This paper examines how extrinsic and intrinsic macroeconomic uncertainty shocks impact the real economy, inflation, and the yield curve. To this end, we build a Markov-switching New Keynesian model with endogenous bond risk premia and a monetary policy rule that experiences stochastic and recurrent shifts between active and passive regimes. Using a novel risk-adjusted log-linear solution method featuring an iterative procedure for solving for endogenous macroeconomic volatility, and estimating the model using Bayesian methods, our results highlight that selecting a parameter space over an indeterminate region allows the model to explain salient features of term premia dynamics. The asymmetric responses of nominal yields to macroeconomic uncertainty across policy regimes also provide sharper identification of the regimes. Counterfactual analysis illustrates how the instability created by indeterminacy led to sharp increase in term premia in the 70s.

*Duke University
†London Business School. hkung@london.edu
‡London Business School
§We thank
1 Introduction

Nominal yields encode information about expectations about inflation, economic growth, and the stance of monetary policy. Persistent episodes where the monetary authority does not aggressively adjust nominal interest rates sufficiently in response to changes in inflation leads to indeterminacy regarding the price level. Long spells at the zero lower bound (ZLB) across the world during the Great Recession are potential examples where the dynamics of real activity and inflation are subject to nonfundamental uncertainty. The propagation of fundamental and nonfundamental uncertainty shocks can significantly differ in passive policy regimes, which can alter the dynamics and co-movement between inflation and output. Therefore, the presence of indeterminacy and sunspot uncertainty are also reflected in the pricing and dynamics of the term structure of interest rates. In this paper, we quantitatively explore how indeterminacy arising from monetary policy and the presence of sunspot fluctuations help to explain the risk properties of nominal yields over various maturities.

We build and estimate a small-scale Markov-switching New Keynesian model with several novel features. First, the representative agent is assumed to have Epstein-Zin preferences. These preferences, in conjunction with low-frequency movements in inflation and long-run growth prospects that are negatively related, produce positive and sizable nominal term premia (e.g., Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2013)). Second, the monetary authority follows an interest rule that is subject to stochastic and recurrent changes between active and passive regimes captured by a Markov chain. The active regime is when monetary policy takes an aggressive stance towards targeting inflation deviations by adjusting the short-term nominal rate. The passive regime is one in which the monetary authority not aggressive adjusting interest rates to stabilize inflation. Importantly, the shifts in regime also induce changes in uncertainty. Third, our model features uncertainty shocks to fundamentals (preference, monetary policy, and technology shocks) and to nonfundamentals (sunspot shocks).

We use a risk-adjusted log-linear solution method that allows for endogenous bond risk premia.\footnote{See, for example, Backus, Routledge, and Zin (2010).} Using the determinacy criteria from Farmer, Waggoner, and Zha (2009) for forward-
looking Markov-switching rational expectations models, we can partition the parameter space into indeterminate and determinate regions.


2 Model

2.1 Household

Assume that the household has recursive utility over streams of consumption \( C_t \) and labor \( L_t \):

\[
U_t = u(C_t, L_t) + \beta_t \left( E_t \left[ U_{t+1}^{1-\theta} \right] \right)^{\frac{1}{1-\psi}}
\]

\[
u(C_t, L_t) = \frac{C_t^{1-1/\psi}}{1-1/\psi} - \tau_0 N_t^{1-1/\psi} L_t^{1+\tau} \frac{1}{1+\tau}
\]

where \( \gamma \) is the coefficient of risk aversion, \( \psi \) is the elasticity of intertemporal substitution, \( \theta \equiv 1 - \frac{1-\gamma}{1-1/\psi} \) is a parameter defined for convenience. The stochastic process \( \beta_t \) captures the agent’s time preference and follows:

\[
\log(\beta_t) = (1 - \rho_\beta) \log(\bar{\beta}) + \rho \log(\beta_{t-1}) + \sigma_{\beta,x_t} \epsilon_{\beta,t}
\]
where $\epsilon_{\beta,t}$ is iid $N(0,1)$. We define variable $b_t = \log \beta_t$, $b_{ss} = \log \bar{\beta}$ which follows

$$b_{t+1} = (1 - \rho_{\beta})b_{ss} + \rho_{\beta} b_t + \sigma_{\beta,v_t+1} \varepsilon_{\beta,t+1}$$

