Bailouts and Speculative Trade in Markets for Aggregate Disaster Risk Insurance

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Abstract

This paper examines how much speculative trade can be sustained in an economy when CRRA-utility agents have heterogeneous beliefs about aggregate consumption disasters but can walk away from their financial commitments. We find that that little or no speculative trade can be sustained in an economy without pledgeable income when agents are reasonably risk averse. In autarchy, the disaster risk premia are set by the pessimists. However, even small bailout subsidies in the disaster state may be sufficient to sustain lots of speculative trade, in turn inducing low disaster risk premia.

1 Introduction

As the Dexia bailout deal closed last week and was approved by the French Parliament, officials overseeing the restructuring say that the bank

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will meet all of its obligations in full. Alexandre Joly, the head of strategy, portfolios and market activities at Dexia, said in an interview that the idea of forcing Dexia’s trading partners to accept a discount on what they are owed ‘is a monstrous idea.’ He added, ‘It is not compatible with rules governing the euro zone, and it has never, ever been considered to our knowledge by any government in charge of the supervision of the banks.’

Before the financial crisis, there was a large amount of trade in the markets for disaster risk insurance, some of which was probably speculative in nature. During the crisis, we learned that a large share of these trades undertaken by financial institutions were not fully collateralized. In some cases, governments intervened to guarantee these trades. Our paper examines the scope for uncollateralized speculative trades with and without ex ante government guarantees.

Supporting speculative trade in markets for aggregate disaster risk insurance is hard in economies without pledgeable income, because the future surpluses generated by trade are not sufficient to make the seller of insurance pay out when a disaster is realized. In fact, for plausibly risk averse investors, we find that no trade can be supported, and that pessimists exclusively price disaster risk.  

However, even small bailout subsidies in the disaster state may be sufficient to support a lot of speculative trade and large reductions in the market price of disaster risk. These trades are implicitly collateralized by the value of future bailouts. When investors are sufficiently risk averse and willing to substitute intertemporally, they are willing to forego large share of current consumption in the disaster state to take advantage of large risk premia in the future. In that case, even small bailout subsidies support a substantial amount of speculative trade.

Our findings have important asset pricing implications. Barro (2006) and Rietz (1988) have argued that rare consumption disasters can account for the equity premium puzzle in an otherwise standard, representative agent model. However, Chen, Joslin, and Tran (2010) point out that this mechanism for generating large equity risk premia does not survive the introduction of a reasonable dose of heterogeneity.

\[1\] If there is pledgeable income in the economy, speculative trade is feasible but all speculative trades are essentially fully collateralized.
in beliefs about the probability of these tail events, because the optimists will short equity disaster risk aggressively, thus driving down the compensation for aggregate disaster risk.

The Chen, Joslin, and Tran (2010) mechanism requires that the optimist be allowed to take huge short positions. What about the optimist’s incentives? Once the tail event is realized, the optimist has an incentive to walk away from its promise. How much speculative trade in disaster insurance markets can be sustained in economy without pledgeable income when the investors can be excluded from trading upon default? Not much, unless the government gets involved.

We investigate how much speculative trade can be sustained in a complete markets economy without pledgeable income. The only motive for trade in this economy is heterogeneity in beliefs about the probability of a macro disaster. Even when agents can be excluded from trading, no trade occurs in equilibrium unless agents have very low risk aversion. The future surplus generated by trade is not enough to induce the optimist to stay in the trading arrangement when a disaster occurs. The pessimists anticipate this and speculative trade breaks down. In autarchy, the disaster risk is priced off the intertemporal marginal rate of substitution of the pessimist. Risk premia are even higher than in a standard representative agent model. This case strengthens the Barro (2006) and Rietz (1988) explanation of the equity premium puzzle.

When there is pledgeable income, all of the speculative trade has to be fully collateralized by assets backed by this pledgeable income. In particular, all of the insurance sold by the optimist is backed by holdings of the safe assets.

We get an approximate Bulow and Rogoff (1989) result in disaster insurance markets with risk averse investors because the threat of exclusion is not sufficient to incentivize the optimists when disaster strikes. Suppose the optimist honors his commitments. Even though the optimist perceives high risk premia in the future, she accumulates wealth at a slow rate in the future, because disaster are rare and because she is not willing to substitute intertemporally. That renders her less willing to accept large consumption drops in the case of a disaster, which precludes speculative trade in disaster risk markets.
However, when the government commits to injecting resources in the trading arrangement in the disaster state, a lot of speculative trade in disaster risk markets is feasible, even in the absence of pledgeable income, because agents use the value of future bailouts as collateral. The surplus needed to keep the optimist in the trading arrangement when a disaster strikes is provided by all of the future bailouts. When investors are highly risk averse and are willing to substitute intertemporally, small bailout subsidies in the disaster state give rise to lots of equilibrium speculative trade and large reductions in the average disaster risk premium.

In the case of additive utility, as risk aversion increases, the optimist perceives larger disaster risk premia, but also becomes less willing to substitute intertemporally. These two effects offset each other. However, suppose we fix the investor’s intertemporal elasticity of substitution. As risk aversion increases, the optimist perceives larger expected returns in non-disaster states and saves more to take advantage of these. As a result, the optimist accumulates wealth at a higher rate, as risk aversion increases. The increased rate of wealth accumulation creates large future surpluses for the optimist and hence makes her willing to accept large consumption drops in case of a disaster without walking away.

**Related Literature**  There is a literature that looks at survival of agents with wrong beliefs. Yan (2008) shows that only agents with beliefs closes to the truth will survive in the long run. This result was qualified by Borovicka (2011) who introduced a distinction between the coefficient of risk aversion and the inverse of the elasticity of intertemporal substitution; when agents are highly willing to substitute intertemporally but sufficiently risk averse, they may survive in the long run even if their beliefs are wrong. The same logic creates room for more speculative trade in disaster risk markets.

**Outline**  The paper is organized as follows. Section 2 describes the environment. Section 3 describes the way we solve for the equilibrium allocations and prices using the optimist’s consumption share as the state variable. Section 4 describes the no trade result in the economy without bailouts. Section 5 describes how we solve
for equilibrium allocations and prices in the economy with bailouts. Section 7 discusses the effects of imputing a preference for early resolution of uncertainty to these investors.

2 Environment

Endowments We consider an endowment economy. We use \( z_t = (s_t, d_t) \) to denote the random event. \( z^t \) denotes the aggregate history. The economy’s aggregate endowment process \( \{e_t\} \) depends only on the aggregate event history. The aggregate endowment evolves stochastically as follows:

\[
e_t(z^t) = e_{t-1}(z^{t-1}) \times \gamma_t(s_t) \text{ in no disaster state,} \tag{1}
\]
\[
e_t(z^t) = e_{t-1}(z^{t-1}) \times \gamma_t(s_t) \times D_t(d_t) \text{ in disaster state.} \tag{2}
\]

The ‘normal’ endowment growth rate \( \gamma_t(s_t) \) only depends on the current shock \( s_t \). The random event \( s_t \) is governed by a Markov process. We use \( \pi(s_t|s_{t-1}) \) to denote the transition probability. Agents know these transition probabilities. However, they disagree about the probability of an aggregate consumption growth disaster. In case of a disaster, \( d_t = 1 \) and \( d_t = 0 \) elsewhere. For tractability, we use the following expression for output loss in case of a disaster \( D_t(1) = \exp(-J), D_t(0) = 1 \). \( J > 0 \) denotes the size of the endowment growth disaster. This is a discrete time version of the model adopted by Chen, Joslin, and Tran (2010). The normal endowment risk is a discrete time approximation of the diffusion component, while the disaster risk is the jump component in aggregate endowment growth.

Agents Each agent is endowed with a claim to \( \{(1/2)e_t(z^t)\} \). They are ex ante identical, except that they have different beliefs about the probability of a disaster. Each agent is indexed by \( \lambda \). The probability of a disaster is perceived to be:

\[
1 - \exp(-\lambda), \tag{3}
\]
where $\lambda$ is a non-negative variable. There are two types of agents: optimists (denoted $o$) and pessimists (denoted $p$). So, we can summarize the agent’s perceived transition probabilities:

$$
\pi_\lambda(s_{t+1}, 0 | s_t) = \pi(s_{t+1} | s_t) \exp(-\lambda_t), \quad (4)
$$

$$
\pi_\lambda(s_{t+1}, 1 | s_t) = \pi(s_{t+1} | s_t) (1 - \exp(-\lambda_t)). \quad (5)
$$

**Preferences** Consumers rank stochastic consumption streams $\{c_t(z^t)\}$ according to the following criterion:

$$
U(c)(\lambda)(z_0) = \sum_{t=0}^{T} \sum_{z^t \geq z^0} \beta^t \pi_\lambda(z^t | z_0) \frac{c_t(z^t)^{1-\gamma}}{1 - \gamma}, \quad (6)
$$

where $\gamma > 0$ is the coefficient of relative risk aversion and $\beta \in (0, 1)$ is the constant time discount factor.

**Continuation Utilities** We define $U(c)(z^t)$ to be the continuation expected lifetime utility from a consumption allocation $c = \{c_t(z^t)\}$ in node $z^t$. This utility can be constructed recursively as follows:

$$
U_t(c)(\lambda)(z^t) = u(c_t(z^t)) + \beta \sum_{z_{t+1}} \pi_\lambda(z_{t+1} | z_t) U_{t+1}(c)(\lambda)(z^t, z_{t+1}),
$$

where we made use of the Markov property the underlying stochastic processes.

### 2.1 Solvency Constraints

Agents trade a complete set of Arrow securities in each period. We use $a_t(z_{t+1}, z^t)$ to denote the net wealth of agent $i$ if state $z_{t+1}$ is realized next period. We use $Q_t(z^t, z_{t+1})$ to denote the price in units of time $t$ consumption of such a claim.
Hence, the agents face the following sequential budget constraint:

$$\sum_{z_{t+1}} a^i_t(z^t, z_{t+1})Q(z^t, z_{t+1}) + c_t(z^t) \leq a^i_{t-1}(z^{t-1}, z_t) + e_t(z^t).$$

In each period, agents face state-contingent solvency constraints:

$$a^i_t(z^t, z_{t+1}) \geq A^i(z^t, z_{t+1}), \text{ for all } z_{t+1}.$$  

Following Alvarez and Jermann (2000), these state-contingent solvency constraints will be just tight enough such that agents remain in the trading arrangements.

Let $J^i_t$ denote the value function of agent $i$ starting the period with net wealth $a^i_{t-1}$:

$$J^i_t(a^i_{t-1}(z^{t-1}, z_t), z^t) = max_{c^i_t, a^i_t} u(c^i_t) + \beta \sum_{z_{t+1}} \pi(z_{t+1}|z_t)J^i_{t+1}(a^i_t(z^t, z_{t+1}), z^{t+1}),$$

subject to the budget constraint:

$$\sum_{z_{t+1}} a^i_t(z_{t+1}, z^t)Q(z^t, z_{t+1}) + c_t(z^t) \leq a^i_{t-1}(z^{t-1}, z_t) + e_t(z^t),$$

and the state-contingent solvency constraints:

$$a^i_t(z_{t+1}, z^t) \geq A^i(z^t, z_{t+1}), \text{ for all } z_{t+1},$$

as well as the appropriate transversality condition:

$$\lim_{t \to \infty} \sup_{z^t} \beta^t u(c^i_t(z^t))a^i_t(z^t, z_{t+1}) = 0.$$  

The asset market is cleared when the value of all contingent positions equals the value of all future bailouts:

$$\sum_i a^i_t(z^t, z_{t+1}) = 0, \text{ for all } z_{t+1}.$$  

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Solvency constraints that are not too tight

Definition 2.1. Alvarez and Jermann (2000) define solvency constraints that are not too tight as follows:

\[ J_t^i(A^i(z_t^{-1}, z_t), z^i_t) = V^{aut,i}(z_t). \]

Note that these could allow for negative net wealth position in some states of the world, because \( A^i(z_t^{-1}, z_t) < 0 \).

No Exclusion If agents cannot be excluded from trading upon default, these solvency constraints rule out negative net wealth positions:

\[ a_t^i(z_t^i, z_{t+1}) \geq 0, \text{for all } z_{t+1}. \]

Valuation We use \( \Pi_t(x)(z_t) \) to denote the value of the claim to \( x \) at time \( t \). This can be computed recursively as follows:

\[ \Pi_t(x)(z_t^i) = x_t(z_t^i) + \sum_{z_{t+1}} Q_t(z_{t+1}|z_t)\Pi_{t+1}(x)(z_t^i, z_{t+1}), \]

The definition of a sequential equilibrium in this environment is standard.

2.2 Bailouts

When the government implements a bailout, this amounts to a subsidy that increases only the disaster endowment, only for those in the trading arrangement:

\[ e_t(z_t^i)(1 + s_t(d_t)), \]

where \( s_t(1) > 0 \), 0 elsewhere. The bailout does not affect the endowment of those outside of the trading arrangement and hence has no bearing on the value of autarchy.
Hence, the agents face the following sequential budget constraint:

$$\sum_{z_{t+1}} a_i^t(z^t, z_{t+1}) Q(z^t, z_{t+1}) + c_t(z^t) \leq a_i^{t-1}(z^{t-1}, z_t) + e_t(z^t),$$

provided that she stays in the trading arrangement. The subsidy does not affect the value of autarchy. The definition of the solvency constraint is unchanged. The (cum subsidy) asset market is cleared when the value of all contingent positions equals the value of all future bailouts:

$$\sum_{i} a_i^t(z^t, z_{t+1}) = \Pi_{t+1}(se)(z^{t+1}), \text{ for all } z_{t+1}.$$

**Definition 2.2.** Alvarez and Jermann (2000) define solvency constraints that are not too tight as follows:

$$J_i^t(A^i(z^{t-1}, z_t), z^t) = V^{aut,i}(z^t).$$

**No Exclusion** If agents cannot be excluded from trading upon default, these solvency constraints rule out negative net wealth positions:

$$a_i^t(z^t, z_{t+1}) \geq 0, \text{ for all } z_{t+1},$$

where net wealth includes the value of the bailouts. The ex subsidy value of net wealth will be negative when the constraint binds.

Instead of computing equilibrium allocations and prices directly in this sequential trading environment, we will switch to an equivalent time zero trading setup. Provided that interest rates are high enough, the sequential equilibrium can be implemented as an equilibrium of the static economy in which all trading occurs at time zero.
3 Solving for Equilibrium Allocations and Prices

3.1 Time Zero Trading

This section considers the case in which agents cannot commit to their promises. Instead, we check in each node that agents have no incentive to walk away from their promises. To do so, we define allocations that are immune to the threat of default, following Kehoe and Levine (1993), Kocherlakota (1996) and Alvarez and Jermann (2000).

