Banks, Liquidity Management and Monetary Policy*

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July 2014

Abstract

We develop a new framework to study the implementation of monetary policy through the banking system. Banks finance illiquid loans by issuing deposits. Deposit transfers across banks must be settled using central bank reserves. Transfers are random and, therefore, create liquidity risk. The degree of liquidity risk determines the supply of credit and the money multiplier. We study how different shocks to the banking system and monetary policy affect the economy by altering the tradeoff between profiting from lending against incurring in greater liquidity risk. We calibrate our model to study, quantitatively, why banks have recently increased their reserve holdings but not expanded loans despite policy efforts. We find that credit demand shocks are the main driving force.

Keywords: Banks, monetary policy, liquidity, capital requirements

*We would like to thank Harjoat Bhamra, John Cochrane, Dean Corbae, Itamar Dreschler, Xavier Freixas, Anil Kashyap, Nobu Kiyotaki, Arvind Krishnamurthy, Ricardo Lagos, Thomas Philippon, Chris Phelan, Tomek Piskorski, Ricardo Reis, Chris Sims, Harald Uhlig, and Mike Woodford for helpful discussions. We also wish to thank seminar participants at Banque de France, Bank of Italy, Bank of Japan, Central Bank of Chile, Central Bank of Peru, Central Bank of Uruguay, Chicago Fed, Columbia University, the European Central Bank, Philadelphia Fed the Riksbank, University of Maryland, University of Chicago, Goethe University, Minneapolis Fed, Universidad Catolica de Chile, SAIF, Yale, and conference participants at the 2nd Rome Junior Conference on Macroeconomics at the Einaudi Institute, the 2013 Barcelona GSE Summer Forum, the 2nd Macro Finance Society Workshop, the Fourth Boston University/Boston Fed Conference on Macro-Finance Linkages, and the CEPR Conference on Banks and Governments in Globalised Financial Markets at Oesterreichische Nationalbank, Workshop on Safe Assets and the Macroeconomy at London Business School, Society of Economic Dynamics. We are grateful for the financial support by the Fondation Banque de France. Emails: jibianchi@wisc.edu and sbigio@columbia.edu.
1 Introduction

The conduct of monetary policy around the world is changing. The past five years have witnessed banking systems that bore unprecedented financial losses and subsequent freezes in interbank markets. Following these events, there was a major reduction in bank lending followed by a protracted recession. In response, central banks in developed economies have reduced policy rates to almost zero and expanded their balance sheets in an open attempt to preserve financial stability and reinvestigate lending. However, in reaction to these unprecedented policy interventions, banks seem to have accumulated central bank reserves without renewing their lending activities as intended. Why? Can central banks do more about this? These remain open questions.

Not surprisingly, the role of banks in the transmission of monetary policy has been at the center of policy debates. However, there are few modern macroeconomic models that take into account that monetary policy is implemented through the banking system, as occurs in practice. Instead, most macroeconomic models assume that central banks control interest rates or monetary aggregates and abstract from how the transmission of monetary policy may depend on the conditions of banks. This paper presents a model that contributes to fill in this gap.

The Mechanism. The building block of our model is a liquidity management problem. Liquidity management is recognized as one of the fundamental problems in banking and can be explained as follows. When a bank grants a loan, it simultaneously creates demand deposits—or credit lines. These deposits can be used by the borrower to perform transactions at any time. Granting a loan is profitable because a higher interest is charged on the loan than what is paid on deposits. However, more lending relative to a given amount of central bank reserves increases a bank’s liquidity risk. When deposits are transferred out of a bank, that bank must transfer reserves to other banks in order to settle transactions. Central bank reserves are critical to clear settlements because loans cannot be sold immediately. Thus, the lower the reserve holdings of a bank, the more likely it is to be short of reserves in the future. This is a source of risk because the bank must incur expensive borrowing from other banks—or the central bank’s discount window—if it falls short of reserves. This friction—the liquidity mismatch—induces a tradeoff between profiting from lending and incurring additional liquidity risks which we call liquidity management. It is by affecting the liquidity management of banks that monetary policy has real effects in the model.

Implementation of Monetary Policy. In the model, the central bank has access to various tools. A first set of instruments are reserve requirements, discount rates, and interests on reserves which influence the cost of being short of reserves. This set of instruments affects the demand for reserves directly. A second set of instruments are open-market operations (OMO) and direct lending to banks. This latter set of instruments alters the effective aggregate amount of reserves in the system. Both types of instruments carry real effects by tilting the liquidity management

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1 As is well known, the Bank of Japan had been facing similar issues since the early nineties.
tradeoff. Macroeconomic effects result from their indirect effect on aggregate lending and interest rates. However, as much as a Central Bank can influence bank decisions, shocks to the banking system may limit the monetary policies ability to induce a certain aggregate lending and output.

**Model Features.** We introduce this liquidity management problem into a dynamic stochastic general equilibrium model with rational profit-maximizing banks. Banks are subject to random deposit transfers. Since loans are illiquid banks use central bank reserves to settle deposit transfers. To accommodate their reserve surpluses or deficits, banks borrow or lend in an over-the-counter (OTC) interbank market. A central bank conducts monetary policy by performing open-market operations altering the quantity of reserves in the system and setting corridor rates. By this, monetary policy directly affects the interbank market rate and the costs associated with the liquidity risk.

Despite the richness in bank portfolio decisions, idiosyncratic withdrawal risk, and an OTC interbank market, we are able to reduce the state space into a single aggregate endogenous state: the aggregate value of bank equity. Moreover, the bank’s problem satisfy portfolio separation. In turn, this allows us to analyze the liquidity management problem through a portfolio problem with non-linear returns that depend only aggregate market conditions. These results make the analysis of the model very transparent and amenable for various extensions and applications.

**Testable Implications.** The model delivers a rich set of testable implications. For individual banks, it explains the behavior of their reserve ratio, leverage ratio and dividend policies. It also provides predictions for aggregate lending, interbank borrowing, equity and excess reserves, as well as for the return on loans and the return on equity. The model also generates endogenous money multipliers and liquidity premia.

This testable implications allow us to address a number of theoretical issues. For example, we study how the transmission of monetary policy depends on the portfolio decisions of banks. We also study the effects of shocks to bank equity, to capital requirements, to the payments system, to credit demand affect lending.

**Quantitative Application.** As an quantitative application of our model, we exploit the lessons derived from the theoretical framework to investigate why banks are not lending despite all the policy efforts. Thanks to its predictions, our model is able to contrast different hypotheses that are informally discussed in policy and academic circles. Through the lens of the model, we evaluate the plausibility of the following:

**Hypothesis 1 - Bank Equity Losses:** Lack of lending responds to an optimal behavior by banks given the equity losses suffered in 2008.

**Hypothesis 2 - Capital Requirements:** The anticipation of higher capital requirements is leading banks to hold more reserves and simultaneously lend less.

**Hypothesis 3 - Increased Precautionary Holdings of Reserves:** Banks hold more reserves because they now face greater liquidity risk.

**Hypothesis 4 - Interest on Excess Reserves:** Interest payments on excess reserves has lead
Hypothesis 5 - Weak Demand: Banks face a weaker effective demand for loans. This hypothesis encompasses a direct shock to the demand for credit or a decline in the effective demand for loans that follows from an increase in credit risk.

We calibrate our model and fit it with shocks associated with each hypothesis that we obtain from the data. We use the model’s predictions to uncover which shocks are more quantitatively relevant to explain the decline in lending while reserves have increased by several multiples. Our model suggests that a combination of shocks best fits the data: In particular, the model suggests that an early disruption in the interbank market followed by a substantial contraction in loan demand is the most quantitatively compelling story.

Organization. The paper is organized as follows. The following section sets the paper in the contexts of the literature. Section 2 presents the model and Section 3 provides theoretical results. Section 4 reports the calibration exercises. There, we study the steady state and policy functions under that calibration. In Section 5, we analyze the transitional dynamics generated after shocks associated with each hypothesis. Finally, in Section 6, we evaluate and discuss the plausibility of each hypothesis.

1.1 Related Literature

A tradition in macroeconomics dating back to at least Bagehot (1873) stresses the importance of analyzing monetary policy in conjunction with banks. A classic mechanical framework to study policy with a full description of households, firms and banks is Gurley and Shaw (1964). With few exceptions, modeling banks was abandoned from macroeconomics for many years. Until the Great Recession, the macroeconomic effects of monetary policy and its implementation through banks were analyzed independently.\(^2\)

In the aftermath of the global financial crisis, there have been numerous calls for constructing models with an explicit role for banks.\(^3\) Some early steps were taken by Gertler and Karadi (2009) and Curdia and Woodford (2009), who show how shocks that disrupt financial intermediation can have important effects on the real economy. Following these papers, a large literature has studied how various policies affect bank equity and macroeconomic outcomes. Our model also belongs to the banking channel view, but emphasize instead how monetary policy affects the tradeoffs banks face in holding assets of different liquidity. In turn, this relates our model to classic models of bank liquidity management and monetary policy.\(^4\) Our contribution to this literature is to bring

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\(^2\)This was a natural simplification by the literature. In the US, the behavior of banks did not seem to matter for monetary policy. In fact, the banking industry was among the most stable industries in terms of returns and the pass through from policy tools to aggregate conditions had little variability.

\(^3\)See for example Woodford (2010) and Mishkin (2011).

\(^4\)Classic papers that study static liquidity management — also called reserve management — by individual banks are Poole (1968) and Frost (1971). Bernanke and Blinder (1988) present a reduced form model that blends reserve management with an IS-LM model. There are many modern textbooks for practitioners that deal with liquidity
the classic insights from the liquidity management literature into a modern, general equilibrium
dynamic model that can be used for the policy analysis and to study banking crises.\footnote{Kashyap and Stein (2000) exploit cross-sectional variation in liquidity holdings by banks and find empirical
evidence for the monetary policy transmission mechanism that we study here. Recently, Jimenez et al. (2012);
Jimnez et al. (2014) exploit both, firm heterogeneity in loan demand and variation in bank liquidity ratios to
identify the presence of the bank lending-channel in Spain.}

We share common elements with recent work by Brunnermeier and Sannikov (2012). Brunner-
meier and Sannikov (2012) also introduce inside and outside money into a dynamic macro model.
Their focus is on the real effects of monetary policy through the redistributive effects of inflation
when there are nominal contracts. The use of reserves for precautionary motives also places our
model close to Stein (2012) and Stein et al. (2013). Those papers study the effects of an increase
in the supply of reserves given an exogenous demand for short-term liquid assets.

Our paper also builds on the search theoretic literature of monetary exchange (see the survey
by Williamson and Wright, 2010). Williamson (2012) studies an environment where assets of
different maturity have different properties as mediums of exchange. Cavalcanti et al. (1999)
provide a theoretical foundation to our setup because reserves there emerge as a disciplining
device to sustain credit creation under moral hazard and guarantee the circulation of deposits. In
turn, we model an interbank market building on earlier work by Afonso and Lagos (2012). They
model the federal fed funds market as an over-the-counter (OTC) market where illiquidity costs
arise endogenously. Our market for reserves is a simplified version of that model.

2 The Model

The description of the model begins with a description of the dynamic decision of banks. The goal
is to derive the supply of loans and the demand for reserves given an exogenous demand for loans,
central bank policies and aggregate shocks. We derive a formal demand for loans and supply of
deposits that closes the model in Appendix D.

2.1 Environment

Time is discrete, is indexed by $t$ and has an infinite horizon. Each period is divided into two
stages: a lending stage (l) and a balancing stage (b). The economy is populated by a continuum
of competitive banks whose identity is denoted by $z \in [0,1]$. Banks face a demand for loans and
a vector of shocks that we describe later. There is an exogenous deterministic monetary policy
chosen by the monetary authority which we refer to as the Fed. There are three types of assets,
deposits, loans and central bank reserves. Deposits and loans are denominated in real terms. Reserves are denominated in nominal terms. Deposits play the role of a numeraire.

