline asset pricing model and the preference specifications. Section 3 analytically shows why asset prices depend so dramatically on the way valuation risk enters the Epstein-Zin utility function. Section 4 empirically quantifies the effects of the valuation risk specification in our baseline model. Section 5 estimates the relative importance of valuation and long-run risk. Section 6 extends our long-run risk model to include valuation risk shocks to cash flow growth and stochastic volatility on cash flow risk. Section 7 concludes.

2 Baseline Asset-Pricing Model

We begin by describing our baseline model. Each period $t$ denotes 1 month. There are two assets: an endowment share, $s_{1,t}$, that pays income, $y_t$, and is in fixed unit supply, and an equity share, $s_{2,t}$, that pays dividends, $d_t$, and is in zero net supply. The agent chooses $\{c_t, s_{1,t}, s_{2,t}\}_{t=0}^{\infty}$ to maximize

$$U^C_t = \left[a^C_t \left(1 - \beta \right) c_t^{(1-\gamma)/\theta} + \beta (E_t[(U^C_{t+1})^{(1-\gamma)}])^{1/\theta/(1-\gamma)}, \ 1 \neq \psi > 0, \right]$$

as used in the current (C) asset pricing literature, or

$$U^R_t = \begin{cases} \left[(1 - a^R_t \beta) c_t^{(1-\gamma)/\theta} + a^R_t \beta (E_t[(U^R_{t+1})^{(1-\gamma)}])^{1/\theta/(1-\gamma)}, \right. \\
\left. c_t^{1 - a^R_t \beta} (E_t[(U^R_{t+1})^{(1-\gamma)}])^{a^R_t \beta/(1-\gamma)}, \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \right. \r

The time-varying weights of the time-aggregator in (3), $a^C_t (1 - \beta)$ and $\beta$, do not sum to 1, whereas the weights in (4), $(1 - a^R_t \beta)$ and $a^R_t \beta$, do sum to 1.
The representative agent’s choices are constrained by the flow budget constraint given by
\[ c_t + p_{y,t}s_{1,t} + p_{d,t}s_{2,t} = (p_{y,t} + y_t)s_{1,t-1} + (p_{d,t} + d_t)s_{2,t-1}, \tag{5} \]
where \( p_{y,t} \) and \( p_{d,t} \) are the endowment and dividend claim prices. The optimality conditions imply
\[ E_t[m_{t+1}^j r_{y,t+1}] = 1, \quad r_{y,t+1} \equiv (p_{y,t+1} + y_{t+1})/p_{y,t}, \tag{6} \]
\[ E_t[m_{t+1}^j r_{d,t+1}] = 1, \quad r_{d,t+1} \equiv (p_{d,t+1} + d_{t+1})/p_{d,t}, \tag{7} \]
where \( j \in \{C, R\} \), \( r_{y,t+1} \) and \( r_{d,t+1} \) are the gross returns on the endowment and equity claim, and
\[ m_{t+1}^C \equiv \beta \left( \frac{a_{t+1}^C}{a_t^C} \right) \left( \frac{c_{t+1}}{c_t} \right)^{-1/\psi} \left( \frac{(V_{t+1}^C)^{1-\gamma}}{E_t[(V_{t+1}^C)^{1-\gamma}]} \right)^{1-\frac{1}{\psi}}, \tag{8} \]
\[ m_{t+1}^R \equiv a_t^R \beta \left( \frac{1 - a_{t+1}^R \beta}{1 - a_t^R \beta} \right) \left( \frac{c_{t+1}}{c_t} \right)^{-1/\psi} \left( \frac{(V_{t+1}^R)^{1-\gamma}}{E_t[(V_{t+1}^R)^{1-\gamma}]} \right)^{1-\frac{1}{\psi}}. \tag{9} \]

To permit an approximate analytical solution, we rewrite (6) and (7) as follows
\[ E_t[\exp(\hat{m}_{t+1}^j + \hat{r}_{y,t+1})] = 1, \tag{10} \]
\[ E_t[\exp(\hat{m}_{t+1}^j + \hat{r}_{d,t+1})] = 1, \tag{11} \]
where \( \hat{m}_{t+1}^j \) is defined in (1) and \( \hat{a}_t \equiv \hat{a}_t^C \approx -\hat{a}_t^R/(1 - \beta) \) so the shocks in the current and revised models are directly comparable. The common time preference shock, \( \hat{a}_{t+2} \), evolves according to
\[ \Delta \hat{a}_{t+2} = \rho_a \Delta \hat{a}_{t+1} + \sigma_a \hat{\varepsilon}_{t+1}, \quad \hat{\varepsilon}_{t+1} \sim \mathcal{N}(0, 1), \tag{12} \]
where \( 0 \leq \rho_a < 1, \sigma_a \geq 0 \) is the shock standard deviation, a hat denotes the log of a variable, and \( \Delta \) denotes a first-difference.\(^3\) We then apply a Campbell and Shiller (1988) approximation to obtain
\[ \hat{r}_{y,t+1} = \kappa_{y0} + \kappa_{y1} \hat{z}_{y,t+1} - \hat{z}_{y,t} + \Delta \hat{y}_{t+1}, \tag{13} \]
\[ \hat{r}_{d,t+1} = \kappa_{d0} + \kappa_{d1} \hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta \hat{d}_{t+1}, \tag{14} \]
where \( \hat{z}_{y,t+1} \) is the price-endowment ratio, \( \hat{z}_{d,t+1} \) is the price-dividend ratio, and
\[ \kappa_{y0} \equiv \log(1 + \exp(\hat{z}_y)) - \kappa_{y1} \hat{z}_y, \quad \kappa_{y1} \equiv \exp(\hat{z}_y)/(1 + \exp(\hat{z}_y)), \tag{15} \]
\[ \kappa_{d0} \equiv \log(1 + \exp(\hat{z}_d)) - \kappa_{d1} \hat{z}_d, \quad \kappa_{d1} \equiv \exp(\hat{z}_d)/(1 + \exp(\hat{z}_d)), \tag{16} \]
are constants that are functions of the steady-state price-endowment and price-dividend ratio.

\(^3\)The revised preferences place a bound on \( a_t \). Specifically, \( 0 < a_t < 1/\beta \). Given the process in (12), \( a_t \) will exceed the upper bound in finite time, since the variance of \( a_t \) is increasing in \( t \). We use (12) to follow the literature. Results with a stationary AR(2) process for \( a_t \) that respects the bound up to a tolerance are available upon request.
To close the model, we assume the processes for endowment and dividend growth are given by

\[ \Delta \hat{y}_{t+1} = \mu_y + \sigma_y \varepsilon^y_{t+1}, \quad \varepsilon^y_{t+1} \sim \mathcal{N}(0, 1), \]

\[ \Delta \hat{d}_{t+1} = \mu_d + \pi dy \varepsilon^y_{t+1} + \psi_d \sigma_y \varepsilon^y_{t+1}, \quad \varepsilon^d_{t+1} \sim \mathcal{N}(0, 1), \]

where \( \mu_y \) and \( \mu_d \) are the growth rates of the assets, \( \sigma_y \geq 0 \) and \( \psi_d \sigma_y \geq 0 \) are the shock standard deviations, and \( \pi dy \) captures the correlation between consumption growth and dividend growth.

Asset market clearing implies

\[ s_{1,t} = 1 \quad \text{and} \quad s_{2,t} = 0, \]

so the resource constraint is given by

\[ \hat{c}_t = \hat{y}_t. \]

Equilibrium includes sequences of quantities \( \{\hat{c}_t\}_{t=0}^{\infty},\{\hat{m}_{t+1}, \hat{z}_{y,t}, \hat{z}_{d,t}, \hat{r}_{y,t}, \hat{r}_{d,t}\}_{t=0}^{\infty} \) and exogenous variables \( \{\hat{y}_t, \hat{d}_t, \hat{a}_t\}_{t=0}^{\infty} \) that satisfy \( (1), (10)-(14), (17), (18), \) and the resource constraint, given the state of the economy, \( \{\hat{a}_{t+1}, \hat{a}_t\} \), and the sequences of shocks, \( \{\varepsilon^y_{t}, \varepsilon^d_{t}, \varepsilon^a_{t}\}_{t=1}^{\infty}. \)

We posit the following solutions for the price-endowment and price-dividend ratios:

\[ \hat{z}_{y,t} = \eta_{y0} + \eta_{y1} \hat{a}_{t+1} + \eta_{y2} \hat{a}_t, \]

\[ \hat{z}_{d,t} = \eta_{d0} + \eta_{d1} \hat{a}_{t+1} + \eta_{d2} \hat{a}_t, \]

where \( \hat{z}_y = \eta_{y0} \) and \( \hat{z}_d = \eta_{d0}. \) We solve the model with the method of undetermined coefficients. Appendix A derives the first-order conditions, SDF, Campbell-Shiller approximation, and solution.

### 3 Intuition

This section develops intuition for why the valuation risk specification has such large effects on the model predictions. To simplify the exposition, we consider different stylized shock processes.