Definition 3.1. An allocation is immune to the threat of default if and only if for each household $\lambda$:

$$U(c)(\lambda)(z^t) \geq U((1/2)e)(\lambda)(z^t) \text{ for all } z^t.$$ (7)

3.2 Static Household Problem

In the limited commitment economy, the household chooses a consumption plan $\{c\}$ to maximize its expected utility:

$$max_{c_t(z^t)} U(c)(z^t),$$

such that the budget constraint is satisfied:

$$\Pi_0[(1/2)e] \geq \Pi_0[c](\lambda),$$

and such that the participation constraints are satisfied in each future node:

$$U(c)(\lambda)(z^t) \geq U((1/2)e)(\lambda)(z^t) \text{ for all } z^t.$$ (8)

Definition 3.2. A Kehoe-Levine equilibrium that is immune to the threat of default is defined in the usual way as a sequence of prices $\{Q_t(z^t|z_0)\}$ and $\{c_t(z^t|z_0)\}$ such that (i) $\{c_t^*(z^t|z_0)\}$ solves the household’s optimization problem, (ii) the goods markets
\[ \sum_i c_i^t(z^t) = e_t(z^t). \]  

(9)

3.3 Bailouts

This section considers what happens when the government bails out the financial sector by injecting outside resources only in the disaster state of the world.

**Definition 3.3.** An allocation is immune to the threat of default if and only if for each household \( \lambda \):

\[ U(c)(\lambda)(z^t) \geq U((1/2)e)(\lambda)(z^t) \text{ for all } z^t, \]

(10)

where

\[ \sum_i c_i^t(z^t) = (1 + s(d_t))e_t(z^t) \text{ with } s(0) = 0, s(1) > 0. \]

(11)

**No Exclusion**  On the other hand, if agents cannot be excluded from the trading arrangement upon default, then net wealth cannot be negative at any node \( z^t \).

**Definition 3.4.** An allocation is immune to the threat of default if and only if for each household \( \lambda \):

\[ \Pi(c)(\lambda)(z^t) \geq \Pi((1/2)e)(z^t) \text{ for all } z^t. \]

(12)

**Lemma 3.1.** In an economy without pledgeable income and without bailouts, no equilibrium speculative trade can be sustained when agents cannot be excluded upon default.

However, in an economy with bailouts in the disaster state, equilibrium speculative trade can be sustained in an economy without pledgeable income.

**Lemma 3.2.** In an economy without pledgeable income and with bailouts, some equilibrium speculative trade may be sustained when agents cannot be excluded upon default.
Even when agents and cannot be excluded from default and there is no pledgeable income, disaster bailouts create virtual collateral that can be used to sustain speculative trade.

3.4 Saddle Point Problem

To solve this time zero household problem, we set up the corresponding Lagrangian. Let $\kappa$ denote the multiplier on the time zero budget constraint, and let $\mu$ denote the multiplier on the participation constraint. We set up the standard saddle point problem:

$$L = \max_{c} \min_{\kappa,\mu} \left( \sum_{t=0}^{\infty} \sum_{z^t} \pi_{\lambda}(z^t|z_0) \beta^t u(c(z^t)) + \kappa \left( \sum_{t=0}^{\infty} \sum_{z^t} Q(z^t|z_0)((1/2)e_t(z^t) - c_t(z^t)) \right) ight)$$

$$+ \sum_{t=0}^{\infty} \sum_{z^t} \mu(z^t) \left( U_t(c)(\lambda)(z^t) - U_t((1/2)e)(\lambda)(z^t) \right).$$

The first order condition for consumption is given by:

$$\pi_{\lambda}(z^t) \beta^t u_c(c(z^t)) \left( 1 + \sum_{z^\tau \leq z^t} \mu(z^\tau) \right) = \kappa Q_t(z^t|z_0).$$

Cumulative Multiplier  To characterize the equilibrium consumption allocations and prices subject to these participation constraints, we use stochastic Pareto-Negishi weights, following Kehoe and Perri (2002), Chien and Lustig (2010) and Chien, Cole, and Lustig (2011). We define the cumulative multiplier

$$\zeta_t = 1 + \sum_{z^\tau \leq z^t} \mu(z^\tau).$$

With this multiplier in hand, we can restate the first order condition as:

$$\pi_{\lambda}(z^t) \beta^t \zeta^t u_c(c(z^t)) = \kappa Q_t(z^t|z_0).$$
**Risk Sharing Rule** Consider the first order condition in state \( z^t = (s^t, d^t) \):

\[
\pi(s^t)\pi_\lambda(d^t)\beta^t\zeta^t(\lambda)u_c(c(z^t)) = \kappa Q_t(z^t|z_0).
\]

Take the ratio of these first order conditions:

\[
\frac{\pi_\lambda(d^t)}{\pi_{\lambda'}(d^t)}\frac{\zeta^t(\lambda)}{\zeta^t(\lambda')}\frac{u_c(c(\lambda; z^t))}{u_c(c(\lambda'; z^t))} = \kappa \frac{\kappa'}{\kappa}.
\]

Define

\[
\xi_t = \frac{\pi_\lambda(d^t)\zeta^t(\lambda)}{\kappa}.
\] (13)

We conjecture a consumption sharing rule:

\[
e_t = \frac{\xi_t^{1/\alpha}}{h_t}e_t,
\]

where \( h_t = \sum_\lambda \xi_t^{1/\alpha}(\lambda) \). It is easy to verify that this risk-sharing rule satisfies the market clearing condition and the first order condition (by construction).

With this risk sharing rule in hand, we can back out an expression for the state prices:

\[
\pi(s^t)|z^t|z_0 = Q_t(z^t|z_0).
\]

Hence, the state price is given by

\[
\pi(s^t)\beta^t h_t^{\alpha} e_t^{-\alpha} = Q_t(z^t|z_0).
\]

**Law of Motion for Stochastic Pareto-Negishi Weights** We can state the law of motion for the stochastic Pareto-Negishi weights.

**Lemma 3.3.** If the participation constraints do not bind for agent \( \lambda \) in state \( z^t \), the multipliers evolve according to:

\[
\xi_t(\lambda)(z^t) = e^{-\lambda}\xi_{t-1}(\lambda)(z^{t-1}), \text{ in case of no disaster}
\]

\[
\xi_t(\lambda)(z^t) = (1-e^{-\lambda})\xi_{t-1}(\lambda)(z^{t-1}), \text{ in case of disaster.}
\]

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provided that

$$U_t(c; \xi_t)(\lambda)(z^t) > U((1/2)e)(\lambda)(z^t)$$

If the participation constraint do bind in some state $z^t$, then $\xi_t(\lambda) = \xi(\lambda)$ is determined such that:

$$U_t(c; \xi)(\lambda)(z^t) = U((1/2)e)(\lambda)(z^t).$$

We transform our growing model into a stationary model with a stochastic time discount rate and a growth-adjusted probability matrix, following Alvarez and Jer-ermann (2001).

3.5 Stationary economy

First, we define growth deflated consumption allocations (or consumption shares) as

$$\hat{c}_t(z^t) = \frac{c_t(z^t)}{e_t(z^t)} \text{ for all } z^t. \quad (14)$$

and we define deflated continuation utilities:

$$\hat{U}_t(c_t(z^t)) = \frac{U_t(c_t(z^t))}{e_t(z^t)^{1-\alpha}}. \quad (15)$$

Next, we define growth-adjusted probabilities and the growth-adjusted discount factor as:

$$\hat{\pi}(s_{t+1}) = \frac{\pi(s_{t+1})\gamma(z_{t+1})^{1-\alpha}}{\sum_{s_{t+1}} \pi(s_{t+1})\gamma(z_{t+1})^{1-\alpha}} \text{ and } \hat{\beta} = \beta \sum_{s_{t+1}} \pi(s_{t+1})\gamma(z_{t+1})^{1-\alpha}.$$

Lemma 3.4. If the ‘normal’ aggregate shocks are i.i.d., the continuation utilities in the stationary economy can be expressed as:

$$\hat{U}_t(\hat{c})(\lambda)(d^t) = \frac{\hat{c}^{1-\alpha}}{1-\alpha} + \hat{\beta} \left[ e^{-\lambda} \hat{U}_{t+1}(d^t, 0) + (1 - e^{-\lambda})e^{-(1-\alpha)\lambda} \hat{U}_{t+1}(d^t, 1) \right],$$
where
\[ \hat{\beta} = \beta \left( \sum_{s_{t+1}} \pi(s_{t+1}) \gamma(s_{t+1})^{1-\alpha} \right). \]

**Autarchy** In the stationary economy, the value of autarchy is constant:
\[ \hat{U} (\hat{\eta}) (\lambda) = u (\hat{\eta}) + \hat{\beta} \sum_{s_{t+1}} \hat{\pi}_\lambda (s_{t+1}) \hat{U} (\hat{\eta}). \] (16)

### 3.6 Recursive Algorithm

To characterize asset prices and wealth dynamics, we develop a recursive algorithm to compute equilibrium allocations and prices in the stationary version of the infinite horizon economy. We will use the optimist’s consumption share as our state variable. To start, we note that the value of autarchy in the stationary economy is defined as:
\[ \hat{V}_{\text{aut}}^o = 0 \]
\[ \hat{V}_{\text{aut}}^p = 0 \] (17)

This follows directly from the definition of continuation utilities in the stationary economy. We can use the optimist’s consumption share, labeled \( \varpi \), as the state variable. Then the law of motion for the optimist’s multiplier is given by:
\[ \xi_o' (\varpi; 0) = \max \left( e^{-\lambda_o} \varpi^\alpha, \xi_o \right), \text{ in case of no disaster} \] (18)
\[ \xi_o' (\varpi; 1) = \max \left( (1 - e^{-\lambda_o}) \varpi^\alpha, \xi_o \right), \text{ in case of disaster}. \] (19)

\[ 15 \]
where $\xi_o$ is such that $\xi_o = \frac{\xi_{o}^{\frac{1}{\alpha}}}{\xi_{o}(\omega; d)\frac{1}{\alpha} + \xi_{o}(1 - \omega; d)\frac{1}{\alpha}}$ satisfies: $\hat{V}_o(\omega) = \hat{V}_o^{aut}$. Then the law of motion for the pessimist’s multiplier is given by:

$$
\xi_{p}^\prime (\omega; 0) = \max \left(e^{-\lambda_p}(1 - \omega)^\alpha, \xi_{p}\right) \text{ in case of no disaster} \quad (20)
$$

$$
\xi_{p}^\prime (\omega; 1) = \max \left((1 - e^{-\lambda_p})(1 - \omega)^\alpha, \xi_{p}\right) \text{ in case of disaster.} \quad (21)
$$

where $\omega_p = \frac{\xi_{p}^{\frac{1}{\alpha}}}{\xi_{o}(\omega; d)\frac{1}{\alpha} + \xi_{o}(\omega; d)\frac{1}{\alpha}}$ satisfies: $\hat{V}_p(1 - \omega_p) = \hat{V}_p^{aut}$. The recursive expression of continuation utilities is in equation (69). The next period’s optimist consumption share is the policy function to:

$$
\omega'(\omega; d) = \frac{\xi_{p}^\prime (\omega; d)^\frac{1}{\alpha}}{h(\omega; d)}
$$

where $h(\omega, d) = \xi_{p}^\prime (\omega; d)^\frac{1}{\alpha} + \xi_{o}^\prime (\omega; d)^\frac{1}{\alpha}$ is defined in the usual way. Hence, we have recovered the law of motion for the state variable. The continuation utilities (scaled by the aggregate endowment) satisfy the following functional equation:

$$
\hat{V}_o(\omega) = \frac{\omega^{1-\alpha}}{1 - \alpha} + \beta \left[e^{(1-\alpha)J} \left(1 - e^{-\lambda_o}\right) \hat{V}_o(\omega'(\omega; 1)) + e^{-\lambda_o} \hat{V}_o(\omega'(\omega; 0))\right], \quad (22)
$$

$$
\hat{V}_p(\omega) = \frac{1 - \omega^{1-\alpha}}{1 - \alpha} + \beta \left[e^{-(1-\alpha)J} \left(1 - e^{-\lambda_p}\right) \hat{V}_p(\omega'(\omega; 1)) + e^{-\lambda_p} \hat{V}_p(\omega'(\omega; 0))\right]. \quad (23)
$$

**State Prices** We can also characterize state prices as a function of the aggregate state variable $\omega$:

$$
\hat{Q}_t(\omega; 0) = \beta h(\omega; 0)^\alpha \quad (24)
$$

$$
\hat{Q}_t(\omega; 1) = \beta h(\omega; 1)^\alpha e^{\alpha J}. \quad (25)
$$

This also implies means we can solve for the net wealth equations (scaled by the
aggregate endowment) recursively:

\[
\hat{P}_o(\varpi) = \varpi - \frac{1}{2} + Q(\varpi; 1)e^{-J} \hat{P}_o(\varpi'(\varpi; 1)) + Q(\varpi; 0)\hat{P}_o(\varpi'(\varpi; 0)),
\]

\[
\hat{P}_p(\varpi) = 1 - \varpi - \frac{1}{2} + Q(\varpi; 1)e^{-J} \hat{P}_p(\varpi'(\varpi; 1)) + Q(\varpi; 0)\hat{P}_p(\varpi'(\varpi; 0)).
\]

It is easy to check that \(\hat{P}_o(\varpi) + \hat{P}_p(\varpi) = 0\) everywhere. The policy function for the optimist’s consumption share \(\varpi'(\varpi; d)\) and the pricing kernel \(Q(\varpi; d)\) provide a complete recursive characterization of a competitive equilibrium. If we start at some initial consumption share for the optimist, we can completely recover the equilibrium as defined in (\).

**No Exclusion** If agents cannot be excluded from trading, we use the same updating rule for the consumption shares, but the optimist’s reservation consumption share is determined such that \(\varpi_o = \frac{\xi_o}{\xi_o(\varpi; d) + \xi_o'(\varpi; d)\varpi}\) satisfies the following condition: \(\hat{P}_o(\varpi) = 0\). Similarly, the pessimist’s reservation consumption share \(\varpi_p = \frac{\xi_p}{\xi_p(\varpi; d) + \xi_p'(\varpi; d)\varpi}\) satisfies: \(\hat{P}_p(1 - \varpi_p) = 0\).