$v_t$ - Markov-switching process, which determines volatility regime. The time $t$ budget constraint of the household is

$$P_t C_t + \frac{B_{t+1}}{R_{t+1}} = D_t + W_t L_t + B_t$$

where $P_t$ is the nominal price of the final good, $B_{t+1}$ are nominal one-period bonds, $R_{t+1}$ is the gross nominal interest rate set at time $t$ by the monetary authority, $D_t$ is nominal dividend income received from the intermediate firms, $W_t$ is the nominal wage rate, and $L_t$ is labor supplied by the household. The household’s intertemporal condition is

$$1 = E_t \left[ M_{t+1} \frac{P_t}{P_{t+1}} \right] R_{t+1}$$

where

$$M_{t+1} = \beta_t \left( \frac{C_{t+1}}{C_t} \right)^{-1/\psi} \left( \frac{U_{t+1}}{E_t U_{t+1}} \right)^{-\theta}$$

is the stochastic discount factor. The intratemporal condition is

$$\frac{W_t}{P_t} = \tau_0 N_t^{1-1/\psi} L_t^\tau$$

See appendix for details.

### 2.2 Final Goods

A representative firm produces the final (consumption) good in a perfectly competitive market. The firm uses a continuum of differentiated intermediate goods $X_{i,t}$ as input in the CES
production technology

\[ Y_t = \left( \int_0^1 X_{i,t}^{\frac{1}{1+\lambda_{p,t}}} \, di \right)^{1+\lambda_{p,t}} \]

where \( \lambda_{p,t} \) determines elasticity of substitution between intermediate goods. \( \lambda_{p,t} \) follows AR(1) process in logs.

\[ \log \lambda_{p,t} - \log \bar{\lambda}_p = \rho_p (\log \lambda_{p,t-1} - \log \bar{\lambda}_p) + \sigma_{p,v_t} \varepsilon_{p,t} \]

Denote \( \tilde{\chi}_t = \log \lambda_{p,t} - \log \bar{\lambda}_p \). Then

\[ \tilde{\chi}_t = \rho_p \tilde{\chi}_{t-1} + \sigma_{p,v_t} \varepsilon_{p,t} \]

The profit maximization problem of the firm yields the following isoelastic demand schedule with price elasticity \( \nu = \frac{1+\lambda_{p,t}}{\lambda_{p,t}} \)

\[ X_{i,t} = Y_t \left( \frac{P_{i,t}}{P_t} \right)^{-\frac{1+\lambda_{p,t}}{\lambda_{p,t}}} \]

where \( P_t \) is the nominal price of the final good and \( P_{i,t} \) is the nominal price of intermediate good \( i \). The inverse demand schedule is

\[ P_{i,t} = P_t Y_t^{\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} X_{i,t}^{-\frac{\lambda_{p,t}}{1+\lambda_{p,t}}} \]

### 2.3 Intermediate Goods

The intermediate goods sector is characterized by a continuum of monopolistic competitive firms. Each intermediate goods firm produces \( X_{i,t} \) using labor \( L_{i,t} \) with constant returns to scale technology:

\[ X_{i,t} = Z_t L_{i,t} \]
where $Z_t$ is an aggregate technology shock that contains transitory and permanent components:

$$Z_t = e^{a_t + n_t}$$

$$a_t = \rho_a a_{t-1} + \sigma_{a,t} \epsilon_{a,t}$$

$$\Delta n_t = \mu + x_{t-1}$$

$$x_{t+1} = \rho x_t + \sigma_{x,t} \epsilon_{x,t+1} + \theta_{\beta,x} \sigma_{x,t} \epsilon_{\beta,t+1} + \theta_{\alpha,x} \sigma_{x,t} \epsilon_{\alpha,t+1} + \theta_{\chi,x} \sigma_{x,t} \epsilon_{\chi,t+1}$$

where $\epsilon_{a,t}, \epsilon_{x,t} \sim iid N(0,1)$, $\mu$ is the unconditional mean of productivity growth, and $\rho_x$ is the persistence parameter of the autoregressive process $x_t$. The intermediate firms face a cost of adjusting the nominal price à la Rotemberg (1982), measured in terms of the final good as

$$G(P_{i,t}, P_{i,t-1}; \Pi, Y_t) = \frac{\phi_R}{2} \left( \frac{P_{i,t}}{\Pi_{ss} P_{i,t-1}} - 1 \right)^2 Y_t$$

where $\Pi_{ss} > 1$ is the steady-state inflation rate and $\phi_R$ is the magnitude of the costs. The source of funds constraint is

$$\mathcal{P}_t D_{i,t} = \mathcal{P}_i X_{i,t} - \mathcal{W}_i L_{i,t} - \mathcal{P}_t G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t),$$

