Uncertainty-Induced Reallocations and Growth

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Abstract

Focusing on U.S. data, we show the existence of a significant positive link between uncertainty and reallocation from private to government capital. This link is important because an increase in the aggregate share of government capital forecasts lower medium-term growth. We rationalize these novel empirical findings in a production economy in which the representative agent has an explicit fear toward uncertainty and government capital is, in equilibrium, a safe asset.

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1 Introduction

Periods of high uncertainty are often associated with sizable and prolonged economic slow downs during which both private consumption and investment decline. Among others, Bloom et al. (2007) and Bloom (2009) suggest that this may be the result of a stronger incentive to wait before proceeding with irreversible production decisions.

In this study, we propose a novel perspective about the propagation of aggregate uncertainty shocks for private economic activity. Specifically, we connect the decline in private activity to the value of the option of accumulating government capital, i.e., investing in government infrastructure.

Focusing on U.S. data, we show the existence of a significant positive link between uncertainty and reallocation from private to government capital. This finding is important because our empirical tests suggest that this reallocation is a leading indicator of sluggish growth.

We rationalize these novel empirical findings in a production economy in which the representative agent has an explicit fear toward uncertainty and government capital is, in equilibrium, a safe asset. In other words, government capital offers low average returns and valuable insurance exactly when uncertainty increases.

We obtain this result by modifying a standard production economy model in three dimensions. First, we use recursive preferences so that uncertainty is directly priced. We calibrate the model in the spirit of Bansal and Yaron (2004) so that the agent dislikes high uncertainty.
Second, we add government capital and allow agents to modify its level subject to a convex adjustment cost specified so that government investment can be negative. We think of this assumption as capturing the ability of the private sector to convert government infrastructure into private productive capital.

Government capital is used for the production of a good that is consumed together with a private good. Since in the data government capital is approximately 25% of total capital, we assume that the representative agent has a bias for the private good.

These two assumptions enable us to think of our government capital as a storage technology. On average, government capital is not very productive, because government goods are not as valuable as private goods. On the other hand, government capital can be cheaply reallocated when needed.

Third, we assume that the government offers its own good in a competitive manner, i.e., charging just its marginal cost, whereas the private sector has monopoly power on the supply of private goods. Since public and private goods are bundled together by a CES aggregator, the demand of private good depends on the elasticity of substitution between private and government goods. In the spirit of Comin and Gertler (2006), we think of private monopolistic rents as a compensation for investment in innovation and hence a fundamental driver of growth. Consistent with the data and with a Comin and Gertler (2006) environment, we postulate a positive link between the relative size of private investments and expected future growth.

Under this assumption, the value of private capital depends on both the present value of its marginal productivity and the present value of future monopoly
rents (Hayashi 1982). With recursive preferences, the latter component is extremely sensitive to uncertainty shocks, implying that even small increases in uncertainty can depress the market value of private capital. This fact explains the reallocation motives in our model: when uncertainty increases, the monopoly rent channel induces the representative agent to reduce private capital and invest more in public capital. At the equilibrium, public capital offers a strong hedge against volatility shocks and is as safe as a risk-free bond. This reallocation, however, comes at the cost of dimming future growth.

From a quantitative point of view, our closed economy model reconciles the observed pattern of savings and private investments. When uncertainty spikes upward, our representative agent reduces private consumption in order to increase her precautionary savings. Simultaneously, private investment declines because the increase in private savings is dominated by a reduction of government savings driven by investment in public capital, consistent with the data.

Given this encouraging fit, we run a counterfactual welfare analysis. According to our setting, the benefits of having access to public capital to hedge exogenous uncertainty shocks can be as high as 10% of life-time consumption. This number should be interpreted as an upper bond, as in our model we abstract away from uncertainty stemming from government policy (see, for example, Pastor and Veronesi (2012, 2013), Kelly et al. (2013), Fernandez-Villaverde et al. (2015), and Baker et al. (2016)), lack of commitment (see, for example, Azzimonti et al. (2009) and Farhi et al. (2013)), and the nature of financing risk (see, among others, Lustig et al. (2008) and Berndt et al. (2012)).

Belo et al. (2013) and Belo and Yu (2013) examine the effects of government investment and spending on asset prices. Our work complements their findings
and highlights a new trade off between growth and government capital in times of higher uncertainty. Both our empirical focus on government capital and growth, and our attention to priced uncertainty shocks are distinct from the work of Baxter and King (1993).

According to our model, it is optimal for the government to expand its size in bad times, an outcome broadly consistent with that of new Keynesian models (for a recent example, see Christiano et al. (2011)). We differ from this literature for our risk-based approach and for our attention to the trade off between long-term growth and government size.

In the next section, we show our main empirical evidence. Section 3 describes the model and its calibration. We summarize our main results in section 4 and run sensitivity analysis in section 5. Section 6 concludes.

2 Empirical Evidence

In this section we show our main empirical findings. We start by looking at stylized patterns of relative investment in private and government capital across booms and recession periods. We then use a VAR approach to isolate the role of uncertainty shocks on investment reallocation. Finally, we demonstrate the link between the relative size of government capital and the long-run productivity growth rate.

Most of our data are well-known and their sources are standard (see Appendix A for a detailed description). Here we find it useful to specify that government capital data are reported in the National Income and Product Account (NIPA),
according to criteria described in Bureau of Economic Analysis (2014). Examples of expenditures included in our government capital measure are provided in table 1. We include both tangible and intangible investment. We also consider local infrastructure like, for example, water and sewer systems.

**TABLE 1: Components of Government Gross Investment**

<table>
<thead>
<tr>
<th>Component</th>
<th>Categories</th>
</tr>
</thead>
<tbody>
<tr>
<td>Structures</td>
<td>Buildings (residential, industrial, educational, hospital, and other)</td>
</tr>
<tr>
<td></td>
<td>Highways and streets</td>
</tr>
<tr>
<td></td>
<td>Sewer systems</td>
</tr>
<tr>
<td></td>
<td>Water systems</td>
</tr>
<tr>
<td></td>
<td>...</td>
</tr>
<tr>
<td>Equipment</td>
<td>Vehicles</td>
</tr>
<tr>
<td></td>
<td>Electronics</td>
</tr>
<tr>
<td></td>
<td>...</td>
</tr>
<tr>
<td>Intellectual property products</td>
<td>Software</td>
</tr>
<tr>
<td></td>
<td>R&amp;D</td>
</tr>
</tbody>
</table>

*Notes:* Component breakdown as seen in NIPA table 3.9.5. Examples are from Bureau of Economic Analysis (2014).

Our data are consistent with other sources explored by Aschauer (1988), Boskin et al. (1989), Peterson (1990), and Kamps (2004).

2.1 **Government Capital and Economic Fluctuations.**

In figure 1, we depict the time-series of the share of gross government investment to total gross investment, as well as the relative share of the stock of government capital to total capital in the economy. Two key features are worthy of notice. First, government capital represents on average about 25% of total capital in the economy, i.e., a sizable portion. Second, the relative share
of government investment is strongly countercyclical, as it tends to quickly increase during recession periods.

To better assess the countercyclicality of government capital, in figure 2 we focus on the average path of government investment and capital stock across recessions. In both panels, we consider data for NBER recession periods starting from 1950 so that we consider the same sample across quarterly and annual data. Including pre-1950 recessions would make our results for quarterly data even stronger. Our plots show that the reallocation toward government capital is both substantial and prolonged, as it often persists beyond the end of the recession.

Furthermore, there is something unique about government investment that goes above and beyond the countercyclical behavior of total government expen-
\[ (a) \text{Relative Gross Investment} \left( \frac{I_g}{I_g + I_p} \right) \]
\[ (b) \text{Relative Capital Stock} \left( \frac{K_g}{K_g + K_p} \right) \]

**FIG. 2. Government Capital During Recessions.** In both panels, we report the average path of the variable of interest across the latest 10 NBER recessions starting from 1950. Time \( t = 1 \) is the first period of the recession period. The left panel shows the average recession path for quarterly gross government investment \( I_g \) as a share of total domestic investment \( I_g + I_p \), which also includes private gross investment \( I_p \). The right panel shows the average recession path for the annual stock of government capital \( K_g \) as a share of the total domestic stock of capital \( K_g + K_p \), which also includes the private capital stock \( K_p \). Our data sources are detailed in Appendix A.