### 3.7 Asset Pricing in the Growing Economy

We can price any collateralizable asset payoff in the growing economy. To do, we can use the pricing kernel for the stationary economy in equation 59. However, we have to convert all prices back to the growing economy. For example, the risk-free rate on a collateralizable asset is given by:

\[
\frac{1}{P_c^f} = \frac{\exp(\mu(-\alpha) + 0.5(\alpha)^2\sigma^2)}{\exp(\mu(1 - \alpha) + 0.5(1 - \alpha)^2\sigma^2)} \left( Q(\varpi; 1)e^{-J} + Q(\varpi; 0) \right)
\]

**Non-collateralizable Assets** However, the price of a non-collateralizable asset is different. First define, the reservation value for this zero-coupon bond priced
obtained by pricing the payoffs off each agent $i$’s IMRS:

$$
\frac{1}{R_{nc}^{i,f}} = \frac{\exp(\mu(-\alpha) + 0.5(\alpha)^2\sigma^2)}{\exp(\mu(1-\alpha) + 0.5(1-\alpha)^2\sigma^2)} \left( Q^i(\varpi; 1)e^{-J} + Q^i(\varpi; 0) \right),
$$

where the state prices:

$$
Q^o(\varpi; 0) = \hat{\beta} \left( \frac{\varpi'(\varpi; 0)}{\varpi} \right)^{-\alpha}
$$

$$
Q^o(\varpi; 1) = \hat{\beta} \left( \frac{\varpi'(\varpi; 0)}{\varpi} \right)^{-\alpha} e^{\alpha J}.
$$

(26)

The price on the non-collateralizable one period discount bond is defined as the maximum of the marginal valuations of the bond by each of the households:

$$
\frac{1}{R_{nc}^{f}} = \max \left( \frac{1}{R_{nc}^{o,f}}, \frac{1}{R_{nc}^{p,f}} \right) \leq \frac{1}{R_c^{f}},
$$

which is smaller than the price of the collateralizable asset. To see why, note that the collateral pricing kernel is given by:

$$
Q(\varpi; 0) = \max (Q^o(\varpi; 0), Q^p(\varpi; 0)).
$$

Proof. If agent $i$ with the highest reservation value takes a long position by reducing consumption at $t$ and buying the non-collateralizable asset, this will not affect his solvency constraints at $t + 1$. His Euler equation are satisfied by construction. If the other agent has a lower reservation value, she would want to short this asset, but she cannot, because of the solvency constraints. 

3.8 Wealth Dynamics

There is a literature that looks at survival of agents with wrong beliefs. Yan (2008) shows that only agents with beliefs closes to the truth will survive in the long run. This result was qualified by Borovicka (2011) who introduced a distinction between the coefficient of risk aversion and the inverse of the elasticity of intertemporal sub-
stitution. We want to examine the ergodic wealth distribution in an environment with limited commitment. In this case, the consumption shares of these agents are bounded below.

**Proposition 3.1.** In an infinite horizon economy without outside wealth, the equilibrium consumption shares of the optimist $\varpi$ lie on the domain $[\varpi_o, 1 - \varpi_p]$, where $V_o(\varpi) = V_o^{aut}$ and $V_p(1 - \varpi_p) = V_p^{aut}$.

*Proof.* This follows immediately from the above. $\square$

Obviously, equilibrium speculative trade is feasible if only if the lower bound for the optimist is smaller than the upper bound implied by the pessimist’s lower bound $\varpi_o < 1 - \varpi_p$. This also defines bounds on net wealth.

**Corollary 3.1.** In an infinite horizon economy without outside wealth, the equilibrium net wealth of the optimist is bounded below by $\varpi$ lie on the domain $P_o(\varpi_o)$ and the equilibrium net wealth of the pessimist is bounded below by $P_p(1 - \varpi_p)$.

*Proof.* This follows immediately from the above. $\square$

Hence the market selection mechanism which drives out agents with beliefs that are wrong breaks down altogether.

## 4 Speculative Trade in the No-Bailout Economy

### 4.1 Full Commitment Economy

We start by considering the case of full commitment. In this case, we do not impose the participation constraints. Section ?? in the appendix describes the case of full commitment in detail.

#### 4.1.1 Consumption Dynamics

Consider a particular path for the stationary version of this economy $d^t$ with $k$ disaster realizations. Then the perceived probabilities of this sequence of events is
given by:

\[ \pi_t(d^t) = e^{-\lambda(t-k)} (1 - e^{-\lambda})^k. \]

**Risk Sharing Rule** Hence the risk-sharing rule is determined by the household’s stochastic Pareto-Negishi weight

\[ \xi_t(d^t) \frac{1}{\alpha} = e^{-\frac{\zeta(t-k)}{\alpha}} \left(1 - e^{-\frac{\zeta}{\alpha}}\right)^k, \]

and the aggregate Pareto-Negishi weight:

\[ h_t(d^t) \frac{1}{\alpha} = \sum_\zeta e^{-\frac{\zeta(t-k)}{\alpha}} \left(1 - e^{-\frac{\zeta}{\alpha}}\right)^\frac{k}{\alpha}. \]

Conditional on \( k \) disasters, the share of consumption allocated to consumer \( \lambda \) is given by:

\[ \hat{c}_t(\lambda; k) = \frac{e^{-\frac{\lambda(t-k)}{\alpha}} \left(1 - e^{-\frac{\lambda}{\alpha}}\right)^\frac{k}{\alpha}}{\sum_\zeta e^{-\frac{\zeta(t-k)}{\alpha}} \left(1 - e^{-\frac{\zeta}{\alpha}}\right)^\frac{k}{\alpha}}. \] \hspace{1cm} (27)

The consumption shares reflect the entire history of disaster realizations.

**State Prices** State prices can simply be expressed as a weighted average of the beliefs:

\[ \hat{Q}_t(k; k) = \beta \left( \frac{\sum_\zeta e^{-\frac{\zeta(t+1-k)}{\alpha}} \left(1 - e^{-\frac{\zeta}{\alpha}}\right)^\frac{k+1}{\alpha}}{\sum_\zeta e^{-\frac{\zeta(t-k)}{\alpha}} \left(1 - e^{-\frac{\zeta}{\alpha}}\right)^\frac{k}{\alpha}} \right)^\alpha \] \hspace{1cm} (28)

\[ \hat{Q}_t(k+1; k) = \beta \left( \frac{\sum_\zeta e^{-\frac{\zeta(t+1-k-1)}{\alpha}} \left(1 - e^{-\frac{\zeta}{\alpha}}\right)^\frac{k+1}{\alpha}}{\sum_\zeta e^{-\frac{\zeta(t-k)}{\alpha}} \left(1 - e^{-\frac{\zeta}{\alpha}}\right)^\frac{k}{\alpha}} \right)^\alpha e^{\alpha J}. \] \hspace{1cm} (29)

**Special Case** When all agents have the same beliefs \( \lambda \), the equilibrium consumption shares are constant over time, and the state prices are given by the standard
Breeden-Lucas-Rubenstein expressions:

\[ \hat{Q}_t(k; k) = \beta e^{-\lambda} \]  
\[ \hat{Q}_t(k + 1; k) = \beta(1 - e^{-\lambda})e^{\alpha J}. \]

4.1.2 Calibrated Economy

We explore the consumption and wealth dynamics in a calibrated version of the economy.

**Calibration** We consider a calibrated version of our economy. We choose the normal endowment growth rate \( \mu \) equal to 0.025, and the standard deviation of the normal endowment growth rate \( \sigma \) equal to 0.02. The size of the aggregate consumption drop when the rare event is realized is \( J = 0.51 \). There is no other source of (aggregate) risk. The pessimist (optimist) is endowed with beliefs \( \lambda \) equal to 0.025 (0.005). The time discount factor \( \beta \) is 0.97. We consider the case of additive utility. The coefficient of relative risk aversion \( \alpha = \rho \) is set to 4.

To solve for the consumption policy function, the actual law of motion for the optimist’s consumption share is not needed. This dispersion of beliefs gives rise to lots of trade in the full commitment economy.

**Consumption Policy Function** Figure 1 plots the optimist consumption share at \( t + 1 \) against the optimist consumption share at \( t \). In the no-disaster states, the optimist’s consumption share increases slightly, because the no-disaster policy line is slightly above the 45-degree line: she earns an insurance premium, but there is large decrease in the disaster state.

**Simulated Path** To get a better sense of the dynamics, we look at a sample of 100 periods with one disaster realizations at \( t = 25 \) and one at \( t = 50 \).

**Consumption and wealth** The top panel of Figure 2 plots the consumption shares (as a fraction of the aggregate endowment) of the optimist and the pessimist.
The optimist’s consumption share starts at 0.6 and increases to 0.64 until a disaster hits at $t = 25$, at which point her consumption share drops to a lower level (0.54). This pattern repeats itself at $t = 50$. After the second drop, her consumption share drops to 0.46. The top panel of Figure 2 also plots the net financial wealth divided by the aggregate endowment for the optimist and the pessimist. The size of these net wealth positions (in absolute value) give a sense of how much trade the heterogeneity in beliefs generates. The optimist starts with zero net wealth, and quickly accumulates wealth until the first disaster strikes. After two disasters, the optimist is shorting three times the aggregate endowment, or 6 times her personal endowment.

**Continuation Utility** The second panel of Figure 2 also plots the continuation utilities for the optimist (on the left) and the pessimist (on the right) against their respective autarchy values. The optimist’s continuation utility climbs above his autarchy value after the first 24 good shocks have been realized, but it drops below it when the first disaster is realized. After this, it starts to climb again, but then it drops below the value of autarchy a second time. This picture shows how ‘hard’ it will be to sustain a lot of trade in an environment with limited commitment on the part of these traders. Anytime the continuation value drops below the value of autarchy, these traders would be tempted to walk away. The optimist would be tempted after every disaster realization.

**Risk Premia** Finally, the bottom panel of figure 2 plots the perceived equilibrium risk premium, both from the perspective of the optimist and the pessimist. As the optimist accumulates more wealth, the risk premia decline, but they increase upon the realization of the disaster in period 50, because the disaster destroys all of the optimist’s net wealth. On average risk premia are small relative to the case of autarchy. The autarchic risk premium in this economy is 6%. However, as figure 2 shows, this market selection mechanism requires the optimist in this case to take huge negative (unsecured) net wealth positions.

Next, we check how much trade is feasible in the economy with limited commit-
4.2 Limited Commitment Economy

Corollary 4.1. In an infinite horizon economy without outside wealth, \( \varpi < 1/2 \) and \( \varpi > 1/2 \) is a necessary condition for equilibrium risk sharing to be feasible.

4.2.1 Calibrated Economy

It turns out that no equilibrium trade can be sustained in our calibrated economy. The necessary condition is not satisfied. To explain why, Figure 3 plots the equilibrium continuation utilities \( \hat{V} \) for the optimist (left panel) and the pessimist (right panel) against their respective autarchy values in the limited commitment economy; we impose that the continuation utility exceed the value at autarchy at all future nodes. It is clear from the graph that the continuation utilities cross the autarchy line almost exactly at 0.5. There is no room for equilibrium trade at all.

4.2.2 General No Trade Result

Why is it hard to sustain trade in this economy? Well, the optimist has to be willing to accept consumption below \( 1/2 \) in the disaster state, incentivized by future surpluses from speculative trade. To check whether risk sharing is feasible without actually solving for an equilibrium, we can compute cutoff consumption shares \( \varpi, \overline{\varpi} \) from the primitives of this economy without actually solving for the equilibrium allocations.

Lemma 4.1. The cutoff consumption shares can be approximated by solving the
following equations:

\[
\frac{\varpi^{1-\alpha}}{1-\alpha} - \frac{0.5^{1-\alpha}}{1-\alpha} \approx \hat{\beta} e^{-\lambda_o} \times \varpi^{-\alpha} \left( \varpi'(\varpi; 0) - \varpi \right) \left( 1 - \alpha \varpi^{-1} (\varpi'(\varpi; 0) - \varpi) \right).
\]

\[
\frac{1 - \varpi^{1-\alpha}}{1 - \alpha} - \frac{0.5^{1-\alpha}}{1 - \alpha} \approx \hat{\beta} e^{-(1-\alpha)j} (1 - e^{-\lambda_p}) \times -(1 - \varpi)^{-\alpha} \left( \varpi'(\varpi; 1) - \varpi \right) (1 + \alpha(1 - \varpi)^{-1} (\varpi'(\varpi; 1) - \varpi)),
\]

where the policy functions solve the following equations:

\[
\frac{\varpi'(\varpi; 0)}{1 - \varpi'(\varpi; 0)} = e^{\frac{\lambda_p - \lambda_o}{\alpha}} \frac{\varpi}{1 - \varpi} \quad \text{and} \quad \frac{\varpi'(\varpi; 1)}{1 - \varpi'(\varpi; 1)} = \left( \frac{1 - e^{-\lambda_o}}{1 - e^{-\lambda_p}} \right)^{1/\alpha} \frac{\varpi}{1 - \varpi}.
\]

Since \( \lambda_p > \lambda_o \), it is easy to verify from these expressions that \( \varpi < 0.5 \) and \( \varpi > 0.5 \). These cutoffs are derived by evaluating the optimist’s continuation utility at the lower bound, and the pessimist’s continuation utility at the upper bound. Note that this cutoff equation only has the non-disaster state on the right hand side for the optimist, simply because in the disaster state she is back at her reservation continuation utility, which equals autarchy. Similarly, the pessimist’s cutoff only involves the disaster state.

### 4.2.3 Calibrated Economy

We return to the same calibrated version of our economy to evaluate these cutoffs. The lower bound for the optimist’s consumption share \( \varpi \) is plotted against the coefficient of relative risk aversion in Figure 4. The top line corresponds to the benchmark case of beliefs given by 0.025 (0.005). There is no scope for speculative trade in this economy, because the lower bound \( \varpi \) is very close to (though smaller than) 1/2. There is not enough surplus generated by the risk-sharing arrangement to induce the optimist to stay in the match at consumption shares well below 0.5, regardless of the level of risk aversion. The reason is the slow rate at which the
optimist accumulates wealth when no disaster is realized. The consumption growth rate in the absence of a disaster determines how large the r.h.s. of the first equation is.

The forces that determine the optimist’s consumption growth rate in the absence of:

\[
\frac{\varpi'(\varpi; 0)}{1 - \varpi'(\varpi; 0)} = e^{\frac{\lambda_p - \lambda_o}{\alpha}} \frac{\varpi}{1 - \varpi}.
\]

First, note that \(\lambda_p - \lambda_o\) is small. Furthermore, this difference is shrunk as \(\alpha\) increases, and the investor’s willingness to substitute over time increases. Hence, the small upward slope of the line. So, even though equilibrium risk premia earned by the optimist increase, the rate of consumption growth declines because she is less willing to substitute intertemporally. As a result, the optimist is not willing to stay in the match when her consumption share decreases well below 1/2. As we increase \(\lambda_p\) from 0.025 to 0.20, the curve shifts down and creates some room for trade at low levels of risk aversion, but then this is no longer a rare event. Note that increasing \(\lambda_o\) tends to shift the curve back up. Hence, to summarize, when risk aversion is higher than 5 and the gap between the beliefs does not exceed 0.02, there is essentially no room for speculative trade.