**Banks.** A bank’s preferences over real dividend streams \( \{DIV_t\}_{t \geq 0} \) are evaluated via an expected utility criterion:

\[
E_0 \sum_{t \geq 0} \beta^t U(DIV_t)
\]

where \( U(DIV) \equiv \frac{DIV^{1-\gamma}}{1-\gamma} \) and \( DIV_t \) is the banker’s consumption at date \( t \).\(^6\) Banks hold a portfolio of loans, \( B_t \), and central bank reserves, \( C_t \), as part of their assets. Demand deposits, \( D_t \), are their only form of liabilities. These holdings are the individual state variables of a bank.

**Loans.** Banks make loans during the lending stage. The flow of new loan issuances is \( I_t \). These loans constitute a promise to repay the bank \( I_t (1-\delta) \delta^n \) in period \( t + 1 + n \) for all \( n \geq 0 \), in units of numeraire. Thus, loans promise a geometrically decaying stream of payments as in the Leland-Toft model—see Leland and Toft (1996). We denote by \( B_t \) the stock of loans held by a bank at time \( t \). Given the structure of payments, the stock of loans has a recursive representation:

\[
B_{t+1} = \delta B_t + I_t.
\]

When banks grant a loan, they provide the borrower a demand deposit account which amount to \( q_l I_t \), where \( q_l \) is the price of the loan. Banks take \( q_l \) as given. Consequently, the bank’s immediate accounting profits are \( (1-q_l) I_t \).

A key feature of our model is that bank loans are illiquid —they cannot be traded— during the balancing stage.\(^7\) The lack of a liquid market for loans in the balancing stage can be rationalized by several market frictions. For example, loans may be illiquid assets if banks specialize in particular customers or if they face agency frictions.\(^8\)

**Demand Deposits.** Deposits earn a real gross interest rate \( R^D = (1 + r^d) \). Behind the scenes, banks enable transactions between third parties. When they obtain a loan, borrowers receive deposits. This means that banks make loans —a liability for the borrower— by issuing their own liabilities —an asset ultimately held by a third party. This swap of liabilities enables borrowers to purchase goods because deposits are effective mediums of exchange. After the transaction, the holder of those deposits may, in turn, transfer those funds again to the accounts of others, make

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\(^6\)Introducing curvature into the objective function is important. This assumption generates smooth dividends and slow-moving bank equity, as observed empirically. Similar preferences are often found in the corporate finance literature. One way to rationalize these preferences is through undiversified investors that hold bank equity. Alternatively, agency frictions may induce equity adjustment costs.

\(^7\)Loans can be sold during the lending stage. This asymmetry between the lending and balancing stage allows us to reduce the state space. In particular, it is not necessary to keep track of the composition but only the size of bank balance sheets thanks to this assumption. Dispensing this assumption would require keeping track of a non-degenerate, cross-sectional distribution for reserves, deposits and loans.

A second key feature of the environment is that deposits are callable on demand. In the balancing stage, banks are subject to random deposit withdrawals $\omega_{t}D_{t}$, where $\omega_{t} \sim F_{t}(\cdot)$ with support in $(-\infty, 1]$. Here, $F_{t}$ is the time-varying cumulative distribution for withdrawals. The operator $E_{\omega}(\cdot)$ is the expectation under $F_{t}$. For simplicity, we assume $F_{t}$ is common to all banks.\(^9\) When $\omega_{t}$ is positive (negative), the bank loses (receives) deposits. The shock $\omega_{t}$ captures the idea above that deposits are constantly circulating when payments are executed or in response to a loss of confidence in a given bank. The complexity of these transfers is approximated by the random process of $\omega_{t}$. For simplicity, we assume that deposits do not leave the banking system:

**Assumption 1 (Deposit Conservation).** Deposits remain within the banking system: $\int_{-\infty}^{1} \omega_{t}dF_{t}(\omega) = 0$, $\forall t$.

This assumption implies that there are no withdrawals of reserves outside of the banking system.\(^10\)

When deposits are transferred across banks, the receptor bank absorbs a liability issued by another bank. Therefore, this transaction needs to be settled with the transfer of an asset. Since bank loans are illiquid, deposit transfers are settled with reserves. Thus, the illiquidity of loans induces a demand for reserves.

**Reserves.** Reserves are special assets issued by the Fed and used by banks to settle transactions. Banks can buy or sell reserves frictionlessly during the lending stage. However, during the balancing stage, they can only borrow or lend reserves in the interbank market we detail below. We denote by $p_{t}$ the price of reserves in terms of deposits. This term is also the inverse of the price level because deposits are in real terms.

By law, banks must hold a minimum amount of reserves within the balancing stage. In particular, the law states that $p_{t}C_{t} \geq \rho D_{t}(1 - \omega_{t})/R^{D}$, where $\rho \in [0, 1]$ is a reserve requirement chosen by the Fed.\(^11\) The case $\rho = 0$ requires banks to finish with a positive balance of reserves—that banks cannot issue these liabilities. Given the reserve requirement, if $\omega_{t}$ is large, reserves may be insufficient to settle the outflow of deposits. In turn, banks that receive a large unexpected inflow will hold reserves in excess of the requirement.

To meet reserve requirements or allocate reserves in excess, banks can lend and borrow from each other or from the Fed. These trades constitute the interbank market. As part of its toolbox, the Fed chooses two policy rates: a lending rate, $r^{\text{DW}}_{t}$, and a borrowing rate, $r^{\text{ER}}_{t}$. The lending rate—or discount window rate—is the rate at which the Fed lends reserves to banks in deficit.

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\(^9\)We could assume that $F$ is a function of the bank’s liquidity or leverage ratio. This would add complexity to the bank’s decisions but would not break any aggregation result. This tractability is lost if $F_{t}$ is a function of the bank’s size.

\(^10\)This assumption can be relaxed to allow for a demand for currency or system-wide bank runs. \(^?\) is a recent paper that studies the endogenous decomposition of the monetary base in currency and reserves.

\(^11\)Some operating frameworks compute reserve balances over a maintenance period. Bank choices in our model would correspond to averages over the maintenance period.
The borrowing rate —the interest on excess reserves—is the interest paid by the Fed to banks who deposit excess reserves at the Fed. These rates satisfy \( r_t^{DW} \geq r_t^{ER} \) and are paid within the period with deposits.\(^{12}\) Banks have the option to trade with the Fed or with other banks.

**Interbank Market.** We assume that the interbank market for reserves is a directed OTC market.\(^{13}\) This interbank market works in the following way. After the realization of withdrawal shocks, banks end with either positive or negative balances relative to their reserve requirements. A bank that wishes to lend a dollar in excess can place a lending order. A bank that needs to borrow a dollar to patch its deficit can place a borrowing order. Orders are placed on a per-unit basis as in Atkeson et al. (2012) on the borrowing or lending sides of the market. After orders are directed to either side, a dollar in excess is randomly matched with a dollar in deficit. Once a match is realized, the lending bank can transfer the unit overnight.

Banks use Nash bargaining to split the surplus of the dollar transfer. In the bargaining problem that emerges, the outside option for the lending bank is to deposit the dollar at the Fed earning \( r_t^{ER} \). For the bank in deficit, the outside option is the discount window rate \( r_t^{DW} \). Because the principle of the loan—the dollar itself—is returned within the period, without loss of generality, banks bargain only about the net rate. We call this net rate the fed funds rate, \( r_t^{FF} \).

The bargaining problem for a match is:

**Problem 1 (Interbank Market Bargaining Problem)**

\[
\max_{r_t^{FF}} \left( m_l r_t^{EF} - m_b r_t^{FF} \right) \xi \left( m_l r_t^{FF} - m_l r_t^{ER} \right)^{1-\xi}.
\]

In the objective function, \( m_l \) is the marginal utility of the bank lending reserves and \( m_b \) id the corresponding term for the bank borrowing reserves. The first-order condition of this problem is:

\[
\frac{\left( r_t^{FF} - r_t^{ER} \right)}{(1 + r_t^{DW}) - (1 + r_t^{FF})} = \frac{(1 - \xi)}{\xi}.
\]

This condition yields an implicit solution for \( r_t^{FF} \). Since \( (1 - \xi) / (\xi) \) is positive, it is clear that \( r_t^{FF} \) will fall within the Fed’s corridor of interest rates, \([r_t^{ER}, r_t^{DW}]\).\(^ {14}\)

The probability that a lending or borrowing order finds a match depends on the relative mass on each side of the market. We denote by \( M^+ \) the mass of lending orders and by \( M^- \) the mass of borrowing orders. The probability that a borrowing order finds a lending order is given by

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\(^{12}\)This determines what in practice is known as the corridor system. We do not model here the reasons why the central bank chooses to have a corridor system, and simply take as given that this is a standard policy instrument to affect credit creation and aggregate demand in the presence of other frictions like nominal rigidities. What is critical for our analysis is the presence of liquidity risk which arises in our model when \( r_t^{DW} > 0 \). In practice, there are other frictions in interbank markets that makes a shortfall of reserves costly like stigma from borrowing at the discount window (see e.g., Armantier et al. (2011) and Ennis and Weinberg (2013)).

\(^{13}\)The features of the interbank market are borrowed from work by Afonso and Lagos (2012).

\(^{14}\)In a Walrasian setting, the interbank rate would equal the discount rate or the excess reserve rates depending on whether there are enough reserves in the system to satisfy the reserve requirements of all banks.
\( \gamma^- = \min\left(1, \frac{M^+}{M^-}\right) \). Conversely, the probability that a lending order finds a borrowing order is \( \gamma^+ = \min\left(1, \frac{M^-}{M^+}\right) \). These probabilities will affect the average cost of being short or long of reserves, which will in turn affects banks’ portfolio decisions and aggregate liquidity. In the quantitative analysis, to capture disturbances in interbank markets, we will also consider shocks that reduce the probability of matching for given \( M^+, M^- \).

There are a few implicit conventions. First, if an order does not find a match, the bank does not lose the opportunity to lend/borrow to/from the Fed. Second, a bank cannot place orders beyond its reserve needs or excess –without this restriction, banks could place higher orders to increase their probabilities of allocating (borrowing) funds. Finally, interests are paid with deposits —this is just a convention since all assets are liquid during the lending stage.

### 2.2 Timing, Laws of Motion and Bank Problems

This section describes the model recursively: we drop time subscripts from now on. We adopt the following notation: If \( Z \) is a variable at the beginning of the period, \( \tilde{Z} \) is its value by the end of the lending stage and the beginning of the balancing stage. Similarly, \( Z' \) denotes its value by the end of the balancing stage and the beginning of the following period. The aggregate state, summarized in the vector \( X \), includes all policy decisions by the Fed, the distribution of withdrawal shocks, \( F \), and a shock to the demand for loans —to be specified below.

**Lending Stage.** Banks enter the lending stage with reserves, \( C \), loans, \( B \), and deposits, \( D \). The bank chooses dividends, \( DIV \), loan issuances, \( I \), and purchases of reserves, \( \varphi \).\(^{15}\) The evolution of deposits follows:

\[
\frac{\dot{D}}{RD} = D + qI + DIV + \varphi p - B(1 - \delta).
\]

Several actions affect this evolution. First, deposits increase when the bank credits \( qI \) deposits in the accounts of borrowers —or whomever they trade with. Second, banks pay dividends to shareholders with deposits. Third, the bank issues \( p\varphi \) deposits to buy \( \varphi \) reserves. Finally, deposits fall by \( B(1 - \delta) \) because loans are amortized with deposits.