#### 3.1 Conventional Model

First, it is useful to review the role of Epstein-Zin preferences and the separation of the RA and IES parameters in matching the risk-free rate and equity premium. For simplicity, we remove valuation risk \( (\sigma_a = 0) \) and assume endowment/dividend risk is perfectly correlated \( (\psi_d = 0; \pi dy = 1). \) The average risk-free rate and average equity premium are given by

\[ E[r_f] = -\log \beta + \frac{\mu_y}{\psi} + \left(\frac{1}{\psi - \gamma}(1 - \gamma) - \gamma^2\right)\sigma_y^2/2, \]

\[ E[ep] = (2\gamma - 1)\sigma_y^2/2, \]

where the first term in \( (21) \) is the subjective discount factor, the second term accounts for endowment growth, and the third term accounts for precautionary savings. Endowment growth \( (\mu_y > 0) \) creates an incentive for agents to borrow in order to smooth consumption. Since the assets are in fixed supply, the risk-free rate must rise to deter borrowing. When the IES, \( \psi, \) is high, agents are willing to accept higher consumption growth so the compensation required to dissuade borrowing

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*We use this specification to illustrate the role of valuation risk. In section 5, we add long-run risk to (17) and (18).*
is lower. Therefore, the model requires a fairly high IES to match the low risk-free rate in the data.

With CRRA preferences, higher RA lowers the IES and pushes up the risk-free rate. With Epstein-Zin preferences, these parameters are independent, so a high IES can lower the risk-free rate without lowering RA. Notice the equity premium only depends on RA. Therefore, the model generates a low risk-free rate and modest equity premium with sufficiently high RA and IES parameter values. Of course, there is an upper bound on what constitute reasonable RA and IES values, which is the source of the risk-free rate and equity premium puzzles. Other prominent features such as long-run risk and stochastic volatility à la Bansal and Yaron (2004) help resolve these puzzles.

3.2 Valuation Risk Model

Now consider an example where we remove cash flow risk \((\sigma_y = 0; \mu_y = \mu_d)\) but keep valuation risk. For simplicity, we assume the time preference shock follows a random walk \((\rho_a = 0)\). Under these assumptions, the return on the endowment and dividend claims are identical so \((\kappa_y0, \kappa_y1, \eta_y0, \eta_y1, \eta_y2) = (\kappa_d0, \kappa_d1, \eta_d0, \eta_d1, \eta_d2) \equiv (\kappa, \kappa1, \eta0, \eta1, \eta2)\).

Current Specification We first solve the model with the current preferences, so the SDF is given by (1) with \(\omega = 1\). In this case, the average risk-free rate and average equity premium are given by

\[
E[r_f] = -\log \beta + \mu_y/\psi + (\theta - 1)\kappa^2\sigma_a^2/2,
\]

\[
E[ep] = (1 - 2\theta)\kappa^2\sigma_a^2/2.
\]

It is also straightforward to show the log-price-dividend ratio is given by \(\hat{z}_t = \hat{z} + \hat{a}_{t+1} - \hat{a}_t\) (i.e., the loadings on \(\hat{a}_{t+1}\) and \(\hat{a}_t\) are 1 and -1). Therefore, when the agent becomes more patient and \(\hat{a}_{t+1}\) rises, the price-dividend ratio jumps one-for-one on impact and then returns to the stationary equilibrium in the next period. Since \(\eta1\) is independent of the IES, there is no endogenous mechanism that prevents the asymptote in \(\theta\) from influencing the risk-free rate or equity premium. It is easy to see from (16) that \(0 < \kappa < 1\). Therefore, \(\theta\) dominates the risk-free rate and equity return when the IES is near 1. The following result describes the comparative statics with the IES:

As \(\psi\) approaches 1 from above, \(\theta\) tends to \(-\infty\). As a result, the average risk-free rate tends to \(-\infty\) while the average equity premium tends to \(+\infty\).

This key finding illustrates why valuation risk seems like such an attractive feature for jointly resolving the risk-free rate and equity premium puzzles. As the IES tends to 1 from above, \(\theta\) becomes increasingly negative, which dominates other determinants of the risk-free rate and equity premium. In particular, with an IES slightly above 1, the asymptote in \(\theta\) causes the average risk-free rate to become arbitrarily small, while making the average equity premium arbitrarily large. Bizarrely, an IES marginally below 1 (a popular value in the macro literature), generates the exact opposite predictions. Even when the IES is above 1 and away from the vertical asymptote, figure 1 shows \(\theta\) can have a meaningful effect on asset prices given a large enough risk aversion parameter.
An IES equal to 1 is a key value in the asset pricing literature. For example, it is the basis of the “risk-sensitive” preferences in Hansen and Sargent (2008, section 14.3). Therefore, it is clearly a desirable property for small perturbations around an IES of 1 to not materially alter the predictions of the model. A well-known example of where this property holds is the standard Epstein-Zin asset pricing model without valuation risk. Even though the log-SDF as written in (2) is undefined when the IES equals 1, both the risk-free rate and the equity premium in (21) and (22) are well-defined.\footnote{Pohl et al. (2018) show the errors from a Campbell-Shiller approximation of the nonlinear model can significantly affect equilibrium outcomes. Appendix B demonstrates that the asymptote does not depend on the approximation.}

**Revised Specification** Next we solve the model with the revised preferences, so the SDF is given by (1) with $\omega = \beta$. In this case, the average risk-free rate and average equity risk premium become

\[
E[r_f] = -\log\beta + \frac{\mu_y}{\psi} + (\theta - 1)(\kappa_1\eta_1)^2\sigma_a^2/2, \tag{25}
\]

\[
E[ep] = (1 - 2\theta)(\kappa_1\eta_1)^2\sigma_a^2/2, \tag{26}
\]

which are the same moments implied by the current specification, except the loading $\eta_1$ appears. This parameter determines the response of the price-dividend ratio to an $\hat{a}_{t+1}$ shock and is no longer invariant to the IES. In particular, $\hat{z}_t = \hat{z} + \eta_1\hat{a}_{t+1} - \hat{a}_t$. Appendix B shows that when the IES equals 1, $\eta_1 = 0$, so the asymptote disappears. Therefore, we obtain the following key result:

*When $\psi = 1$, the price-dividend ratio does not respond on impact to time preference shocks. Therefore, valuation risk generates no equity premium.*

Why does the influence of the asymptote disappear when the IES equals 1? The response of the price-dividend ratio to an anticipated change in $\hat{a}_{t+1}$ is determined by the relative strength of the substitution and wealth effects. First, consider the substitution effect. A higher $\hat{a}_{t+1}$ means the agent values present consumption more relative to the future and therefore wants to consume more today by reducing saving.\footnote{With the revised preferences, a rise in $\hat{a}_{t+1}$ corresponds to a fall in $\hat{a}_{t+1}^{R}$, so the agent becomes more impatient.} This effect lowers current asset demand and the price-dividend ratio.

The wealth effect operates in the opposite direction. When $\hat{a}_{t+1}$ is higher, the rise in the agent’s value of $c_t$ is less than the fall in the value of future certainty equivalent consumption since consumption is expected to grow. Therefore, the agent feels poorer, causing current asset demand and the price-dividend ratio to rise. When the IES equals 1, the substitution and wealth effects cancel out. This means the price-dividend ratio and the *ex-post* return on equity does not react on impact to an anticipated change in $\hat{a}_{t+1}$, which eliminates the effects of the asymptote. When the IES exceeds 1, as is typically the case in asset pricing models, the substitution effect dominates and reduces current asset demand on impact, causing the price-dividend ratio to fall. In the special case when there is no consumption growth, there are no wealth effects of time preference shocks, and substitution effects do not occur until the change in $\hat{a}_{t+1}$ materializes and lowers the discount rate.
3.3 **Graphical Illustration**

Our analytical results show the way a time preference shock enters Epstein-Zin recursive utility determines whether the asymptote in $\theta$ shows up in equilibrium outcomes. Figure 2 illustrates our results by plotting the average risk-free rate, the average equity premium, and the price-dividend ratio loading on the preference shock as a function of $\psi$. We focus on the setting in section 3.2 and plot the results under both preferences with and without growth.

With the current preferences, the average risk-free rate and average equity premium exhibit a vertical asymptote when the IES equals 1, regardless of whether $\mu_y$ is positive. As a result, the risk-free rate approaches positive infinity as the IES approaches 1 from below and negative infinity as the IES approaches 1 from above. The equity premium has the same comparative statics, except with the opposite sign. These results occur because $\eta_1 = 1$, regardless of the value of the IES. Therefore, the volatility of the return on equity is independent of the IES, while the volatility of the stochastic discount factor becomes infinitely large. This means the paradoxical agent with these preferences will sacrifice an infinite amount of consumption in order to hold an asset with zero risk.

In contrast, with the revised preferences the risk-free rate and equity premium are continuous in the IES, regardless of the value of $\mu_y$. When $\mu_y = 0$, the endowment stream is constant. This means there is no incentive to smooth consumption, the average risk-free rate is independent of the IES, and there is no immediate response of the price-dividend ratio to a time-preference shock. As a result, there are no unanticipated changes in the equity return and the average equity premium is zero.