where $D_{i,t}$ is the real dividend paid by the firm. The objective of the firm is to maximize shareholder’s value $V_t^{(i)} = V^{(i)}(\cdot)$ taking the pricing kernel $M_t$, the competitive nominal wage $\mathcal{W}_t$, and the vector of aggregate state variables $\Psi_t = (\mathcal{P}_t, Z_t, Y_t)$ as given:

$$V^{(i)}(\mathcal{P}_{i,t-1}; \Psi_t) = \max_{P_{i,t}, \mathcal{L}_{i,t}} \{ D_{i,t} + E_t[M_{t+1} V^{(i)}(\mathcal{P}_{i,t+1}; \Psi_{t+1})] \},$$

subject to

$$D_{i,t} = \frac{P_{i,t}}{\mathcal{P}_t} X_{i,t} - \frac{\mathcal{W}_i}{\mathcal{P}_t} L_{i,t} - G(\mathcal{P}_{i,t}, \mathcal{P}_{i,t-1}; \mathcal{P}_t, Y_t),$$

$$\frac{P_{i,t}}{\mathcal{P}_t} = \left( \frac{X_{i,t}}{Y_t} \right)^{-\frac{\lambda_{p,t}}{1+\lambda_{p,t}}}$$
The Lagrangian for intermediate firm \( i \) is:

\[
V^{(i)}(P_{i,t-1}; \Psi_t) = \frac{P_{i,t}X_{i,t}}{P_t} - W_tL_t - G(P_{i,t}, P_{i,t-1}; P_t, Y_t) + E_t [M_{t+1} V^{(i)}(P_{i,t}; \Psi_{t+1})]
\]

\[
+ \gamma_{it} \left( \frac{P_{it}}{P_t} - \left( \frac{X_{it}}{Y_t} \right)^{\frac{-\lambda_{p,t} + \frac{1}{1 + \lambda_{p,t}}}{-\frac{1}{1 + \lambda_{p,t}}}} \right)
\]

Or,

\[
V^{(i)}(P_{i,t-1}; \Psi_t) = Y_t^{\frac{\lambda_{p,t}}{1 + \lambda_{p,t}}} Z_t^{1 - \frac{\lambda_{p,t}}{1 + \lambda_{p,t}}} L_{it}^{1 - \frac{\lambda_{p,t}}{1 + \lambda_{p,t}}} - \frac{W_t}{P_t} L_{it} - \frac{\phi R}{2} \left( \frac{P_{it}}{\Pi_{ss} P_{i,t-1}} - 1 \right)^2 Y_t + E_t [M_{t+1} V^{(i)}(P_{i,t}; \Psi_{t+1})] + \gamma_{it} \left( \frac{P_{it}}{P_t} - \left( \frac{Z_t L_{it}}{Y_t} \right)^{\frac{-\lambda_{p,t}}{1 + \lambda_{p,t}}} \right)
\]

The corresponding first-order conditions are:

\[
\frac{W_t}{P_t} = \left( 1 - \frac{\lambda_{p,t}}{1 + \lambda_{p,t}} \right) Y_t^{\frac{\lambda_{p,t}}{1 + \lambda_{p,t}}} Z_t^{1 - \frac{\lambda_{p,t}}{1 + \lambda_{p,t}}} L_{it}^{1 - \frac{\lambda_{p,t}}{1 + \lambda_{p,t}}} - \frac{W_t}{P_t} L_{it} - \frac{\phi R}{2} \left( \frac{P_{it}}{\Pi_{ss} P_{i,t-1}} - 1 \right)^2 Y_t + E_t [M_{t+1} V^{(i)}(P_{i,t}; \Psi_{t+1})]
\]

\[
\frac{\gamma_{it}}{P_t} = \phi R \left( \frac{P_{it}}{\Pi_{ss} P_{i,t-1}} - 1 \right) Y_t - E_t [M_{t+1} V^{(i)}(P_{i,t}; \Psi_{t+1})]
\]

The envelope condition is:

\[
V_{p,t}^{(i)} = -\phi R \left( \frac{P_{it}}{\Pi_{ss} P_{i,t-1}} - 1 \right) \left( \frac{-P_{it} Y_{i,t}}{\Pi_{ss} P_{i,t-1}^2} \right)
\]