At the end of the lending stage reserves are the sum of the previous stock plus purchases of reserves, \( \tilde{C} = C + \varphi \). Loans evolve according to \( \hat{B} = \delta B + I \). Banks choose \( \{I, DIV, \varphi\} \) subject to these laws of motion and a capital requirement constraint. The capital requirement constraint imposes an upper bound, \( \kappa \), on the stock of deposits relative to equity —marked-to-market.\(^{16}\)

Denoting by \( V^l \) and \( V^b \) the bank’s value function during the lending and balancing stages, we

\(^{15}\)The purchase of reserves \( \varphi \) occurs during the lending stage. Thus, this is a different flow than the flow that follows from loans in the interbank market which occurs during the balancing stage.

\(^{16}\)On the technical side, the capital requirement constraint bounds the bank’s problem and prevents a Ponzi scheme. It is important to note that if the bank arrives to a node with negative equity, the problem is not well defined. However, when choosing its policies, the bank will make decisions that guarantee that it does not run out of equity. Implicitly, it is assumed that if the bank violates any constraint, it goes bankrupt, which has a large negative value.
have the following recursive problem in the lending stage:

**Problem 2** *In the lending stage, banks solve:*

\[
V^l(C, B, D; X) = \max_{I, DIV, \varphi} U(DIV) + \mathbb{E}\left[V^b(\tilde{C}, \tilde{B}, \tilde{D}; \tilde{X})\right]
\]

\[
\frac{\tilde{D}}{RD} = D + qI + DIV + p\varphi - B(1 - \delta)
\]

\[
\tilde{C} = C + \varphi
\]

\[
\tilde{B} = \delta B + I
\]

\[
\frac{\tilde{D}}{RD} \leq \kappa \left(q\tilde{B} + p\tilde{C} - \frac{\tilde{D}}{RD}\right); \tilde{B}, \tilde{C}, \tilde{D} \geq 0.
\]

**Balancing Stage.** During the balancing stage, withdrawal shocks shift deposits and reserves across the banking system, leading to a distribution of reserve deficits and surpluses. Let \(x\) be the reserve deficit for an individual bank. Given that withdrawals are settled with reserves, this deficit is:

\[
x = \rho \left(\frac{\tilde{D} - \omega\tilde{D}}{RD}\right) - \left(\frac{\tilde{C}p - \omega\tilde{D}}{RD}\right).
\]

Given the structure of the OTC market described above, a bank with a reserve surplus obtains a return of \(r^{FF}\) if it lends a unit of reserves in the interbank market and \(r^{ER}\) if it lends to the Fed. Notice that for any Nash bargaining parameter \(r^{FF} > r^{ER}\), banks always attempt to lend first in the interbank market. Thus, they place lending orders for every dollar in excess. In equilibrium, only a fraction \(\gamma^+\) of those orders are matched and earn a return of \(r^{FF}\). The rest earns the Fed’s borrowing rate \(r^{ER}\). Thus, the average return on excess reserves is:

\[
\chi_l = \gamma^+r^{FF} + (1 - \gamma^+)r^{ER}_t.
\]

Analogously, a bank with a reserve deficit borrows from the interbank market before borrowing from the Fed because \(r^{FF} < r^{DW}_t\). The cost of reserve deficits is:

\[
\chi_b = \gamma^-r^{FF} + (1 - \gamma^-)r^{DW}_t.
\]

The difference between \(\chi_l\) and \(\chi_b\) is an endogenous wedge between the marginal value of excess reserves and the cost of reserve deficits. The simple rule that characterizes orders in the interbank market problem yields a value function for the bank during the balancing stage:
Problem 3 The value of the bank’s problem during the balancing stage is:

\[
V^b(\tilde{C}, \tilde{B}, \tilde{D}; \tilde{X}) = \beta \mathbb{E} \left[ V^l(C', B', D'; X') | \tilde{X} \right]
\]

\[
D' = \tilde{D}(1 - \omega) + \chi(x)
\]

\[
B' = \tilde{B}
\]

\[
x = \rho \left( \frac{\tilde{D} - \omega \tilde{D}}{R_D} \right) - \left( \tilde{C}p - \frac{\omega \tilde{D}}{R_D} \right)
\]

\[
C' = \tilde{C} - \frac{\omega \tilde{D}}{p}.
\]

Here \( \chi \) represents the illiquidity cost, the return/cost of excess/deficit of reserves:

\[
\chi(x) = \begin{cases} 
\chi_l x & \text{if } x \leq 0 \\
\chi_b x & \text{if } x > 0
\end{cases}
\]

We can collapse the problem of a bank for the entire period through a single Bellman equation by substituting \( V^b \) into \( V^l \):

Problem 4 The bank’s problem during the lending stage is:

\[
V^l(C, B, D, X) = \max_{\{I, DIV, \varphi\}} U(DIV) \ldots \tag{1}
\]

\[
+ \beta \mathbb{E} \left[ V^l \left( \tilde{C} - \frac{\omega' \tilde{D}}{p}, \tilde{B}, \tilde{D}(1 - \omega') + \chi \left( \frac{(\rho + \omega'(1 - \rho))\tilde{D} - \tilde{C}p}{R_D} \right) \right) ; X' | \tilde{X} \right]
\]

\[
\tilde{D} \frac{R_D}{R_D} = D + qI + DIV_t + p\varphi - B(1 - \delta)
\]

\[
\tilde{B} = \delta B + I
\]

\[
\tilde{C} = \varphi + C
\]

\[
\tilde{D} \frac{R_D}{R_D} \leq \kappa \left( \tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R_D} \right).
\]

The following section provides a characterization of this problem.

2.3 Characterization of the Bank Problem

The recursive problem of banks can be characterized through a single state variable, the banks’ equity value after loan amortizations, \( E = pC + (\delta q + 1 - \delta)B - D \). Substituting the laws of motion for reserves and loans \( \tilde{C} = \varphi + C \) and \( \tilde{B} = \delta B + I \), into the law of motion for deposits 2.2, we have that the evolution of deposits takes the form of a budget constraint:

\[
E = q\tilde{B} + \tilde{C}p + DIV - \tilde{D}/R_D.
\]
In this budget constraint $E$ is the value of the bank’s available resources, which is predetermined. We use an updating rule for $E$ that depends on the bank’s current decisions to express the bank’s value function through a single-state variable:

**Proposition 1** (Single-state Representation)

$$V(E) = \max \{\tilde{c}, \tilde{B}, \tilde{D}, DIV\} U(DIV) + \beta E[V(E')|X]$$

$$E = q\tilde{B} + p\tilde{C} + DIV - \frac{\tilde{D}}{R^D}$$

$$E' = (q'\delta + 1 - \delta)\tilde{B} + p'\tilde{C} - \tilde{D} - \chi \left(\frac{(\rho + \omega'(1-\rho))\tilde{D}}{R^D} - \tilde{C}p\right)$$

$$\frac{\tilde{D}}{R^D} \leq \kappa \left(\tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D}\right).$$

This problem resembles a standard consumption-savings problem subject to a leverage constraint. Dividends play the role of consumption; the bank’s savings are allocated into loans, $\tilde{B}$, and reserves, $\tilde{C}$, and it can lever its position issuing deposits $\tilde{D}$. Its choice is subject to a capital requirement constraint—the leverage constraint. The budget constraint is linear in $E$ and the objective is homothetic. Thus, by the results in Alvarez and Stokey (1998), the solution to this problem exists, is unique, and policy functions are linear in equity. Formally,

**Proposition 2** (Homogeneity—$\gamma$) The value function $V(E; X)$ satisfies

$$V(E; X) = v(X) E^{1-\gamma}$$

where $v(\cdot)$ satisfies

$$v(X) = \max_{\tilde{c}, \tilde{b}, d, div} U(div) + \beta E[v(X')|X] E_{\omega'}(e')^{1-\gamma}$$

subject to

$$1 = q\tilde{b} + p\tilde{c} + div - \frac{\tilde{d}}{R^D}$$

$$e' = (q'\delta + (1-\delta))\tilde{b} + p'\tilde{c} - \tilde{d} - \chi \left(\rho + \omega'(1-\rho)\right)\frac{\tilde{d}}{R^D} - p\tilde{c}$$

$$\frac{\tilde{d}}{R^D} \leq \kappa \left(q\tilde{b} + \tilde{c}p - \frac{\tilde{d}}{R^D}\right).$$

Moreover, the policy functions in (2) satisfy $[\tilde{C} \; \tilde{B} \; \tilde{D}] = [\tilde{c} \; \tilde{b} \; \tilde{d}] \cdot E$.

---

\(^{17}\text{From here on, we use the terms cash and reserves interchangeably. This is not to be confused with cash holdings by firms which may refer to deposits.}\)
According to this proposition, the policy functions in (2) can be recovered from (3) by scaling them by equity, i.e., if \( c^* \) is the solution to (3), we have that \( C = Ec^* \), and the same applies for the rest of the policy functions. An important implication is that two banks with different equity are scaled versions of a bank with one unit of equity.\(^{18}\) This also implies that the distribution of equity is not a state variable, but rather only the aggregate value of equity. Moreover, although there is no invariant distribution for bank equity —the variance of distribution grows over time— the model yields predictions about the cross-sectional dispersion of equity growth.

An additional useful property of the bank’s problem is that it satisfies portfolio separation. In particular, the choice of dividends can be analyzed independently —through a consumption savings problem with a single asset— from the portfolio choices between deposits, reserves and loans. We use the principle of optimality to break the Bellman equation (3) into two components.

**Proposition 3** (Separation) *The value function \( v(\cdot) \) defined in (3) solves:

\[
v(X) = \max_{\text{div} \in \mathbb{R}_+} U(\text{div}) + \beta \mathbb{E}[v(X') | X] \Omega(X)^{1-\gamma} (1 - \text{div})^{1-\gamma}.
\]

(4)

Here \( \Omega(X) \) is the value of the certainty-equivalent portfolio value of the bank. \( \Omega(X) \) is the outcome of the following liquidity management portfolio problem:

\[
\Omega(X) \equiv \max_{\{w_b, w_c, w_d\} \in \mathbb{R}_+^3} \left\{ \mathbb{E}_{\omega'} \left[ R^B_X w_b + R^C_X w_c - w_d R^D_X - R^X_X(w_d, w_c) \right]^{1-\gamma} \right\}^{1/(1-\gamma)}
\]

\[w_b + w_c - w_d = 1\]
\[w_d \leq \kappa (w_b + w_c - w_d)\]

(5)

with \( R^B_X \equiv \frac{q' \delta + (1-\delta)}{q} \), \( R^C_X \equiv \frac{p'}{p} \), \( R^X_X \equiv \chi((\rho + \omega'(1 - \rho)) w_d - w_c) \).

Once we solve the policy functions of this portfolio problem, we can reverse the solution for \( \tilde{c}, \tilde{b}, \tilde{d} \) that solve (3) via the following formulas: \( \tilde{b} = (1 - \text{div}) w_b/q, \tilde{c} = (1 - \text{div}) w_c/p \) and \( \tilde{d} = (1 - \text{div}) w_d R^D \).

The maximization problem that determines \( \Omega(X) \) consists of choosing portfolio shares among assets of different risk, liquidity and return. This problem is a liquidity management portfolio problem with the objective of maximizing the certainty equivalent return on equity, where the return on equity is given by:

\[
R^E(\omega'; w_b, w_d, w_c) \equiv R^B w_b + R^C w_c - R^D w_d - R^X(w_d, w_c, \omega')
\]

This portfolio problem is not a standard portfolio problem because it features non-linear returns. The return on loans is linear and equals the sum of the coupon payment plus the resale price

---

\(^{18}\)Studying differences between big and small banks is beyond the scope of this paper. See Corbae and D’Erasmo (2013) and Corbae and D’Erasmo (2014) for recent contributions on this dimension. For our purpose, an important fact is that reserves are widely distributed across the banking sector, as documented by Wolman and Ennis (2011).
of loans: $R^B \equiv (\delta q + (1 - \delta))/q$. The return on reserves and deposits can be separated into independent—intrinsic—return components and a joint return component. The intrinsic return on reserves is the deflation rate $R^C \equiv p'/p$. The independent return of deposits is the interest on deposits, $R^D$. The joint return component, which depends on $\omega'$, captures the cost—or benefit—of running out of reserves. This illiquidity cost depends on the conditions of the interbank market and is given by:

$$R^X(w_d, w_c, \omega') \equiv \chi ((\rho + (1 - \rho) \omega') w_d - w_c).$$

The risk and return of each asset varies with the aggregate state, making the solution to the liquidity management portfolio problem time varying. In addition, the solution for the dividend rate and marginal values of bank equity satisfy a system of equations described below.