When $\mu_y > 0$, the agent has an incentive to smooth consumption, so the SDF and the return on the equity become correlated. When $\psi > 1$, the substitution effect from the preference shock dominates the wealth effect. This causes the price-dividend ratio to fall ($\eta_1 < 0$) when the SDF falls and leads to a positive equity premium. In this case, the comparative static effect of the equity premium to a change in the IES has the opposite sign in the revised valuation risk model compared to the current valuation risk model. In the revised model, the equity premium is rising in the IES,
whereas in the current model it is falling in the IES. This is because the asymptote dominates the
determination of asset prices in the current model, even for large IES values. When \( \psi < 1 \), the
wealth effect dominates the substitution effect, so the price-dividend ratio rises (\( \eta_1 > 0 \)) when the
SDF falls. The negative covariance generates a negative valuation risk equity premium. Finally,
when \( \psi = 1 \), the substitution and wealth effects cancel out, leaving the price-dividend ratio un-
changed (\( \eta_1 = 0 \)). As a result, valuation risk generates no equity premium when the IES equals 1.

4** Estimated Baseline Model**

This section returns to the baseline model in section 2, which has valuation risk and stochastic en-
dowment and dividend growth. We estimate the model with both the current and revised preference
specifications and then report on how the parameter estimates and key asset pricing moments differ.

4.1 **Data and Estimation Method** We construct our data using the procedure in Bansal and
Yaron (2004), Beeler and Campbell (2012), Bansal et al. (2016), and Schorfheide et al. (2018). The
moments are based on five time series: real per capita consumption expenditures on nondurables
and services, the real equity return, real dividends, the real risk-free rate, and the price-dividend ra-
tio. Nominal equity returns are calculated with the CRSP value-weighted return on stocks. We ob-
tain data with and without dividends to back out a time series for nominal dividends. Both series are
converted to real using the consumer price index (CPI). The nominal risk-free rate is based on the
CRSP yield-to-maturity on 90-day Treasury bills. We first convert the nominal series to real using
the CPI. Then we construct an *ex-ante* real rate by regressing the *ex-post* real rate on the nominal
rate and inflation over the last year. The consumption data is annual. We convert the monthly asset
pricing data to annual series using data from the last month of each year. The model is estimated
using annual data from 1929 to 2017—the longest time span available without combining sources.

We estimate each model in two stages. In the first stage, we use Generalized Method of Mo-
ments (GMM) to obtain point estimates and a variance-covariance matrix of key moments in the
data. In the second stage, we use Simulated Method of Moments (SMM) to search for the param-
eter vector, \( \theta \), that minimizes the squared distance between the GMM point estimates, \( \bar{\Psi}_D \), and
median short-sample model moments, \( \bar{\Psi}_M \). The weighting matrix, \( W_D \), is the inverse diagonal of
the GMM estimate of the variance-covariance matrix, \( \bar{\Sigma}_D \). The objective function, \( J \), is given by

\[
J(\theta) = \frac{[\bar{\Psi}_M(\theta) - \bar{\Psi}_D]^{\prime} W_D [\bar{\Psi}_M(\theta) - \bar{\Psi}_D]}{N_M},
\]

where we normalize by the number of moments, \( N_M \), so \( J \) reflects the average distance from the
moments in \( \bar{\Psi}_D \). We use simulated annealing and then recursively apply Matlab’s `fminsearch`
to minimize \( J \) since gradient-based methods alone did not sufficiently search the parameter space.

Following Albuquerque et al. (2016), our algorithm matches the following 19 moments: the
mean and standard deviation of consumption growth, dividend growth, real stock returns, the real risk-free rate, and the price-dividend ratio, the correlation between dividend growth and consumption growth, the correlation between equity returns and both consumption and dividend growth at a 1-, 5-, and 10-year horizon, and the autocorrelation of the price-dividend ratio and real risk-free rate. Appendix C and Appendix D provide more information about our data and estimation method.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Current</th>
<th>Revised</th>
<th>Parameter</th>
<th>Current</th>
<th>Revised</th>
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(a) Parameter estimates. Current specification: $J = 1.11$; Revised specification: $J = 3.40$.

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<td>0.99</td>
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</table>

(b) Unconditional short-sample moments given the parameter estimates for each model. “All Shocks” simulates the model with all of the shocks, “Only CFR” simulates the model with only the cash flow risk shocks, and “Only VR” simulates the model with only the valuation risk shocks.

Table 1: Baseline model estimates

4.2 Parameter Estimates and Moments Table 1a shows the estimated parameter values and table 1b reports selected data and model moments under the current and revised valuation risk specifications. The current estimates are very similar to the values reported in Albuquerque et al. (2016), despite differences in the data construction. The model fits the data extremely well, with a lower $J$ value than the revised model. The current model requires a remarkably low RA value (1.6). The low RA value is due to the asymptote in the current valuation risk specification. An IES close to 1 raises the equity premium to an arbitrarily large extent, while IES values further from 1 cause the equity premium to asymptote at a value above what the revised specification generates. Therefore, the current model is able to maintain very low RA and still match moments in the data.
Revised valuation risk behaves more like cash flow risk, in that both the risk-free rate and equity premium are increasing in the IES. As a result, the revised valuation risk IES (8.0) is much higher than the current valuation risk IES (1.9). The higher IES diminishes the consumption smoothing motive and lowers the risk-free rate. Since the higher IES does not sufficiently raise the covariance between the SDF and the equity return, the data requires much higher RA. Our RA estimate (218.3) is an order of magnitude larger than what is usually accepted in the asset pricing literature.\(^7\) Despite much higher IES and RA values, the revised model is unable to generate a low enough risk-free rate or a high enough equity premium to match the data. The elevated parameter values also cause the revised model to underpredict the variance of the equity return and overpredict the variance of the risk-free rate. However, the other moments presented in the table match the data fairly closely.

Next, we decompose the relative role of valuation risk and cash flow risk in explaining the various asset pricing moments. Table 1b reports the model moments corresponding to counterfactual simulations that either remove valuation risk (“Only CFR”) or cash flow risk (“Only VR”) from the model. In each case, we re-solve the current and revised models after setting \(\sigma_a = \rho_a = 0\) for “Only CFR” and \(\sigma_y = 0\) for “Only VR”, so agents make decisions subject to only one type of risk.\(^8\)

With the current preferences, cash flow risk by itself generates a slightly negative equity premium and almost no precautionary savings demand because the RA parameter is so low. Therefore, the average risk-free rate is much higher than in the data. Without serial correlation in cash flow growth, cash flow risk alone cannot generate movements in the risk-free rate. As a result, it is valuation risk and the effects of the embedded asymptote that match all of the asset pricing moments.

With the revised preferences, valuation risk plays a much smaller role in explaining asset pricing moments. For example, with only valuation risk the average risk-free rate and equity premium are 3.74% and 2.80%, compared with 0.18% and 5.65% under the current preferences. Valuation risk still helps the model match asset pricing moments, particularly some of the volatilities and autocorrelations, but its ability to match the equity premium and risk-free rate is greatly diminished.\(^9\)

5 ESTIMATED LONG-RUN RISK MODEL

The baseline model is too simple to explain the data with the revised preferences. Also, valuation risk appears to have a more prominent role in matching some moments than cash flow risk. However, these results are not much of a surprise given that we have abstracted from long-run cash flow risk, which is a well-known potential resolution of many asset pricing puzzles. Therefore, this section introduces long-run risk to our baseline model and re-examines the role of valuation risk.

\(^7\)Mehra and Prescott (1985) suggest restricting RA to a maximum of 10. The acceptable range for the IES is less clearly defined in the literature, but values above 3 are atypical. Both revised estimates are well outside of these ranges.

\(^8\)We re-solve the model because expectations affect asset prices. If we only zero out the preference shock, the solution still includes valuation risk and would give the appearance that cash flow shocks generate a low risk-free rate.

\(^9\)Appendix E shows our results are robust to removing the long-run correlations and extending the data sample.
In order to introduce long-run risk, we modify (17) and (18) as follows:

$$
\Delta \hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y \varepsilon^y_{t+1}, \varepsilon^y_{t+1} \sim N(0, 1),
$$

$$
\Delta \hat{d}_{t+1} = \mu_d + \phi_d \hat{x}_t + \pi_{dy} \varepsilon^y_{t+1} \sigma_y \varepsilon^d_{t+1}, \varepsilon^d_{t+1} \sim N(0, 1),
$$

$$
\hat{x}_{t+1} = \rho_x \hat{x}_t + \psi_x \sigma_y \varepsilon^x_{t+1}, \varepsilon^x_{t+1} \sim N(0, 1),
$$

where the specification of the persistent component, \( \hat{x}_t \), which is common to both the endowment and dividends growth processes, follows Bansal and Yaron (2004). We apply the same estimation procedure as the baseline model, except we estimate three additional parameters, \( \phi_d, \rho_x, \) and \( \psi_x \).

Table 2a shows the estimated parameters and table 2b reports key asset pricing moments for the model with long-run risk. In the current model, the presence of long-run risk does not help the model fit the data any better (the \( J \) value is 1.11 in both models). Both the RA (1.6) and IES (1.9) parameter values are nearly identical to the baseline model. Long-run risk plays a minor role since the asymptote resulting from the valuation risk specification continues to dominate the determination of asset prices. Valuation risk by itself explains almost all of the asset pricing moments, including the near-zero risk free rate and 6.5% equity premium. Without valuation risk, the model generates no equity premium, a risk-free rate of 3.6%, and equity return volatility well below the data.