During recession periods, government expenditure increases relative to total private expenditure (i.e., gross private investment plus consumption) mainly through the public investment channel (figure 3(a)).

This dynamic behavior has been even more pronounced during the Great Recession, with almost no sign of reversal three years after the beginning of the recession (figure 3(b)). In order to better understand what drives this reallocation, in the next section we examine the role of uncertainty shocks through a VAR approach.
FIG. 3. Reallocation During Recessions. In the left panel, we report the average path of the variables of interest across the latest 10 NBER recessions starting from 1950. Time $t = 1$ is the first quarter of the recession. The right panel focuses on the Great Recession only (2007:Q4–2009:Q2). Total federal expenditure is denoted by $G$. Total private expenditure is the sum of private consumption ($C$) and gross private investment ($I_p$). The subcomponent of government expenditure associated to gross government investment is denoted as $I_g$. Our data sources are detailed in Appendix A.

2.2 Government Capital and Uncertainty Shocks.

A broad measure of uncertainty. In order to evaluate the sources of the aforementioned capital reallocation, we start by estimating the following VAR(1):

$$Y_t = \mu_Y + \Phi Y_t + \Sigma u_t$$

in which

$$Y_t = \begin{bmatrix} \Delta a_t \\ iV \omega_l t \\ I_{g,t}/(I_{p,t} + I_{g,t}) \end{bmatrix},$$

where $\Delta a_t$, $iV \omega_l t$, and $I_{g,t}/(I_{p,t} + I_{g,t})$ are variables of interest.
where \( I_{g,t}/(I_{p,t} + I_{g,t}) \), \( \Delta a_t \), and \( iVol_t \), denote government gross investment share, productivity growth, and integrated volatility for stock market returns, respectively. Integrated volatility is a broad measure of uncertainty with two relevant advantages: (i) it is easy to compute (see Appendix A for details), and (ii) it is available on long samples. Productivity captures shocks to the level of economic activity.

Throughout this study, we do not need to take a stand on causality! across uncertainty and level shocks. We identify impulse responses through a lower diagonal Cholesky decomposition and point out that their pattern does not change whether level shocks or volatility shocks are ranked first. For the purpose of our analysis, both methods produce similar orthogonalized level and volatility shocks. Using our estimated VAR, we trace the response of the government investment share to both productivity and volatility shocks in figure 4(a). For robustness, we consider both quarterly and annual data.

Positive productivity shocks imply a relative shift of resources away from the public sector, whereas the opposite is true for adverse uncertainty shocks. Both of these adjustments are sizable and statistically significant over a 4-year horizon. Most importantly, focusing on the magnitude of the dynamic reallocation we note that uncertainty shocks are as relevant as level shocks.

The results depicted in figure 4(a) are based on a sample starting in 1969. We choose this starting date because our alternative measure of uncertainty, i.e., productivity volatility, is not available prior to that year. In Appendix B, we show that our results are unchanged when we use our longer quarterly sample starting in 1947 (see appendix figure B1).
This figure shows the response of gross government ($I_g$) and private investment ($I_p$) to both productivity growth and volatility shocks. The left panels refer to government investment as a share of total domestic investment. All results are based on the VAR specified in equations (1)–(2), in which we use stock market integrated volatility to measure uncertainty. In the right panels, investment series are HP filtered and replace the government investment share in our VAR. Our sources are detailed in Appendix A. Our sample starts in 1969 and ends in 2011. Data are quarterly unless otherwise specified. Confidence intervals are adjusted for heteroscedasticity.

Turning our attention to figure 4(b), we note that volatility shocks are associated to a significant increase in government investment and a simultaneous decline in private investment. Hence our reallocation evidence is not driven solely by private investment adjustments. With respect to positive level shocks, instead, both government and private investment increase. In this case, the decline of the government investment share is just a reflection of the stronger adjustment in private investment.
Productivity uncertainty. Integrated stock market volatility results from many different economic phenomena that are not solely related to uncertainty shocks. As an example, integrated volatility may be driven by sentiment shocks, or time-varying market frictions. In order to focus on a fundamental measure of economy activity uncertainty, we extract time-varying volatility from productivity growth.

Specifically, in the spirit of Bansal and Shaliastovich (2013), we form the following array of forecasting variables

\[ F_t = [y_t(1), y_t(2), ... y_t(6), inf_t, pd_t, iVol_t], \tag{3} \]

where \( y(m) \) is the yield of a US Treasury bond with maturity \( m \), \( inf \) denotes inflation, and \( pd \) refers to the price-dividend ratio. We demean productivity growth (\( \Delta a \)) through the following forecasting regression

\[ \Delta a_{t+1} = \mu + x_t + \epsilon_{a,t+1} \tag{4} \]
\[ x_t = b_x F_t \tag{5} \]

and filter expected volatility by specifying the following projection:

\[ |\epsilon_{a,t+1}| = vol_t + resid_{t+1} \tag{6} \]
\[ vol_t = b_0^v + b_{vol} F_t. \tag{7} \]

We jointly estimate the system of equations (3)–(7) and report summary results for both quarterly and annual data in table 2. A standard Wald test rejects the null hypothesis that there is no predictability in productivity volatility.
**Table 2: Productivity Uncertainty**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>($\rho_v$)</td>
<td>($\sigma_v$)</td>
<td>($H_0 : b^i_v = 0 \forall i$)</td>
</tr>
<tr>
<td>Quarterly</td>
<td>0.75</td>
<td>0.15</td>
<td>25.55</td>
</tr>
<tr>
<td></td>
<td>(0.15)</td>
<td>(0.05)</td>
<td>[0.00]</td>
</tr>
<tr>
<td>Annual</td>
<td>0.65</td>
<td>0.40</td>
<td>20.63</td>
</tr>
<tr>
<td></td>
<td>(0.32)</td>
<td>(0.52)</td>
<td>[0.01]</td>
</tr>
</tbody>
</table>

*Notes:* This table reports results from estimating the system of equations (3)–(7) augmented with the following representation for log-volatility

$$\log(\text{vol}_{t}) = c_v + \rho_v \log(\text{vol}_{t-1}) + \sigma_v \epsilon_{v,t} + b_v |sr\epsilon_{a,t} + b_v |lr\epsilon_{x,t},$$

in which $\epsilon_{v,t}$ refers to a standardized volatility-specific shocks, as we control for both short-run productivity shocks ($\epsilon_{a,t}$) and growth news shocks ($\epsilon_{x,t}$). Numbers in parentheses are Newey-West adjusted standard errors. Numbers in square brackets are $p$-values for the null hypotheses that productivity volatility is constant ($H_0 : b^i_v = 0 \forall i = 1, \ldots, 9$).

Both with quarterly and annual data, our volatility measure is persistent and volatile.

We show our fitted volatility processes in figure 5 and make three remarks. First, our estimates replicate the time-pattern documented in the literature for other macro quantities, as we capture both the post-1980 Great Moderation and the subsequent turbulence period. Second, productivity volatility is countercyclical. Third, when we use annual data we detect more time-varying volatility than in the estimation with quarterly data. We conjecture that this is driven by the fact that annual data are less noisy than quarterly data. We find these results reassuring as they confirm that our productivity-based measure of uncertainty is reliable.
FIG. 5. Productivity uncertainty. This figure shows quarterly and annual conditional volatility of productivity growth. We recover these measures by jointly estimating the system of equation (3)–(7) by GMM. Our sources are detailed in Appendix A. Our sample ranges from 1969:Q1 to 2011:Q1 (1969-2011) for our quarterly (annual) data.

We proceed by estimating the VAR specified in equation (1) using the following vector of variables

\[
Y_t = \begin{bmatrix}
\Delta a_t \\
x_t \\
v_{ol_t} \\
I_{g,t}/(I_{p,t} + I_{g,t})
\end{bmatrix},
\]

where the productivity long-run component \( x_t \) is added solely to control for growth news shocks.

As in the case of integrated volatility, uncertainty shocks promote a strong and persistent reallocation of resources toward public capital. This effect is even more pronounced when we focus on annual data, as productivity volatility swings are more sizable at an annual frequency. The prediction of our annual data VAR are subject to larger confidence intervals than those from our
Response of Government Investment Share (productivity volatility).