### 4.3 Autarchic Prices

**Proposition 4.1.** If no equilibrium trade can be sustained, then the autarchic state prices are just given by the highest IMRS in each state:

\[
\hat{Q}^o(0) = \hat{\beta} e^{-\lambda_o},
\]

\[
\hat{Q}^o(1) = \hat{\beta} e^{-\lambda_p e^{\alpha J}}.
\]

In the no-trade case, the pessimist prices disaster risk. Hence, if we feed in the same path of shocks, then we get no variation in the equilibrium risk premium, simply because the consumption shares do not respond to the shocks. The equilibrium risk premium (see equation 4.1) is constant. Furthermore, the equilibrium risk premium
is very high: around 6.4% per annum for the pessimist, compared to the around 3% in the case of perfect risk sharing. Since the optimist is no longer allowed to take short positions to provide insurance to the pessimist, the equity risk premium is almost complete determined by the pessimist. Furthermore, the equilibrium risk-free rate is -4% in this economy. This reflects the scarcity of ‘collateral’.

In the version of this economy without pledgeable income, no equilibrium trade can be sustained, and disaster risk is priced exclusively by the pessimists. The government can subsidize disaster risk insurance by committing to injecting resources in the disaster state.

5 Solving Equilibrium Allocations and Prices in the Bailout Economy

To start, we note that the value of autarchy in the stationary economy is defined as before (see equation 17): The value of autarchy is not affected by the bailouts. We can use the optimist’s consumption share, labeled \( \bar{\omega} \), as the state variable. Then the law of motion for the optimist’s multiplier is given by:

\[
\xi'_o(\bar{\omega};0) = \max \left( e^{-\lambda_o \bar{\omega}^\alpha} \xi_o(0) \right), \text{ in case of no disaster} \tag{33}
\]

\[
\xi'_o(\bar{\omega};1) = \max \left( (1 - e^{-\lambda_o}) \bar{\omega}^\alpha, \xi_o(1) \right), \text{ in case of disaster} \tag{34}
\]

where \( \xi_o(d) \) is such that \( \bar{\omega}_o(d) = \frac{\xi_o}{\xi'_o(\bar{\omega};d) \bar{\omega}^\alpha + \xi'_o(\bar{\omega};d) \bar{\omega}^\alpha} \) is chosen such that: \( \tilde{V}_o(\bar{\omega}(d), d) = \tilde{V}_o^{\text{aut}} \). Then the law of motion for the pessimist’s multiplier is given by:

\[
\xi'_p(\bar{\omega};0) = \max \left( e^{-\lambda_p (1 - \bar{\omega})^\alpha} \xi_p(0) \right) \text{ in case of no disaster} \tag{35}
\]

\[
\xi'_p(\bar{\omega};1) = \max \left( (1 - e^{-\lambda_p}) (1 - \bar{\omega})^\alpha, \xi_p(1) \right) \text{ in case of disaster} \tag{36}
\]
where \( \varpi_p(d) = \frac{\xi_p(d)^{\frac{1}{\alpha + 1}}}{\xi_o(d)^{\frac{1}{\alpha + 1}} + \xi_p(d)^{\frac{1}{\alpha + 1}}} \) is chosen such that: \( \hat{V}(1 - \varpi_p(d), d) = \hat{V}^a \). The recursive expression of continuation utilities is in equation (69). The next period’s optimist consumption share is the policy function to:

\[
\varpi'(d; \varpi) = \frac{\xi_p(d)^{\frac{1}{\alpha + 1}}}{h(d)}
\]

where \( h(d) = \xi_p(d)^{\frac{1}{\alpha + 1}} + \xi_o(d)^{\frac{1}{\alpha + 1}} \) is defined in the usual way. Hence, we have recovered the law of motion for the state variable. The continuation utilities (scaled by the aggregate endowment) satisfy the following functional equation:

\[
\hat{V}_o(d; \varpi) = \frac{((1 + s(d)) \varpi)^{1-\alpha}}{1-\alpha} + \hat{\beta} \left[ e^{-(1-\alpha)J} \left( 1 - e^{-\lambda_o} \right) \hat{V}_o(\varpi'; 1, 1) + e^{-\lambda_o} \hat{V}_o(\varpi'; 0, 0) \right], \tag{37}
\]

\[
\hat{V}_p(d; \varpi) = \frac{((1 + s(d)(1 - \varpi))^{1-\alpha}}{1-\alpha} + \hat{\beta} \left[ e^{-(1-\alpha)J} \left( 1 - e^{-\lambda_p} \right) \hat{V}_p(\varpi'; 1, 1) + e^{-\lambda_p} \hat{V}_p(\varpi'; 0, 0) \right]. \tag{38}
\]

**State Prices** We can also characterize state prices as a function of the aggregate state variable \( \varpi \):

\[
Q_t(\varpi, d; 0) = \hat{\beta} h(\varpi; 0)^\alpha \left( \frac{1}{1 + s(d)} \right)^{-\alpha} \tag{39}
\]

\[
Q_t(\varpi, d; 1) = \hat{\beta} h(\varpi; 1)^\alpha e^{\alpha J} \left( \frac{1 + s(1)}{1 + s(d)} \right)^{-\alpha}. \tag{40}
\]

This also implies means we can solve for the net wealth equations (scaled by the aggregate endowment) recursively:

\[
\hat{P}_o(d; \varpi) = \varpi(1 + s(d)) - \frac{1}{2} + Q(\varpi, d; 1)e^{-J} \hat{P}_o(\varpi'; 1, 1) + Q(\varpi, d; 0) \hat{P}_o(\varpi'; 0, 0),
\]

\[
\hat{P}_p(d; \varpi) = (1 - \varpi)(1 + s(d)) - \frac{1}{2} + Q(\varpi, d; 1)e^{-J} \hat{P}_p(\varpi'; 1, 1) + Q(\varpi, d; 0) \hat{P}_p(\varpi'; 0, 0).
\]
We can also define the market value of their net wealth positions:

\[
\hat{P}^{\text{market}}_o(\varpi, d) = \varpi - \frac{1}{2} + Q(\varpi, d; 1)e^{-J}\hat{P}^{\text{market}}_o(\varpi'; 1) + Q(\varpi, d; 0)\hat{P}^{\text{market}}_o(\varpi'; 0, 0),
\]

\[
\hat{P}^{\text{market}}_p(\varpi, d) = (1 - \varpi) - \frac{1}{2} + Q(\varpi, d; 1)e^{-J}\hat{P}^{\text{market}}_p(\varpi'; 1) + Q(\varpi, d; 0)\hat{P}^{\text{market}}_p(\varpi'; 0, 0).
\]

The cost of the bailouts is given by the following expression:

\[
\hat{P}^{\text{cost}}(\varpi, d) = s(d) + Q(\varpi, d; 1)e^{-J}\hat{P}^{\text{cost}}(\varpi'; 1) + Q(\varpi, d; 0)\hat{P}^{\text{cost}}(\varpi'; 0, 0).
\]

The sum of the net wealth positions equals the value of the bailouts:

\[
\hat{P}_o(\varpi, d) + \hat{P}_p(\varpi, d) = \hat{P}^{\text{cost}}(\varpi, d).
\]

**Equilibrium** The policy function for the optimist’s consumption share \(\varpi'(\varpi; d)\) and the pricing kernel \(Q(\varpi; d)\) provide a complete recursive characterization of a competitive equilibrium. If we start at some initial consumption share for the optimist, we can completely recover the equilibrium as defined in ()..

**No Exclusion** If agents cannot be excluded from trading, we use the same updating rule for the consumption shares, but the optimist’s reservation consumption share is determined such that \(\varpi_o(d) = \frac{\xi_{1/\alpha_o}}{\xi_{1/\alpha_o}(\varpi; d)^{1/\alpha_o} + \xi_{1/\alpha_o}(\varpi; d)^{1/\alpha_o}}\) satisfies the following condition: \(\hat{P}_o(\varpi, d) = 0\). Similarly, the pessimist’s reservation consumption share \(\varpi_p(d) = \frac{\xi_{1/\alpha_p}}{\xi_{1/\alpha_p}(\varpi; d)^{1/\alpha_p} + \xi_{1/\alpha_p}(\varpi; d)^{1/\alpha_p}}\) satisfies: \(\hat{P}_p(1 - \varpi_p, d) = 0\). Even when agents cannot be excluded, there is room for speculative trade, because we can have positive net wealth for one agent, without negative net wealth for the other agent.

**Cutoff Shares** To check whether risk sharing is feasible, we can compute cutoff consumption shares from the primitives of this economy without actually solving for the equilibrium allocations.

**Lemma 5.1.** The cutoff consumption shares can be solved from the following equa-
\[
\frac{(\varpi(1+s(1)))^{1-\alpha}}{1-\alpha} - \frac{0.5^{1-\alpha}}{1-\alpha} = \beta e^{-\lambda_0} + \varpi^{-\alpha} (\varpi'(\varpi; 0) - \varpi) \\
(1 - \alpha \varpi^{-1})(\varpi'(\varpi; 0) - \varpi).
\]

where the policy functions solve the following equations:

\[
\frac{\varpi'(\varpi; 0)}{1 - \varpi'(\varpi; 0)} = e^{\lambda_p - \lambda_0} \frac{\varpi}{1 - \varpi} \quad \text{and} \quad \frac{\varpi'(\varpi; 1)}{1 - \varpi'(\varpi; 1)} = \left(1 - e^{-\lambda_0}\right)^{1/\alpha} \frac{\varpi}{1 - \varpi}.
\]

6 Speculative Trade in the Bailout Economy

Next, we consider a version of the calibrated economy in which the government commits to a bailout policy. We maintain the same calibration, but we consider the case in which the subsidy in the disaster state amounts to 5% (\(\chi\)) of total income in the disaster state. This delivers an average ratio of the total cost (at true market prices) to the total endowment of 24%. The pessimist (optimist) is endowed with beliefs \(\lambda\) equal to 0.025 (0.005).

**Consumption Policy Function** The domain for the optimist’s consumption share \([\varpi, \overline{\varpi}]\) is given by [0.47, 0.53]. There is definitely scope for speculative trade in this economy. Figure 6 plots the optimist consumption share \(\varpi\) at \(t + 1\) against the optimist consumption share at \(t\). In the no-disaster states, the optimist’s consumption share increases slightly as long as it is smaller than the upper bound, but there is a large decrease in the disaster state.

Why do we get room for speculative trade? Figure 7 plots the continuation utility \(\hat{V}\) of the optimist (pessimist) against the optimist consumption share \(\varpi\) at \(t\). As is clear from the graph, the optimist’s disaster continuation utility crosses autarchya below \(\varpi = 0.5\), while the pessimist’s disaster continuation utility crosses his autarchy line above \(\varpi = 0.5\), because of the subsidy in the disaster state.
**Simulated Path**  We look at a sample of 100 periods with one disaster realizations at $t = 25$ and $t = 50$.

**Consumption**  The top panel of Figure 8 plots the consumption shares (as a fraction of the aggregate endowment) of the optimist and the pessimist. The optimist’s consumption share starts at 0.5 and increases to 0.53 until a disaster hits at $t = 25$, at which point her consumption share drops to a lower level (0.47). This pattern repeats itself at $t = 50$.

**Wealth**  The top panel of Figure 8 also plots the net financial wealth divided by the aggregate endowment for the optimist and the pessimist. The size of these net wealth positions (in absolute value) give a sense of how much trade the heterogeneity in beliefs generates. The optimist starts out with a zero net wealth position. As long as the good draws continue, her market value of wealth rises to hit 20% of the aggregate endowment at $t = 24$, just before the disaster hits. At $t = 25$, her net wealth declines to a slightly negative position. Basically, the optimist’s promises were almost fully collateralized. If they were fully collateralized, his net wealth would hit zero when the constraint binds. However, the optimist’s market wealth net of subsidies reaches -0.20% in the disaster state. Hence, the collateral is being provided by the value of current and future bailouts. Figure 9 plots the market value of wealth for the optimist and the pessimist. The ex subsidy value of the optimist’s financial wealth is negative in the disaster state, but the cum subsidy value is not.

**Continuation Utility**  The middle panel of Figure 8 plots the continuation utilities for the optimist (left panel) and the pessimist (right panel) against their respective autarchy values. The optimist’s continuation utility drops to his autarchy value when the disaster is realized and the participation constraint binds. After this, it starts to climb again. The pessimist’s constraint does not bind in this simulated path.
Equilibrium Risk Premium  The bottom panel of Figure 8 plots the perceived equilibrium risk premium, both from the perspective of the optimist and the pessimist. In the economy with binding solvency constraints, the equilibrium risk premium is much more volatile. The equilibrium risk premium starts off at 2.5%, and then gradually decreases to 0.005% as the optimist accumulates more wealth. When the disaster is realized, the pessimist essentially prices the disaster risk and the risk premium increases to almost 2.5%.

6.1 Consumption and Wealth Distribution

Without subsidies, there is no scope for speculative trade. This section considers what happens to the equilibrium distribution of consumption and wealth when we the government subsidizes disaster risk by committing to a bailout policy. To generate these moments, we assume that the pessimist has the right beliefs.

**Consumption**  Panel A of Table 1 reports the stats for the consumption and wealth distribution in the economy with bailouts. As we increase the size of the subsidy, the lower bound $\varpi$ gradually increases. As a result, the optimist can take larger short positions by selling aggregate disaster risk insurance. For example, when the subsidy is 10% of income in the disaster state, The lower bound is 0.45, while the upper bound is 0.54. Judging by its impact on consumption and wealth, the subsidy mainly benefits the optimist. The average consumption share of the optimist is 0.52. The standard deviation of his consumption share is 2%.

**Wealth**  Similarly, the average net wealth position (divided by the aggregate endowment) of the optimist $\hat{P}_o$ is 0.24 when we include the subsidy (0.12 ex subsidy). In the disaster state, the optimist’s net wealth including the subsidy (ex subsidy) drops to -4% (-36%) of the total endowment. Hence, there is substantial uncollateralized trade in equilibrium. Of course, volatility increases as well with the size of the subsidy. The standard deviation of net wealth (relative to the aggregate endowment) is 16%.
Figure 10 plots a histogram for optimist consumption share and net wealth. Even though the pessimist has the right beliefs, the pessimist spends 2/3 of the time at the lower bound for net wealth and consumption.

No exclusion  Table 2 reports the consumption and wealth distribution statistics when investors cannot be excluded from default. Because these solvency constraints are tighter, less speculative trade is feasible. Panel A shows that the range of consumption shares is smaller, especially when the IES is smaller than the coefficient of risk aversion. As a result, the optimist’s average consumption share tends to be smaller as well. In this case, negative net wealth positions (including the subsidy) are not allowed, as is clear from panel B. However, the ex subsidy net wealth position of the optimist are negative (Panel C). The optimist’s short positions are fully collateralized by the value of future bailouts. Note that the market value of bailouts relative to GDP is very high in the case of recursive utility; this is the collateral that allows for speculative trade.

6.2 Asset Prices

Table 3 reports sample moments obtained by simulating the model for 150,000 periods. We vary the fraction of the total endowment that is pledgeable. Panel A reports aggregate risk premia, B reports risk-free returns on collateralizable assets, while panel C reports returns on non-collateralizable assets.