The results change dramatically with the revised specification. There are four key results. One, the model with long-run risk provides a much better fit of the data over the baseline model (the \( J \) value falls from 3.40 to 0.32), which is consistent with the message in Bansal and Yaron (2004).

Two, with long-run risk, the revised specification fits the data better than the current specification (\( J \) of 0.32 compared to 1.11), in contrast with the results from the baseline model. Therefore, without other features, the asymptote hurts the ability of the long-run risk model to match the data.

Three, RA declines from 218.3 in the baseline model to 30.1 in the model with long-run risk, closer to the acceptable range in the literature. With our formal GMM-SMM estimation procedure, it is not clear how to incorporate a penalty on parameter values falling outside the acceptable range in the literature. Instead, we decided to locally search for the optimal parameterization with a RA inside the acceptable range in the literature. We found RA and IES values of 3.5 and 5.1. However, the \( J \) value increased to 0.41, which is higher than in table 2a but lower than in the baseline model.

Four, valuation and cash flow risk no longer have similar roles in explaining asset pricing moments. In contrast with the current model, cash flow risk by itself generates an equity premium close to the data, even with lower RA. Valuation risk alone only generates a 2.6% equity premium, similar to the baseline model. Interestingly, however, valuation risk still plays an important role because it explains the volatility and autocorrelation of the risk-free rate. The standard deviation

\(^{10}\)Long-run risk adds one additional state variable, \( \hat{x}_t \). Following the guess and verify procedure applied to the baseline model, we use Mathematica to solve for unknown coefficients in the price-endowment and price-dividend ratios.
and autocorrelation of the risk-free rate in the data are 3.6% and 0.66, whereas long-run risk alone generates values of 0.02% and 0.93. In short, there is still a role for valuation risk, but it no longer has the ability to unilaterally resolve several long-standing asset pricing puzzles in its revised form.

**The Correlation Puzzle** Another important asset pricing puzzle pertains to the correlation between equity returns and fundamentals (Cochrane and Hansen (1992)). In the data, the correlation between equity returns and consumption growth is near zero, regardless of the horizon. The correlation between equity returns and dividend growth is small over short horizons but increases over longer horizons. The central issue is that many asset-pricing models predict too strong of a correlation between stock returns and fundamentals relative to the data. Clearly, if valuation risk generates meaningful volatility in asset returns and yet is uncorrelated with consumption and dividend growth (as in the model in section 2), then valuation risk has the potential to resolve the correlation puzzle.

Table 3 shows the correlations between equity returns and fundamentals over 1-, 5-, and 10-year horizons in the data and the model. We also consider a counterfactual with only cash flow risk

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Current</th>
<th>Revised</th>
<th>Parameter</th>
<th>Current</th>
<th>Revised</th>
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(a) Parameter estimates. Current specification: $J = 1.11$; Revised specification: $J = 0.32$. 

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>All Shocks</th>
<th>Only CFR</th>
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<th>All Shocks</th>
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</table>

(b) Unconditional short-sample moments given the parameter estimates for each model. “All Shocks” simulates the model with all of the shocks, “Only CFR” simulates the model with only the cash flow risk shocks, and “Only VR” simulates the model with only the valuation risk shocks.
The correlations with consumption growth are similar across the current and revised valuation risk specifications. Consistent with the data, the model predicts a weak correlation over all horizons. With both specifications, cash flow risk is sufficient for the model to match the data. The correlations with dividend growth are also similar across the two specifications, but their sources differ. With the current specification, the correlations are driven by valuation risk, whereas cash flow risk plays the primary role with the revised specification. The intuition for these results is straightforward. In a model with long-run risk, most of the volatility in equity returns comes from changes in consumption and dividend growth, while valuation risk is relegated to a secondary role. These results demonstrate that valuation risk is also insufficient for resolving the correlation puzzle.

### 6 Extended Long-Run Risk Models

This section further examines the role of valuation risk by extending our model with long-run risk in two independent ways. First, we consider an extension where valuation risk shocks directly affect consumption and dividend growth, in addition to their effect on asset prices through the SDF (henceforth, the “demand shock model”). This feature is similar to a discount factor shock in a dynamic stochastic general equilibrium model. For example, in the workhorse New Keynesian model, an increase in the discount factor looks like a typical negative demand shock that lowers both inflation and consumption. Therefore, it provides another potential mechanism for valuation risk to help fit the data. Following Albuquerque et al. (2016), we augment (27) and (28) as follows:

\[
\Delta \hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y \varepsilon_{y, t+1}^y + \pi_{ya} \sigma_a \varepsilon_{a, t+1}^a, \\
\Delta \hat{d}_{t+1} = \mu_d + \phi_d \hat{x}_t + \pi_{da} \sigma_y \varepsilon_{y, t+1}^y + \psi_d \sigma_d \varepsilon_{d, t+1}^d + \pi_{da} \sigma_a \varepsilon_{a, t+1}^a,
\]

where \( \pi_{ya} \) and \( \pi_{da} \) determine the correlations between valuation risk shocks and cash flow growth.

Second, we introduce stochastic volatility to cash flow risk following Bansal and Yaron (2004) (henceforth, the “SV Model”). This feature directly affects the price-dividend ratio and helps
match its unconditional volatility. Furthermore, when we remove valuation risk from our baseline long-run risk model, it generates very little volatility and a nearly perfect autocorrelation in the risk-free rate, in contrast with the data. An important question is whether the presence of SV will further diminish the role of valuation risk. To add SV to the model, we modify (27)-(29) as follows:

\[
\Delta \hat{y}_{t+1} = \mu_y + \hat{x}_t + \sigma_y \varepsilon_{y,t+1},
\]

\[
\Delta \hat{d}_{t+1} = \mu_d + \phi_d \hat{x}_t + \pi_y \sigma_y \varepsilon_{y,t+1} + \psi_d \sigma_y \varepsilon_{d,t+1},
\]

\[
\hat{x}_{t+1} = \rho_x \hat{x}_t + \psi_x \sigma_y \varepsilon_{x,t+1},
\]

where

\[
\sigma_{y,t+1}^2 = \sigma_y^2 + \rho_{\sigma_y} (\sigma_{y,t}^2 - \sigma_y^2) + \nu_y \varepsilon_{y,t+1}.
\]

\(\sigma_y^2\) is mean volatility, \(\rho_{\sigma_y}\) is the persistence, and \(\nu_y\) is the standard deviation of the volatility shock.

Table 4a shows the parameter estimates for both models with the revised valuation risk specification.\(^{11}\) The demand shock model fits the data better than the baseline long-run risk model (the \(J\) value declines from 0.32 to 0.25). Furthermore, RA falls substantially from 30.1 to 7.7, so it is now inside the acceptable range in the literature. In contrast, the SV model provides a smaller improvement to the empirical fit (the \(J\) value declines to 0.30) and RA increases to 62.4. When we search for a parameterization with a RA value that is in line with the asset pricing literature, the lowest \(J\) value is 0.34, which is slightly higher than in the baseline long-run risk model. While previous sections clearly show that valuation risk by itself is not able to resolve the equity premium puzzle, these results demonstrate that it has an important role even in the presence of long-run risk.

Table 4b compares the estimated moments in each model to the data. The demand shock model fits the data better because it generates a larger volatility of dividend growth. All else equal, lower RA causes the volatility of equity returns to rise. With the additional source of volatility in the demand shock model, the estimated response of dividend growth to a long-run cash flow risk shock is much smaller than in the baseline long-run risk model (\(\phi_d\) declines from 17.3 to 5.1). Therefore, the model is able to closely match both equity return and dividend growth volatility with much lower RA. These benefits are evident in the counterfactual simulations that isolate the effects of each shock type. When the demand shock model only includes valuation risk, there is now positive dividend growth volatility (2.35%) and much lower equity return volatility (10% instead of 16.5%).\(^{12}\)

The addition of SV has a smaller effect on the results. The estimates of the valuation risk persistence (\(\rho_a\)) and standard deviation (\(\sigma_a\)) are roughly the same in the models with and without SV. This suggests the presence of SV, which gives cash flow risk another mechanism to affect the data, does not diminish the role of valuation risk. Furthermore, the counterfactuals show that with only

\(^{11}\)In these extended models, the results with the current specification are similar to previous sections. We focus on the revised specification, since previous sections already show the undesirable properties of the current specification.

\(^{12}\)We do not show the correlations between equity returns and cash flow growth because they are similar to table 3.
valuation risk, which is an additional source of volatility that affects our estimates.

10 years of data since we match long-run correlations and use a balanced sample. Third, we include sample excludes the Great Depression period. While our raw data starts in 1930 as in BKY, we lose returns as well as the volatility and autocorrelation of the risk-free rate. Second, our effective moments. Our estimation includes long-run correlations between cash flow growth and equity cash flow risk, the SV model would significantly under-predict the volatility of the risk-free rate.