The figure in panel (a) shows the response of the government investment share to both productivity growth and volatility shocks. Gross government investment ($I_g$) is divided by total domestic investment ($I_g + I_p$), which also includes private gross investment ($I_p$). All results are based on the VAR specified in equations (1) and (8), in which we measure uncertainty by productivity volatility estimated as in equations (3)–(7). The panels on the left (right) are based on quarterly (annual) data. In panel (b), we replace expected long-run volatility with the residual of equation (6). Our sources are detailed in Appendix A. Our sample starts in 1969 and ends in 2011. Confidence intervals are adjusted for heteroscedasticity.

quarterly data, but this is not surprising given our short-sample. Both with quarterly and annual data, adverse uncertainty shocks promote a reallocation that is significant at least over a 2-year horizon.

In figure 6(b), we replace our measure of expected volatility with the residual of the estimation of equations (6)–(7). We note that there is no significant reallocation in this case, implying that what really matters for investment flows is the extent of expected long-term uncertainty. Most importantly, in figure 7 we
show that our reallocation results apply also to the subcomponent of aggregate investment related to R&D activities. This result is obtained after augmenting our VAR with the Baa spread, a common index of credit conditions. To the extent to which R&D fosters innovation and growth, these results suggest that expansions of government capital may come at the cost of slowing down future growth.

Given these considerations, in the next section we explore the link between relative size of government and private capital and long-term growth. We are interested in understanding whether there is a correlation between uncertainty shocks, relative contraction of the private sector, and sluggish growth.

### 2.3 Relative Government Size and Long-Run Growth.

In this section, we investigate whether the relative size of government capital is related to future productivity growth. Let $\Delta a_{t,t+10}$ be the 10-year-ahead cumulative aggregate productivity growth at time $t$. The results depicted in figure 8 suggest the existence of a negative correlation between government capital share and future long-term growth.

In order to formally test this negative link, we run the following forecasting regression:

$$
\frac{1}{10} \Delta a_{t,t+9} = \mu + b_x F_t + b_y \frac{k_{g,t}}{k_{tot,t}} + FinControls_t + \epsilon_{a,t+1} \tag{9}
$$

where $F_t$ refers to the forecasting variables described in equation (3), and $\frac{k_{g,t}}{k_{tot,t}}$ denotes the public-to-total capital stock ratio. The goal of this auxiliary re-
Fig. 7. Reallocation and R&D. This figure shows the response of government investment ($I_g$) and private R&D investment ($I_{R&D}$). All results are based on the VAR specified in equations (1) and (8), in which (i) we measure uncertainty by productivity volatility estimated as in equations (3)–(7), (ii) replace the last variable with HP-filtered investment levels, and (iii) we control for credit condition by adding the Baa spread. Our sources are detailed in Appendix A. Our quarterly sample starts in 1969 and ends in 2011. Confidence intervals are adjusted for heteroscedasticity.

Regression is to unveil a possible connection between the size of public capital and 10-year growth above and beyond what is captured by the procedure in Bansal and Shaliastovich (2013). Furthermore, we control for credit and liquidity conditions by considering also the national financial conditions indexes of the Chicago Fed and either the Aaa or the Baa spread.
The vertical axis refers to 10-year forward moving average for aggregate productivity growth ($\Delta a_{t+10}$). Our data sources are detailed in Appendix A.

Table 3 shows the results from estimating (9) across different combinations of controls. Across all cases, the public-to-total capital ratio has strong and negative predictive power, meaning that periods in which public capital is expanded relative to total capital lead times of sluggish long-term growth. Furthermore, the increment in the adjusted $R^2$ due to the government capital ratio is always substantial.

In an attempt to corroborate further the relevance of the link between government capital size and productivity growth, we also estimate a VAR with our 10-year-ahead cumulative aggregate productivity growth and the government-to-total capital ratio over the 1952:q1–2005:q4 sample. We then use the VAR to form dynamic forecasts of long-term productivity growth under two scenarios.

In the first scenario, we take the level of the government capital share from the data over the remaining of our sample (2006:q1–2015:q4). In the second
### TABLE 3: Relative Government Size and Long-Run Growth

<table>
<thead>
<tr>
<th>Variables</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{k_g}{k_{total}} )</td>
<td>(-0.28^{***})</td>
<td>(-0.24^{***})</td>
<td>(-0.26^{***})</td>
<td>(-0.12^{***})</td>
</tr>
<tr>
<td></td>
<td>(0.05)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td>(0.04)</td>
</tr>
<tr>
<td>Credit Tightness Control: Aaa10y</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Credit Tightness Control: Baa10y</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Predicting Factors</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Adj ( R^2 )</td>
<td>0.49</td>
<td>0.51</td>
<td>0.32</td>
<td>0.22</td>
</tr>
<tr>
<td>Adj ( R^2 ) w/o ( \frac{k_g}{k_{total}} )</td>
<td>0.26</td>
<td>0.41</td>
<td>0.12</td>
<td>0.00</td>
</tr>
</tbody>
</table>

Notes: This table reports the estimated coefficients \( \hat{b}_g \) for the regression described in equation (9). \( \frac{k_{gt}}{k_{total,t}} \) denotes the public-to-total capital stock ratio. The other forecasting variables in \( F_t \) are specified in equation (3). Our controls for financial conditions always include the national financial conditions indexes of the Chicago Fed. Our sources are detailed in Appendix A. Our annual sample starts in 1969 and ends in 2011. Confidence intervals are adjusted for heteroscedasticity. One, two, and three asterisks denote 10%, 5%, and 1% significance, respectively.

In the case, instead, we set the capital ratio from 2006:q1 onward equal to its unconditional mean so that it plays no role in the future forecasted pattern. As shown in figure 9, the increased size of government capital observed during the Great Recession capital may be a leading indicator of a sizable and persistent growth slow-down.

These empirical findings have a broad and significant implication, namely, they suggest that the reallocation to the government sector may be costly because it is typically followed by prolonged growth slow-downs. In the next section, we explain these findings in a production economy in which the representative agent has an explicit fear toward uncertainty and government capital is, in equilibrium, a safe asset which offers less volatility at the cost of lower growth.
The dashed line refers to 10-year forward moving average for aggregate productivity growth ($\frac{\Delta a_{t,t+10}}{10}$) over the 1952:q1–2005:q4 sample. The solid line shows the prediction of a VAR with aggregate productivity growth, government capital share, and augmented with and financial controls (the national financial conditions indexes of the Chicago Fed and the Baa spread). The dotted line refers to a counterfactual scenario in which the government capital share is set to its own long-run mean in 2006:q1. Our data sources are detailed in Appendix A.

3 **The Model**

We start by describing the representative household problem and then describe both the private and public production sectors.
3.1 Household problem

The objective of the representative agent is to maximize her utility

\[ U_t = \left[ (1 - \delta) \tilde{C}_t^{1 - \frac{1}{\psi}} + \delta \left( E_t[U_{t+1}^{1-\gamma}] \right)^{1 - \frac{1}{1-\gamma}} \right]^{\frac{1}{1-\gamma}} \tag{10} \]

where the consumption bundle \( \tilde{C}_t \) is

\[ \tilde{C}_t = C_t - \bar{\omega}_{l,p} \frac{S L_t N_{p,t}^{\omega_l}}{\omega_l} - \bar{\omega}_{l,g} \frac{S L_t N_{g,t}^{\omega_l}}{\omega_l}, \]

in which \( C_t \) denotes the consumption of the final good, \( N_{p,t} \) is the labor supply in private sector, and \( N_{g,t} \) is the labor supply in government sector. To ensure balance growth with Greenwood et al. (1988) preferences, we introduce an exogenous preference shock process, \( S L_t \), cointegrated with productivity. Specifically, we define:

\[ sla_t := \frac{S L_t}{A_t} \]

and assume

\[ sla_t = (1 - \theta_{sla})\mu + (1 - \theta_{sla})(sla_{t-1} - \Delta a_t). \]

We set \( \theta_{sla} \approx 1 \) so that \( S L_t \) mimics an exogenous linear trend.

The budget constraint of the representative household is:

\[ C_t + T_t + S_t V_t^{ex} = S_{t-1}(V_{p,t}^{ex} + D_t) + (w_{p,t} N_{p,t} + w_{g,t} N_{g,t}) / \bar{p}_t, \tag{11} \]

where \( T_t \) is a lump-sum transfer from the household to the government; \( S_t \in [0, 1] \) represents the percentage ownership of private capital; \( V_t^{ex} \) is the ex-dividends value of private capital; \( D_t \) is the corporate payout; \( w_p \) is the wage
paid by the private firm; and \( w_g \) is the wage paid by the government. As described in detail in the next section, \( \tilde{p}_t \) is the relative price of the final good with respect to the numeraire (government) good. This change of unit is required because all variables are expressed in terms of the final good.