**Proposition 4** (Solution for dividends and bank value) *Given the solution to the portfolio problem (5) the dividend ratio and value of bank equity are given by:*

$$\text{div}(X) = \frac{1}{1 + \left[\beta(1 - \gamma) \mathbb{E}[v(X') | X] \Omega^*(X)^{1-\gamma}\right]^{1/\gamma}}$$

and

$$v(X) = \frac{1}{1 - \gamma} \left[1 + (\beta(1 - \gamma) \Omega^*(X)^{1-\gamma} \mathbb{E}[v(X') | X])^{1/\gamma}\right]^{\gamma}.$$  

The policy functions of banks determine the loan supply and demand for reserves. This concludes the partial equilibrium analysis of the bank’s portfolio decisions. We now describe the demand for loans and the actions of the Fed.

### 2.4 Loan Demand

We consider a downward sloping demand for loans with respect to the loan rate, i.e., increasing on the price. In particular, we consider a constant elasticity demand function:

$$q_t = \Theta_t \left(I_t^D\right)^\epsilon, \epsilon > 0, \Theta_t > 0.$$  

(7)

where $\epsilon$ is the inverse of the semi-elasticity of credit demand with respect to the price, which could capture the extent to which non-financial firms can substitute bank loans for other forms of liabilities. The term $\Theta_t$ captures possible credit demand shifters. Below, we analyze a microfoundation for this loan demand function based on a working capital constraint for firms.

### 2.5 The Fed’s Balance Sheet and its Operations

This section describes the Fed’s balance sheet and how the Fed implements monetary policy. The Fed’s balance sheet is analogous to that of commercial banks with an important exception: the
Fed does not issue demand deposits as liabilities, it issues reserves instead. As part of its assets, the Fed holds commercial bank deposits, $D^\text{Fed}_t$, and private sector loans, $B^\text{Fed}_t$. As liabilities, the Fed issues $M^0_t$ reserves—high power money. The Fed’s assets and liabilities satisfy the following laws of motion:

\[
M^0_{t+1} = M^0_t + \varphi^\text{Fed}_t
\]

\[
\frac{D^\text{Fed}_{t+1}}{R^D} = D^\text{Fed}_t + p_t \varphi^\text{Fed}_t + (1 - \delta) B^\text{Fed}_t - q_t I^\text{Fed}_t + \chi^\text{Fed}_t - T_t
\]

\[
B^\text{Fed}_{t+1} = \delta B^\text{Fed}_t + I^\text{Fed}_t.
\]

The laws of motion for these state variables are very similar to the laws of motion for banks. Here, $\varphi^\text{Fed}_t$ represents the Fed’s purchase of deposits by issuing reserves to commercial banks. Its deposits are affected by the purchase or sale of loans, $I^\text{Fed}_t$, and the coupon payments of previous loans, $(1 - \delta) B^\text{Fed}_t$. In addition, the Fed’s deposits vary with, $T_t$, the transfers to or from the fiscal authority —the analogue of dividends. Finally, $\chi^\text{Fed}_t$ represents the Fed’s income revenue that stems from its participation in the fed funds market:

\[
\chi^\text{Fed}_t = r^{\text{DW}}_t \left(1 - \gamma^-\right) M^- - r^{\text{ER}}_t \left(1 - \gamma^+\right) M^+.
\]

\begin{tabular}{l}
\hline
Earnings from Discount Loans & \hline
\end{tabular}

\begin{tabular}{l}
\hline
Losses from Interest Payments on Excess Reserves & \hline
\end{tabular}

The Fed’s balance sheet constraint is obtained by combining the laws of motion for reserves, loans and deposits:

\[
p_t \left(M^0_{t+1} - M^0_t\right) + (1 - \delta) B^\text{Fed}_t + \chi^\text{Fed}_t = D^\text{Fed}_{t+1}/R^D - D^\text{Fed}_t + q_t (B^\text{Fed}_{t+1} - \delta B^\text{Fed}_t) + T_t. \quad (8)
\]

The Fed has a monopoly over the supply of reserves, $M^0_t$, and alters this quantity through several operations.

**Unconventional Open-Market Operations.** Since there are no government bonds, only unconventional open-market operations are available.\(^{19}\) An unconventional OMO involves the purchase of loans and the issuance of reserves. This operation does not affect the stock of commercial bank deposits held by the Fed. To keep the amount of deposits constant, the Fed issues $M^0$ buying deposits from banks, but then sells those deposits to purchase loans.

**Open-Market Liquidity Facilities.** Liquidity facilities are deposits of reserves by the Fed

\(^{19}\)Incorporating Treasury Bills (T-bills) and conventional open-market operations into our model is relatively straightforward. If T-bills are illiquid in the balancing stage, T-Bills and loans become perfect substitutes from a bank’s perspective and the model becomes equivalent to our baseline model—with an additional market-clearing condition for T-bills. If T-Bills are perfectly liquid, we can show that banks that have a deficit in reserves sell first their holdings of T-Bills before accessing the interbank market. In the intermediate case where T-Bills are imperfect substitutes, the price of T-Bills would depend on the distribution of assets in the economy.
at commercial banks.

**Fed Profits and Transfers.** In equilibrium, the Fed can return surpluses or losses. These operational results follow from the return on the Fed’s loans and its profits/losses in the interbank market $\chi_t^{Fed}$. We assume that the Fed transfers losses or profits immediately.

### 2.6 Market Clearing, Evolution of Bank Equity and Equilibrium

**Bank Equity Evolution.** Define $\bar{E}_t \equiv \int_0^1 E_t(z) \, dz$ as the aggregate of equity in the banking sector. The equity of an individual bank evolves according to $E_{t+1}(z) = e_t(\omega) E_t(z)$. Here, $e_t(\omega)$ is the growth rate of bank equity of a bank with withdrawal shock $\omega$. The measure of equity holdings at each bank is denoted by $\Gamma_t$. Since the model is scale invariant, we only need to keep track of the evolution of average equity, $\bar{E}_t$ which by independence grows at the rate $E_t [e_t(\omega)]$.\(^{20}\)

**Loans Market.** Market clearing in the loans market requires us to equate the loan demand $I_t^D$ to the supply of new loans made by banks and the Fed. Hence, equilibrium must satisfy:

$$I_t^D \equiv \left( q_t / \Theta_t \right) \frac{1}{\epsilon} = B_{t+1} - \delta B_t + B_{t+1}^{Fed} - \delta B_t^{Fed}. \quad (9)$$

**Money Market.** Reserves are not lent outside the banking system; there is no use of currency. This implies that the aggregate holdings of reserves during the lending stage must equal the supply of reserves issued by the Fed:

$$\int_0^1 \tilde{c}_t(z) E_t(z) \, dz = M_{0t} \quad \rightarrow \tilde{c}_t \bar{E}_t = M_{0t}. \quad (10)$$

**Interbank Market.** The equilibrium conditions for the interbank market depend on $\gamma^+$ and $\gamma^-$, the probability of matches in the reserve market. These probabilities, in turn, depend on $M^-$ and $M^+$, the mass of reserves in deficit and surplus. During the lending stage, banks are identical replicas of each other scaled by equity. Thus, for every value of $E_t(z)$, there’s an identical distribution of banks short and long of reserves. The shock that leads to $x = 0$ is $\omega^* = (\tilde{C}/p - \rho \tilde{D}) / (1 - \rho)$. This implies that the mass of reserves in deficit is given by:

$$M^- = E [x(\omega) | \omega > \omega^*] \left( 1 - F \left( \frac{\tilde{C}/p - \rho \tilde{D}}{1 - \rho} \right) \right) \bar{E}_t$$

and the mass of surplus reserves is,

$$M^+ = E [x(\omega) | \omega < \omega^*] F \left( \frac{\tilde{C}/p - \rho \tilde{D}}{1 - \rho} \right) \bar{E}_t.$$

\(^{20}\)A limiting distribution for $\Gamma_t$ is not well-defined unless one adapts the process for equity growth.
**Money Aggregate.** Deposits constitute the monetary creation by banks, \( M_t^1 \equiv \int_0^1 \hat{d}(z) E_t(z) \, dz \). The endogenous money multiplier is \( \mu_t = \frac{M_t^1}{M_0^t} \).

**Equilibrium.** The definition of equilibrium is as follows.

**Definition.** Given \( M_0^t, D_0^t, B_0^t \), a competitive equilibrium is a sequence of bank policy rules \( \{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, \text{div}_t\}_t \geq 0 \), bank values \( \{v_t\}_t \geq 0 \), government policies \( \{\rho_t, D_{Fed}^t, B_{Fed}^t, M_0^t, T_t, \kappa_t, r_{ER}^t, r_{DW}^t\}_t \geq 0 \), aggregate shocks \( \{\Theta_t, F_t\}_t \geq 0 \), measures of equity distributions \( \{\Gamma_t\}_t \geq 0 \), measures of reserve surpluses and deficits \( \{M^+, M^-\}_t \geq 0 \) and prices \( \{q_t, p_t, r_{FedFunds}^t\}_t \geq 0 \), such that: (1) Given price sequences \( \{q_t, p_t, r_{FedFunds}^t\}_t \geq 0 \) and policies \( \{\rho_t, D_{Fed}^t, B_{Fed}^t, M_0^t, \kappa_t, r_{ER}^t, r_{DW}^t\}_t \geq 0 \), the policy functions \( \{\tilde{c}_t, \tilde{b}_t, \tilde{d}_t, \text{div}_t\}_t \geq 0 \) are solutions to Problem 4. Moreover, \( v_t \) is the value in Proposition 3. (2) The money market clears: \( \tilde{c}_t \cdot E_t = M_0^t \). (3) The loan market clears: \( I_t^D = \Theta_t^{-1} q_t^1 \). (4) \( \Gamma_t \) evolves consistently with \( e_t(\omega) \). (5) The masses \( \{M^+, M^-\}_t \geq 0 \) are also consistent with policy functions and the sequence of distributions \( F_t \). All the policy functions of Problem 4 satisfy \( \left[ \tilde{C} \tilde{B} \tilde{D} \right] = \left[ \tilde{c} \tilde{b} \tilde{d} \right] \cdot E \).

Before proceeding to the analysis of particular parameterizations of the model, we discuss a possible microfoundation for the demand for loans and the supply of deposits.

### 2.7 Non-Banking Sector

The competitive equilibrium defined above assumes an exogenous demand for loans, given by (7), and an exogenous supply of deposits — the banking system faces a perfectly elastic supply of deposits at rate \( R^D \). In Appendix D we provide a simple microfoundation for the demand for loans and the supply of deposits. This microfoundation has the following features.

**Loans Supply.** We introduce a continuum of households with quasi-linear utility. Deposits are their only savings instruments. They face convex disutility from labor and linear utility from consumption. The linearity in consumption leads to a perfectly elastic supply of savings where \( R^D \) equals the inverse of the discount factor of households, \( 1/\beta^D \). The lump-sum tax \( T_t \) on the Fed’s budget constraint is levied from these households. This assumption guarantees that taxes do not affect the supply of deposits or the demand for loans.