The limited role of SV may seem surprising in light of the results in Bansal et al. (2016) (henceforth, BKY). We attribute the differences to three important factors. One, we match different moments. Our estimation includes long-run correlations between cash flow growth and equity returns as well as the volatility and autocorrelation of the risk-free rate. Second, our effective sample excludes the Great Depression period. While our raw data starts in 1930 as in BKY, we lose 10 years of data since we match long-run correlations and use a balanced sample. Third, we include valuation risk, which is an additional source of volatility that affects our estimates. Appendix F shows we obtain similar estimates to BKY when our setups are more closely aligned. However, the model performs very poorly at matching the moments that are not included in their estimation.\footnote{There are also differences in the weighting matrix (BKY recursively update it based on model estimates, instead of fixing it to the data) and how the moments are calculated (BKY use theoretical instead of short-sample moments).}

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Demand</th>
<th>SV</th>
<th>SV Alt</th>
<th>Parameter</th>
<th>Demand</th>
<th>SV</th>
<th>SV Alt</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>7.74202</td>
<td>62.38565</td>
<td>9.48497</td>
<td>$\rho_a$</td>
<td>0.95499</td>
<td>0.95770</td>
<td>0.96331</td>
</tr>
<tr>
<td>$\psi$</td>
<td>3.84415</td>
<td>6.21599</td>
<td>3.36354</td>
<td>$\phi_d$</td>
<td>5.10257</td>
<td>16.31730</td>
<td>4.13180</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99881</td>
<td>0.99796</td>
<td>0.99855</td>
<td>$\rho_c$</td>
<td>0.99814</td>
<td>0.99716</td>
<td>0.99816</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>0.00257</td>
<td>0.00060</td>
<td>0.00223</td>
<td>$\psi_x$</td>
<td>0.01860</td>
<td>0.00457</td>
<td>0.01701</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>0.00159</td>
<td>0.00167</td>
<td>0.00162</td>
<td>$\pi_{ya}$</td>
<td>-3.70415</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\mu_d$</td>
<td>0.00146</td>
<td>0.00155</td>
<td>0.00159</td>
<td>$\pi_{da}$</td>
<td>9.57472</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$\psi_d$</td>
<td>5.59763</td>
<td>3.78774</td>
<td>4.47096</td>
<td>$\rho_d$</td>
<td>-</td>
<td>0.17137</td>
<td>0.42457</td>
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<tr>
<td>$\pi_{dy}$</td>
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<td>-0.43604</td>
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<td>$\nu_y$</td>
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<td>0.00002</td>
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<tr>
<td>$\sigma_a$</td>
<td>0.00072</td>
<td>0.00069</td>
<td>0.00062</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(a) Parameter estimates. Demand: $J = 0.25$; Stochastic Volatility (SV): $J = 0.30$; Alternative Stochastic Volatility (Alt SV): $J = 0.34$.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Demand Shock Specification</th>
<th>Stochastic Volatility Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\Delta c]$</td>
<td>2.00</td>
<td>All Shocks</td>
<td>Only CFR</td>
</tr>
<tr>
<td>$E[\Delta d]$</td>
<td>2.06</td>
<td>1.95</td>
<td>1.95</td>
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<tr>
<td>$E[r_d]$</td>
<td>6.68</td>
<td>6.73</td>
<td>7.06</td>
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<tr>
<td>$E[r_f]$</td>
<td>0.18</td>
<td>0.49</td>
<td>0.84</td>
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<td>$E[z_d]$</td>
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<td>3.45</td>
<td>3.13</td>
</tr>
<tr>
<td>$E[\epsilon]$</td>
<td>6.51</td>
<td>6.24</td>
<td>6.22</td>
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<tr>
<td>$SD[\Delta c]$</td>
<td>1.36</td>
<td>1.39</td>
<td>1.05</td>
</tr>
<tr>
<td>$SD[\Delta d]$</td>
<td>7.25</td>
<td>6.23</td>
<td>5.72</td>
</tr>
<tr>
<td>$SD[r_d]$</td>
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<td>17.52</td>
<td>14.78</td>
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<td>2.56</td>
<td>0.14</td>
</tr>
<tr>
<td>$SD[z_d]$</td>
<td>0.44</td>
<td>0.47</td>
<td>0.39</td>
</tr>
<tr>
<td>$Corr[\Delta c, \Delta d]$</td>
<td>0.02</td>
<td>-0.05</td>
<td>0.30</td>
</tr>
<tr>
<td>$AC[r_f]$</td>
<td>0.66</td>
<td>0.68</td>
<td>0.95</td>
</tr>
<tr>
<td>$AC[z_d]$</td>
<td>0.92</td>
<td>0.93</td>
<td>0.93</td>
</tr>
</tbody>
</table>

(b) Unconditional short-sample moments given the parameter estimates for each model. “All Shocks” simulates the model with all of the shocks, “Only CFR” simulates the model with only the cash flow risk shocks, and “Only VR” simulates the model with only the valuation risk shocks.

Table 4: Extended long-run risk model estimates
7 Conclusion

The way valuation risk enters Epstein-Zin recursive utility has important implications. Under the current specification, an undesirable asymptote in the parameter space permeates equilibrium outcomes. As a consequence, a positive equity premium paradoxically appears in a stylized model where logic implies one should not exist. Empirically, the presence of the asymptote perversely allows valuation risk by itself to explain the historically low risk-free rate and high equity premium.

Once we revise the preferences to remove the asymptote, valuation risk has a much smaller role in explaining asset pricing moments. In particular, it is no longer able to unilaterally resolve the equity premium, risk-free rate, and correlation puzzles. However, valuation risk still plays an important role in matching the volatility and autocorrelation of the risk-free rate. Furthermore, allowing valuation risk shocks to directly affect cash flow growth introduces an important source of volatility that significantly reduces the risk aversion necessary to match data. In short, valuation risk is not nearly as important as the current literature suggests, but it still has a consequential role.

References


A PRICING KERNEL DERIVATION AND MODEL SOLUTION

The value function for preference specification \( j \in \{C, R\} \) is given by

\[
V_t^j = \max[w_{1,t}^{C}(1-\gamma)/\theta + w_{2,t}^{C}(E_t[(V_{t+1}^{j})^{\gamma}])^{1/\theta}/(1-\gamma) - \lambda_t(c_t + p_{d,t}s_{1,t} + p_{y,t}s_{2,t} - (p_{d,t} + d_t)s_{1,t-1} - (p_{y,t} + y_t)s_{2,t-1}),
\]

where \( w_{1,t}^{C} = a_t^{C}(1-\beta), w_{1,t}^{R} = 1 - a_t^{R}\beta, w_{2,t}^{C} = \beta, \) and \( w_{2,t}^{R} = a_t^{R}\beta. \) Optimality implies

\[
w_{1,t}^{C}(V_t^{j})^{1/\psi}c_t^{-1/\psi} = \lambda_t, \tag{36}
\]

\[
w_{2,t}^{C}(V_t^{j})^{1/\psi}(E_t[(V_{t+1}^{j})^{\gamma}])^{1/\theta-1}E_t[(V_{t+1}^{j})^{\gamma}(\partial V_{t+1}^{j}/\partial s_{1,t})] = \lambda_t p_{y,t}, \tag{37}
\]

\[
w_{2,t}^{R}(V_t^{j})^{1/\psi}(E_t[(V_{t+1}^{j})^{\gamma}])^{1/\theta-1}E_t[(V_{t+1}^{j})^{\gamma}(\partial V_{t+1}^{j}/\partial s_{2,t})] = \lambda_t p_{d,t}, \tag{38}
\]

where \( \partial V_{t+1}^{j}/\partial s_{1,t-1} = \lambda_t(p_{y,t} + y_t) \) and \( \partial V_{t+1}^{j}/\partial s_{2,t-1} = \lambda_t(p_{d,t} + d_t) \) by the envelope theorem. Updating the envelope conditions and combining (36)-(38) yields (8) and (9) in the main text.
Following Epstein and Zin (1991), we posit the following minimum state variable solution:

\[ V_t^j = \xi_{1,t} s_{1,t-1} + \xi_{2,t} s_{2,t-1} \quad \text{and} \quad c_t = \xi_{3,t} s_{1,t-1} + \xi_{4,t} s_{2,t-1}. \]  \hspace{1cm} (39)

where \( \xi \) is a vector of unknown coefficients. The envelope conditions combined with (36) imply

\[ \xi_{1,t} = w_{1,t}^j (V_t^j)^{1/\psi} c_t^{-1/\psi} (p_{y,t} + y_t), \]  \hspace{1cm} (40)

\[ \xi_{2,t} = w_{1,t}^j (V_t^j)^{1/\psi} c_t^{-1/\psi} (p_{d,t} + d_t). \]  \hspace{1cm} (41)

Multiplying the respective conditions by \( s_{1,t-1} \) and \( s_{2,t-1} \) and then adding yields

\[ V_t^j = w_{1,t}^j (V_t^j)^{1/\psi} c_t^{-1/\psi} (p_{y,t} + y_t) s_{1,t-1} + (p_{d,t} + d_t) s_{2,t-1}, \]  \hspace{1cm} (42)

which after plugging in the budget constraint, (5), can be written as

\[ (V_t^j)^{(1-\gamma)/\theta} = w_{1,t}^j c_t^{-1/\psi} (c_t + p_{y,t} s_{1,t} + p_{d,t} s_{2,t}) = w_{1,t}^j c_t^{-1/\psi} (c_t + p_{y,t}). \]  \hspace{1cm} (43)

Therefore, the optimal value function is given by

\[ w_{1,t}^j c_t^{-1/\psi} p_{y,t} = w_{2,t}^j (E_t[(V_{t+1}^j)^{1-\gamma}])^{1/\theta}. \]  \hspace{1cm} (44)

Solving (43) for \( V_t^j \) and (44) for \( E_t[(V_{t+1}^j)^{1-\gamma}] \) and then plugging into (8) and (9) implies

\[ m_{t+1} = \beta (x_t^j)^{\theta} (c_{t+1}/c_t)^{-\theta/\psi} f_{y,t+1}^{\theta}, \]  \hspace{1cm} (45)

where \( x_t^C \equiv a_{t+1}^C/a_t^C \) and \( x_t^R \equiv a_t^R (1 - a_{t+1}^R)/(1 - a_t^R \beta) \). Taking logs of (45) yields (1), where

\[ \hat{x}_t^C = \hat{a}_{t+1}^C - \hat{a}_t^C, \]

\[ \hat{x}_t^R = \hat{a}_t^R + \log(1 - \beta \exp(\hat{a}_t^R)) - \log(1 - \beta \exp(\hat{a}_t^R)) \approx -(\beta \hat{a}_{t+1}^R - \hat{a}_t^R)/(1 - \beta). \]

We define \( \hat{a}_t \equiv \hat{a}_t^C = -\hat{a}_t^R/(1 - \beta) \), so the preference shocks in the two models are directly comparable. It follows that \( \hat{x}_t^j = \omega_j \hat{x}_t + \hat{a}_t \) just like in (1), where \( \omega_C = 1 \) and \( \omega_R = \beta \).