**Optimality.** The optimal investment strategy implies that:

\[
V_t^{ex} = E_t \left[ M_{t+1}(V_{t+1}^{ex} + D_{t+1}) \right],
\]

where \( M_{t+1} \) is the IMRS of the agent in final consumption units. The optimal supply of labor in private and public sectors implies:

\[
w_{p,t}/\tilde{p}_t = \bar{\omega}_{l,p} SL_t N_{p,t}^{\omega_l - 1}
\]

\[
w_{g,t}/\tilde{p}_t = \bar{\omega}_{l,g} SL_t N_{g,t}^{\omega_l - 1}.
\]

### 3.2 Final good producer

The final good in the economy is a bundle of private goods, \( Y_{p,t} \), and public goods, \( Y_{g,t} \)

\[
Y_t = \left[ \omega_t Y_{p,t}^{\frac{1}{\tau} - \frac{1}{\tau}} + (1 - \omega_t) Y_{g,t}^{\frac{1}{\tau} - \frac{1}{\tau}} \right]^{\frac{1}{\frac{1}{\tau} - \frac{1}{\tau}}}. \tag{12}
\]

The elasticity of substitution between these two goods is determined by \( \tau \). The relative demand of the private good with respect to the public good is also determined by the possibly time-varying preference process \( \omega_t \). For parsimony, we assume that

\[
\omega_t = \omega e^{\phi_t \nu_{t-1}},
\]

21
where $v_{t-1}$ is the time-varying volatility of productivity, and $\phi_v$ is a non-positive constant. This expression captures the idea that public goods may be more desirable in high-uncertainty periods. In our sensitivity analysis, we show that most of our results do not require the presence of a preference shocks, i.e., they hold even when $\phi_v = 0$.

We assume the existence of a competitive producer that solves the following profit maximization problem taking prices as given:

$$\max_{Y_{p,t},Y_{g,t}} \tilde{p}_t Y_t - p_t Y_{p,t} - Y_{g,t},$$

(13)

where the price of the government good is normalized to one (numeraire). Optimality implies:

$$\frac{\omega_t}{1 - \omega_t} \left( \frac{Y_{p,t}}{Y_{g,t}} \right)^{-\frac{1}{\tau}} = p_t.$$  

(14)

The relative price of the final good w.r.t the numeraire good:

$$\tilde{p}_t \equiv \frac{\partial Y_{g,t}}{\partial Y_t} = \frac{1}{1 - \omega_t} \left( \frac{Y_{g,t}}{Y_t} \right)^{\frac{1}{\tau}}.$$  

\subsection{3.3 Productivity}

Private firm productivity growth ($\Delta a_p$) is modeled as follows:

$$\Delta a_{p,t} = \mu + x_{t-1} + e^{v_{t-1}} \sigma \epsilon_{a,t} + \phi_{growth} [(k_{p,t} - k_{g,t}) - (\bar{k}_p - \bar{k}_g)]$$

(15)

$$x_t = \rho x_{t-1} + e^{v_{t-1}} \sigma \epsilon_x,t$$

$$v_t = \rho_v v_{t-1} + \sigma_v \epsilon_{v,t} + \beta_{v,a} \epsilon_{a,t} + \beta_{v,x} \epsilon_x,t$$

$$\epsilon_{a,t}, \epsilon_{x,t}, \epsilon_{v,t} \sim i.i.d. N(0, 1),$$
where $\epsilon_{a,t}$ is a short-run shock, $x_t$ is long-run component that is responsible for small but persistent fluctuations in the drift of productivity, and $e^{\nu t}$ introduces time-varying volatility. Let $k_{p,t}$ and $k_{g,t}$ denote the logarithm of private and government capital, respectively. Similarly, let $\bar{k}_p$ and $\bar{k}_g$ indicate their steady state values. By setting the parameter $\phi_{growth}$ to a positive value we can capture our empirical evidence on the negative link between relative size of government capital and long-term growth.

We think of this specification as the reduced form of an endogenous growth model in which long-term growth is promoted solely through innovations generated by the private sector. In this kind of setting, reallocating investment to government capital would endogenously produce a growth slow down. To keep our analysis focused on the role of government capital, we abstract away from endogenizing R&D decisions, but we believe that an adaptation of the Comin and Gertler (2006) model would deliver qualitatively similar results.

We complete this part of the model by letting the government productivity process, $A_{g,t}$, be a slow moving process cointegrated with $A_{p,t}$ so that

$$ar_t \equiv \log \frac{A_{g,t}}{A_{p,t}} = (1 - \rho_{ar})ar_{t-1} + (1 - \rho_{ar})(\mu - \Delta a_{p,t}) + \rho_{ar}ar_c,$$

and hence

$$\Delta a_{g,t} = \mu + \rho_{ar}(\Delta a_{p,t} - \mu) + \rho_{ar}(ar_c - ar_{t-1}) + \rho_{ar}\Delta a_{p,t},$$

where $ar_c$ denotes the average productivity gap between the two sectors. Setting $\rho_{ar} \approx 1$ implies $\Delta a_{g,t} \approx \Delta a_{p,t}$, that is, the two sectors are subject to the
same extent to the same shocks. When $\rho_{ar} \approx 0^+$, government productivity $a_{g,t}$ grows at the steady rate $\mu$.

### 3.4 Public Firm

The public firm provides the public good to the final producer in a competitive fashion, i.e., by taking the price of its own good as given. The public firm can use private goods to accumulate capital and solves the following dynamic problem:

$$V_{g,t} = \max_{Y_{g,t}, K_{g,t+1}, N_{g,t}, I_{g,t}} Y_{g,t} - \tilde{p}_t I_{g,t} - w_{g,t} N_{g,t} + E_t[M_{t+1} V_{g,t+1}]$$

subject to

$$Y_{g,t} \leq F_{g,t} = K_{g,t}^{\alpha_g} (A_{g,t} N_{g,t})^{1-\alpha_g} \quad (\lambda_{g,t})$$

$$K_{g,t+1} \leq \left( 1 - \delta + \Gamma_{g,t} \left( \frac{I_{g,t}}{K_{g,t}} \right) \right) K_{g,t} \quad (q_{g,t}) ;$$

where $\lambda_{g,t}$ is the shadow price of the government good, and $q_{g,t}$ is the shadow value of government capital. The adjustment cost function is defined as follows,

$$\Gamma_{g,t} \left( \frac{I_{g,t}}{K_{g,t}} \right) = \frac{\alpha_{g,1}}{1 - \frac{1}{\xi_g} \left( \frac{I_{g,t}}{K_{g,t}} + 1 \right)}^{1 - \frac{1}{\xi_g}} + \alpha_{g,0} ,$$

and allows for reversibility of government investment.\(^1\) This assumption captures the ability of the private sector to use infrastructure generated by the government.

\(^1\)The constant $\alpha_{g,0}$ is set so that at the deterministic steady state $\frac{I_{g,t}}{K_{g,t}} = \Gamma_g$. The coefficient $\alpha_{g,1}$ is set so that at the deterministic steady state $\Gamma'_g = 1$.  