**Derivation of Loan Demand.** The demand for loans (7) emerges from the decisions of firms that need to borrow working capital to hire workers. Hiring decisions are made once, but production is realized slowly, in a way that delivers the maturity structure of debt that we described above.
3 Theoretical Analysis

3.1 Liquidity Premia and Liquidity Management

This section provides more insights about the implementation of monetary policy in the model. First, we derive an expression for a liquidity premium of reserves relative to loans. This liquidity premium has two components: the direct marginal benefit of avoiding borrowing in the interbank market and a risk premium. We then consider the case of risk-neutral banks. That exercise illustrates that there are real effects of monetary policy as long there is a kink in $\chi(\cdot)$. We then analyze the model when there are no withdrawals. In this case, excess reserves are zero and hence, there are limited effects from monetary policy. Finally, we analyze equilibria when when $r^{DW} = r^{ER} = 0$, a version of the zero lower bound (ZLB). For that case, lending is determined by the banking system’s equity, the capital requirements, and demand shocks, but not by withdrawal risks.

**Bank Portfolio Problem.** Fix a state $X$. To spare notation, we suppress the $X$ argument from prices and policy functions and leave this reference as implicit. We rewrite Problem 5 by inserting the budget constraint into the objective:

$$
\Omega = \max_{w_d \in [0,\kappa]} \left( E_{\omega'} \left[ \left( \frac{R^B}{\text{Return on Loans}} - (R^B - R^C) w_d + \frac{(R^B - R^D) w_d - R^x (w_d, w_c, \omega')}{\text{Arbitrage}} \right)^{1-\gamma} \right] \right)^{\frac{1}{1-\gamma}}.
$$

This objective can be read as follows. If banks hold no reserves nor issue deposits, they obtain a return on equity of $R^B$. Issuing additional deposits provides a direct arbitrage of $R^B - R^D$, but also exposes the bank to greater liquidity costs $R^x (w_d, w_c, \omega')$. In turn, banks can reduce these liquidity costs by holding more reserves, although they must forgo an opportunity cost, the spread between loans and reserves, $R^B - R^C$.

**Liquidity Premium.** First-order conditions with respect to reserves and deposits yield:

$$
w_C :: \quad R^B - R^C = -\frac{E_{\omega'} \left[ (R^E_{\omega'})^{-\gamma} R^x (w_d, w_c, \omega') \right]}{E_{\omega'} (R^E_{\omega'})^{-\gamma}} \tag{10}
$$

and

$$
w_D :: \quad R^B - R^D = \frac{E_{\omega'} \left[ (R^E_{\omega'})^{-\gamma} (R^x (w_d, w_c, \omega')) \right] + \mu}{E_{\omega'} (R^E_{\omega'})^{-\gamma}}, \tag{11}
$$

where $\mu$ is the multiplier associated with the capital requirement constraint.\(^{21}\) We rearrange (10)

\(^{21}\)We ignore the non-negativity constraints on deposits and loans because they are not binding in equilibrium. In addition, we assume that reserves are strictly positive.
and define the stochastic discount factor \( m' \equiv \text{div} (X') \frac{E_{\omega'} [(R^S_{\omega'})^{-\gamma}] E[1-\text{div}(X)]}{E[\text{div}(X)]} \) to obtain:

\[
R^B - R^C = -\frac{E_{\omega'} [m' \cdot R^X_c (w_d, w_c, \omega')]}{E_{\omega'} [m']}
= -\frac{E_{\omega'} [R^X_c (w_d, w_c, \omega')]}{E_{\omega'} [m']} + \frac{COV_{\omega'} [m', R^X_c (w_d, w_c, \omega')]}{E_{\omega'} [m']}
\]

The left-hand side of this expression is the liquidity premium, i.e., the difference between the return on loans and reserves. This liquidity premium equals the direct benefit of holding additional reserves, \(-E_{\omega'} [R^X_c (w_d, w_c, \omega')]\), adjusted by a liquidity risk premium. The direct benefit, \(-E_{\omega'} [R^X_c (w_d, w_c, \omega')]\), is the expected marginal reduction in expected interest payments in the interbank market by holding additional reserves. The liquidity risk premium emerges because the stochastic discount factor varies with \( \omega' \).

We obtain a similar expression for the spread between loans and deposits:

\[
R^B - R^D \geq \frac{E_{\omega'} [R^X_d (w_d, w_c, \omega')]}{E_{\omega'} [m']} - \frac{COV_{\omega'} [m', R^X_d (w_d, w_c, \omega')]}{E_{\omega'} [m']}
\]

which holds at equality if \( w_d < \kappa \).

This expression states that the direct arbitrage obtained by lending, \( R^B - R^D \), equals the expected marginal increase in liquidity costs of additional deposits, \( E_{\omega'} [R^X_d (w_d, w_c, \omega')] \), plus a liquidity risk premium. In addition, when the capital requirement constraint is binding, this excess return is larger.\(^{22}\)

Define a bank’s reserve rate as \( L \equiv (w_c/w_d) \). The following lemma states that liquidity costs are linear in \( \{w_d, w_c\} \):

**Lemma 1 (Linear Liquidity Risk)** \( E_{\omega'} [R^X_c (w_d, w_c, \omega')] \) is homogeneous of degree \( \{w_d, w_c\} \).

Moreover, we have an exact expression for the expected marginal benefit of additional reserves:

**Lemma 2 (Marginal Liquidity Cost)** The marginal value of liquidity is:

\[
-\mathbb{E}_{\omega'} [R^X_c (1, L, \omega')] = \chi_b \Pr \left[ \omega' \geq \frac{L - \rho}{(1 - \rho)} \right] + \chi_l \Pr \left[ \omega' \leq \frac{L - \rho}{(1 - \rho)} \right].
\]

\(^{22}\)These expressions are similar to other standard asset-pricing equations with portfolio constraints except for the liquidity adjustment. This expression may be useful for empirical investigations. For example, during the financial crises of 2008-2009, interest rate spreads widened. This increase has been attributed to greater credit risks and tighter capital requirements. The formulae above suggest that liquidity risks could also explain part of these spreads and the expression may be useful to distill these effects.
This lemma implies that the marginal value of additional liquidity, \( \omega^d \mathbb{E}_{\omega'} [R^x_c (1, L, \omega')] \), equals the expected interest payments from the interbank market. Finally, recall that the lemma above implies:

**Corollary 1** If there is no spread in the corridor system, \( r^{ER} = r^{DW} \), then \( r^{FF} = r^{ER} = r^{DW} \) and the marginal value of liquidity is constant and equal to \( R^x_c = r^{FF} \).

We will use this corollary and the previous lemma to derive additional results below.

### 3.2 Limit Case I: Risk-Neutral Banks (\( \gamma = 0 \)).

For \( \gamma = 0 \), the bank’s objective is to maximize expected returns. Thus, for this case:

\[
\Omega = R^B + \max_{\{w_d, w_c\}} \left( R^B - R^D \right) w_d - \left( R^B - R^C \right) w_c - \mathbb{E}_{\omega'} [R^x (w_d, w_c)].
\]

By Lemma 1, we can factor \( w_d \) and transform the problem above to:

\[
\Omega = R^B + \max_{w_d} w_d \begin{pmatrix}
\left( R^B - R^D \right) + \max_L \left\{ - \left( R^B - R^C \right) L - \mathbb{E}_{\omega'} [R^x (1, L)] \right\}
\end{pmatrix}
\]

subject to \( \omega^d \in [0, \kappa] \) and \( L \in \left[ 0, \frac{1 + \omega^d}{\omega^d} \right] \).

This reformulation shows that the portfolio problem of risk-neutral bankers can be separated into two. First, the bank must solve an optimal liquidity management problem. Second, given a choice for \( L \), the return per unit of leverage becomes linear and the bank must choose a leverage scale.

The choice of leverage obeys the following trade-off. Issuing deposits yields a direct return of \( (R^B - R^D) \). However, the \( L \) fraction of deposits are used to purchase reserves optimally. The optimal reserve ratio trades off the opportunity cost of obtaining liquidity against the reduction in the expected illiquidity cost. Let \( L^* \) be the optimal reserve ratio. \( L^* \) satisfies:

\[
\left( R^B - R^C \right) = - \mathbb{E}_{\omega'} [R^x_c (1, L^*)] \quad (12)
\]

which is consistent with the first-order condition (10) when \( m = 1 \). Given \( L^* \), the problem is linear in \( w_d \) if \( L^* < \frac{1 + w_d}{w_d} \). In equilibrium, \( L \leq \frac{1 + w_d}{w_d} \) is non-binding, otherwise an equilibrium features no loans. This, in turn, is ruled out by the shape of the loan demand. Since \( -\mathbb{E}_{\omega'} [R^x_c (1, L, \omega')] \in [r^{ER}_t, r^{DW}_t] \), the first-order condition above implies a relationship between the liquidity premium and the rates of the corridor system:

**Proposition 5** In equilibrium, \( R^C + r^{ER}_t \leq R^B \leq R^C + r^{DW}_t \).
The proposition shows that the Fed’s corridor rates impose restrictions on the equilibrium spread between loans and reserves. In particular, this spread is bounded by the width of the corridor rates.\textsuperscript{23} There are several insights that follow from the proposition. First, equation (12) captures a first-order effect of monetary policy. The choice of reserve holdings affects the expected penalties incurred in the interbank market. Thus, although risk aversion may reinforce this effect, monetary policy has effects in a risk-neutral environment through this channel. Second, if $r^{DW} = r^{ER}$, the marginal value of liquidity is independent of $\omega$. This implies that under risk neutrality, changes in second, or higher order moments of $F_t$, do not affect portfolio choices. Moreover, the proposition also underscores the role of the kink in $\chi$: When $r^{DW} = r^{ER}$, $\chi$ has no kink. This means that the Fed cannot target $R^B$ and $R^C$ simultaneously because the bank’s portfolio and all interest rates are determined uniquely by the choice of $r^{DW} = r^{ER}$. There is no scope for open-market operations.

Now, defining the return to an additional unit of leverage—the bank’s levered returns is:

$$R^L = \left( R^B - R^D \right) - \left( (R^B - R^C) L^* + E\omega [R^x (1, L^*)] \right).$$

An equilibrium for $\gamma = 0$ is characterized by:

**Proposition 6** (Linear Characterization) When $\gamma = 0$, in equilibrium, $\Omega = R^B + \max \{ \kappa R^L, 0 \}$, and:

$$w^* = \begin{cases} 0 & \text{if } R^L < 0 \\
[0, \kappa] & \text{if } R^L = 0 \text{ and } \text{div} = \begin{cases} 0 & \text{if } \beta \nu \Omega > 1 \\
[0, 1] & \text{if } \beta \nu \Omega = 1 \\
1 & \text{if } \beta \nu \Omega = 1 
\end{cases} \\
\kappa & \text{if } R^L > 0 \end{cases}$$

In a steady state, $\beta \nu \Omega = 1$, $\text{div} = \Omega - 1$. A steady state falls into one of the following cases:\textsuperscript{24}

**Case 1** (non-binding leverage constraint steady state ($\mu = 0$)). The steady-state value of equity, $E_{ss}$, is sufficiently large such that $R_{ss}^B = 1/\beta$ is feasible and the following conditions hold:

$$R^L = (1/\beta - 1/\beta^D) - ((1/\beta - R^C) L^* + R^x (1, L^*)) = 0, R^E = \frac{1}{\beta}.$$  

**Case 2** (binding leverage constraint steady state ($\mu > 0$)). $E^{ss}$ is such that for $w^* = \kappa$: $R_{ss}^B > 1/\beta$ and,

$$R^L = (R^B - 1/\beta^D) - ((R^B - R^C) L^* + R^x (1, L^*)) > 0, \ (R^B + \kappa R^L) = \frac{1}{\beta}. \quad (13)$$

\textsuperscript{23}Under risk aversion, a risk premium adjustment would emerge and the loan-reserve spread could exceed the width of the bands. However, the corridor system would still impose bounds on the interest spread because the liquidity risk premium is also affected by the width of the bands.