The Campbell and Shiller (1988) approximation to the return on the endowment is given by

\[ \hat{r}_{y,t+1} = \log(y_{t+1}(p_{y,t+1}/y_{t+1}) + y_{t+1}) - \log(y_t(p_{y,t}/y_t)) \]

\[ = \log(\exp(\hat{z}_{y,t+1}) + 1) - \hat{z}_{y,t} + \Delta \hat{y}_{t+1} \]

\[ \approx \log(\exp(\hat{z}_y) + 1) + \exp(\hat{z}_y)(\hat{z}_{y,t+1} - \hat{z}_y)/(1 + \exp(\hat{z}_y)) - \hat{z}_{y,t} + \Delta \hat{y}_{t+1} \]

\[ = \kappa_y(\hat{z}_{y,t+1} - \hat{z}_{y,t}) + \Delta \hat{y}_{t+1}. \]

The derivation for the equity return, \( \hat{r}_{d,t+1} \), is analogous to the return on the endowment.
We solve the model using a guess and verify method. For the endowment claim, we obtain

\[ 0 = \log(E_t[\exp(\tilde{m}_{t+1} + \hat{r}_{y,t+1})]) \]

\[ = \log\left( E_t \left[ \exp \left( \theta \hat{\beta} + \theta \omega^j \hat{a}_{t+1} - \hat{a}_t + (\theta(1-1/\psi) - 1)\Delta \hat{y}_{t+1} + \Delta \hat{d}_{t+1} \right) \right] \right) \]

\[ = \log\left( E_t \left[ \exp \left( \theta \hat{\beta} + \theta \omega^j \hat{a}_{t+1} - \hat{a}_t + (\theta(1-1/\psi) - 1)\Delta \hat{y}_{t+1} + \Delta \hat{d}_{t+1} \right) \right] \right) \]

\[ = \log\left( E_t \left[ \exp \left( \theta \hat{\beta} + \theta \omega^j \hat{a}_{t+1} - \hat{a}_t + (\theta(1-1/\psi) - 1)\Delta \hat{y}_{t+1} + \Delta \hat{d}_{t+1} \right) \right] \right) \]

where \( \hat{\rho} = 1 + \rho_a \). The last equality follows from the log-normality of \( \exp(\varepsilon_y) \) and \( \exp(\varepsilon_a) \).

After equating coefficients, we obtain the following exclusion restrictions:

\[ \dot{\beta} + (1-1/\psi)\mu_y + (\kappa y_0 + \eta y_0(\kappa y_1 - 1)) + \frac{\theta}{2}(1-1/\psi)^2 \sigma_y^2 + \kappa_{y1}^2 \sigma_a^2 = 0, \]

\[ \omega^j + \eta y_1(\kappa y_1 \hat{\rho} - 1) + \eta y_2 \kappa y_1 = 0, \]

\[ 1 + \eta y_2 + \kappa y_1 \eta y_1 \rho_a = 0. \]

For the dividend claim, we obtain

\[ 0 = \log(E_t[\exp(\tilde{m}_{t+1} + \hat{r}_{d,t+1})]) \]

\[ = \log\left( E_t \left[ \exp \left( \theta \hat{\beta} + \theta \omega^j \hat{a}_{t+1} - \hat{a}_t + (\theta(1-1/\psi) - 1)\Delta \hat{y}_{t+1} + \Delta \hat{d}_{t+1} \right) \right] \right) \]

\[ = \log\left( E_t \left[ \exp \left( \theta \hat{\beta} + \theta \omega^j \hat{a}_{t+1} - \hat{a}_t + (\theta(1-1/\psi) - 1)\Delta \hat{y}_{t+1} + \Delta \hat{d}_{t+1} \right) \right] \right) \]

\[ = \log\left( E_t \left[ \exp \left( \theta \hat{\beta} + \theta \omega^j \hat{a}_{t+1} - \hat{a}_t + (\theta(1-1/\psi) - 1)\Delta \hat{y}_{t+1} + \Delta \hat{d}_{t+1} \right) \right] \right) \]
Once again, equating coefficients implies the following exclusion restrictions:

\[
\theta \hat{\beta} + (\theta(1 - 1/\psi) - 1)\mu_y + \mu_d + (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) + (\kappa_{d0} + \eta_{d0}(\kappa_{d1} - 1)) \\
+ \frac{1}{2}(\pi_{dy} - \gamma)^2 \sigma_y^2 + ((\theta - 1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1})^2 \sigma_a^2 + \psi^2 \sigma_y^2 = 0, \tag{49}
\]

\[
\theta \omega^d + (\theta - 1)[(\rho\kappa_{y1} - 1)\eta_{y1} + \kappa_{y1}\eta_{y2}] + (\rho\kappa_{d1} - 1)\eta_{d1} + \kappa_{d1}\eta_{d2} = 0, \tag{50}
\]

\[
\theta + (\theta - 1)\eta_{y2} + \eta_{d2} + ((\theta - 1)\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1})\rho_a = 0. \tag{51}
\]

Equations (46)-(51), along with (15) and (16), form a system of 10 equations and 10 unknowns.

**B ASSET PRICES**

Given the model solution, we can solve for the risk free rate. The Euler equation implies

\[
r_{f,t} = -\log(E_t[\exp(m_{t+1})]) = -E_t[m_{t+1}] - \frac{1}{2} \text{Var}_t[m_{t+1}],
\]

since the risk-free rate is known at time-\( t \). The pricing kernel is given by

\[
m_{t+1} = \theta \hat{\beta} + (\omega^i\hat{a}_{t+1} - \hat{a}_t) - (\theta/\psi)\Delta \hat{y}_{t+1} + (\theta - 1)\hat{r}_{y,t+1} \\
= \theta \hat{\beta} + (\omega^i\hat{a}_{t+1} - \hat{a}_t) - \gamma \Delta \hat{y}_{t+1} + (\theta - 1)(\kappa_{y0} + \kappa_{y1}\hat{\epsilon}_{y,t+1} - \hat{\epsilon}_{y,t}) \\
= \theta \hat{\beta} + (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) - \gamma \mu_y + (\theta \omega^d + (\theta - 1)[(\rho\kappa_{y1} - 1)\eta_{y1} + \kappa_{y1}\eta_{y2}])\hat{a}_{t+1} \\
- (\theta + (\theta - 1)(\eta_{y2} + \kappa_{y1}\eta_{y1}\rho_a))\hat{a}_t + (\theta - 1)\kappa_{y1}\eta_{y1}\sigma_a \hat{a}_{t+1} + \kappa_{d1}\eta_{d1}\hat{a}_{t+1} - \hat{a}_t,
\]

where the last line follows from imposing (47) and (48). Therefore, the risk-free rate is given by

\[
r_{f,t} = \gamma \mu_y - \theta \hat{\beta} - (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) - (\omega^i\hat{a}_{t+1} - \hat{a}_t) - \frac{1}{2}(\gamma \sigma_y^2) - \frac{1}{2}((\theta - 1)\kappa_{y1}\eta_{y1}\sigma_a)^2.
\]

The unconditional expected risk-free rate is given by

\[
E[r_f] = \gamma \mu_y - \theta \hat{\beta} - (\theta - 1)(\kappa_{y0} + \eta_{y0}(\kappa_{y1} - 1)) - \frac{1}{2}(\gamma \sigma_y^2) - \frac{1}{2}((\theta - 1)\kappa_{y1}\eta_{y1}\sigma_a)^2. \tag{52}
\]

Plugging (46) into (52) implies

\[
E[r_f] = -\hat{\beta} + \mu_y/\psi + \frac{1}{2}(\theta - 1)\kappa_{y1}\eta_{y1}\sigma_a^2 + \frac{1}{2}((1/\psi - \gamma)(1 - \gamma) - \gamma^2)\sigma_y^2. \tag{53}
\]