24
Here $V_{g,t}$ is expressed in terms of the numeraire good, i.e., the government good. Consistent with this change of units,

$$M_{t+1}^g \equiv M_{t+1} \frac{\partial Y_{t+1}}{\partial Y_{g,t+1}} \frac{\partial Y_{g,t+1}}{\partial Y_{g,t}}.$$ 

The optimality condition with respect to $I_{g,t}$ pins down the marginal value of public capital:

$$q_{g,t} = \frac{\tilde{p}_t}{\Gamma_{g,t}},$$

(17)

where $\tilde{p}_t$ accounts for the fact that investment is made using the final good. The optimality condition with respect to $Y_g$ implies that

$$\lambda_{g,t} \equiv 1,$$

i.e., the marginal cost must be equal to the price of the good. As a result, the optimal demand of labor implies

$$w_{g,t} = F_{g,N_{g,t}},$$

and the optimality with respect to $K_{g,t+1}$ is

$$q_{g,t} = E_t \left[ M_{t+1}^g \frac{\partial V_{g,t+1}}{\partial K_{g,t+1}} \right]$$

$$= E_t \left[ M_{t+1}^g \left( F_{g,K_{g,t+1}} + F_{g,A_{g,t+1}} \frac{\partial A_{g,t+1}}{\partial K_{g,t+1}} + \left( 1 - \delta + \Gamma_{g,t+1} - \frac{I_{g,t+1}}{K_{g,t+1} \Gamma_{g,t+1}} q_{g,t+1} \right) \right) \right];$$

where

$$\frac{\partial A_{g,t+1}}{\partial K_{g,t+1}} := -A_{p,t+1} e^{ar_{t+1}} \rho_{ar} \phi_{growth} \frac{\phi_{growth}}{K_{g,t+1}}.$$
In our no-arbitrage equation, the term $F_{g,A_{g,t+1}}\frac{\partial A_{g,t+1}}{\partial K_{g,t+1}}$ captures the negative effect of an increase of the relative size of the government capital stock on TFP growth.

### 3.5 Private Firm

The private firm has monopolistic power over the private good that she produces. The private firm has also access to the same technology of the public firm and hence it could produce the public good as well. Since the price of the public good is set equal to its marginal cost, there is no scope for positive profits creation, i.e., marginal profits are null and hence the firm is indifferent between utilizing the public technology or not. Without loss of generality, we assume that the private firm specializes in the production of the private good.

We assume that the private firm buys production inputs, investment goods, and labor, in a competitive way, i.e., by taking their price as given. Hence the problem of the private firm is as follows:

$$V_{p,t} = \max_{N_{p,t}, I_{p,t}, Y_{p,t}, K_{p,t+1}} \left[ \omega_t \left( \frac{Y_{p,t}}{Y_{g,t}} \right)^{-\frac{1}{\tau}} Y_{p,t} - \tilde{p}_t I_{p,t} - w_{p,t} N_{p,t} + E_t[M_{g,t+1}^q V_{p,t+1}] \right]$$

subject to

$$Y_{p,t} \leq F_{p,t} = K_{p,t}^{\alpha p} (A_{p,t} N_{p,t})^{1-\alpha p}$$

$$K_{p,t+1} \leq \left( 1 - \delta + \Gamma_{p,t} \left( \frac{I_{p,t}}{K_{p,t}} \right) \right) K_{p,t}$$
where \( \lambda_{p,t} \) is the shadow marginal cost, and \( q_{p,t} \) is the shadow value of private capital. The adjustment cost function is specified as in Jermann (1998):

\[
\Gamma_{p,t} = \frac{\alpha_{p,1}}{1 - \frac{1}{\xi_p}} \left( \frac{I_{p,t}}{K_{p,t}} \right)^{1 - \frac{1}{\xi_p}} + \alpha_{p,0},
\]

and the optimality condition with respect to \( I_{p,t} \) pins down the marginal value of private capital:

\[
q_{p,t} = \frac{\tilde{p}_t}{\Gamma_{p,t}}.
\]  

(20)

Optimizing with respect to \( Y_p \), yields the following:

\[
p_t = \frac{1}{1 - 1/\tau} \lambda_{p,t},
\]

i.e., there is a mark-up between the price of the private good and its marginal costs. As a result, the optimal demand of labor implies

\[
\frac{w_{p,t}}{p_t} = (1 - 1/\tau) F_{p,N_{p,t}},
\]

i.e., labor compensation is lower than labor productivity.

The determination of the optimal amount of \( K_{p,t+1} \) implies

\[
q_{p,t} = E_t \left[ M_{t+1}^q \frac{\partial V_{p,t+1}}{\partial K_{p,t+1}} \right] = E_t \left[ M_{t+1}^q \left( (1 - 1/\tau)p_t F_{p,K_{p,t+1}} + \frac{\partial p_{t+1}}{\partial A_{p,t+1}} \frac{\partial A_{p,t+1}}{\partial K_{p,t+1}} \right) \right] + E_t \left[ M_{t+1}^q \left( 1 - \delta + \Gamma_{p,t+1} - \frac{I_{p,t+1}}{K_{p,t+1}} \Gamma'_{p,t+1} \right) q_{p,t+1}, \right]
\]  

(21)
where

$$\frac{\partial p_{t+1} F_{p,t+1}}{\partial A_{p,t+1}} \frac{\partial A_{p,t+1}}{\partial K_{p,t+1}} = (1 - 1/\tau(1 - \rho_{ar}))(1 - \alpha_{p}p_{t+1}) \frac{\phi_{growth} F_{p,t+1}}{K_{p,t+1}}.$$

The term $\frac{\partial p_{t+1} F_{p,t+1}}{\partial A_{p,t+1}} \frac{\partial A_{p,t+1}}{\partial K_{p,t+1}}$ captures the positive effect of an increase in private capital on TFP growth. In addition, the marginal product of capital, $p_{t} F_{p,K_{p,t+1}}$, is reduced by $1/\tau$, i.e., the recursion accounts for the fact that an extra unit of capital simultaneously increases quantities produced and decreases their unit price $p_{t}$.

If we let $V_{t}^{R}$ be the present value of the monopolistic rents,

$$V_{t}^{R} = \frac{1}{\tau}p_{t} F_{p,t} + E_{t} \left[ M_{t+1}^{g} V_{t+1}^{R} \right],$$

and $V_{t}^{s}$ be the present value of the productivity spillovers,

$$V_{t}^{s} = \frac{\partial p_{t} F_{p,t}}{\partial A_{p,t}} \frac{\partial A_{p,t}}{\partial K_{p,t}} + E_{t} \left[ M_{t+1}^{g} V_{t+1}^{s} \right],$$

the ex-dividends value of the private firm (in final-good units) can be recovered as in Hayashi (1982),

$$V_{p,t}^{ex} = \frac{1}{p_{t}} \left[ q_{p,t} + E_{t} \left[ \frac{M_{t+1}^{g} V_{t+1}^{R}}{K_{p,t+1}} \right] - E_{t} \left[ M_{t+1}^{g} V_{t+1}^{s} \right] \right] K_{p,t+1}.$$
3.6 Market Clearing and Payout Flows

The final good sector clears when

\[ Y_t = C_t + I_{p,t} + I_{g,t}. \]  \hspace{1cm} (22)

In the equity market, the following holds:

\[ S_t = 1, \]  \hspace{1cm} (23)

and the corporate pay-out (in numeraire units) is

\[
\tilde{p}_t D_t = p_t Y_{p,t} - w_{p,t} N_{p,t} - \tilde{p}_t I_{p,t} \\
= p_t F_{p,K_{p,t}} K_{p,t} + \frac{1}{\tau} p_t F_{p,N_{p,t}} N_{p,t} - \tilde{p}_t I_{p,t} \\
= p_t F_{p,K_{p,t}} K_{p,t}(1 - 1/\tau) + \frac{1}{\tau} p_t F_{p,t} - \tilde{p}_t I_{p,t}.
\]  \hspace{1cm} (24)

The government payout is positive when the tax flow is negative, i.e., when the agent receives a net subsidy. In numeraire units, we have

\[
-\tilde{p}_t T_t = Y_{g,t} - w_{g,t} N_{g,t} - \tilde{p}_t I_{g,t} \\
= \alpha_g Y_{g,t} - \tilde{p}_t I_{g,t}. \]  \hspace{1cm} (25)
3.7 Calibration and Solution Method

We report our quarterly calibration in table 4. Many of the parameters are standard. The preference parameters are set in the spirit of the long-run risk literature. See, among others, Bansal and Yaron (2004).

Both the capital income share and the depreciation rate of capital are set to the same values across sectors. We choose numbers as in Croce (2014). We also set the elasticity of the adjustment cost functions to be the same. We choose a moderate value that let investment be as volatile as in the data.

In the bundle that aggregates private and government goods, the weight $\omega$ is chosen to match the relative size of private and government investment. The preference shock coefficient $\phi_v$ is set to match as much as possible the impulse responses obtained from our empirical VAR. The elasticity of substitution $\tau$ is set to have a profit share comparable to the data.