\textsuperscript{24}Unless leverage constraints are binding, a transition toward a steady state is instantaneous as in other models with linear bank objectives—i.e., Bigio (2010) for example, If dividends cannot be negative and equity is low, banks would retain earnings until they reach a $E_{ss}$, consistent with the proposition.
Proposition 6 characterizes two potential classes of steady states. If at steady state, capital requirements do not bind, the choice of \( \{ r_{ss}^{ER}, r_{ss}^{DW} \} \) and \( M_{0ss} \) can affect \( R^C \) but not \( R^B \). If instead capital requirements are binding, different combinations of \( \{ r_{ss}^{ER}, r_{ss}^{DW} \} \) and \( M_{0ss} \) can affect \( R^C \) separately from \( R^B \), as long as these rates satisfy (13).

3.3 Limit Case II: No Withdrawal Shocks (\( \Pr (\omega = 0) = 1 \)).

A special case that provides additional insights is when there are no withdrawal shocks, \( \Pr [\omega = 0] = 1 \). For this case, there is no difference between the portfolio decisions of risk-neutral and a risk-averse bankers —although their dividend policies may differ because the intertemporal elasticity of substitution may vary. Without uncertainty, the value of the portfolio problem is:

\[
R^B + \max_{w_d \in [0, \kappa]} w_d \left[ (R^B - R^D) + \left( \max_{L \in [0, \frac{1+\omega_d}{1-\omega_d}]} - \left( (R^B - R^C) L + \chi (L - \rho) \right) \right) \right].
\]

An equilibrium with deterministic shocks satisfies the following analogue of Proposition 6:

**Proposition 7** In equilibrium, \( R^C + r_t^{ER} \leq R^B \leq R^C + r_t^{DW} \). Moreover, in an equilibrium with positive reserve holdings, \( L^* = \rho \). The value of the bank’s portfolio is \( \Omega = R^B + \kappa \max \left\{ \left( (R^B - R^D) - (R^B - R^C) \rho \right) \right\} \) and the banker’s policies are:

\[
w_d^* = \begin{cases} 
0 & \text{if } R^B < R^D + (R^B - R^C) \rho \\
[0, \kappa] & \text{if } R^B = R^D + (R^B - R^C) \rho \\
\kappa & \text{if } R^B > R^D + (R^B - R^C) \rho 
\end{cases}
\]

An equilibrium with deterministic shocks satisfies the following analogue of Proposition 6:

According to this proposition, in a monetary equilibrium —\( M_{0t} > 0 \), a banker sets the reserve ratio to \( \rho \). Since \( L^* = \rho \) is independent of \( \{ r_t^{ER}, r_t^{DW} \} \), as long as this implementability constraint is satisfied, changes in \( \{ r_t^{ER}, r_t^{DW} \} \) have no effects on allocations. This is an important observation because it underscores the role of liquidity risk: the corridor rates affects equilibrium allocations only if there is liquidity risk. The reason is that \( r_t^{DW} (r_t^{ER}) \) acts like a punishment (prize) for holding reserves below (above) \( \rho \). Without risk, increasing \( r_t^{DW} \) is like increasing the punishment of a constraint that is already satisfied for a lower punishment. A similar insight holds for \( r_t^{ER} \).

Overall, for this limit case, since banks hold a liquidity ratio of \( L^* = \rho \) per deposit, reserve requirements act like a tax on financial intermediation: for every deposit, banks must maintain \( \rho \)

\[\text{When shocks are deterministic, banks control the amount of liquidity holdings by the end of the period. In that case, they choose zero holdings of reserves if they can either borrow them cheaply from the discount window, } r_t^{DW} \leq R_t^B - R_t^C, \text{ or they would not hold loans if the interest rate on excess reserves exceeds } R_t^B - R_t^C. \text{ In equilibrium, reserves and loans are made so } r_t^{ER} \leq R_t^B - R_t^C \leq r_t^{DW} \text{ is an implementability condition for the Fed’s policy.}\]
in reserves—which earn no return as opposed to loans. The rest of the equilibrium is characterized
by Propositions 3 and 4.

3.4 Limit Case III: Zero Lower Bound ($r^{DW}_t = r^{ER}_t = 0$).

Consider the ZLB as states where there is no liquidity risk, i.e. $\chi_t(\cdot) = 0$. We focus on the case
where $r^{DW}_t = r^{ER}_t = 0$.26,27

Thus, $\Omega$ becomes:

$$\Omega = R^B + \max_{wd} \left( (R^B - R^D) + \left( \max_{L \in [0, 1+\omega d]} (R^B - R^C) L \right) \right).$$

An equilibrium with strictly positive holdings of both loans and reserves requires $R^B = R^C$, as
reserves are only valued due to their monetary return. Because risk of withdrawals play no role, the
asset composition of the individual bank’s balance sheet is indeterminate. If in addition, capital
requirements do not bind, then $R^B = R^C = R^D$ so $\Omega = R^B + \kappa \max \{ R^B - R^D, 0 \}$. In summary,

**Proposition 8** A monetary equilibrium at the ZLB, $r^{DW}_t = r^{ER}_t = 0$, satisfies, $R^B_t = R^C_t \geq R^D_t$.
The inequality is strict if and only if capital requirements are binding.

Notice that at the ZLB, the Fed has effects on lending if the capital requirement is binding. By carrying out open-market operations and varying the relative return on reserves, the Fed can affect lending.

4 Calibration

4.1 Dispersion of Deposit Growth ($F_t$)

Our model requires a specification of the random withdrawal process for deposits, $F_t$. To obtain
an empirical counterpart for this distribution, we use information from individual US commercial-
bank Call Reports. The Call Reports contain balance sheet information obtained from regulatory
filings collected by the Federal Deposit Insurance Corporation (FDIC). This information is com-

26 Absence of liquidity risk also arises when banks are not subject to capital requirements, $R^C \geq R^D$ and
$r^{ER} = 0$. In this case, banks accumulate enough reserves so that they always have enough reserves to cover
deposits withdrawals. As long as deposits have a higher return or capital requirements bind, banks remain exposed
to liquidity risk and there is a determined portfolio for individual banks, unlike standard monetary models (see
e.g. Buera and Nicolini (2013) for a model with financial frictions and a cash in advance constraint on households
consumption).

27 The bounds on $r^{DW}_t, r^{ER}_t \geq 0$ arise naturally in this setup. If $r^{DW}_t = r^{ER}_t$, one could argue that banks could
request to hold currency—as opposed to electronic reserves. If $r^{DW}_t < 0$, banks would make infinite profits by
borrowing from the Fed.
piled quarterly, so we define a period in our model as one quarter. We use information from 2000Q1-2010Q4.

In our model, all banks experience the same expected growth rates in deposits during the lending stage. Deviations from the average growth during the lending stage are directly associated with $\omega$, the withdrawal shocks in the model. Hence, the distribution of the deviations from average deposit growth rates is directly associated with $F_t$. Thus, we calibrate $F_t$ to that distribution.

Now, there is no obvious empirical counterpart for demand deposits, the only liability in our model. In practice, commercial banks have other liabilities that include bonds and interbank loans, long-term deposits such as time and savings deposits, in addition to demand deposits. To obtain an empirical counterpart of $F_t$, we use total deposits which include time and saving deposits and demand deposits. There are several reasons for this choice. The first reason is practical: total deposits feature a trend similar to the growth of all bank liabilities. This is not true when we use demand deposits. A second reason is that we do not want to attribute all deposit funding to demand deposits. Demand deposits feature substantially more dispersion than total deposits, which could exaggerate the liquidity costs associated with monetary policy changes. Finally, although total deposits feature less dispersion than demand deposits, their is still substantial dispersion in growth rates of deposits as Figure 1 shows.

The histogram in Figure 1 reports the empirical frequencies of the cross-sectional deviations of growth rates from the mean growth rates of the cross-section, for each bank-quarter observation. The bars in Figure 1 report the pre-crisis frequencies for the 2000Q1-2007Q4 sample of cross-sectional dispersion in deposit growth rates. The solid curve is the analogue for a post-crisis sample, 2008Q1-2010Q4. The dispersion in growth rates in Figure 1 suggests that total deposits are consistent with substantial liquidity risk, according to our model. However, the comparison among both samples shows only a minor change in the distribution during the crises —with a slightly more concentrated mass on the left.\footnote{We use this information and a shutdown in the interbank market to study when we investigate hypothesis 3.}

Given the constructed empirical distribution, we fit a logistic distribution $F(\omega, \mu^\omega, \sigma^\omega)$ with $\mu = -0.0029$ and $\sigma^\omega = 0.022$. We conduct a Kolmogorov-Smirnov goodness-of-fit hypothesis test. We cannot reject that the empirical distribution is logistic —with a 50 percent confidence. Appendix , provides additional details on how we construct the empirical distribution of deposit growth-rate deviations. That appendix also investigates the empirical soundness of other features of our model.\footnote{Our model predicts that the growth of equity is highly correlated —though not perfectly correlated— with the behavior of deposits. In the appendix, we show a positive correlation of about 0.17. This should be expected since our model does not capture credit risks , variations in security prices, differences in dividend policies, or shifts in operating costs. We also discuss the validity of the time-independence of $\omega$. We show that our deposit growth measures show a positive but small autocorrelation —of about 0.17.}
Figure 1: Histogram of Deviations from Cross-Sectional Mean Growth Rates for Total Deposits. 

Note: For every bank-quarter observation, the histogram reports frequencies for deviations of the growth rate of total deposits relative to the cross-sectional average growth of total deposits in a given quarter.
4.2 Parameter Values

The values of all parameters are listed in Table 1. We need to assign values to the following parameters \( \{\kappa, \rho, \beta, \delta, \gamma, \epsilon, R_d\} \). We set the capital requirement, \( \kappa = 15 \), and the reserve requirement, \( \rho = 0.05 \), to be consistent with actual regulatory parameters: this choice corresponds to a required capital ratio of 9 percent and a reserve ratio of 5 percent. We set \( \delta = 0 \) so that loans become one-period loans. We set risk aversion to \( \gamma = 0.5 \). The value of the loan demand elasticity given by the inverse of \( \epsilon \) is set to 1.8, which is an estimate of the loan demand elasticity by Bassett et al. (2010).\(^{30}\) Finally, we set the discount factor so as to match a return on equity of 8 percent a year. This implies \( \beta = 0.98 \). The interest rate on deposits is set to \( R_D = 1 \).

<table>
<thead>
<tr>
<th>Value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital requirement</td>
<td>( \kappa = 10 )</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>( \beta = 0.985 )</td>
</tr>
<tr>
<td>Risk Aversion</td>
<td>( \gamma = 0.5 )</td>
</tr>
<tr>
<td>Loan Maturity</td>
<td>( \delta = 0 )</td>
</tr>
<tr>
<td>Reserve Requirement</td>
<td>( \rho = 0.05 )</td>
</tr>
<tr>
<td>Loan Demand Elasticity</td>
<td>( 1/\epsilon = 1.8 )</td>
</tr>
<tr>
<td>Discount Window Rate (annual)</td>
<td>( R_{DW} = 2.5 )</td>
</tr>
<tr>
<td>Interest on Reserves (annual)</td>
<td>( r^{ER} = 0 )</td>
</tr>
</tbody>
</table>

Table 1: Parameter Values

We also fix the steady-state values of \( r^{ER}, R_{DW} \) and \( R^C \) —our policy target. We set \( r^{ER} = 0 \), which is the pre-crisis interest rate on reserves paid by the Federal Reserve. The interest rate on discount window rate is set to 3 percent expressed at annualized rates. These choices deliver a fed funds rate of 1.25 percent.\(^{31}\) Finally, we assume that the Fed targets price stability so \( R^C = 1 \).