Plugging in the envelope condition into the foc with respect to \( P_{it} \):

\[
\frac{\gamma_{it}}{P_t} = \phi R \left( \frac{P_{it}}{\Pi_{ss} P_{i,t-1}} - 1 \right) Y_t - E_t \left[ M_{t+1} \left( -\phi R \left( \frac{P_{i,t+1}}{\Pi_{ss} P_{i,t}} - 1 \right) \left( -\frac{P_{i,t+1} Y_{t+1}}{\Pi_{ss} P_{i,t}^2} \right) \right) \right]
\]
2.4 Central Bank

The central bank follows a modified Taylor rule that depends on the lagged interest rate, as well as output and inflation deviations:

\[
\ln \left( \frac{\mathcal{R}_t}{\mathcal{R}_{ss}} \right) = \rho_r \ln \left( \frac{\mathcal{R}_{t-1}}{\mathcal{R}_{ss}} \right) + (1 - \rho_r) \left( \rho_{\pi,si} \ln \left( \frac{\Pi_t}{\Pi_{ss}} \right) + \rho_y \ln \left( \frac{\hat{Y}_t}{\hat{Y}_{ss}} \right) \right) + \sigma_{\xi,v} \xi_t,
\]

where \( \mathcal{R}_t \) is the gross nominal short rate, \( \hat{Y}_t \equiv \frac{Y_t}{Z_t} \) is detrended output, and \( \xi_t \sim N(0, 1) \) is an i.i.d. monetary policy shock. Variables with an \( ss \) subscript denote steady-state values.

2.5 Symmetric Equilibrium

We choose a symmetric equilibrium, where all intermediate firms make identical decisions. Also, \( B_t = 0 \). The aggregate resource constraint is

\[
Y_t = C_t + \phi_R \left( \frac{\Pi_t}{\Pi_{ss}} - 1 \right)^2 Y_t
\]

where \( \Pi_t \equiv \frac{\mathcal{P}_t}{\mathcal{P}_{t-1}} \) is the gross inflation rate.

2.6 Model solution

The model is log-linearized with risk-adjustment.

1. Modified expected utility

\[
\frac{1}{\beta_{\mu(1-\psi)}} \tilde{u}_t + \frac{e^{-u_{ss}}}{\beta_{\mu(1-\psi)}} (\tau_0 - 1) \tilde{c}_t = E_t \bar{u}_{t+1} + \frac{e^{-u_{ss}}}{\beta_{\mu(1-\psi)}} \tau_0 \tilde{a}_t + \tilde{b}_t + (1 - 1/\psi) \bar{x}_t + \frac{(1-\theta)}{2} Var_t \left[ \bar{u}_{t+1} \right]
\]
2. Firm’s FOC

\[-(\nu - 1)(\frac{1}{\psi} + \tau)\tilde{c}_t + \Phi_R\tilde{p}_t = \Phi_R\beta e^{\mu(1-\frac{1}{\psi})}E_t\tilde{p}_{t+1} - (\nu - 1)(1 + \tau)\tilde{a}_t + \frac{\nu - 1}{\nu}\tilde{\chi}_t + \frac{\Phi_R}{2}\beta e^{\mu(1-\frac{1}{\psi})}\left(2Cov_t\left[-\theta\tilde{u}_{t+1} + (1 - \frac{1}{\psi})\tilde{c}_{t+1}, \tilde{p}_{t+1}\right] + 3Var_t[\tilde{p}_{t+1}]\right)\]

3. Asset pricing equation for short-term nominal interest rate (Note \(\rho_r = 0\))

\[-\theta \tilde{u}_{t+1} - \frac{1}{\psi}\tilde{c}_{t+1} - \tilde{\pi}_{t+1}\] 

\[E_t\left[-\theta\tilde{u}_{t+1} - \frac{1}{\psi}\tilde{c}_{t+1} - \tilde{\pi}_{t+1}\right] + (1 - \theta)\tilde{b}_t - \frac{\theta \beta e^{-\psi\tau_0}}{\beta e^{\mu(1-\frac{1}{\psi})}}\tilde{a}_t - \gamma\tilde{x}_t + \tilde{\xi}_t + \frac{1}{2}Var_t\left[-\theta\tilde{u}_{t+1} - \frac{1}{\psi}\tilde{c}_{t+1} - \tilde{\pi}_{t+1}\right]\]