Productivity is calibrated according to our quarterly data. Specifically, we jointly estimate the system of equations (3)–(7) augmented with the following equations

$$\log(\text{vol}_t) = c_v + \rho_v \log(\text{vol}_{t-1}) + \sigma_v \epsilon_{v,t} + b_v|sr\epsilon_{a,t} + b_v|lr\epsilon_{x,t}$$  \hspace{1cm} (26)  

$$x_t = \rho x_{t-1} + \epsilon_{x,t} + b_x|sr\epsilon_{a,t}.$$  \hspace{1cm} (27)

The parameters relevant for short- and long-run risk are consistent with prior studies (Croce 2014) and fit our estimates. The parameter $b_v, sr$ accounts for the negative correlation between relative volatility and short-run shocks in the
data. Both the persistence and the magnitude of time-varying volatility are consistent with our findings in table 2.

The persistence of the long-run component, $\rho$, is estimated to be 0.88 with a standard error of 0.04. The relative standard deviation of the long-run shocks, $\sigma_x/\sigma_a$, is estimated to be 0.28 with a standard error of 0.05. We adopt a conservative calibration in this dimension. Both $b_{x,cr}$ and $b_{v,lr}$ are estimated to be small and play no crucial role. The growth coefficient $\phi_{growth}$ is set in order to replicate the long-run response of output detected in our empirical VAR.

Private and public productivity share the same dynamics, although public productivity is set to a lower average level. This is consistent with the data provided by the BEA. The model is solved with a third-order perturbation method.
<table>
<thead>
<tr>
<th>Preferences</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Relative Risk Aversion</td>
<td>(γ) 10</td>
</tr>
<tr>
<td>Intertemporal Elasiticity of Substitution</td>
<td>(ψ) 2</td>
</tr>
<tr>
<td>Subjective Discount Rate</td>
<td>(β) 0.973^{1/4}</td>
</tr>
<tr>
<td>Labor Elasticity</td>
<td>(ω_l) 1.5</td>
</tr>
<tr>
<td>Cointegration Labor Preference Shock</td>
<td>(θ_{sla}) 0.1</td>
</tr>
<tr>
<td>Public Good Preference Shock Coefficient</td>
<td>(φ_v) -0.015</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Production</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Private Sector Capital Share</td>
<td>(α_p) 0.3</td>
</tr>
<tr>
<td>Public Sector Capital Share</td>
<td>(α_g) 0.3</td>
</tr>
<tr>
<td>Private Capital Depreciation Rate</td>
<td>(δ_n) 0.06/4</td>
</tr>
<tr>
<td>Public Capital Depreciation Rate</td>
<td>(δ_s) 0.06/4</td>
</tr>
<tr>
<td>Private Sector Adjustment Cost Elasticity</td>
<td>(ξ_p) 3.5</td>
</tr>
<tr>
<td>Public Sector Adjustment Cost Elasticity</td>
<td>(ξ_g) 3.5</td>
</tr>
</tbody>
</table>

| Final Good Production | | |
|-----------------------|-------|
| Private Good Bias | (ω) 0.6 |
| Private/Public Elasticity of Substitution | (τ) 5 |

| Private Productivity | | |
|----------------------|-------|
| Unconditional Growth | (μ) 0.018/4 |
| Short-Run Vol. | (σ_a) 0.045/2 |
| Relative Long-Run Vol. | (σ_x/σ_a) 0.2 |
| Volatility of Log-Volatility | (σ_v) 0.15 |
| Long-Run Growth Persistence | (ρ) 0.94 |
| Relative Log-Volatility Persistence | (ρ_v) 0.74 |
| Relative Log-Volatility Short Run Exposure | (β_v,a) -3.5 |
| Relative Log-Volatility Long Run Exposure | (β_v,x) 0 |
| Growth Coefficient | (φ_{growth}) 0.05 |

| Public Productivity | | |
|---------------------|-------|
| Cointegration Speed | (ρ_{ar}) 1 |
| Average Level Gap | (a_{rc}) -0.43 |

*Notes:* This table reports our benchmark quarterly calibration.
4 Results

In this section, we use our benchmark model to study both new unconditional moments and to quantify the benefits of government capital through a counterfactual analysis. We also show the relevance of both recursive preferences and volatility shocks to generate the reallocation observed in the data.

4.1 Benchmark Model

The mechanism. In figure 10, we depict the response of variables of interest to both short-run productivity shocks and volatility shocks. We note several points. First, with respect to a short-run shock, our model behaves similarly to a standard business cycle model, as private consumption, investment, and output simultaneously expand. As in the data, a positive short-run shock produces a reallocation toward the private sector that, upon impact, increases (depresses) the shadow value of private (public) capital.

An adverse volatility shock, instead, produces a contraction in private economic activity and promotes a reallocation toward government capital. Consistent with the data, both private consumption and private investment fall. In this case, thanks to the reallocation effect the value of public capital increases and hence provides an hedge against volatility. The value of private capital, in contrast, decreases sharply because rents are very sensitive to volatility shocks. This explains why private capital average-$q$ declines so much compared to marginal-$q$. 
**Fig. 10. Impulse Responses.** This figure shows percentage deviations from steady state. Our benchmark calibration is reported in table 4. The dashed line refers to the model with no time-varying volatility ($\sigma_v = 0$).

To better quantify the performance of our model, in table 5 we show a comprehensive list of moments generated through simulations. The top portion of the table shows standard moments for private macro aggregates. Our model matches very well all these well known figures.
<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Benchmark</th>
<th>NP</th>
<th>No Vol</th>
<th>CRRA, NP</th>
<th>CRRA, NP</th>
<th>Small Gov</th>
<th>Small, NP</th>
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<tbody>
<tr>
<td>$\sigma(\Delta y)$ (%)</td>
<td>4.85 (0.90)</td>
<td>4.85</td>
<td>4.60</td>
<td>4.33</td>
<td>6.15</td>
<td>4.63</td>
<td>9.92</td>
<td>5.96</td>
</tr>
<tr>
<td>$\sigma(\Delta c)/\sigma(\Delta y)$</td>
<td>0.61 (0.11)</td>
<td>0.72</td>
<td>0.70</td>
<td>0.69</td>
<td>1.12</td>
<td>0.72</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>$\sigma(\Delta i_{tot})/\sigma(\Delta y)$</td>
<td>2.14 (0.11)</td>
<td>2.50</td>
<td>2.53</td>
<td>2.55</td>
<td>11.66</td>
<td>2.16</td>
<td>3.47</td>
<td>1.75</td>
</tr>
<tr>
<td>$E[I_p/Y]$ (%)</td>
<td>15.06 (0.79)</td>
<td>13.56</td>
<td>13.52</td>
<td>13.52</td>
<td>12.99</td>
<td>15.64</td>
<td>17.09</td>
<td>19.36</td>
</tr>
<tr>
<td>$\rho(\Delta c, \Delta i)$</td>
<td>0.84 (0.41)</td>
<td>0.72</td>
<td>0.71</td>
<td>0.69</td>
<td>0.82</td>
<td>0.75</td>
<td>0.58</td>
<td>0.93</td>
</tr>
<tr>
<td>$ACF(\Delta c)$</td>
<td>0.35 (0.12)</td>
<td>0.52</td>
<td>0.57</td>
<td>0.57</td>
<td>0.50</td>
<td>0.59</td>
<td>0.50</td>
<td>0.47</td>
</tr>
<tr>
<td>$E[I_p/Y_{tot}]$ (%)</td>
<td>26.37 (2.36)</td>
<td>28.09</td>
<td>28.11</td>
<td>28.30</td>
<td>-48.18</td>
<td>27.65</td>
<td>8.96</td>
<td>1.05</td>
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<tr>
<td>$\sigma(I_p/Y_{tot})$ (%)</td>
<td>12.43 (4.06)</td>
<td>11.10</td>
<td>10.98</td>
<td>9.95</td>
<td>226.91</td>
<td>7.69</td>
<td>46.80</td>
<td>3.55</td>
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<tr>
<td>$E[I_p/Y]$ (%)</td>
<td>5.35 (0.51)</td>
<td>5.71</td>
<td>5.71</td>
<td>5.68</td>
<td>-2.49</td>
<td>6.21</td>
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<td>0.24</td>
</tr>
<tr>
<td>$\sigma(I_p/Y)$ (%)</td>
<td>2.79 (1.00)</td>
<td>2.60</td>
<td>2.59</td>
<td>2.42</td>
<td>3.97</td>
<td>2.18</td>
<td>2.30</td>
<td>0.66</td>
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<tr>
<td>$E\left[\frac{\Delta K}{\Delta Y_{tot}}\right]$ (%)</td>
<td>101.84</td>
<td>101.70</td>
<td>101.90</td>
<td>116.85</td>
<td>82.05</td>
<td>6.47</td>
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<tr>
<td>$\sigma(\Delta K/\Delta Y_{tot})$ (%)</td>
<td>10.37</td>
<td>10.22</td>
<td>9.41</td>
<td>42.49</td>
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<td>3.64</td>
<td>0.98</td>
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<tr>
<td>$E\left[\frac{\Delta K}{\Delta K_{tot}}\right]$ (%)</td>
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<td>30.88</td>
<td>30.80</td>
<td>30.64</td>
<td>28.88</td>
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<td>1.57</td>
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<tr>
<td>$E\left[\frac{\Delta K}{\Delta K_{tot}}\right]$ (%)</td>
<td>5.04</td>
<td>3.66</td>
<td>3.61</td>
<td>3.76</td>
<td>-13.51</td>
<td>0.81</td>
<td>2.63</td>
<td>1.83</td>
</tr>
<tr>
<td>$\sigma(r_{p, ex})$ (%)</td>
<td>19.72 (1.78)</td>
<td>14.44</td>
<td>14.40</td>
<td>14.24</td>
<td>21.25</td>
<td>13.85</td>
<td>14.69</td>
<td>14.01</td>
</tr>
<tr>
<td>$E[r_{p, ex}]$ (%)</td>
<td>-0.04</td>
<td>-0.04</td>
<td>-0.03</td>
<td>-1.33</td>
<td>0.00</td>
<td>0.00</td>
<td>-0.82</td>
<td>-0.36</td>
</tr>
<tr>
<td>$\sigma(r_{p, ex})$ (%)</td>
<td>0.19</td>
<td>0.19</td>
<td>0.19</td>
<td>1.34</td>
<td>0.23</td>
<td>1.67</td>
<td>1.67</td>
<td>1.03</td>
</tr>
<tr>
<td>$E[r_{f}]$ (%)</td>
<td>0.49 (0.50)</td>
<td>2.02</td>
<td>2.03</td>
<td>1.99</td>
<td>3.64</td>
<td>3.66</td>
<td>2.07</td>
<td>2.39</td>
</tr>
<tr>
<td>$\sigma(r_{f})$ (%)</td>
<td>2.75 (0.48)</td>
<td>0.98</td>
<td>0.78</td>
<td>0.72</td>
<td>2.38</td>
<td>0.91</td>
<td>5.39</td>
<td>1.87</td>
</tr>
</tbody>
</table>