4.3 Steady State Equilibrium Portfolio

We start with an analysis of the equilibrium portfolio at steady state and investigate the effects of withdrawal shocks on banks’ balance sheets. The equilibrium portfolio corresponds to the solution of the Bellman equation (1) evaluated at the loan price that clears the loans market, according to condition (9), and the equilibrium probability of matching in the interbank market.

The left panel of Figure 2 shows the probability distribution of the reserve deficits during the balancing stage, and the penalty associated with each deficit —the mass of the probability

\(^{30}\)This value for the elasticity of loan-demand is consistent with the microfoundation provided in Appendix D, based on estimates of the elasticity of labor supply in the lower range.

\(^{31}\)Since we consider a steady state without inflation, this is also the real interest rate.
distribution is rescaled to fit in the same plot. The penalty function \( \chi \) has a kink at zero, because \( r_{\text{DW}} > r_{\text{ER}} \). Notice that the distribution of the reserve deficits inherits the distribution of the withdrawal shock, as the reserve deficit depends linearly on the withdrawal realization. Because in equilibrium, there is an average excess surplus, the distribution’s mean is above zero.

The right panel of Figure 2 shows the distribution of equity growth as function \( \omega \). In equilibrium, banks that experience deposit inflows will increase their equity, whereas those that experience outflows see their equity shrink. Because the penalty inflicts relatively higher losses to outflows than to the benefits from inflows, the distribution of equity growth is skewed to the left. In particular, there is a fat tail with probabilities of losing about 2 percent of equity in a given period, while the probability of growing more than 1 percent in a period is close to nil.

![Figure 2: Portfolio Choices and Effects of Withdrawal Shocks](image)

### 4.4 Policy Functions at Given Prices

We start with a partial equilibrium analysis of the model by showing banks’ policy functions for different loan prices. Figure 3 reports decisions for reserves, loans and dividends, as well as liquidity and leverage ratios, the value of the asset portfolio, liquidity risks, expected returns, and expected equity growth rates for different loan prices \( q \). The policies correspond to the solution to the Bellman equation (4) for different values of \( q \), and fixing the probability of a match in the interbank markets at steady state. The solid dots in Figure 3 are the values associated with the equilibrium \( q \).

As Figure 3 shows, the supply of loans is decreasing in \( q \) —i.e., increasing in the return on loans. Instead, reserve rates are increasing in \( q \). As the loan prices decrease, loans become more profitable which leads banks to keep a lower fraction of their assets in low return assets, i.e., reserves. For a sufficiently low price of loans, the non-negativity constraint on reserves becomes
Figure 3: Policy Function for Different Loan Prices

binding — banks only borrow reserves from the Fed and pay them back by the end of the balancing stage.

In addition, dividends are increasing in $q$ due to a substitution effect: when returns on loans are high, banks cut dividend payments to allocate more funds to profitable lending. Exposure to liquidity risk, measured as the standard deviation of the cost of rebalancing the portfolio $\chi(x)x$, is also decreasing in loan prices, reflecting the fact that banks’ asset portfolios become relatively more illiquid when loan prices decrease.
5 Transitional Dynamics

This section studies the transitional dynamics of the economy in response to different shocks associated with hypotheses 1–5. The shocks we consider are equity losses, a tightening of capital requirements, an increase in the dispersion of withdrawals, a shutdown of the interbank market, credit demand shocks, and changes in the discount window and interest rate on reserves. Shocks are unanticipated upon arrival at \( t = 0 \) but their paths are deterministic for \( t > 0 \).\(^{32}\) Throughout the experiments, we consider a monetary policy regime such that the Fed has a zero inflation target \( R^C = 0 \), i.e., the Fed performs open-market operations —altering \( MO_t \)— to maintain price stability: \( p_t = p \).\(^{33}\)

5.1 Equity Losses

We begin with a shock that translates into a sudden unexpected decline in bank equity. This shock captures an unexpected rise in non-performing loans, security losses or off-balance sheet losses left out of the model.\(^{34}\) Figure 4 illustrates how bank balance sheets shrink in response to 2 percent equity losses. The top panel shows the evolution of total lending, total reserves, and liquidity risk, and the bottom panel shows the level of equity, return on loans and the dividend rate.

To understand these dynamics, recall that all bank policy functions are linear in equity. Thus, holding prices fixed, a loss in equity should lead to a proportional 2 percent decline in loans and reserves. However, the contraction in loan supply also generates a drop in loan prices on impact —through a movement along the loan demand. The reduction in \( q \) leads to an increase in loan returns through the transition. As a consequence of the higher profitability on loans, reserve holdings fall relatively more than loans. Banks shift their portfolios toward loans while willingly exposing themselves to more liquidity risk. The overall return to the banks’s portfolio also increases after the shock. With this, dividends fall as their opportunity cost increases. The increase in bank returns and lower dividends leads to a gradual recovery of initial equity losses. As equity recovers, the economy converges to the initial steady state and the transition is quick; the effects of the shock cannot be observed after six quarters.

When \( \delta > 0 \), there is an additional amplification effect not shown here. The reduction in the supply of credit further lowers \( q \), and this in turn, lowers marked-to-market equity, \( E \), beyond the initial impact of the shock. All other responses are therefore amplified.

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\(^{32}\)The assumption of unanticipated shocks is mainly for pedagogical purposes. In fact, it is relatively straightforward to compute the model to allow for aggregate shocks, which are anticipated. Due to scale invariance, we would not have to keep track of the cross-sectional distribution of equity anyway.

\(^{33}\)We assume this not only for illustrative reasons but also because in the context of the Great Recession, the core personal consumption expenditures index (PCE) remained close to 1 percent. It is straightforward to consider alternative monetary policy regimes.

\(^{34}\)One way to incorporate this explicitly in the model would be to consider specific shocks to loan default rates. To the extent that equity is the only state variable, the analysis of the transitional dynamics is analogue to studying the evolution of the model under a richer structure for loans.
5.2 Capital Requirements

The effects of a sudden and transitory tightening of capital requirements, i.e., a reduction in $\kappa$, are shown in Figure 5. The shock is a 10 percent decrease in $\kappa$ which is associated with a 1 percent increase in the capital ratio of banks for the calibrated level of leverage.

The short-run behavior of the transition is very similar to the behavior after equity losses. As with equity losses, the contraction in capital requirements reduces the supply of loans because as the constraint gets tighter, banks must operate as if they had less equity—the capital requirement is binding in the steady state.

In the medium term, equity begins to exceed its steady state value. This happens because the return on loans increases and banks pay less dividends. Eventually, the increase in equity overcomes the increase in capital requirements. Ultimately, the economy converges back to a steady state level of equity as the capital requirement shock converges back to its original level.
5.3 Interbank Market Shocks

5.3.1 Bank-Run Shocks

Here we study the possibility of a bank run. We consider a 5 percent probability that all the deposits are withdrawn from a given bank—that is, $\omega = 1$. This bank runs are on individual institutions as we maintain the assumption that deposits are not withdrawn from the banking system as a whole.\textsuperscript{35} We assume that this bank run probability follows a deterministic AR(1) process such that the shock lives for about two years. The effects of this shock are illustrated in Figure 6.

The risk of a bank run generates an increase in liquidity risk, leading banks to hoard reserves. Because reserve requirements are constant, this means that banks accumulate more excess reserves. Notice that the liquidity risk is still about three times larger than in the steady state although banks hold more reserves. As the Fed’s objective is a zero inflation target, the Fed supplies reserves to meet this target. Naturally, higher liquidity costs induce a decline in the supply of loans, as banks substitute loans for reserves. In equilibrium, this leads to an increase in the price of loans and a decline in the aggregate volume of lending.

In tandem, banks respond to the risk of a bank run by cutting dividend payments. Although

\textsuperscript{35}Thus we adjust $F$ accordingly by assuming a 5 percent probability of a large inflow of deposits. It is also possible to extend the model to study system-wide bank runs, as in Uhlig (2010) (see also Robatto (2013)).
higher liquidity costs are associated with lower returns, the contraction in loan supply generates a more-than-compensating increase in expected bank returns. This leads to an increase in equity over time. As equity grows, this mitigates the fall in lending ratios. Eventually, lending rises above its steady state value. This is because several quarters after the shock is realized, the effect on bank equity compensates for the portfolio effect, as the shock begins to decay.

### 5.3.2 Interbank Market Shutdown

Disruptions in the interbank market can be studied through shocks that drive the probability of a match in the interbank market to zero.\(^{36}\) Hence, reserves are borrowed (lent) only from (to) the Fed. In particular, banks that face a reserve deficit borrow directly at \(r^{DW}\). Thus, this shock increases expected liquidity costs. The effects of the interbank market freeze are shown in Figure 7. Overall, the effects are similar to the bank run shock we describe earlier.

\(^{36}\)A recent macroeconomic model of endogenous interbank market freezing due to asymmetric information with one-period lived banks is Boissay et al. (2013).
5.4 Credit Demand

The effects of negative credit demand shocks are captured through a decline in $\Theta_t$. Figure 8 illustrates the effects of a negative temporary shock to $\Theta_t$. We assume the shock follows a deterministic AR(1) process that lasts for about seven years.

The effects of credit demand shocks contrast sharply with the effect of the shocks considered above, because all of the prior shocks cause a contraction in the supply of loans. In contrast, demand shocks cause a decline in the return on loans — and a shift along the supply curve. As a result, banks shift their portfolios toward reserves — as the opportunity cost of holding reserves lowers. The liquidity risk almost vanishes. Initially, banks respond by paying higher dividends due to the overall decline in the their portfolio returns. The reduction in returns and dividend increments brings equity below the steady state. As the shock decays — around a year and a half later — the economy follows a similar transition as with the shock to equity, slowly increasing lending rates and reducing dividend rates until equity returns to the steady state.
5.5 Policy Rates

5.5.1 Discount Window

We now analyze the effects of interest rate policy shocks. In the experiment, we study a positive shock of 100 basis points (bps) to the discount window rate, expressed at annualized rates. The effects of an increase in the discount window rate are depicted in Figure 9. Banks respond to this increase by reducing lending. Policy effects are similar to the effects of shocks that increase liquidity costs. Notice that the pass-through from the policy rate to the return on loans is almost perfect.

5.5.2 Interest on Reserves

A shock to the interest on excess reserves works similarly to an increase in the discount window rate since both increase the return of holding reserves. We study a shock that raises this rate from 0 bps to 100 bps (annualized), a shock that corresponds to the recent Fed policy of remunerating excess reserves. The effect of this policy is illustrated in Figure 10. The shock makes reserves relatively more attractive. In response, banks reallocate their portfolio from loans toward reserves.\textsuperscript{37}

\textsuperscript{37}Notice that liquidity risk does not decline despite the increase in cash holdings by banks. This occurs because the increase in the interest rate on excess reserves leads to larger differences in returns between banks on surplus and banks on deficit.

Figure 8: Impulse Response to Credit Demand Shock

Figure 9: Impulse Response to Discount Window Shock

Figure 10: Impulse Response to Interest on Reserves Shock
Figure 9: Impulse Response to a Rise in the Discount Window Rate

Figure 10: Impulse Response to a Shock to the Interest Rate on Excess Reserves
5.6 Unconventional Open-Market Operations

Finally, we study loan purchases by the Fed. We study the effects of loan purchases amounting to 2 percent of the outstanding stock of loans at the steady state. We also assume that the Fed gradually reverses the operation in about four years. Unconventional open-market operations boost total lending in the economy, as shown in Figure 11. However, there is a partial crowding-out effect. The Fed purchases lower the return on loans, which in turn leads private banks to lend less. In equilibrium, banks also hold more reserves. As a result, the transitions are similar to the transitions after a negative credit demand shock, with the difference that total bank lending increases because of the Fed’s holdings. The reason being that the Fed’s OMO reduce the effective demand for loans that banks face, as the Fed takes over part of their activity.