Equations describing dynamics of exogenous variables \(b_t, a_t, x_t, \xi_t, \chi_t\)

\[\tilde{b}_{t+1} = \rho_\beta \tilde{b}_t + \sigma_{\beta,v_{t+1}} \varepsilon_{\beta,t+1}\]

\[\tilde{a}_{t+1} = \rho_a \tilde{a}_t + \sigma_{a,v_{t+1}} \varepsilon_{a,t+1}\]

\[\tilde{x}_{t+1} = \rho_x \tilde{x}_t + \sigma_{x,v_{t+1}} \varepsilon_{x,t+1} + \theta_{\beta,x} \sigma_{x,v_{t+1}} \varepsilon_{\beta,t+1} + \theta_{a,x} \sigma_{x,v_{t+1}} \varepsilon_{a,t+1} + \theta_{\chi,x} \sigma_{x,v_{t+1}} \varepsilon_{\chi,t+1}\]

\[\tilde{\xi}_{t+1} = \rho_\xi \tilde{\xi}_t + \sigma_{\xi,v_{t+1}} \varepsilon_{\xi,t+1}\]

\[\tilde{\chi}_{t+1} = \rho_\chi \tilde{\chi}_t + \sigma_{\chi,v_{t+1}} \varepsilon_{\chi,t+1}\]

The three expectational equation can be rewritten in a matrix form:

\[
\begin{pmatrix}
\tilde{u}_t \\
\tilde{c}_t \\
\tilde{\pi}_t
\end{pmatrix} = \tilde{A}E_t
\begin{pmatrix}
\tilde{u}_{t+1} \\
\tilde{c}_{t+1} \\
\tilde{\pi}_{t+1}
\end{pmatrix} + \tilde{D}
\begin{pmatrix}
\tilde{b}_t \\
\tilde{a}_t \\
\tilde{x}_t \\
\tilde{\xi}_t \\
\tilde{\chi}_t
\end{pmatrix} + \tilde{C}_{s,v_t}
\]
or

\[
\begin{pmatrix}
\tilde{u}_t \\
\tilde{c}_t \\
\tilde{\pi}_t
\end{pmatrix} = A_{st} E_t \begin{pmatrix}
\tilde{u}_{t+1} \\
\tilde{c}_{t+1} \\
\tilde{\pi}_{t+1}
\end{pmatrix} + D_{st} \begin{pmatrix}
\tilde{a}_t \\
\tilde{x}_t \\
\tilde{\xi}_t
\end{pmatrix} + C_{st, vt}
\]

where \( A_{st} = \Gamma_{st}^{-1} \tilde{A} \), \( D_{st} = \Gamma_{st}^{-1} \tilde{D} \), and \( C_{st, vt} = \Gamma_{st}^{-1} \tilde{C}_{st, vt} \).

\( C_{st, vt} \) and \( \tilde{C}_{st, vt} \) are risk-adjusting terms containing variances of endogenous variables.

We solve the system following methodology of Farmner, Waggoner, Zha (2011). Solution:

\[
\begin{pmatrix}
\tilde{u}_t \\
\tilde{c}_t \\
\tilde{\pi}_t
\end{pmatrix} = Q_{st} \begin{pmatrix}
\tilde{b}_t \\
\tilde{a}_t \\
\tilde{x}_t \\
\tilde{\xi}_t
\end{pmatrix} + S_{st, vt} + w_t
\]

(1)

\[
w_t = \Lambda_{st-1, s_t} w_{t-1} + V_{s_t} V'_{s_t} \eta_t
\]

(2)

\( w_t \) - non-fundamental part of solution. \( \eta_t \) - sunspot shock.

To construct \( \Lambda \) we follow proposition 1 in Farmer, Waggoner, and Zha (2009). According to theorem 1 in Farmer, Waggoner, and Zha (2009) \( \Lambda_{st, st+1} = V_{s_t} \Phi_{st, st+1} V'_{s_t} \). Where \( \Phi_{st, st+1} \) is some \( k_{st} \times k_{st+1} \) matrix, \( V_{s_t} \) is \( n \times k_{st} \) and \( 0 \leq k_{st} \leq n \). Following proposition 1, we construct one possible \( \Lambda_{st, st+1} \) assuming that \( \Phi_{st, st+1} \) is a scalar and \( V_{s_t} \) is a \( n \times 1 \) vector (\( n \) - number of equations. here \( n = 3 \)). we assume \( V_{s_t} = \frac{v_{s_t}}{\|v_{s_t}\|} \) and \( \Phi_{st, st+1} = \frac{\|v_{s_{t+1}} c(s_t)\}}{\|v_{s_t}\|} \), where \( c_{st} \) is a scalar. We find \( c_{st} \) such that

\[
det(diag(\Gamma_i) - ((diag(c_i) P) \otimes I_n)) = 0
\]

\[
r_{\sigma}(diag(c_i^2) P) < 1
\]