When the size of the subsidy is .5%, the average risk premium is 5.91% (from the optimist’s point of view). The standard deviation is very small. Very little equilibrium trade is feasible. As we increase the subsidy, the price of disaster risk decreases and average risk premia decile, while the standard deviation decreases from 0.09% % to 0.50%. When the size of the subsidy is 10% of income in the disaster state, the total present value cost of the subsidies is 39% of income. A lot of equilibrium trade can be sustained, and a result, the optimists manages to push equilibrium risk premia down. The average risk premium as perceived by the optimist is -2.23%, while the standard deviation is high (0.50%). The smallest risk premium is -2.86 %.
The highest risk premium is -0.62%. The risk premia measured from the pessimist’s point of view are very similar.\(^2\) The average risk-free rate is high at 2.32%.

The risk-free rate on collateralizable assets, reported in Panel B, increases from -2.37% to 5.33%. As safe assets become scarcer and agents become more constrained, the conditional volatility of marginal utility growth increases. This force pushes down the risk-free rate. Panel C reports the returns on non-collateralizable assets. These returns decrease from -1.06% to 5.57%. As a result, the spread, reported in panel D, shrinks from 1.30% to 0.24%, as we increase the size of the subsidy from 0% to 10%. This spread measures the convenience yield of collateral assets. The convenience yield is very elastic with respect to the supply of safe assets. This model reproduces the relation between Treasuries/GDP and the convenience yield that was documented in U.S. data by Krishnamurthy and Vissing-Jorgensen (2012).

No Exclusion

7 Recursive Utility

Without bailouts and pledgeable income, no speculative trade can be sustained, even in the case of recursive utility. However, for a given size bailout, much more speculative trade occurs in equilibrium in the case of recursive utility provided that investors have a preference for early resolution of uncertainty.

7.1 Preferences

Consumers rank stochastic consumption streams \(\{c_t(z_t)\}\) according to the following criterion:

\[
V_t(\lambda) = [(1 - \beta)c_t^{1-\rho} + \beta(R_t^\lambda V_{t+1}^{1-\rho})^{1-\rho}],
\]

\(\text{(41)}\)

\(^2\)The average risk premium as perceived by the pessimist is equally low (2.29%), while the standard deviation is 1.99%. The smallest risk premium is -20 bps. The highest risk premium is 5.85%.
where $\mathcal{R}^\lambda$ denotes the following operator:

$$\mathcal{R}^\lambda_t V_{t+1} = \left( E_t^\lambda V_{t+1}^{1-\alpha} \right)^{1/1-\alpha}$$

$\rho^{-1}$ controls the intertemporal elasticity of substitution, while $\alpha$ controls risk aversion. If $\alpha = \rho$, preferences are time-separable.

**Continuation Utilities** We define $U_t(c)(z_t)$ to be the continuation expected lifetime utility from a consumption allocation $c = \{c_t(z_t)\}$ in node $z_t$. This utility can be constructed recursively as follows:

$$U_t(c)(\lambda)(z_t) = \left[ (1 - \beta) c_t^{1-\rho} + \beta \left( \sum_{z_{t+1}} \pi_{\lambda}(z_{t+1}|z_t) \left( U_{t+1}(c)(\lambda)(z^{t+1}) \right)^{1-\alpha} \right) \right]^{\frac{1}{1-\alpha}},$$

where we made use of the Markov property the underlying stochastic processes.

Section A in the separate appendix derives the recursive characterization of equilibrium allocations and prices using the optimist consumption share as the state variable.

### 7.2 Consumption and Wealth Distribution

Panel II of Table 1 reports the stats for the consumption and wealth distribution in the economy with bailouts. We fixed $\rho$ equal to 0.75, and $\alpha$ is 5. When the subsidy is zero, the only equilibrium is the autarchic one. As we increase the size of the subsidy, the lower (upper) bound $\overline{\omega}$ decreases (increases) dramatically. Even with a small subsidy of 0.5%, the lower bound $\overline{\omega}$ drops to 0.26, while the upper bound $\overline{\omega}$ to 0.64.

The amount of trade is an order of magnitude larger than in the case of additive utility. Even in the case of the 0.5% subsidy, the optimist is allowed a negative (positive) net wealth position of -1.46 (+2.14) times total income in the disaster state (normal state). So, there is a large amount of equilibrium leverage that is
sustained by future surpluses to be earned by the optimist in the disaster insurance market.

As before, the subsidy mainly benefits the optimist. Her average consumption share is 0.58 in the case of the smallest subsidy, and her average net wealth position (including the subsidy) is 1.67 times the aggregate endowment.

The standard deviation of the optimist consumption shares is much larger (around 10% in all cases), because the higher willingness to substitute intertemporally gives rise to much higher consumption growth rates for the optimists, when she is unconstrained, and much larger consumption drops when the disaster state is realized. This explains why the optimist is willing to accept much lower consumption shares in the disaster state. Similarly, the standard deviation of the optimist’s net wealth (divided by total income) is much higher as well (close to 90%).

Cutoff Consumption Shares To understand these results, it helps to analyze the cutoff consumption shares again. As before, we compute cutoff consumption shares from the primitives of this economy, but in this case the cutoffs depend on the continuation utilities.

Lemma 7.1. The cutoff consumption shares can be approximated by solving the following equations:

$$\bar{\omega}^{1-\rho} - 0.5^{1-\rho} \approx -\tilde{\beta}(V_o^{\text{aut}})^{1-\rho} \left[ e^{-\lambda_o \phi_o^{1-\alpha}} + (1 - e^{-\lambda_o}) e^{-(1-\alpha)J} \right]^{\frac{1-\rho}{1-\alpha}}$$

$$+ \tilde{\beta}(V_p^{\text{aut}})^{1-\rho} \left[ e^{-\lambda_p + (1 - e^{-\lambda_p}) e^{-(1-\alpha)J}(\phi_p)^{1-\alpha}} \right]^{\frac{1-\rho}{1-\alpha}}$$

$$\left(1 - \bar{\omega}\right)^{1-\rho} - 0.5^{1-\rho} \approx -\tilde{\beta}(V_o^{\text{aut}})^{1-\rho} \left[ e^{-\lambda_o + (1 - e^{-\lambda_p}) e^{-(1-\alpha)J}(\phi_p)^{1-\alpha}} \right]^{\frac{1-\rho}{1-\alpha}}$$

$$+ \tilde{\beta}(V_p^{\text{aut}})^{1-\rho} \left[ e^{-\lambda_p + (1 - e^{-\lambda_p}) e^{-(1-\alpha)J}(\phi_p)^{1-\alpha}} \right]^{\frac{1-\rho}{1-\alpha}},$$

where the policy functions solve the following equations:

$$\frac{\omega'(\bar{\omega}; 0)}{1 - \omega'(\bar{\omega}; 0)} = e^{\frac{\lambda_o - \lambda_o}{\rho}} \left( \frac{V_o(\omega'(\bar{\omega}; 0))}{RV_o(\bar{\omega})} \frac{RV_p(\bar{\omega})}{V_p(\omega'(\bar{\omega}; 0))} \right)^{(\rho-\alpha)/\rho} \bar{\omega}.$$
and

\[
\frac{\varpi'(\varpi; 1)}{1 - \varpi'(\varpi; 1)} = \left(1 - e^{-\lambda_o}\right)^{1/\rho} \left(\frac{V_o(\varpi'(\varpi; 1))}{R V_o(\varpi)} \frac{R V_p(\varpi)}{V_p(\varpi'(\varpi; 1))}\right)^{(\rho - \alpha)/\rho} \frac{\varpi}{1 - \varpi},
\]

and where \(\phi^o\) and \(\phi^p\) are defined as:

\[
\phi^o \approx \frac{V^o_{\text{aut}} + \varpi^o(\varpi'(\varpi; 0) - \varpi)(1 - \rho \varpi^{-1} (\varpi'(\varpi; 0) - \varpi))}{V^o_{\text{aut}}}
\]

\[
\phi^p \approx \frac{V^p_{\text{aut}} - \varpi^p(\varpi'(\varpi; 1) - \varpi)(1 - \rho \varpi^{-1} (\varpi'(\varpi; 1) - \varpi))}{V^p_{\text{aut}}}
\]

These cutoff shares actually depend on the equilibrium continuation values and hence cannot be computed simply from the primitives of the economy. However, assume (as was the case for additive utility) that no trade is feasible. Given this assumption, we can approximate the law of motion as follows:

\[
\frac{\varpi'(\varpi; 0)}{1 - \varpi'(\varpi; 0)} \approx e^{\frac{\lambda_p - \lambda_o}{\rho}} \left(\frac{V^o_{\text{aut}}}{R V^o_{\text{aut}}} \frac{R V^p_{\text{aut}}}{V^p_{\text{aut}}}\right)^{(\rho - \alpha)/\rho} \frac{\varpi}{1 - \varpi}
\]

\[
\frac{\varpi'(\varpi; 1)}{1 - \varpi'(\varpi; 1)} \approx \left(1 - e^{-\lambda_o}\right)^{1/\rho} \left(\frac{V^o_{\text{aut}}}{R V^o_{\text{aut}}} \frac{R V^p_{\text{aut}}}{V^p_{\text{aut}}}\right)^{(\rho - \alpha)/\rho} \frac{\varpi}{1 - \varpi}
\]

**Calibrated Example**  We consider the same calibrated version of our economy. The parameter \(\rho\) is set to 0.75. In the top panel of Figure 11, the lower bound for the optimist’s consumption share \(\varpi\) is plotted against the coefficient of relative risk aversion in figure 11. The lower bound \(\varpi\) drops well below 1/2. As we increase risk aversion and equilibrium risk premia increase, the optimist is willing to accept consumption levels well below 1/2. There is enough surplus generated by the risk-sharing arrangement to induce the optimist to stay in the match at consumption shares well below 0.5, regardless of the level of risk aversion, because she anticipates large wealth gains. This follows from her high willingness to defer consumption and
save these higher returns to accumulate wealth. The absence of a disaster lowers the marginal utility of the pessimist more when agents have a preference for early resolution of uncertainty, because \( \left( \frac{V^\text{aut}_{R}}{R V^\text{aut}_{o}} \right) < 1 \) in this expression:

\[
\frac{\varpi'(\varpi; 0)}{1 - \varpi'(\varpi; 0)} = e^{\frac{\lambda_p - \lambda_o}{\rho}} \left( \frac{V^\text{aut}_{o}}{R V^\text{aut}_{o}} \frac{RV^\text{aut}_{p}}{V^\text{aut}_{p}} \right)^{(\rho - \alpha)/\rho} \frac{\varpi}{1 - \varpi},
\]

thus increasing the optimist’s consumption growth rate as \( \alpha \) increases. In the bottom panel of Figure 11, we plot the lower bound for the optimist’s consumption share \( \varpi \) is plotted against the \( \rho \) in figure 11. The coefficient of risk aversion is fixed at 4. As \( \rho \) increases, the lower bound tends to 0.5.

### 7.3 Asset Prices

Panel II of Table 3 reports sample moments obtained by simulating the model for 150,000 periods. We vary the fraction of the total endowment that is pledgeable. Panel A reports aggregate risk premia, B reports risk-free returns on collateralizable assets, while panel C reports returns on non-collateralizable assets.

The top panel shows the aggregate risk premia. Given the large short positions adopted by the optimist, even with a subsidy of .5%, there is a decline in the average equity risk premium as perceived by the optimist from 11.05% in autarchy to 3.87%. The risk premia are quite volatile. The standard deviation is 0.45%. The risk-free rate on collateralizable assets is 0.16%.

### References


Table 1: Consumption and Wealth in Economy with Bailouts

<table>
<thead>
<tr>
<th></th>
<th>Panel I: $\rho = 5$</th>
<th>Panel II: $\rho = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(1)$</td>
<td>0.005 0.01 0.025 0.05 0.075 0.01 0.005 0.01 0.025 0.05 0.075 0.01</td>
<td></td>
</tr>
<tr>
<td>$cost$</td>
<td>0.04 0.07 0.15 0.24 0.32 0.39 0.02 0.04 0.10 0.20 0.29 0.37</td>
<td></td>
</tr>
</tbody>
</table>

Panel A: Moments of Optimist Consumption Share

<table>
<thead>
<tr>
<th></th>
<th>$E[	ilde{c}]$</th>
<th>$std[	ilde{c}]$</th>
<th>$min[	ilde{c}]$</th>
<th>$max[	ilde{c}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.50 0.51 0.51 0.52 0.52 0.52 0.58 0.58 0.58 0.59 0.59 0.59</td>
<td>0.00 0.01 0.01 0.02 0.02 0.02 0.10 0.10 0.10 0.10 0.10 0.10</td>
<td></td>
<td></td>
</tr>
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</table>

Panel B: Moments of Optimist Net Wealth

<table>
<thead>
<tr>
<th></th>
<th>$E[	ilde{P}_o]$</th>
<th>$std[	ilde{P}_o]$</th>
<th>$min[	ilde{P}_o]$</th>
<th>$max[	ilde{P}_o]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.04 0.07 0.13 0.18 0.22 0.24 1.67 1.69 1.74 1.87 1.96 2.07</td>
<td>0.01 0.03 0.06 0.10 0.14 0.16 0.86 0.87 0.89 0.91 0.93 0.97</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel C: Moments of Optimist Net Market Wealth

<table>
<thead>
<tr>
<th></th>
<th>$E[	ilde{P}_{mn}]$</th>
<th>$std[	ilde{P}_{mn}]$</th>
<th>$min[	ilde{P}_{mn}]$</th>
<th>$max[	ilde{P}_{mn}]$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.02 0.04 0.06 0.09 0.11 0.12 1.66 1.68 1.71 1.82 1.88 1.97</td>
<td>0.02 0.03 0.06 0.11 0.15 0.18 0.86 0.87 0.89 0.90 0.93 0.96</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Tables lists moments of consumption shares and net wealth (divided by the aggregate endowment) in economy with subsidy $s(1)$. The second row reports the value of the safe assets relative to the total endowment. Sample moments reported for 150,000 simulations using $\lambda_p$ as the true probability. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005).
Simulations using \( p \) as the true probability. Possibilist (optimist) with a equal to 0.95 (0.05). The second row reports the value of the safe assets relative to the total endowment. Sample moments reported for 150,000.