We can also derive an expression for the equity premium, given by,

\[
E_t[e_{p,t+1}] = E_t[r_{d,t+1} - r_{f,t}] = -\frac{1}{2} \text{Var}_t[\hat{r}_{d,t+1}] - \text{Cov}_t[\hat{m}_{t+1}, \hat{r}_{d,t+1}], \tag{54}
\]

since the Euler equation implies that

\[
E_t[\hat{m}_{t+1} + \hat{r}_{d,t+1}] + \frac{1}{2} \text{Var}_t[\hat{m}_{t+1} + \hat{r}_{d,t+1}] = 0.
\]
We first solve for the return on dividends, which is given by

\[
\hat{r}_{d,t+1} = \kappa_{d0} + \kappa_{d1}\hat{z}_{d,t+1} - \hat{z}_{d,t} + \Delta\hat{d}_{t+1}
\]

\[
= \kappa_{d0} + \kappa_{d1}(\eta_{d0} + \eta_{d1}\hat{a}_{t+2} + \eta_{d2}\hat{a}_{t+1}) - (\eta_{d0} + \eta_{d1}\hat{a}_{t+1} + \eta_{d2}\hat{a}_{t}) + \Delta\hat{d}_{t+1}
\]

\[
= (\mu_d + \kappa_{d0} + (\kappa_{d1} - 1)\eta_{d0}) + ((\bar{\rho}\kappa_{d1} - 1)\eta_{d1} + \kappa_{d1}\eta_{d2})\hat{a}_{t+1} - (\kappa_{d1}\eta_{d1}\rho_a + \eta_{d2})\hat{a}_t
\]

\[+ \kappa_{d1}\eta_{d1}\sigma_a \epsilon_{a,t+1} + \pi_{dy}\sigma_y \epsilon_{y,t+1} + \psi_d \sigma_y \epsilon_{d,t+1}.
\]

Therefore, given (54), the unconditional equity premium can be written as

\[
E[ep] = \frac{1}{2}(2\gamma - \pi_{dy})\pi_{dy}\sigma_y^2 - \frac{1}{2}\psi_d^2\sigma_y^2 - \frac{1}{2}(2(\theta - 1))\kappa_{y1}\eta_{y1} + \kappa_{d1}\eta_{d1}\sigma_a^2.
\] (55)

**B.1 Special Case 1 (\(\sigma_a = \psi_d = 0\) & \(\pi_{dy} = 1\))** In this case, there is no valuation risk (\(\hat{a}_t = 0\)) and endowment/dividend risk is perfectly correlated (\(\Delta\hat{y}_{t+1} = \mu_y + \sigma_y \epsilon_{y,t+1}^y; \Delta\hat{d}_{t+1} = \mu_d + \sigma_y \epsilon_{d,t+1}^y\)).

Under these assumptions, it is easy to see that (53) and (55) reduce to (21) and (22) in the main text.

**B.2 Special Case 2 (\(\sigma_y = 0\), \(\rho_a = 0\), & \(\mu_y = \mu_d\))** In this case, there is no cash flow risk (\(\Delta\hat{y}_{t+1} = \Delta\hat{d}_{t+1} = \mu_y\)) and the preference shock follows a random walk (\(\Delta\hat{a}_{t+2} = \sigma_a \epsilon_{d,t+1}^a\)).

Under these assumptions, the return on the endowment and dividend claims are identical, so \(\{\kappa_{y0}, \kappa_{y1}, \eta_{y0}, \eta_{y1}, \eta_{y2}\} = \{\kappa_{d0}, \kappa_{d1}, \eta_{d0}, \eta_{d1}, \eta_{d2}\} \equiv \{\kappa_0, \kappa_1, \eta_0, \eta_1, \eta_2\}\).

Therefore, (53) and (55) reduce to (25) and (26) in the main text. The exclusion restrictions, (15) and (46)-(48), simplify to

\[
0 = \hat{\beta} + (1 - 1/\psi)\mu_y + (\kappa_0 + \eta_0(\kappa_1 - 1)) + \frac{\kappa}{2}\eta_1^2\eta_1^2\sigma_a^2,
\] (56)

\[
0 = \omega^j + \eta_1(\kappa_1 - 1) + \eta_2\kappa_1,
\] (57)

\[
\eta_2 = -1,
\] (58)

\[
\kappa_0 = \log(1 + \exp(\eta_0)) - \kappa_1\eta_0,
\] (59)

\[
\kappa_1 = \exp(\eta_0)/(1 + \exp(\eta_0)).
\] (60)

First, notice that \(0 < \kappa_1 < 1\). Therefore, with the current preferences (\(\omega^C = 1\)), it is easy to see that \(\eta_1 = 1\). With revised preferences (\(\omega^R = \beta\)), the solution for \(\eta_1\) is more complicated. However, for \(\psi = 1\), we guess and then verify that \(\eta_1 = 0\). In this case, from (57) \(\kappa_1 = \beta\) and (56) reduces to

\[
0 = \log \beta + \kappa_0 - (1 - \beta)\eta_0,
\] (61)

This restriction implies that \(\eta_0 = \log \beta - \log(1 - \beta)\) and \(\kappa_0 = -(1 - \beta)\log(1 - \beta) - \beta \log \beta\).

**Unanticipated Valuation Risk** With a small change in the timing of the valuation risk shock, we can derive a closed-form expression for the risk-free rate without relying on a Campbell-Shiller approximation to show that the asymptote is not due to the approximation. Building on special case 2, we assume \(\Delta\hat{a}_{t+1} = \sigma \epsilon_{t+1}^a\) instead of \(\Delta\hat{a}_{t+1} = \sigma \epsilon_{t}^a\), so \(\Delta\hat{a}_{t+1}\) is no longer anticipated.
Preferences are given by (3), so the equilibrium condition that prices asset $i$ is given by

$$1 = \beta E_t[(a_{t+1}/a_t)(c_{t+1}/c_t)^{-1/\psi}(V_{t+1}^{1-\gamma}/E_t[V_{t+1}^{1-\gamma}])^{1-1/\theta} r_{i,t+1}],$$

(62)

where we dropped the preference specific superscripts. We begin by conjecturing that the value function takes the form $V_t/c_t = \eta a_t^{\theta/(1-\gamma)}$. Substituting the guess into the value function implies

$$\eta = \frac{1 - \beta}{1 - \beta \exp((1 - \gamma)\mu_y + \sigma^2/2)}.$$  

(63)

which verifies our conjecture. Therefore, we can substitute into (62) to obtain

$$\hat{r}_{f,t} = -\log \beta + \mu_y/\psi - \log(E_t[(a_{t+1}/a_t)^\theta]) / \theta,$$

$$= -\log \beta + \mu_y/\psi - \log(E_t[\exp(\theta \Delta a_{t+1})]) / \theta,$$

$$= -\log \beta + \mu_y/\psi - \theta \sigma^2/2.$$  

This result shows that the risk-free rate will inherit the asymptote in $\theta$.

C DATA SOURCES

We drew from the following data sources to estimate our models:


We applied the following transformations to the above series:

1. Annual Per Capita Real Consumption Growth (annual frequency):

$$\Delta \hat{c}_t = 100 \log(RCONS_t/RCONS_{t-1})$$
2. **Annual Real Dividend Growth (monthly frequency):**

\[
P_{1928M1} = 100, \quad P_t = P_{t-1}(1 + RETX_t), \quad D_t = (RETD_t - RETX_t)P_{t-1}, \\
\hat{d}_t = \sum_{i=t-11}^t D_i / CPI_t, \quad \Delta \hat{d}_t = 100 \log(d_t/d_{t-12})
\]

3. **Annual Real Equity Return (monthly frequency):**

\[
\pi_t^m = \log(CPI_t/CPI_{t-1}), \\
\hat{r}_{d,t} = 100 \sum_{i=t-11}^t (\log(1 + RETD_i) - \pi_t^m)
\]

4. **Annual Real Risk-free Rate (monthly frequency):**

\[
rfr_t = RFR_t - \log(CPI_{t+3}/CPI_t), \quad \pi_t^q = \log(CPI_t/CPI_{t-12})/4, \\
\hat{r}_{fr,t} = 400(\hat{\beta}_0 + \hat{\beta}_1 RFR_t + \hat{\beta}_2 \pi_t^q),
\]

where \(\hat{\beta}_j\) are the OLS estimates in a regression of the *ex-post* real rate, \(rfr\), on the nominal rate, \(RFR\), and lagged inflation, \(\pi^q\). The fitted values are estimates of the *ex-ante* real rate.

5. **Price-Dividend Ratio (monthly frequency):**

\[
\hat{z}_{d,t} = \log(P_t/\sum_{i=t-11}^t D_i)
\]

We use December of each year to convert the monthly time series to an annual frequency.