Given \( c_{st} \), vector \( v_{s_t} \) is an eigenvector which corresponds to a zero eigenvalue of

\[
diag(\Gamma_i) - ((diag(c_i) P) \otimes I_n)
\]

10
Given the way we construct solution, sunspot shock is effectively one dimensional. Indeed any sunspot shock $\eta_t$ can be projected onto $V_{st}$:

$$
\eta_t = V_{st} \zeta_t + V_{st}^\perp \zeta_t^\perp
$$

where $V_{st}^\perp$ is 3 by 2 matrix with columns orthogonal to $V_{st}$, such that columns of $V_{st}$ and $V_{st}^\perp$ form a basis in 3 dimensional space. Note that $V_{st}^t V_{st}^\perp = 0_{1 \times 2}$ Plugging this expression for $\eta_t$ in (2), we get:

$$
w_t = \Lambda_{s_{t-1},s_t} w_{t-1} + V_{st} V_{st}^t \eta_t
$$

$$
= \Lambda_{s_{t-1},s_t} w_{t-1} + V_{st} V_{st}^t (V_{st} \zeta_t + V_{st}^\perp \zeta_t^\perp)
$$

$$
= \Lambda_{s_{t-1},s_t} w_{t-1} + V_{st} \zeta_t
$$

So, $\zeta_t$ is “effective” $1 \times 1$, sunspot shock.

### 3 Calibration

The model is calibrated on a quarterly frequency. Table 1 summarizes calibrated parameter values. Values of $\psi$ and $\gamma$ are chosen to be 1.5 and 10 correspondingly, as standard in the literature. $\mu$ is calibrated to match average GDP growth rate in the data and $\beta$ is calibrated to match average level of short term interest rate. Persistence of transitional and permanent productivity shocks $\rho_a$, $\rho_x$ and correlation between them ($\theta_{a,x}$) are calibrated to match average term premium in the data. Price elasticity of demand $\nu$ is set to 6, which corresponds to an average markup of 20%. Taylor rule parameters in active regime are chosen to be $\rho_\pi = 1.5$, $\rho_y = 0.1$ and are in the range of estimates from the literature. In a passive regime $\rho_\pi$ is calibrated to 0.77, which falls in an indeterminacy region of parameter space. Active monetary policy regime is calibrated to be more persistent than the passive one: corresponding quarterly probabilities to stay in the same regime is 0.98 for active regime and 0.75 for a passive one. Value of steady state inflation $\pi_{ss}$ is chosen to be consistent with average inflation rate in the data. Volatility regimes are assumed to be quite persistent -
probability to stay in the same regime next quarter is 0.98 for both regimes.

Table 2 demonstrates some basic macroeconomic and asset pricing moments computed from the model and the data. To compute moments in the data we used Fama - Bliss Discount Bonds data for the yield curve. Data for effective Fed Fund Rate, real GDP and GDP Deflator are from FRED St. Louis Fed database. Time frame used is Q3 1954 - Q4 2015. Overall model is able to match basic asset pricing and macroeconomic moments we observe in the data.

4 Quantitative Results

Figures 1 and 2 demonstrate impulse response to a transitional productivity shock. Like in Kung (2015) productivity shock decreases future marginal costs for firms and hence inflation today decreases. Monetary policy responds to drop in inflation by lowering interest rate. Since in response to the shock model produces persistent movements in inflation and short interest rate, the rest of yield curve moves down as well, as long-term yields are expectations of future short term interest rates. Shock to transitional component of productivity in the model also have impact on permanent component of productivity $x_t$. As a result in response to transitional productivity shock expected consumption growth is increasing. These mechanism creates negative co-movement of inflation and expected consumption growth and is responsible for making long-term bonds risky and hence for generating term premium (Kung (2015), Piazzesi and Schneider (2007) and Bansal and Shaliastovich (2013)). Note that in passive monetary policy regime effect on inflation and yield curve are larger than in the active one.

Figures 3 and 4 demonstrate impulse response to a volatility shock in the model. These graphs show a transitional dynamic of the model if it starts in high volatility regime. Like in Basu and Bundick (2012) increase in volatility increases precautionary savings: households want to consume less, save more and work more. In equilibrium for households to consume less, labor input must decrease as well, so despite labor supply increases equilibrium wages must fall to make equilibrium labor input to fall as well. Decrease in wages leads to a fall in marginal cost of producing output for intermediate firms and hence it leads to a fall in
inflation. Central bank responds to fall in inflation by lowering short term interest rate, which also leads to a decrease in longer maturity yields. In passive monetary policy regime effect on inflation and consumption are significantly higher.