Table 2: Consumption and Wealth in Economy with Bailouts: No Exclusion

<table>
<thead>
<tr>
<th>Panel A: Moments of Optimist Consumption Share</th>
<th>Panel B: Moments of Optimist Net Wealth</th>
<th>Panel C: Moments of Optimal Net Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \bar{c} )</td>
<td>( \bar{w} )</td>
<td>( \bar{w}_o )</td>
</tr>
<tr>
<td>( \bar{c}_{\text{max}} )</td>
<td>( \bar{w}_{o\text{max}} )</td>
<td>( \bar{w}_{i\text{max}} )</td>
</tr>
<tr>
<td>( \bar{c}_{\text{min}} )</td>
<td>( \bar{w}_{o\text{min}} )</td>
<td>( \bar{w}_{i\text{min}} )</td>
</tr>
<tr>
<td>( \bar{c}_{\text{std}} )</td>
<td>( \bar{w}_{o\text{std}} )</td>
<td>( \bar{w}_{i\text{std}} )</td>
</tr>
<tr>
<td>( \bar{c}_{\text{E}} )</td>
<td>( \bar{w}_{o\text{E}} )</td>
<td>( \bar{w}_{i\text{E}} )</td>
</tr>
<tr>
<td>( \bar{c}_{\text{v}} )</td>
<td>( \bar{w}_{o\text{v}} )</td>
<td>( \bar{w}_{i\text{v}} )</td>
</tr>
<tr>
<td>( \bar{c}_{\text{( \lambda )}} )</td>
<td>( \bar{w}_{o\text{( \lambda )}} )</td>
<td>( \bar{w}_{i\text{( \lambda )}} )</td>
</tr>
</tbody>
</table>
Table 3: Asset Prices in Economy with Bailouts

<table>
<thead>
<tr>
<th></th>
<th>Panel I: $\rho = 5$</th>
<th>Panel II: $\rho = 0.75$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$s(1)$</td>
<td>cost</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Panel A: Aggregate Risk Premia</td>
<td>$E \left( E_{t}^{c, aut\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$6.32$</td>
</tr>
<tr>
<td></td>
<td>$E \left( E_{t}^{c, aut\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$5.54$</td>
</tr>
<tr>
<td></td>
<td>$E \left( E_{t}^{\sigma\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$5.91$</td>
</tr>
<tr>
<td></td>
<td>$\sigma \left( E_{t}^{\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$0.09$</td>
</tr>
<tr>
<td></td>
<td>$\min \left( E_{t}^{\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$5.86$</td>
</tr>
<tr>
<td></td>
<td>$\max \left( E_{t}^{\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$6.22$</td>
</tr>
<tr>
<td></td>
<td>$E \left( E_{t}^{\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$5.10$</td>
</tr>
<tr>
<td></td>
<td>$\sigma \left( E_{t}^{\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$0.10$</td>
</tr>
<tr>
<td></td>
<td>$\min \left( E_{t}^{\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$5.05$</td>
</tr>
<tr>
<td></td>
<td>$\max \left( E_{t}^{\left[ R_{c,t+1}^{e} \right]} \right)$</td>
<td>$5.44$</td>
</tr>
<tr>
<td>Panel B: Risk-free Rates on Collateralizable Assets</td>
<td>$E\left[r_{f}^{c}\right]$</td>
<td>$-2.37$</td>
</tr>
<tr>
<td></td>
<td>$\sigma\left[r_{f}^{c}\right]$</td>
<td>$0.45$</td>
</tr>
<tr>
<td>Panel C: Risk-Free Rates on Non-collateralizable Assets</td>
<td>$E\left[r_{f}^{nc}\right]$</td>
<td>$-1.06$</td>
</tr>
<tr>
<td></td>
<td>$\sigma\left[r_{f}^{nc}\right]$</td>
<td>$0.93$</td>
</tr>
<tr>
<td>Panel D: Spreads</td>
<td>$E\left[r_{f}^{nc} - r_{f}^{c}\right]$</td>
<td>$1.30$</td>
</tr>
<tr>
<td></td>
<td>$\sigma\left[r_{f}^{nc} - r_{f}^{c}\right]$</td>
<td>$0.69$</td>
</tr>
</tbody>
</table>

Tables lists moments of risk premia and risk-free rates in economy with subsidy $s(1)$. The second row reports the value of the safe assets relative to the total endowment. Sample moments reported for 150,000 simulations using $\lambda_{p}$ as the true probability. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005). Panel A reports aggregate risk premia (on fully collateralizable assets). Panel B reports risk-free returns on fully collateralizable assets. Panel C reports risk-free returns on non-collateralizable assets. Panel D reports the spread between returns on collateralizable and non-collateralizable assets.
Table 4: Asset Prices in Economy with Bailouts

<table>
<thead>
<tr>
<th>Year</th>
<th>Panel A: Aggregate Risk Premium</th>
<th>Panel B: Risk-free Rates on Non-collateralizable Assets</th>
<th>Panel C: Risk-free Rates on Collateralizable Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>2005</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
</tr>
<tr>
<td>2006</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
</tr>
<tr>
<td>2007</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
</tr>
<tr>
<td>2008</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
</tr>
<tr>
<td>2009</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
</tr>
<tr>
<td>2010</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
<td>0.00 0.00 0.00 1.00</td>
</tr>
</tbody>
</table>

Panel D reports the spread between returns on collateralizable and non-collateralizable assets. Panel E reports the standard deviation of returns on non-collateralizable assets. Panel F reports the standard deviation of risk-free returns in the economy.
Figure 1: Consumption Policy Function in Full Commitment Economy

Plot of optimist consumption share of aggregate endowment. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005).
Figure 2: Consumption in Full Commitment Economy

Plot of consumption share of aggregate endowment. Pessimist (optimist) with \( \lambda \) equal to 0.025 (0.005). 100 periods with disasters in period 50.
Figure 3: Continuation Utility in Limited Commitment Economy

Simulation of 100 periods with disaster in period 50. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005).
Figure 4: Lower Bound

Plot of the lower bound the optimist’s consumption share $\bar{\omega}$ against $\rho = \alpha$ from 1.5 to 10. Pessimist (optimist) with $\lambda$ equal to [0.025, 0.05, 0.10, 0.20] (0.005).
Figure 5: Lower Bound

Plot of the lower bound the optimist’s consumption share $\varpi$ against $\rho = \alpha$ from 1.5 to 10. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005). Subsidies range from 0 to 0.20.
Figure 6: Consumption Policy Function in Bailout Economy: Additive Utility

Plot of optimist consumption share of aggregate endowment. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005). The subsidy in the disaster state is 5% of the total endowment.
Figure 7: Continuation Utility Function in Bailout Economy: Additive Utility

Plot of continuation utility $\hat{V}$ against optimist consumption share of aggregate endowment. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005). The subsidy in the disaster state is 5% of the total endowment.
Figure 8: Simulation in Bailout Economy: Additive Utility.

Plot of consumption share \( \varpi \) of aggregate endowment. Pessimist (optimist) with \( \lambda \) equal to 0.025 (0.005). 100 periods with disasters in period 50. The subsidy in the disaster state is 5% of the total endowment.
Figure 9: Net Wealth in Bailout Economy

Simulation of 100 periods with disasters in period 25/50. Pessimist (optimist) with \( \lambda \) equal to 0.025 (0.005). The subsidy in the disaster state is 5% of the total endowment. The ‘o’ label indicates the ex subsidy value of net wealth.
Figure 10: Histogram for Optimist in Limited Commitment Economy

Simulation of 150,000 periods. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005). $\lambda_p$ is the true probability. The subsidy in the disaster state is 5% of the aggregate endowment.
Figure 11: Lower Bound

Plot of the lower bound for the optimist’s consumption share $\varpi$ against the coefficient of risk aversion $\alpha$. The parameter $\rho$ is set to 0.75. Pessimist (optimist) with $\lambda$ equal to $[0.025, 0.05, 0.10, 0.20] (0.005)$. 
Figure 12: Histogram for Optimist in Limited Commitment Economy

Simulation of 150,000 periods. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005). $\lambda_p$ is the true probability. 1% of the total endowment is pledgeable.
Figure 13: Simulation in Bailout Economy: Recursive Utility.

Plot of consumption share $\varpi$ of aggregate endowment. Pessimist (optimist) with $\lambda$ equal to 0.025 (0.005). 100 periods with disasters in period 50. The subsidy in the disaster state is 5% of the total endowment.
A Recursive Utility

To solve for the equilibrium allocations and prices, we solve a static version of the investor problem. Let $\kappa$ denote the multiplier on the time zero budget constraint, and let $\mu$ denote the multiplier on the participation constraint. We set up the standard saddle point problem:

$$
L = \max_{c_t(z^t)} \min_{\kappa, \mu} \sum_{t=0}^{\infty} U_0(c)(\lambda)(z^t) + \kappa \left( \sum_{t=0}^{\infty} \sum_{z^t} Q(z^t|z_0)((1/2)e_t(z^t) - c_t(z^t)) \right) 
+ \sum_{t=0}^{\infty} \sum_{z^t} \mu(z^t) \left( U_t(c)(\lambda)(z^t) - U_t((1/2)e)(\lambda)(z^t) \right).
$$

The first order condition for optimality is given by:

$$
\frac{\partial U_0(c)(\lambda)}{\partial c_t(z^t)} + \sum_{z^t \leq z^t} \mu(z^t) \frac{\partial U_t(c)(\lambda)}{\partial c_t(z^t)} = \kappa Q_t(z^t|z_0).
$$

Cumulative Multiplier To characterize the equilibrium consumption allocations and prices subject to these participation constraints, we use stochastic Pareto-Negishi weights, following Kehoe and Perri (2002), Chien and Lustig (2010) and Chien, Cole, and Lustig (2011). We start by defining some notation. $M_{\lambda,0,t}(z^t)$ is defined as the product of the one-period gradients $\prod_{z^t \leq z^t} M_t(z^t)$; the one-period-ahead gradient is given by:

$$
M_{\lambda,t+1}(z^{t+1}) = \beta \pi_{t+1}(z^{t+1}|z_t) \left( \frac{V_{t+1}(z^{t+1})}{R_{t+1}V_{t+1}} \right)^{\rho-\alpha}, \quad t \geq 1.
$$

Next, consider the first order condition for consumption in state $s^t$:

$$
(1 - \beta) \left( c_t(z^t) \right)^{-\rho} \left( M_{0,t}^\lambda(z^t) + \sum_{z^t \leq z^t} \mu(z^t) M_{\lambda,t}^\lambda(z^t) \right) = \kappa Q_t(z^t|z_0),
$$

Take the ratio of these first order conditions for two households, $\lambda$ and $\lambda'$:

$$
\frac{(M_{0,t}^\lambda(z^t) + \sum_{z^t \leq z^t} \mu(z^t) M_{\lambda,t}^\lambda(z^t)) (c_t(\lambda; z^t))^{-\rho}}{(M_{0,t}^{\lambda'}(z^t) + \sum_{z^t \leq z^t} \mu(z^t) M_{\lambda,t}^{\lambda'}(z^t)) (c_t(\lambda'; z^t))^{-\rho}} = \frac{\kappa}{\kappa'}.
$$

56
Define the Pareto-Negishi weight $\xi$ as follows:

$$\xi_t(\lambda)(z^t) = \left( M_{0,t}^\lambda(z^t) + \sum_{z^\tau \preceq z^t} \mu(z^\tau) M_{t,\tau}^\lambda(z^t) \right) / \kappa. \tag{42}$$

We conjecture a consumption sharing rule:

$$c_t = \xi^{1/\rho}_t h_t e_t,$$ \tag{43}

where $h_t = \sum \xi^{1/\rho}_t(\lambda)$. This risk-sharing rule satisfies the market clearing condition and the first order condition (by construction).

We can back out an expression for the state prices:

$$\pi(s^t|s_0)^{\beta^t h_t^\rho e_t^-} = Q_t(z^t|z_0).$$

Hence, the state price is given by, after normalization, by:

$$\frac{\pi(s^t|s_0)^{\beta^t h_t^\rho e_t^-}}{\pi_0^{\beta^t h_0^\rho e_0^-}} = \frac{Q_t(z^t|z_0)}{Q_0}.$$

**Law of Motion for Stochastic Pareto-Negishi Weights**

**Lemma A.1.** If the participation constraints do not bind for agent $\lambda$ in node $z^t$, then the multiplier evolve according to:

$$\xi_t(\lambda)(z^t) = e^{-\lambda} \xi_{t-1}(\lambda)(z^{t-1}) \left( \frac{V_t(z^t)}{R_{t-1} V_t} \right)^{\rho - \alpha}, \text{ in case of no disaster}$$

$$\xi_t(\lambda)(z^t) = (1 - e^{-\lambda}) \xi_{t-1}(\lambda)(z^{t-1}) \left( \frac{V_t(z^t)}{R_{t-1} V_t} \right)^{\rho - \alpha}, \text{ in case of disaster.}$$

provided that

$$U_t(c; \xi_t(\lambda)(z^t)) > U((1/2)e)(\lambda)(z^t)$$

If the participation constraint do bind in some state $z^t$, then $\xi_t(\lambda) = \xi_\lambda(\lambda)$ is determined such that:

$$U_t(c; \xi_\lambda(\lambda)(z^t)) = U((1/2)e)(\lambda)(z^t).$$
A.1 Stationary Economy

We transform our model with stochastic growth into a stationary model with a stochastic time discount rate and a growth-adjusted probability matrix, following Alvarez and Jermann (2001) and Krueger and Lustig (2010). First, we define growth deflated consumption allocations (or consumption shares) as

$$\hat{c}_t(s^t) = \frac{c_t(s^t)}{e_t(z^t)} \text{ for all } s^t. \tag{44}$$

Next, we define growth-adjusted probabilities and the growth-adjusted discount factor as:

$$\hat{\pi}(s_{t+1}|s_t) = \frac{\pi(s_{t+1}|s_t)\gamma(z_{t+1})^{1-\alpha}}{\sum_{s_{t+1}} \pi(s_{t+1}|s_t)\gamma(z_{t+1})^{1-\alpha}} \text{ and } \hat{\beta}(s_t) = \beta \sum_{s_{t+1}} \pi(s_{t+1}|s_t)\gamma(z_{t+1})^{1-\alpha}. \text{ Note that } \hat{\pi} \text{ is a well-defined Markov matrix in that } \sum_{s_{t+1}} \hat{\pi}(s_{t+1}|s_t) = 1 \text{ for all } s_t \text{ and that } \hat{\beta}(s_t) \text{ is stochastic as long as the original Markov process is not iid over time.}

Lemma A.2. If the ‘normal’ aggregate shocks are i.i.d., the continuation utilities can be expressed as:

$$\hat{V}_t(\hat{c})(\lambda)(z^t) = \frac{V_t(c^t)}{e_t(z^t)} = \left[ (1 - \hat{\beta})\hat{c}_t^{1-\rho} + \hat{\beta}(R_t\hat{V}_{t+1})^{1-\rho} \right]^{\frac{1}{1-\rho}},$$

where

$$\hat{\beta} = \beta \left( \sum_{s_{t+1}} \pi(s_{t+1})\gamma(s_{t+1})^{1-\alpha} \right)^{\frac{1-\rho}{1-\alpha}}.$$ 

IID Shocks Suppose that the normal endowment growth rate in logs is normally distributed: $\log \gamma \sim N(\mu, \sigma)$. Then it is easy to verify the adjusted discount rate is simply

$$\hat{\beta} = \beta \exp \left( \mu(1 - \rho) + 0.5(1 - \alpha)(1 - \rho)\sigma^2 \right).$$

This delivers the standard Hansen and Singleton (1983) expression for the risk-free rate in an economy without disaster shocks. The ‘Gaussian component’ of aggregate endowment growth will only affect the rate at which agents discount future utility flows.
In the appendix we prove that this transformation does not alter the agents’ ranking of different consumption streams.