**D Estimation Method**

The estimation method is conducted in two stages. The first stage estimates moments in the data using a 2-step Generalized Method of Moments (GMM) estimator with a Newey and West (1987) weighting matrix with 10 lags. The second stage implements a Simulated Method of Moments (SMM) procedure that searches for a parameter vector that minimizes the distance between the GMM estimates in the data and short-sample predictions of the model, weighted by the diagonal of the GMM estimate of the variance-covariance matrix. The following steps outline the algorithm:

1. Use GMM to estimate the data moments, \(\hat{\Psi}_D\), and variance-covariance matrix, \(\hat{\Sigma}_D\).
2. Specify a guess, \(\hat{\theta}_0\), for the \(N_e\) estimates parameters and the parameter variance-covariance matrix, \(\Sigma_P\), which is initialized as a diagonal matrix. Note that \(\theta\) is model dependent.
3. Use simulated annealing to minimize the distance between the data and model moments.
    (a) For all \(i \in \{0, \ldots, N_d\}\), perform the following steps:
i. Draw a candidate vector of parameters, \( \tilde{\theta}_{i}^{\text{cand}} \), where

\[
\tilde{\theta}_{i}^{\text{cand}} \sim \begin{cases} 
\hat{\theta}_0 & \text{for } i = 0, \\
N(\hat{\theta}_{i-1}, c\Sigma_P) & \text{for } i > 0.
\end{cases}
\]

We set \( c \) to target an overall acceptance rate of roughly 30%.

ii. Solve the Campbell-Shiller approximation of the model given \( \tilde{\theta}_{i}^{\text{cand}} \).

iii. Simulate the monthly model 1,000 times for the same length as the data plus a burn-in period. We burn off 10,000 months so that the first period closely approximates a representative draw from the model’s ergodic distribution. For each simulation \( j \), calculate the moments, \( \Psi_{M,j}(\tilde{\theta}_{i}^{\text{cand}}) \), analogous to those in the data.

iv. Calculate the median moments across the short-sample simulations, \( \bar{\Psi}_{M}(\tilde{\theta}_{i}^{\text{cand}}) = \text{median} \left( \{ \Psi_{M,j}(\tilde{\theta}_{i}^{\text{cand}}) \}_{j=1}^{1000} \right) \), and evaluate the objective function given by

\[
J_{i}^{\text{cand}} = \left[ \bar{\Psi}_{M}(\tilde{\theta}_{i}^{\text{cand}}) - \tilde{\Psi}_{D} \right]'W_D[\bar{\Psi}_{M}(\tilde{\theta}_{i}^{\text{cand}}) - \tilde{\Psi}_{D}] / N_M,
\]

where \( W_D \) is the inverse diagonal of the GMM estimate of the matrix, \( \tilde{\Sigma}_D \).

v. Accept or reject the candidate draw according to

\[
(\hat{\theta}_i, J_i) = \begin{cases} 
(\tilde{\theta}_{i}^{\text{cand}}, J_{i}^{\text{cand}}) & \text{if } i = 0, \\
(\tilde{\theta}_{i}^{\text{cand}}, J_{i}^{\text{cand}}) & \text{if } \min(1, \exp(J_{i-1} - J_{i}^{\text{cand}})/t) > \hat{u}, \\
(\hat{\theta}_{i-1}, J_{i-1}) & \text{otherwise,}
\end{cases}
\]

where \( t \) is the temperature and \( \hat{u} \) is a draw from a uniform distribution. The lower the temperature, the more likely it is that the candidate draw is rejected.

(b) Find the parameter draw \( \hat{\theta}_0^{\text{min}} \) that corresponds to \( J_{\text{min}}^{\text{guess}} \), and update \( \Sigma_P \).

i. Discard the first \( N_d/2 \) draws. Stack the remaining draws in a \( N_d/2 \times N_e \) matrix, \( \hat{\Theta} \), and define \( \hat{\Theta} = \hat{\Theta} - \sum_{i=1}^{N_d/2} \hat{\theta}_{i,j} / (N_d/2) \).

ii. Calculate \( \Sigma_{P}^{\text{up}} = \hat{\Theta}'\hat{\Theta} / (N_d/2) \).

4. Repeat the previous step \( N_{SM} \) times, initializing at draw \( \hat{\theta}_0 = \hat{\theta}_0^{\text{min}} \) and covariance matrix \( \Sigma_P = \Sigma_{P}^{\text{up}} \). Gradually decrease the temperature each time. Pool all the draws and \( J \) values together and find the minimum \( J \), denoted \( J_{\text{guess}} \), and corresponding parameter draw, \( \theta_{\text{guess}} \).

5. Run MATLAB’s \texttt{fminsearch}, using \( \theta_{\text{guess}} \) as an initial guess. We set the tolerance on \( \theta \) to 0.1. The resulting minimum is \( \hat{\theta}_0^{\text{min}} \) and the corresponding \( J \) value is \( J_{\text{min}}^{\text{guess}} \).

6. Repeat the previous step, each time updating the guess, until \( J_{\text{guess}} - J_{\text{min}}^{\text{guess}} < 0.001 \).
E Robustness of Baseline Model Estimates

The estimation procedure that generates the results in the main paper matches long-run correlations between equity returns and cash flow growth. We decided to include these moments for two reasons. One, they are used in Albuquerque et al. (2016), who estimate similar asset pricing models. Two, it allows us to re-examine whether valuation risk helps resolve the correlation puzzle. However, there is one main drawback of matching long-run correlations. It forces us to remove the Great Depression period to maintain a balanced sample. For example, since we include the correlation between equity returns and consumption growth over the last 10 years, our effective sample runs from 1940 to 2017, even though our raw data starts in 1930. Therefore, the decision of whether to include these long-run correlations changes some of the other moments we are trying to match. The most significant change is to the standard deviation of dividend growth, which is nearly twice as large in the sample that includes the Great Depression period. Therefore, this section tests the robustness of our baseline estimates by removing the long-run correlations from the estimation.
Table 5 shows the estimates for the baseline model. Our qualitative results are unchanged, despite differences in many of the data moments. The current specification fits the data very well with small RA and IES values and the results are driven almost entirely by valuation risk. In contrast, the revised specification fits the data much worse. In particular, it underpredicts the equity premium and overpredicts the risk-free rate because valuation risk plays a much smaller role. Furthermore, the estimates of the RA and IES are well outside the acceptable range in the asset pricing literature.

F Long-Run Risk Model without Valuation Risk

As a validation exercise, table 6 provides estimates from our long-run risk model with SV in a setting that is similar to Bansal et al. (2016). Specifically, we remove valuation risk from the model and no longer match the following moments: the volatility and autocorrelation of the risk-free rate and all correlations between equity returns and both dividend and consumption growth. By removing these moments, we are able to include the Great Depression in our sample as in BKY.

There are three key takeaways. One, the model fits this subset of moments extremely well (the $J$ value is 0.07). Two, RA falls from 62.4 in our model with valuation risk to 6.6. The estimated equity premium and risk-free rate are 5.4% and 0.3%, so the model appears to solve the equity premium and risk-free rate puzzles. Three, the model fits the moments that we removed from the estimation quite poorly. It underpredicts the variance and autocorrelation of the risk-free rate and overpredicts the correlations between equity returns and cash flow growth. In short, we are able to generate estimates close to what BKY report at the expense of failing to match some key moments.

Table 6: Long-run risk model estimates with stochastic volatility but no valuation risk

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Parameter</th>
<th>Estimate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>6.55646</td>
<td>$\mu_d$</td>
<td>0.00146</td>
<td>$\rho_x$</td>
<td>0.99645</td>
</tr>
<tr>
<td>$\psi$</td>
<td>1.82937</td>
<td>$\psi_d$</td>
<td>7.24388</td>
<td>$\psi_x$</td>
<td>0.04277</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99880</td>
<td>$\pi_{dy}$</td>
<td>3.18658</td>
<td>$\rho_{\sigma}$</td>
<td>0.21524</td>
</tr>
<tr>
<td>$\sigma_y$</td>
<td>0.00161</td>
<td>$\phi_d$</td>
<td>2.53709</td>
<td>$\nu_y$</td>
<td>0.00003</td>
</tr>
<tr>
<td>$\mu_y$</td>
<td>0.00153</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

(a) Parameter estimates. No valuation risk: $J = 0.07$.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Data</th>
<th>Model</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E[\Delta c]$</td>
<td>1.89</td>
<td>1.93</td>
<td>$SD[r_d]$</td>
<td>19.15</td>
</tr>
<tr>
<td>$E[\Delta d]$</td>
<td>1.47</td>
<td>1.94</td>
<td>$SD[z_d]$</td>
<td>0.45</td>
</tr>
<tr>
<td>$E[r_d]$</td>
<td>6.51</td>
<td>5.66</td>
<td>$Corr[\Delta c, \Delta d]$</td>
<td>0.54</td>
</tr>
<tr>
<td>$E[r_f]$</td>
<td>0.25</td>
<td>0.29</td>
<td>$AC[z_d]$</td>
<td>0.89</td>
</tr>
<tr>
<td>$E[ep]$</td>
<td>3.42</td>
<td>3.41</td>
<td>$SD[r_f]$</td>
<td>2.72</td>
</tr>
<tr>
<td>$E[ep]$</td>
<td>6.26</td>
<td>5.37</td>
<td>$Corr[\Delta c, r_d]$</td>
<td>0.05</td>
</tr>
<tr>
<td>$SD[\Delta c]$</td>
<td>1.99</td>
<td>2.00</td>
<td>$Corr[\Delta d, r_d]$</td>
<td>0.07</td>
</tr>
<tr>
<td>$SD[\Delta d]$</td>
<td>11.09</td>
<td>10.81</td>
<td>$AC[r_f]$</td>
<td>0.68</td>
</tr>
</tbody>
</table>

(b) Unconditional short-sample moments. Bolded moments are not included in the SMM algorithm.