Figures 5 and 6 demonstrate impulse response to a sunspot shock. In response to the shock short term interest rate moves 25 bps up, while the rest of yield curve moving down with 1 year yield going down by 100 - 150 bps and 5-year yield going down by 20 bps. So sunspot shock have a big effect on 1-year yield, while having much smaller effect on short term interest rate. Note that effect on consumption has different sign in active and passive monetary policy regimes and hence creates different correlation between consumption and inflation. This potentially can generate a different response of term premia to sunspot shocks in active and passive monetary regimes.

5 Estimation

6 Conclusion
References


Figure 1: IRF to a. Blue line - active monetary policy regime. Red line - passive monetary policy regime. Plots show average IRFs across 10000 simulations.
Figure 2: IRF to a. Blue line - active monetary policy regime. Red line - passive monetary policy regime. Plots show average IRFs across 10000 simulations.
Table 1: Quarterly calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>A. Preferences</strong></td>
<td></td>
<td></td>
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<tr>
<td>( \overline{\beta} )</td>
<td>Mean of subjective discount factor</td>
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<tr>
<td>( \rho_\beta )</td>
<td>Persistence of subjective discount factor</td>
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<td>( \sigma_{\beta}(v = 1) )</td>
<td>Volatility of ( b_t ) in 1(^{st} ) vol. regime</td>
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<td>( \sigma_{\beta}(v = 2) )</td>
<td>Volatility of ( b_t ) in 2(^{nd} ) vol. regime</td>
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<td>( \psi )</td>
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</tr>
<tr>
<td>( \gamma )</td>
<td>Risk aversion</td>
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<tr>
<td>( L )</td>
<td>Upper bound on labor supply</td>
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<tr>
<td><strong>B. Productivity</strong></td>
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<td>( \rho_a )</td>
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<tr>
<td>( \sigma_a(v = 2) )</td>
<td>vol. of ( a_t ) in 2(^{nd} ) vol. regime</td>
<td>0.0050</td>
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<tr>
<td>( \mu )</td>
<td>Average productivity growth</td>
<td>0.0050</td>
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<tr>
<td>( \rho_\chi )</td>
<td>Persistence of permanent productivity shocks</td>
<td>0.9900</td>
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<tr>
<td>( \sigma_\chi(v = 1) )</td>
<td>vol. of ( x_t ) in 1(^{st} ) vol. regime</td>
<td>0.000010</td>
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<tr>
<td>( \sigma_\chi(v = 2) )</td>
<td>vol. of ( x_t ) in 2(^{nd} ) vol. regime</td>
<td>0.000007</td>
</tr>
<tr>
<td><strong>C. Technology</strong></td>
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<tr>
<td>( \phi_R )</td>
<td>Magnitude of price adjustment costs</td>
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<tr>
<td>( \nu )</td>
<td>Price elasticity for intermediate goods</td>
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<tr>
<td>( \overline{\chi} )</td>
<td>( 1/(\nu - 1) )</td>
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<td>( \rho_\chi )</td>
<td>Persistence of markup shocks</td>
<td>0.9000</td>
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<td>( \sigma_\chi(v = 1) )</td>
<td>vol. of ( \chi_t ) in 1(^{st} ) vol. regime</td>
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<td>( \sigma_\chi(v = 2) )</td>
<td>vol. of ( \chi_t ) in 2(^{nd} ) vol. regime</td>
<td>0.0500</td>
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<td><strong>D. Monetary Policy</strong></td>
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<tr>
<td>( \rho_r )</td>
<td>Degree of monetary policy inertia</td>
<td>0.0000</td>
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<td>( \rho_{\pi,1} )</td>
<td>Sens. of interest rate to inflation in 1(^{st} ) regime</td>
<td>1.5000</td>
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<tr>
<td>( \rho_{\pi,2} )</td>
<td>Sens. of interest rate to inflation in 2(^{nd} ) regime</td>
<td>0.7700</td>
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<tr>
<td>( \rho_y )</td>
<td>Sensitivity of interest rate to output</td>
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<td>( \sigma_\zeta(v = 1) )</td>
<td>Volatility of ( \xi_t ) in 1(^{st} ) vol. regime</td>
<td>0.0030</td>
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<td>( \sigma_\zeta(v = 2) )</td>
<td>Volatility of ( \xi_t ) in 2(^{nd} ) vol. regime</td>
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<td>( \rho_\xi )</td>
<td>Persistence of monetary policy shocks</td>
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<td>( \Pi_{ss} )</td>
<td>Inflation in deterministic steady state</td>
<td>0.0178</td>
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<td><strong>E. Probability of vol. regimes</strong></td>
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<td>( p_{v,11} )</td>
<td>Probability to stay in 1(^{st} ) vol. regime</td>
<td>0.9800</td>
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<tr>
<td>( p_{v,22} )</td>
<td>Probability to stay in 2(^{nd} ) vol. regime</td>
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<td><strong>F. Probability of monetary regimes</strong></td>
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<tr>
<td>( p_{11} )</td>
<td>Probability to stay in 1(^{st} ) regime</td>
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<tr>
<td>( p_{22} )</td>
<td>Probability to stay in 2(^{nd} ) regime</td>
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<td><strong>G. Correlations</strong></td>
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<tr>
<td>( \theta_{a,x} )</td>
<td>Impact of shocks to ( a_t ) on ( x_t )</td>
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<tr>
<td>( \theta_{\beta,x} )</td>
<td>Impact of shocks to ( b_t ) on ( x_t )</td>
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</tr>
<tr>
<td>( \theta_{\chi,x} )</td>
<td>Impact of shocks to ( \chi_t ) on ( x_t )</td>
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<tr>
<td>( \sigma_\zeta )</td>
<td>Vol. of sunspot shock</td>
<td>0.000500</td>
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</table>