### A.2 Recursive Algorithm

To characterize asset prices and wealth dynamics, we develop a recursive algorithm to compute equilibrium allocations and prices in a stationary version of the infinite horizon economy. To start, we note that the value of autarchy in the stationary economy is defined as:

\[
\hat{V}_{\text{aut}}^o = \left[ (1 - \beta)0.5^{1-\rho} + \beta(\hat{\mathcal{R}}\hat{V}_{\text{aut}}^o)^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{45}
\]

\[
\hat{V}_{\text{aut}}^p = \left[ (1 - \beta)0.5^{1-\rho} + \beta(\hat{\mathcal{R}}\hat{V}_{\text{aut}}^p)^{1-\rho} \right]^{\frac{1}{1-\rho}}, \tag{46}
\]

where the expected continuation utilities \( \hat{\mathcal{R}}\hat{V}_{\text{aut}}^i \) are given by:

\[
\hat{\mathcal{R}}\hat{V}_{\text{aut}}^o = \hat{V}_{\text{aut}}^o \left( e^{-(1-\alpha)J} \left( 1 - e^{-\lambda_o} \right) + e^{-\lambda_o} \right)^{\frac{1}{1-\alpha}},
\]

\[
\hat{\mathcal{R}}\hat{V}_{\text{aut}}^p = \hat{V}_{\text{aut}}^p \left( e^{-(1-\alpha)J} \left( 1 - e^{-\lambda_p} \right) + e^{-\lambda_p} \right)^{\frac{1}{1-\alpha}}.
\]

We can use the optimist’s consumption share, labeled \( \varpi \), as the state variable. Then the law of motion for the optimist’s multiplier is given by:

\[
\xi'_{o}(\varpi; 0) = \max \left( e^{-\lambda_o} \varpi^\alpha \left( \frac{\hat{V}_{o}(\varpi'; 0)}{\hat{\mathcal{R}}\hat{V}_{o}(\varpi')} \right)^{\rho-\alpha} ; \xi_o \right),
\]

in case of no disaster.

\[
\xi'_{o}(\varpi; 1) = \max \left( (1 - e^{-\lambda_o}) \varpi^\alpha \left( \frac{\hat{V}_{o}(\varpi'; 1)}{\hat{\mathcal{R}}\hat{V}_{o}(\varpi')} \right)^{\rho-\alpha} ; \xi_o \right),
\]

in case of disaster. \( \tag{48} \)

where \( \xi_o \) is such that \( \varpi_o = \frac{\xi_o^\frac{1}{\beta}}{\xi_o^\frac{1}{\beta} + \xi_o'} \) satisfies: \( \hat{V}_o(\varpi) = \hat{V}_{o,\text{aut}}^o \). Then the law of
motion for the pessimist’s multiplier is given by:

\[
\xi_p' (\varpi; 0) = \max \left( e^{-\lambda_p} (1 - \varpi)^{\rho} \left( \frac{\hat{V}_p(\varpi'(\varpi'; 0))}{\mathcal{R} \hat{V}_p(\varpi')} \right)^{\rho - \alpha}, \xi_p \right),
\]

in case of no disaster.

\[
\xi_p' (\varpi; 1) = \max \left( (1 - e^{-\lambda_p})(1 - \varpi)^{\rho} \left( \frac{\hat{V}_p(\varpi'(\varpi'; 1))}{\mathcal{R} \hat{V}_p(\varpi')} \right)^{\rho - \alpha}, \xi_p \right),
\]

in case of disaster.

(49)

where \( \varpi_p = \frac{\xi^\frac{1}{\lambda}}{\xi_o(\varpi; d) + \xi_p(\varpi; d)} \) satisfies: \( \hat{V}_p(1 - \varpi_p) = \hat{V}_p^{aut} \). The recursive expression of continuation utilities is in equation (69). The next period’s optimist consumption share is the policy function to:

\[
\varpi'(\varpi; d) = \frac{\xi_o'(\varpi; d)^{\frac{1}{\lambda}}}{h(\varpi; d)}
\]

where \( h(\varpi, d) = \xi_o'(\varpi; d)^{\frac{1}{\lambda}} + \xi_o'(\varpi; d)^{\frac{1}{\lambda}} \) is defined in the usual way. Hence, we have recovered the law of motion for the state variable. The continuation utilities (scaled by the aggregate endowment) satisfy the following functional equation:

\[
\hat{V}_o(\varpi) = \left[ (1 - \beta) \varpi^{1-\rho} + \hat{\beta} (\mathcal{R} \hat{V}_o(\varpi'))^{1-\rho} \right]^{\frac{1}{1-\rho}}
\]

\[
\hat{V}_p(\varpi) = \left[ (1 - \beta) \varpi^{1-\rho} + \hat{\beta} (\mathcal{R} \hat{V}_p(\varpi'))^{1-\rho} \right]^{\frac{1}{1-\rho}},
\]

(50)

(51)

where the expected continuation utilities \( \mathcal{R} \hat{V}_i \) are given by:

\[
\mathcal{R} \hat{V}_o(\varpi') = \left[ e^{-(1-\alpha)J} (1 - e^{-\lambda_o}) \hat{V}_o^{1-\alpha}(\varpi'(\varpi'; 1)) + e^{-\lambda_o} \hat{V}_o^{1-\alpha}(\varpi'(\varpi'; 0)) \right]^{\frac{1}{1-\alpha}},
\]

\[
\mathcal{R} \hat{V}_p(\varpi') = \left[ e^{-(1-\alpha)J} (1 - e^{-\lambda_p}) \hat{V}_p^{1-\alpha}(\varpi'(\varpi'; 1)) + e^{-\lambda_p} \hat{V}_p^{1-\alpha}(\varpi'(\varpi'; 0)) \right]^{\frac{1}{1-\alpha}}.
\]
**State Prices** We can also characterize state prices as a function of the aggregate state variable $\varpi$:

\[ Q_t(\varpi; 0) = \hat{\beta} h(\varpi; 0)^\rho \]  \hspace{1cm} (52)

\[ Q_t(\varpi; 1) = \hat{\beta} h(\varpi; 1)^\rho e^{\alpha J} \]  \hspace{1cm} (53)

We use $(\hat{P}_o, \hat{P}_p)$ to denote the ratio of net wealth to the aggregate endowment for the optimist (pessimist). This also implies means we can solve for the net wealth equations (scaled by the aggregate endowment) recursively:

\[ \hat{P}_o(\varpi) = \varpi + Q(\varpi; 1)e^{-J}\hat{P}_o(\varpi'(\varpi; 1)) + Q(\varpi; 0)\hat{P}_o(\varpi'(\varpi; 0)), \]

\[ \hat{P}_p(\varpi) = 1 - \varpi + Q(\varpi; 1)e^{-J}\hat{P}_p(\varpi'(\varpi; 1)) + Q(\varpi; 0)\hat{P}_p(\varpi'(\varpi; 0)). \]

The policy function for the optimist’s consumption share $\varpi'(\varpi; d)$ and the pricing kernel $Q(\varpi; d)$ provide a complete recursive characterization of a competitive equilibrium. If we start at some initial consumption share for the optimist, we can completely recover the equilibrium as defined in (5).

### A.3 Bailouts

This section considers what happens when we introduce bailouts; the bailout is an injection of resources $(s(d_t) > 0)$, but only in the disaster state. As we will show, this is equivalent to a subsidy of the disaster risk price by the government. To characterize asset prices and wealth dynamics, we develop a recursive algorithm to compute equilibrium allocations and prices in a stationary version of the infinite horizon economy.

To characterize asset prices and wealth dynamics, we develop a recursive algorithm to compute equilibrium allocations and prices in a stationary version of the infinite horizon economy. To start, we note that the value of autarchy in the stationary economy is defined as:

\[ \hat{V}_o^{aut} = \left( (1 - \beta)0.5^{1-\rho} + \hat{\beta}(R\hat{V}_o^{aut})^{1-\rho} \right)^{1/(1-\rho)}, \]

\[ \hat{V}_p^{aut} = \left( (1 - \beta)0.5^{1-\rho} + \hat{\beta}(R\hat{V}_p^{aut})^{1-\rho} \right)^{1/(1-\rho)}, \]
where
\[
R\hat{\hat{V}}_{o}^{aut} = \hat{\hat{V}}_{o}^{aut} (e^{-(1-\alpha)J} (1 - e^{-\lambda_o}) + e^{-\lambda_o})^{\frac{1}{1-\alpha}},
\]
\[
R\hat{\hat{V}}_{p}^{aut} = \hat{\hat{V}}_{p}^{aut} (e^{-(1-\alpha)J} (1 - e^{-\lambda_p}) + e^{-\lambda_p})^{\frac{1}{1-\alpha}}.
\]

We can use the optimist’s consumption share, labeled \(\varpi\), as the state variable. Then the law of motion for the optimist’s multiplier is given by:
\[
\xi'_{o}(\varpi; 0) = \max \left( e^{-\lambda_o} \varpi^\alpha \left( \frac{\hat{\hat{V}}_{o}(\varpi' (\varpi; 0))}{R\hat{\hat{V}}_{o}(\varpi')} \right)^{\rho-\alpha}, \xi_{o}(0) \right),
\]
in case of no disaster.
\[
\xi'_{o}(\varpi; 1) = \max \left( (1 - e^{-\lambda_o}) \varpi^{\alpha} \left( \frac{\hat{\hat{V}}_{o}(\varpi' (\varpi; 1))}{R\hat{\hat{V}}_{o}(\varpi')} \right)^{\rho-\alpha}, \xi_{o}(1) \right),
\]
in case of disaster.

(54)

where \(\xi_{o}(d)\) is such that \(\varpi_{o}(d) = \frac{\xi_{o}^{\frac{1}{\alpha}}}{\varpi_{o}(d)^{\frac{1}{\alpha}} + \xi_{o}(d)^{\frac{1}{\alpha}}}\) satisfies: \(\hat{\hat{V}}_{o}(\varpi(d), d) = \hat{\hat{V}}_{o}^{aut}\). Then the law of motion for the pessimist’s multiplier is given by:
\[
\xi'_{p}(\varpi; 0) = \max \left( e^{-\lambda_p} (1 - \varpi)^\rho \left( \frac{\hat{\hat{V}}_{p}(\varpi' (\varpi; 0))}{R\hat{\hat{V}}_{p}(\varpi')} \right)^{\rho-\alpha}, \xi_{p}(0) \right),
\]
in case of no disaster.
\[
\xi'_{p}(\varpi; 1) = \max \left( (1 - e^{-\lambda_p}) (1 - \varpi)^\rho \left( \frac{\hat{\hat{V}}_{p}(\varpi' (\varpi; 1))}{R\hat{\hat{V}}_{p}(\varpi')} \right)^{\rho-\alpha}, \xi_{p}(1) \right)
\]
in case of disaster.

(55)

where \(\varpi_{p}(d) = \frac{\xi_{p}^{\frac{1}{\rho}}}{\varpi_{p}(d)^{\frac{1}{\rho}} + \xi_{p}(d)^{\frac{1}{\rho}}}\) satisfies: \(\hat{\hat{V}}_{p}(1 - \varpi_{p}(d), d) = \hat{\hat{V}}_{p}^{aut}\). The recursive expression of continuation utilities is in equation (69). The next period’s optimist consumption share is the policy function to:
\[
\varpi'(\varpi; d) = \frac{\xi_{p}(\varpi; d)^{\frac{1}{\rho}}}{h(\varpi; d)}
\]
where \(h(\varpi, d) = \xi_{p}(\varpi; d)^{\frac{1}{\rho}} + \xi_{o}(\varpi; d)^{\frac{1}{\rho}}\) is defined in the usual way. Hence, we have
recovered the law of motion for the state variable. The continuation utilities (scaled by the aggregate endowment) satisfy the following functional equation:

\[
\hat{V}_o(\varpi, d) = \left[ (1 - \beta) (\varpi (1 + s(d)))^{1 - \rho} + \hat{\beta} (\mathcal{R}\hat{V}_o(\varpi'))^{1 - \rho} \right]^{\frac{1}{1 - \rho}}
\]

(56)

\[
\hat{V}_p(\varpi, d) = \left[ (1 - \beta) (\varpi (1 + s(d)))^{1 - \rho} + \hat{\beta} (\mathcal{R}\hat{V}_p(\varpi'))^{1 - \rho} \right]^{\frac{1}{1 - \rho}}.
\]

(57)

where the expected continuation utilities are given by:

\[
\mathcal{R}\hat{V}_o(\varpi') = \left[ e^{-(1-\alpha)J} \left( 1 - e^{-\lambda_o} \right) \hat{V}_o^{1-\alpha}(\varpi'(\varpi; 1), 1) + e^{-\lambda_o} \hat{V}_o^{1-\alpha}(\varpi'(\varpi; 0), 0) \right]^{\frac{1}{1 - \alpha}},
\]

\[
\mathcal{R}\hat{V}_p(\varpi') = \left[ e^{-(1-\alpha)J} \left( 1 - e^{-\lambda_p} \right) \hat{V}_p^{1-\alpha}(\varpi'(\varpi; 1), 1) + e^{-\lambda_p} \hat{V}_p^{1-\alpha}(\varpi'(\varpi; 0), 0) \right]^{\frac{1}{1 - \alpha}}.
\]

State Prices  We can also characterize state prices as a function of the aggregate state variable \( \varpi \):

\[
Q_t(\varpi; 0) = \hat{\beta} h(\varpi; 0)^\rho
\]

(58)

\[
Q_t(\varpi; 1) = \hat{\beta} h(\varpi; 1)^\rho e^{\alpha J}.
\]

(59)

We use \((\hat{P}_o, \hat{P}_p)\) to denote the ratio of net wealth to the aggregate endowment for the optimist (pessimist). This also implies means we can solve for the net wealth equations (scaled by the aggregate endowment) recursively:

\[
\hat{P}_o(\varpi) = \varpi + Q(\varpi; 1) e^{-J} \hat{P}_o(\varpi'(\varpi; 1)) + Q(\varpi; 0) \hat{P}_o(\varpi'(\varpi; 0)),
\]

\[
\hat{P}_p(\varpi) = 1 - \varpi + Q(\varpi; 1) e^{-J} \hat{P}_p(\varpi'(\varpi; 1)) + Q(\varpi; 0) \hat{P}_p(\varpi'(\varpi; 0)).
\]

The policy function for the optimist’s consumption share \( \varpi'(\varpi; d) \) and the pricing kernel \( Q(\varpi; d) \) provide a complete recursive characterization of a competitive equilibrium. If we start at some initial consumption share for the optimist, we can completely recover the equilibrium as defined in (\).