**Notes:** Empirical moments are computed using annual data from 1929 to 2014. All data sources are discussed in Appendix A. Numbers in parentheses are standard errors adjusted for heteroscedasticity. The entries for the model are obtained by repetitions of small samples. Our baseline calibration is detailed in table 4. The utility-productivity ratio, $U/A$, is measured at both the stochastic (sss) and the deterministic (dss) steady state. NP denotes the model with no preference shocks. NI denotes the model with no spillover of private capital on future expected productivity growth. Small refers to a configuration with a nearly null government sector.
In the second portion of this table, we focus on moments that are specific to government capital. The model delivers an investment flow that is on average slightly higher than in the data but statistically plausible. In terms of volatility, the model produces results aligned to our empirical evidence.

Thanks to the fact that government capital provides insurance against volatility shocks, its implied market value is substantial and predicted to be approximately 102% of total output. As it can be observed in the bottom portion of this table, private capital is very risky and requires a levered premium of 3.7%, whereas government capital is overall as safe as a risk-free bond. This result is driven by the interplay of reallocation motives and riskiness of the present value of the monopolistic rents.

**Impulse Responses.** To better compare our model to the data, in figures 11 and 12 we compare the VAR and the model impulse responses to an adverse volatility shock.

The reallocation toward government capital is broadly consistent with both quarterly and annual data. When we focus on government-to-total investment at a quarterly frequency, our model predicts an excessive reaction. Since the mean reversion in the model is stronger than in the data, time-aggregating our simulated data to annual frequency produces a smoother response.

Figure 12 shows that the reallocation produced by our model can replicate the paths for output obtained with our VAR approach. More precisely, our model can produce a significant slow-down in both aggregate and private output over the short- and medium-term.
**FIG. 11. Impulse Responses to a Volatility Shock (I).** This figure shows the response of detrended private consumption and investment to volatility shocks. All results are based on the VAR specified in equations (1) and (8), in which we measure uncertainty by productivity volatility estimated as in equations (3)–(7). The panels on the left (right) are based on quarterly (annual) data. Our sources are detailed in Appendix A. Our sample starts in 1969 and ends in 2011. Confidence intervals are adjusted for heteroscedasticity. The entries from our quarterly model are time-aggregated to an annual frequency in the panels on the right side.

5 Sensitivity Analysis.

**No Preference Shock.** We depict the impulse responses of our model with no preference shock ($\phi_v = 0$) in figure C2, in the appendix. In the benchmark model, the preference shock helps us to better match the data by promoting an immediate response upon the arrival of the uncertainty shock, as it promotes reallocation of labor.
FIG. 12. Impulse Responses to a Volatility Shock (II). This figure shows the response of detrended aggregate output and private output to volatility shocks. All results are based on the VAR specified in equations (1) and (8), in which we measure uncertainty by productivity volatility estimated as in equations (3)–(7). The panels on the left (right) are based on quarterly (annual) data. Our sources are detailed in Appendix A. Our sample starts in 1969 and ends in 2011. Confidence intervals are adjusted for heteroscedasticity. The entries from our quarterly model are time-aggregated to an annual frequency in the panels on the right side.

As shown in figure C3, without preference shock the model still features a relevant uncertainty-induced reallocation toward government capital. From a quantitative perspective, however, the magnitude of the reallocation toward government capital is smaller than under the benchmark calibration.

Further, figure C3 shows that our model with no preference shock can still replicate part of the long term slow down in macroeconomic quantities that we observe in the data. Indeed, with recursive preferences the representative household is willing to trade off future growth for lower immediate uncertainty...
by investing in safe government capital. These considerations are consistent with the results reported in table 5.

**CRRA preferences.** We depict the impulse responses of our model with CRRA and no preference shock in figure C4. The figure on the left shows that our results are not just driven by risk-aversion. Having an IES greater than one is important to replicate key features of the data. Indeed, when the agents have CRRA preference with high risk aversion and low IES, the model cannot explain the fall in private investment due to high uncertainty. Further, this calibration predicts that private capital should be a good hedge against uncertainty shocks, a counterfactual result.

The panels on the right hand side of figure C4 show another important tension. If we set the IES to 2 and adopt CRRA preferences, the model output resembles the case in which there are no volatility shocks. The reason is that with a relative risk aversion of 1/2, the concerns for risk are basically null. These considerations explain why the moments produced under CRRA preferences deteriorate compared to those of the benchmark model (see table 5).
The role of government capital. The last two columns of table 5 focus on the economy in which we assume that there is complete bias toward private goods, i.e., government capital is null. Let $WC$ be the welfare cost of uncertainty in life-time consumption units. In our model, $WC$ can be computed as follows:

$$WC := \log \frac{U^{dss}}{A} - \log \frac{U^{sss}}{A}$$

where $U/A^{dss} (U/A^{sss})$ is the utility ratio computed at the deterministic (stochastic) steady state.

When we compare $WC$ with and without government capital, we find a difference of 10% of life-time consumption in the benchmark model with preference shocks. Specifically, the reallocation of resources toward government capital promoted by uncertainty shocks improves welfare by 10%. This number declines to 5% without the preference shock, suggesting that the benefits of the pure hedging attainable through government capital are valuable but limited.

6 Conclusions

We propose a novel way to think about economic slow downs associated to high-uncertainty periods. Specifically, focusing on U.S. data we show the existence of a significant positive link between uncertainty and reallocation from private to government capital. Our empirical tests suggest that this reallocation is a leading indicator of sluggish growth.

We rationalize these novel empirical findings in a production economy in which (a) the representative agent has an explicit fear toward uncertainty; and (b)
in periods of high uncertainty, the agent exercises the option to devote more resources to government capital.