This table reports the parameter values used in the quarterly calibration of the model.
Table 2: Macroeconomic and Asset Pricing Moments

<table>
<thead>
<tr>
<th></th>
<th>Log-Linearization</th>
<th>Data</th>
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<tbody>
<tr>
<td><strong>A. Macroeconomic moments</strong></td>
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<tr>
<td>$E(\Delta y)$</td>
<td>0.0201</td>
<td>0.0195</td>
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<tr>
<td>$\text{Std}(\Delta y)$</td>
<td>0.0203</td>
<td>0.0237</td>
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<tr>
<td>$E(\pi)$</td>
<td>0.0305</td>
<td>0.0338</td>
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<tr>
<td>$\text{Std}(\pi)$</td>
<td>0.0260</td>
<td>0.0277</td>
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<tr>
<td><strong>B. Asset pricing moments</strong></td>
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<tr>
<td>$E(r)$</td>
<td>0.0482</td>
<td>0.0500</td>
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<tr>
<td>$\text{Std}(r)$</td>
<td>0.0202</td>
<td>0.0357</td>
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<tr>
<td>$E(y_1)$</td>
<td>0.0482</td>
<td>0.0500</td>
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<tr>
<td>$\text{Std}(y_1)$</td>
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<td>0.0357</td>
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<tr>
<td>$E(y_4)$</td>
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<td>0.0500</td>
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<tr>
<td>$\text{Std}(y_4)$</td>
<td>0.0692</td>
<td>0.0315</td>
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<td>$E(y_8)$</td>
<td>0.0534</td>
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<tr>
<td>$\text{Std}(y_8)$</td>
<td>0.0363</td>
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<td>$E(y_{12})$</td>
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<tr>
<td>$\text{Std}(y_{12})$</td>
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<td>$E(y_{16})$</td>
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<tr>
<td>$\text{Std}(y_{16})$</td>
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<td>0.0296</td>
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<tr>
<td>$E(y_{20})$</td>
<td>0.0610</td>
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<tr>
<td>$\text{Std}(y_{20})$</td>
<td>0.0169</td>
<td>0.0289</td>
</tr>
</tbody>
</table>

The model is calibrated at a quarterly frequency and the reported statistics are annualized.
Figure 3: IRF to volatility shock. Blue line - active monetary policy regime. Red line - passive monetary policy regime. Plots show average IRFs across 10000 simulations.
Figure 4: IRF to volatility shock. Blue line - active monetary policy regime. Red line - passive monetary policy regime. Plots show average IRFs across 10000 simulations.
Figure 5: IRF to sunspot shock. Blue line - active monetary policy regime. Red line - passive monetary policy regime. Plots show average IRFs across 10000 simulations.
Figure 6: IRF to sunspot shock. Blue line - active monetary policy regime. Red line - passive monetary policy regime. Plots show average IRFs across 10000 simulations.