B  Proofs

• Proof of Lemma 3.1
Proof. Suppose that the following strict inequality holds
\[
\Pi(c)(\lambda_j)(z^t) > \Pi((1/2)e)(z^t),
\]
for some $z^t$. From the linearity of the pricing functional, we know that
\[
\Pi(c)(\lambda_o)(z^t) + \Pi(c)(\lambda_p)(z^t) = \Pi(e)(z^t),
\]
which implies, when combined with the previous equation, that for $i \neq j$:
\[
\Pi(c)(\lambda_i)(z^t) < \Pi((1/2)e)(z^t),
\]
but we can rule that out, because that violates $i$’s solvency constraint. Hence, no speculative trade is feasible. 

• Proof of Lemma 3.2

Proof. Suppose that the following strict inequality holds
\[
\Pi(c)(\lambda_j)(z^t) > \Pi((1/2)e)(z^t),
\]
for some $z^t$. From the linearity of the pricing functional, we know that
\[
\Pi(c)(\lambda_o)(z^t) + \Pi(c)(\lambda_p)(z^t) = \Pi(e(1 + s(d)))(z^t) > \Pi(e)(z^t),
\]
which when combined with the previous equation, does not imply that for $i \neq j$:
\[
\Pi(c)(\lambda_i)(z^t) < \Pi((1/2)e)(z^t),
\]
Hence, some speculative trade is feasible. 

• Proof of Lemma 3.3:

Proof. When the constraint in node $z^t$ does not bind, the Lagrangian multiplier is zero ($\mu_t(z^t) = 0$). From equation (13), equation 3.3 immediately follows. When the constraint in node $z^t$ does bind and $\mu_t(z^t) > 0$, the constraint holds with equality:
\[
\mu_t(z^t) \left( U_t(c; \xi_t)(\lambda)(z^t) - U((1/2)e)(\lambda)(z^t) \right).
\]

\[\]
• Proof of Lemma 3.4:

**Proof.** We let \( \hat{U}(\hat{c})(s^t) \) denote the lifetime expected continuation utility in node \( s^t \), under the new transition probabilities and discount factor, defined over consumption shares \( \{\hat{c}_t(s^t)\} \)

\[
\hat{U}(\hat{c})(z^t) = u(\hat{c}_t(z^t)) + \hat{\beta}(s_t) \sum_{s_{t+1}} \hat{\pi}(s_{t+1}) \left( e^{-\lambda} \hat{U}(\hat{c})(z^t, s_{t+1}, 0) + (1 - e^{-\lambda})e^{-(1-\alpha)J} \hat{U}(\hat{c})(z^t, s_{t+1}, 1) \right).
\]

We assume that the normal shocks are i.i.d. We conjecture that the equilibrium consumption shares do not depend on the history of aggregate normal shocks \( s^t \). In that case, we can state:

\[
\hat{U}(\hat{c})(d^t) = u(\hat{c}_t(d^t)) + \hat{\beta} \left( e^{-\lambda} \hat{U}(\hat{c})(d^t, 0) + (1 - e^{-\lambda})e^{-(1-\alpha)J} \hat{U}(\hat{c})(d^t, 1) \right),
\]

where we have used \( \sum_{s_{t+1}} \hat{\pi}(s_{t+1}) = 1 \). Given that the \( s^t \) shocks do not affect the ranking of consumption bundles, the equilibrium consumption shares will not depend on \( s^t \). \( \square \)

• Proof of Lemma A.1:

**Proof.** When the constraint in node \( z^t \) does not bind, the Lagrangian multiplier is zero \( (\mu_t(z^t) = 0) \). From equation 42, it follows that,

\[
\xi_t(z^t) = \xi_{t-1}(z^{t-1}) \beta \pi_{t+1}(z_t|z_{t-1}) \left( \frac{V_t(z^t)}{R_{t-1} V_t} \right)^{\beta-\alpha}.
\]

Note that \( \beta \pi(s_t|s_{t-1}) \) is common for all households and hence can be omitted. Hence, we get the expression in (A.1). When the constraint at \( t \) does bind and \( \mu_t(z^t) > 0 \), the constraint holds with equality. \( \square \)

• Proof of Lemma A.2:

**Proof.** We let \( \hat{V}(\hat{c})(s^t) \) denote the lifetime expected continuation utility in node \( s^t \), under the new transition probabilities and discount factor, defined over
consumption shares $\{\hat{c}_t(s^t)\}$:

$$
\hat{V}_t(\hat{c})(\lambda)(z^t) = \frac{V_t(c^t)}{e_t(z^t)} = \left[ (1 - \beta)\hat{c}_t^{1 - \rho} + \beta \left[ \mathcal{R}_t \frac{V_{t+1}}{e_t} \right]^{1 - \rho} \right]^{\frac{1}{1 - \rho}},
$$

where $\mathcal{R}$ denotes the following operator:

$$
\mathcal{R}_t \frac{V_{t+1}}{e_t} = \left( E^t \left[ \frac{e_{t+1}}{e_t} \hat{V}_{t+1} \right]^{1 - \alpha} \right)^{1/1 - \alpha}
$$

Hence, it follows that:

$$
\mathcal{R}_t \frac{V_{t+1}}{e_t} = \left[ \sum_{s_{t+1}} \pi(s_{t+1}) \gamma(s_{t+1})^{1 - \alpha} \left( e^{-\lambda} \hat{V}(\hat{c})(z^t, s_{t+1}, 0)^{1 - \alpha} + (1 - e^{-\lambda}) e^{-(1 - \alpha)J} \hat{V}(\hat{c})(z^t, s_{t+1}, 1)^{1 - \alpha} \right) \right]^{1/1 - \alpha}
$$

We conjecture that the equilibrium consumption shares do not depend on the history of aggregate normal shocks $s^t$. In that case, we can state the continuation utilities only as a function of the history of aggregate disasters. This implies that we can state the following expression:

$$
\hat{\mathcal{R}}_t \hat{V}_{t+1} = \beta \left( \sum_{s_{t+1}} \pi(s_{t+1}) \gamma(s_{t+1})^{1 - \alpha} \right)^{1/1 - \alpha}
$$

\[\times \left[ e^{-\lambda} \hat{V}(\hat{c})(z^t, 0)^{1 - \alpha} + (1 - e^{-\lambda}) e^{-(1 - \alpha)J} \hat{V}(\hat{c})(z^t, 1)^{1 - \alpha} \right]^{1/1 - \alpha}.
\]

\[\square\]

• Proof of Corollary 4.1:

\[\textit{Proof.}\] This follows immediately from the above. In autarchy, the pessimist gets $1/2$ in all future states. If $\varpi \leq 1/2$, the pessimist gets less than or exactly $1/2$ in all future states from staying in the risk sharing arrangement. Same argument applies to the optimist.

\[\square\]

• Proof of Lemma 4.1:

\[\textit{Proof.}\] We start from the definition of the lower and upper bounds on the optimist’s consumption shares and from the definition of autarchy. We know that
in case of a disaster realization, the optimist’s constraint binds next period, when she is currently in autarchy. We also know that in case of no disaster realization, the pessimist’s constraint binds next period, when she is currently in autarchy. This can be easily verified from the law of motion for the weights in equations 19 and 21. Hence, we obtain the following expressions for the difference between the continuation utility and the value of autarchy:

\[
\hat{V}_o(\varpi) - \hat{V}_o^{aut} = \frac{\varpi^{1-\alpha}}{1-\alpha} - \frac{0.5^{1-\alpha}}{1-\alpha} + \hat{\beta}e^{-\lambda_o} \left[ \hat{V}_o(\varpi'(\varpi; 0)) - \hat{V}_o^{aut} \right].
\]

(66)

\[
\hat{V}_p(\varpi) - \hat{V}_p^{aut} = \frac{1 - \varpi^{1-\alpha}}{1-\alpha} - \frac{0.5^{1-\alpha}}{1-\alpha} + \hat{\beta}e^{-(1-\alpha)J} (1 - e^{-\lambda_p}) \left[ \hat{V}_p(\varpi'(\varpi; 1)) - \hat{V}_p^{aut} \right].
\]

(67)

where we have used the result that the optimist’s constraint binds in the next period in case of a disaster realization, and similarly, the pessimist’s constraint binds in case of no disaster realization in the next period, when they start at autarchy in the current period. Note that when the constraint binds, the corresponding value equals autarchy. In case the constraint of the optimist does not bind, we know that the optimist’s consumption share evolves according to:

\[
\frac{\varpi'(\varpi; 0)}{1 - \varpi'(\varpi; 0)} = e^{\lambda_o - \lambda_o} \frac{\varpi}{1 - \varpi}.
\]

In case the constraint of the pessimist does not bind in the disaster state, we know that the optimist’s consumption share evolves according to:

\[
\frac{\varpi'(\varpi; 1)}{1 - \varpi'(\varpi; 1)} = \left( 1 - e^{-\lambda_o} \right)^{1/\alpha} \frac{\varpi}{1 - \varpi}.
\]

To solve for the lower bound from the first equation, we use a second order Taylor expansion of the optimist value function around the cutoff value:

\[
\hat{V}_o(\varpi'(\varpi; 0)) \approx \hat{V}_o(\varpi) + \hat{V}_o'(\varpi) (\varpi'(\varpi; 0) - \varpi) + \hat{V}_o''(\varpi) (\varpi'(\varpi; 0) - \varpi)^2
\]

\[
\hat{V}_p(\varpi'(\varpi; 0)) \approx \hat{V}_p(\varpi) + \hat{V}_p'(\varpi) (\varpi'(\varpi; 1) - \varpi) + \hat{V}_p''(\varpi) (\varpi'(\varpi; 1) - \varpi)^2
\]

We know that \(\hat{V}_o'(\varpi) = \varpi^{-\alpha}\), and that \(\hat{V}_p'(\varpi) = -(1-\varpi)^{-\alpha}\). Similarly, we know
that $\hat{V}''(\bar{\omega}) = -\alpha \bar{\omega}^{-1}$, and that $\hat{V}''(\bar{\omega}) = -\alpha (1 - \bar{\omega})^{-1}$. This delivers the result. The same procedure can be used to derive the pessimist’s equation. □

- **Proof of Lemma 7.1:**

  **Proof.** We start from the definition of the lower and upper bounds on the optimist’s consumption shares and from the definition of autarchy. We know that in case of a disaster realization, the optimist’s constraint binds next period, when she is currently in autarchy. We also know that in case of no disaster realization, the pessimist’s constraint binds next period, when she is currently in autarchy. This can be easily be verified from the law of motion for the weights in equations 19 and 21. Hence, we obtain the following expressions for the difference between the continuation utility and the value of autarchy:

  \[
  V_o(\bar{\omega})^{1-\rho} - (V^a_o)^{1-\rho} = \bar{\omega}^{1-\rho} - 0.5^{1-\rho} \\
  + \hat{\beta}(V^a_o)^{1-\rho} \left[ e^{-\lambda_o (V_o(\bar{\omega}'(0))/V^a_o)^{1-\alpha}} + (1 - e^{-\lambda_o}) e^{-(1-\alpha)J} \right]^{\frac{1-\rho}{\rho - \alpha}} \\
  - \hat{\beta}(V^a_o)^{1-\rho} \left[ e^{-\lambda_o} + (1 - e^{-\lambda_o}) e^{-(1-\alpha)J} \right]^{\frac{1-\rho}{\rho - \alpha}}. \\
  \tag{68}
  \]

  where we have used the result that the optimist’s constraint binds in the next period in case of a disaster realization.

  \[
  V_o(\bar{\omega})^{1-\rho} - (V^a_o)^{1-\rho} = \bar{\omega}^{1-\rho} - 0.5^{1-\rho} \\
  + \hat{\beta}(V^a_o)^{1-\rho} \left[ e^{-\lambda_o (V_o(\bar{\omega}'(0))/V^a_o)^{1-\alpha}} + (1 - e^{-\lambda_o}) e^{-(1-\alpha)J} \right]^{\frac{1-\rho}{\rho - \alpha}} \\
  - \hat{\beta}(V^a_o)^{1-\rho} \left[ e^{-\lambda_o} + (1 - e^{-\lambda_o}) e^{-(1-\alpha)J} \right]^{\frac{1-\rho}{\rho - \alpha}}. \\
  \tag{69}
  \]

  Note that when the constraint binds, the corresponding value equals autarchy. In case the constraint of the optimist does not bind, we know that the optimist’s consumption share evolves according to:

  \[
  \frac{\bar{\omega}'(\bar{\omega}; 0)}{1 - \bar{\omega}'(\bar{\omega}; 0)} = e^{\frac{\lambda_o - \lambda_o}{\rho}} \left( \frac{V_o(\bar{\omega}'(\bar{\omega}; 0))}{RV^a_o(\bar{\omega})} \right)^{(\rho - \alpha)/\rho} \frac{R V^p_o(\bar{\omega})}{V^p_o(\bar{\omega}'(\bar{\omega}; 0))} \\
  \frac{1}{1 - \bar{\omega}}. 
  \]

  In case the constraint of the pessimist does not bind in the disaster state, we

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know that the optimist’s consumption share evolves according to:

\[
\frac{\varpi'(\varpi; 1)}{1 - \varpi'(\varpi; 1)} = \left(\frac{1 - e^{-\lambda_o}}{1 - e^{-\lambda_p}}\right)^{1/\rho} \left(\frac{V_o(\varpi'(\varpi; 1))}{RV_o(\varpi)} + \frac{RV_p(\varpi)}{V_p(\varpi'(\varpi; 1))}\right)^{(\rho-a)/\rho} \frac{\varpi}{1 - \varpi}.
\]

To solve for the lower bound from the first equation, we use a second order Taylor expansion of the optimist value function around the cutoff value:

\[
\hat{V}_o(\varpi'(\varpi'; 0)) \approx \hat{V}_o(\varpi) + \hat{V}'_o(\varpi)(\varpi'(\varpi'; 0) - \varpi) + \hat{V}''_o(\varpi)(\varpi'(\varpi'; 0) - \varpi)^2
\]

We know that \(\hat{V}'_o(\varpi) = \varpi - \rho\), and that \(\hat{V}''_o(\varpi) = -\rho(1 - \varpi)^{-1}\). Similarly, we know that \(\hat{V}''_p(\varpi) = -\rho(1 - \varpi)^{-1}\). This implies that:

\[
\hat{V}_o(\varpi'(\varpi'; 0)) = V_o^\text{aut} + \varpi^{-\rho}(\varpi'(\varpi'; 0) - \varpi)(1 - \rho^{-1}(\varpi'(\varpi'; 0) - \varpi))
\]

This delivers the result. The same procedure can be used to derive the pessimist’s equation. \qed