In normal times, investing in government capital is costly because government services are less valuable than the private ones. During periods of high uncertainty, in contrast, private capital is perceived as extremely risky, as the private sector has monopoly power and the present value of monopoly rents is highly exposed to uncertainty shocks. The resulting reallocation of resources toward government capital generates a temporary decline in private activity consistent with the data.

Future work should focus on the interplay between government investment and distortionary taxation. Furthermore, since government capital is related to uncertainty, it should be used to explain the cross-section of equity returns.
References


Appendix A: Data Description

NIPA. The national income and product accounts (NIPAs) are a set of economic accounts produced by the Bureau of Economic Analysis (BEA). See Bureau of Economic Analysis (2014) for more underlying details on the construction of the data series.

Government Investment \((I_g)\). Data are from the NIPA table 3.1. The quarterly data series begins in 1947:Q1 and our downloaded sample ends in 2015:Q2. Government gross investment consists of spending by both general government and government enterprises for fixed assets that benefit the public or that assist government agencies in their productive activities. Put another way, government gross investment is a measure of the additions to, and replacements of, the stock of government-owned fixed assets. It consists of investment by both general government and government enterprises in structures (such as highways and schools), in equipment (such as military hardware), and in intellectual property products (software and research and development), and it includes own-account investment by government. See Bureau of Economic Analysis (2014) for more details.

Private Investment \((I_p)\). Fixed private investment data are from the NIPA table 1.1.5. The quarterly data series begins in 1947:Q1 and our downloaded sample ends in 2015:Q2. See Bureau of Economic Analysis (2014) for more details.

Government Capital \((K_g)\) and Private Capital \((K_p)\). Capital stock data are from the NIPA table 5.10. The annual data series begins in 1951 and our downloaded sample ends in 2013. We specifically use the data series for fixed
assets (structures, equipment, and intellectual property products) and thus our total capital stock \((K_g + K_g)\) does not include inventories. Capital stocks are accumulated totals computed from gross investment, consumption of fixed capital, and other adjustments. See Bureau of Economic Analysis (2014) for more details.

**Perconsal Consumption Expenditures** \((C)\). Data are from the NIPA table 1.1.5. The quarterly data series begins in 1947:Q1 and our downloaded sample ends in 2015:Q3. The annual data series begins in 1929 and our downloaded sample ends in 2014.

**Government Expenditures and Investment** \((G)\). Data are from the NIPA table 1.1.5. The quarterly data series begins in 1947:Q1 and our downloaded sample ends in 2015:Q3. The annual data series begins in 1929 and our downloaded sample ends in 2014. Compared to government investment, this data series also includes government expenditures.

**Gross Domestic Product** \((Y)\). Data are from the NIPA table 1.1.5. The quarterly data series begins in 1947:Q1 and our downloaded sample ends in 2015:Q3. The annual data series begins in 1929 and our downloaded sample ends in 2014.

**Total Factor Productivity Growth** \((\Delta a)\). Business sector TFP data are from the Federal Reserve Bank of San Francisco. The quarterly data series begins in 1947:Q2 and our downloaded sample ends in 2015:Q2.

**Integrated Volatility.** We compute our quarterly integrated volatility measure as 
\[
\sqrt{66 \times \frac{1}{N} \sum_{i=1}^{N} (r_{m,i} - r_{f,i})^2}
\]
where \(N\) is the number of daily observations in a given quarter and \(r_{m,i} - r_{f,i}\) is the market excess return for a given day.
The resulting quarterly series spans 1926:Q3 to 2014:Q3. Market excess return data were downloaded from the Kenneth R. French Data Library.

**Real Personal Consumption Expenditures.** Data are from NIPA table 1.1.6. Units are billions of chained 2009 dollars. The quarterly data are seasonally adjusted. The quarterly data series begins in 1947:Q1 and our downloaded sample ends in 2015:Q3. The annual data series begins in 1929 and our downloaded sample ends in 2014.

**Real Private Investment.** Data are from the NIPA table 1.1.6. Units are billions of chained 2009 dollars. The quarterly data are seasonally adjusted. The quarterly data series begins in 1947:Q1 and our downloaded sample ends in 2015:Q3. The annual data series begins in 1929 and our downloaded sample ends in 2014. So we use the total gross investment series, which includes both fixed and inventory investment. The fixed investment series, directly comparable to \( I_p \), is only available from 1999.

**Real Government Investment.** Data are from the NIPA table 3.9.1. Units are percent change from the previous period. The quarterly data are seasonally adjusted. The quarterly data series begins in 1947:Q1 and our downloaded sample ends in 2015:Q3. The annual data series begins in 1929 and our downloaded sample ends in 2014. Neither data series in chained 2009 dollars is available prior to 1999, so we use these percent change data series to construct the series of levels.

**Growth Rates for Real Private Investment and Real Government Investment.** We compute annual real growth rates using constructed real investment series. The only real investment series available from the BEA with
data back to 1929 is for total (i.e. the sum of fixed and inventory) private investment. We convert the annual series for nominal private fixed investment and nominal government investment, which both start in 1929, to real figures using the implied deflator between nominal and real total private investment.

**Price-Dividend Ratio.** Price and dividend data are from Robert Shiller’s website [http://www.econ.yale.edu/~shiller/data.htm](http://www.econ.yale.edu/~shiller/data.htm). These monthly data series begin in 1871:M1 and our downloaded sample ends in 2014:M6. We compute a quarterly price-dividend ratio data series by dividing the third month’s price by the sum of dividends in each quarter. See the website for more details on the underlying data construction.


**Inflation.** Data are from Ibbotson Associates. The monthly data begins in 1926:M1 and our downloaded sample ends in 2013:M12. We compound the monthly figures in each quarter to create a quarterly series.

**Private Research and Development (R&D) Investment.** Data are from the NIPA table 1.5.5. Units are billions of dollars. The quarterly data are seasonally adjusted. The quarterly data series begins in 1947:Q1 and our downloaded sample ends in 2015:Q3. The annual data series begins in 1929 and our downloaded sample ends in 2014.
Real Private Research and Development (R&D) Investment. The nominal series values are converted to real values using the implied deflator between the reported nominal and real private investment.

Moody’s Seasoned Aaa/Baa Corporate Bond Yield Spreads. Data are from Federal Reserve Bank of St. Louis Economic Database (FRED). The spreads are computed relative to the 10-year Treasury Constant Maturity. The monthly data series begin in 1953:M4 and our downloaded sample ends in 2016:M9. The quarterly data series are computed as the quarterly averages of the monthly series.

Chicago Fed National Financial Conditions Index (NFCI) and Adjusted National Financial Conditions Index (ANFCI). Data are from Federal Reserve Bank of Chicago website (https://www.chicagofed.org/publications/nfci/index). The NFCI is a weighted average of a large number of variables (105 measures of financial activity) each expressed relative to their sample averages and scaled by their sample standard deviations. The ANFCI removes the variation in the individual indicators attributable to economic activity and inflation before computing the index. The weekly data series begin on January 5, 1973 and our downloaded sample ends on February 10, 2017. The quarterly data series are computed as the quarterly averages of the weekly series.
Appendix B: Additional Empirical Results

(a) Response of $\frac{I_g}{I_g + I_p}$ to Productivity Shocks

(b) Response of $\frac{I_g}{I_g + I_p}$ to Uncertainty Shocks

**Fig. B1.** Response of Government Investment Share (integrated volatility). See note for Fig. 4(a). The only difference is that the sample starts in 1947.
Appendix C: Additional Figures

**Fig. C2. Impulse Responses.** This figure shows percentage deviations from steady state. Our benchmark calibration is reported in table 4. The dashed line refers to the model with no preference shock ($\phi_v = 0$).
This figure shows the response of the government-to-private investment (panel a), and private and total output (panel b) to uncertainty shocks occurred during the Great Recession (i.e., shocks fitted after 2007:Q4 (2007) from quarterly (annual) data). The entries for the data are based on the VAR specified in equations (1) and (8), in which we measure uncertainty by productivity volatility estimated as in equations (3)–(7). We adopt both quarterly and annual data. Our sources are detailed in Appendix A. Our sample starts in 1969 and ends in 2011. Confidence intervals are adjusted for heteroscedasticity. The entries from our quarterly model are time-aggregated to an annual frequency when compared to annual data.
**Fig. C4. Impulse Responses with CRRA.** This figure shows percentage deviations from steady state. Our benchmark calibration is reported in table 4. The dashed line refers to the model with CRRA preferences and no preference shock.