6 Application - Which Hypotheses Fit the Facts?

This section explores the possible driving forces that explain the holdings of excess reserves without a corresponding increasing in lending by banks sector during the US financial crisis. Here, we discuss how the different shocks we studied in the preceding section fit the patterns we observe for the data. We first revisit some key fact about monetary policy, monetary aggregates and banking indicators during the recession, which motivate our application.

6.1 Monetary Facts

Fact 1: Anomalous Fed-Funds Rate Behavior. The panel (a) of Figure 12 plots the daily series for several interest rates. The fed funds rate is the series that fluctuates around the Fed’s target—the step-like series in the middle. The Fed’s corridor rates, the overnight discount rate and the interest rate on excess reserves, are the data analogues of \( \{rl^{DW}, rER\} \). Prior to the Great Recession, the fed funds rate was consistently below the discount window rate. During the midst of the crisis, the fed funds rate exceeded the discount rate during several days. This anomalous behavior reflects the disruptions that went on in the interbank market and that we try to capture with the bank run shock and interbank market shutdown shocks. Since the beginning of the recession, the fed funds rate dropped to its lowest historical levels for almost 5 years reflecting a reduction in the costs of illiquidity for banks.

Fact 2: Fed Balance-Sheet expansion. Panel (b) shows the assets held in the Balance Sheet of the Fed. The panel shows a substantial increase in these asset holdings. This increase corresponds to the large scale open-market operations programs carried out after the collapse of Lehman Brothers. During its initial face, most of the increment was due to assets purchases from direct lending to banks. The subsequent programs included unconventional open-market operations such as the purchase of long-term bonds and mortgage-backed securities. Panel (c)
shows the increase in the Fed’s assets from direct lending to banks and mortgage-backed securities—liquidity facilities and MBS, see the data Appendix for more details. The series is reported in terms relative to the total of bank credit of US commercial banks. Whereas prior to the crisis, this figure was close to 0 percent, this ratio reached 18 percent by the middle of 2014. The counterpart of this figure is a proportional increase in Fed’s liabilities, reserves. The model captures these open-market operations through the Fed’s purchases of loans.

**Fact 3: Excess-Reserve Holdings.** Panel (d) shows an increment in holdings of excess reserves by the banking system. Whereas prior to the crisis there were virtually no excess reserves, during its aftermath, excess reserves amount almost 16 times greater than the amount of required reserves.

**Fact 4: Depressed Lending Activities.** Panel (e) shows the decline in commercial and industrial (C&I) lending during the crisis. The figure shows the raw series and a series that subtracts the decline in the stock of C&I loan commitments from the original series. This figure shows that while there was a large monetary expansion by the Fed during the middle of the crisis, there was a simultaneous substantial decline in lending to firms.

**Fact 5: Drop in Money Multiplier.** The large drop in the money multiplier for M1 is a summarizes facts 2, 3, and 4. This is shown in Panel (f).
Figure 12: Monetary Facts
6.2 Banking Facts

Our model yields predictions about the behavior of several banking indicators for different shocks. We will use this information to gain further insights about the nature of the shocks that affected banks during the crisis. Here, we report four indicators computed from Commercial Bank Call Reports. These ratios are reported as simple averages and averages weighted by assets for the cross-section at a given point in time. We summarize the main facts we want to underscore:

**Fact 6: Decline in Book-Value Leverage.** The upper-left panel of Figure 13 shows the decline in the tangible leverage—a measure that subtracts tangible assets from the book value of equity. From its peak at the middle of the crisis to 2010Q4, the average tangible leverage falls from 16 to about 12.

**Fact 7: Increase Liquidity Ratio.** The upper-right panel of Figure 13 shows the behavior of the liquidity ratio, the ratio of liquid assets over total assets. Here we take liquid assets to be the sum of reserves plus Treasury bills. The data shows an increase from 6 percent to 12 percent for the same period. Interestingly, this implies the increase in reserves was not offset by the reduction in other liquid assets.

**Fact 8: Bank-Equity Losses.** The bottom-left panel of Figure 13 shows the behavior of the realized returns on equity. The figure shows a sharp decline that begins in 2007Q4 from its historical average to a value of about 0. By the middle of the crisis, we observe an even further decline, especially concentrated among the largest banks. During the peak of the crisis, losses exceeded 5 percent on average and almost reach 10 percent for the average weighted by assets.

**Fact 9: Dividends.** The bottom-right panel of Figure 13 plots the evolution of the dividend-per-equity of banks. Dividends already showed a declining trend prior to the Great Recession. This further declines all throughout the duration of the recession, but erratically increases thereafter. Although the model also yields predictions for the dividend yield in the model, it is worth noting that dividends were also constrained by policy during the crisis.

The following section discusses which combination of shocks—and hypothesis—are more consistent with the banking and monetary facts we have described so far.
Figure 13: Banking Indicators: The figure reports four indicators of banking activity for the universe of commercial banks in the US. All the series correspond to ratios of variables reported by computing simple averages and averages weighted by assets. Tangible leverage in (a) is total liabilities relative to equity minus intangible assets. The liquidity ratio in (b) is the sum of vault cash, reserves, and Treasury securities relative to total assets. The return on equity in (c) is the ratio of net operating revenues over equity. The dividend ratio in (d) is the sum of common and preferred dividends over equity. More details are found in Appendix E.
6.3 Which Hypotheses Fit the Facts?

We can now use our model as a laboratory to investigate how important are hypothesis 1–5 in explaining the facts described above. In particular, we can shed light on what are the possible drivers of the increase in liquidity and the persistent drop in lending that occurred in October 2008 after the failure of Lehman Brothers. We first describe the qualitative prediction of our model and then turn to a quantitative evaluation.

6.3.1 Discussion

The shocks that we study can be classified into supply and demand shocks. Within supply shocks, we have two classes of shocks. First, equity losses and increments in capital requirements constrain the entire portfolio of the bank. As analyzed above, by reducing the supply of loans and raising the relative return on loans relative to reserves, banks substitute reserves for lending. Hence, these two hypotheses can explain the collapse in lending but are inconsistent with the observed increase in liquidity ratios by banks. Second, we have shocks to higher uncertainty in the withdrawal process $F_t$, the shutdown of the interbank market, and the increase in corridor rates. These shocks do not affect on impact the funding capacity of the bank but reduce the relative return on loans. As a result, banks substitute lending for reserves, and in addition, pay less dividends, which is consistent with the pattern that we see in the midst of the crisis. However, as of 2010 more flexible lending facilities by the Fed, probably reduce the importance of these supply shocks. In fact, a reduction of the discount window rate in our model, as observed since 2009, significantly reduce the quantitative importance of shocks leading to either higher uncertainty or higher disruptions in interbank markets. Finally, interest on reserves also deliver effects consistent with the data, but it has quantitatively mild effects.

With regards to demand shocks, a negative credit demand shock reduces return on loans and leads banks to substitute reserves for lending. Therefore, this shock is consistent with the decline in bank lending and the increase in reserves holdings —through a substitution effect. This shock is, however, inconsistent with the path for dividends that were sharply reduced at the beginning of the crisis. Finally, the low lending rates of 2010 also favors the hypothesis of a persistent negative credit demand shock.

To summarize, the model supports hypothesis 3 associated with disruptions in the interbank markets and hypothesis 5 associated with a persistent negative credit demand shock. The short-run effects after each shock are summarized by the arrows in the table below.

---

38 The path for dividends was also influenced by government policies, in particular for those financial institutions that participated in government recapitalization programs.

39 Some caution also has to be taken from using low lending rates as evidence for credit demand shocks for at least two reasons. First, there is a change in the composition of credit towards less risky loans. Second, besides interest rates, banks seem to have tightening lending standards, e.g. by requiring higher downpayments in mortgages.
6.3.2 Quantitative Evaluation

The goal of this section is to assess the model’s ability to quantitatively account for the monetary and banking facts we described in the previous section. For this purpose, we compute the model’s transitional dynamics after a sequence of deterministic shocks assuming the economy was in steady state in 2007.Q3. In this experiment, we feed in a sequence of supply shocks that we interpret as observables and impute demand shocks as a residual to match the decline in lending.

**Shocks.** We feed the following shocks, in line with the hypothesis discussed above. First, we consider a shock to bank equity losses of 2 percent, which is equivalent to 0.2 percent of total assets — this shock assumes a corresponding decline in loans keeping liabilities constant. The magnitude of this shock corresponds to the unexpected losses of AAA-rated subprime MBS tranches, estimated by Park (2011). Second, we consider a shock that anticipates higher capital requirements along the new prescriptions of Basel III. In particular, we assume that agents anticipate that the maximum leverage ratio will be permanently reduced from $\kappa = 15$ to $\kappa = 12$ starting 2013.Q1. This is in line with new regulations that require a gradual increase in capital requirements of 2 percent between 2013.Q1 and 2015.Q4 (see BIS (2010)). Third, we consider a freezing of interbank market during 2008.Q3 and 2008.Q4, in line with the evidence of a sharp decrease in the Fed funds market and the increase in discount window operations which reached 400 billion dollars at the peak of the crisis. As explained above, this implies the probability of a match in the interbank markets in the balancing stage becomes zero, so that banks only trade with the Fed.

The two remaining observable shocks we consider are policy responses by the Fed. First, we feed the sequence for discount window rates and interest on excess reserves shown in panel (a) of Figure ?? . We also assume that these policies reverse gradually to steady state starting in 2015.

---

40The amount of Fed funds sold was reduced by a factor of 4 in 2008Q3, and never recover from pre-crisis levels. We consider only two periods for this shock because as described in Section 6.1, interbank market rates normalized following various policies by the Fed. Moreover, because discount window rates were significantly reduced in 2009.Q3, results would be very similar with more persistent shocks.
Second, we feed the sequence of loan purchases as part of the unconventional OMO carried out by the FED, as described in Fact 2. We assume that these operations are gradually reversed starting in 2020, except for the interest on reserves that we assume they stay at a constant level of 0.2 percent (annualized).

Finally, we estimate a credit demand shock. Given the observed time series for loans shown in Figure ??, we consider a shock to $\Theta_t$ to match the decline in lending. Specifically, $\Theta_t^{-1}$ is reduced by about 1.1 percent in every quarter from 2008.Q3 to 2010.Q4 and stays permanently at a level of 12 percent below the steady state.

Results. Figures ??-?? show the evolution of loans, liquidity ratios, dividend rates and the return on loans in the data and in the model for different simulation scenarios. Figure ?? shows that the model that includes all shocks can account reasonably well for the key patterns in the data. As it turns out, the credit demand shock is the most important one. To see this, consider the third panel from Figure ?? which feeds all shocks considered except for the credit demand shock.

Notice also that the money multiplier is also falling, ...

![Figure 14: All Shocks](image)

\footnote{For the model simulations, variations in lending are taken with respect to the steady state. For the data, the percentage change in lending takes September 2007 as the basis value. Return on loans in the data correspond to the real return on one-year mortgage rates.}
7 Conclusions

Modern monetary-macro models have developed independently from banking models. Recent crises in the US, Europe and Japan, however, have revealed the need of a model to study monetary policy in conjunction with the banking system. This model should be used as a tool to address many issues that emerge in current policy debates.

This paper presents a dynamic macro model to study the implementation of monetary policy through the liquidity management of banks. We used the model to understand the effects of various shocks to the banking system. As an application, we employ the model to contribute to one policy question: why have banks held on to so many reserves and not expanded their lending? We argued that an early interbank market freeze may have been important at the early beginning of the Great Recession. However, a persistent decline in the demand for loans seems the most plausible story to explain the increase in the holding of reserves and the decline in lending since 2008 onwards. This result is suggestive of phenomena where an initial contraction in the loans supply eventually translates into a subsequent strong and permanent contraction in demand for credit.

We believe the model can be used to answer a number of other issues present in policy debates. For example, the model can be used to study the Fed’s exit strategy and the fiscal implications. In addition, it can also be used to analyze changes in policy tools used in monetary policy implementation in the past. There are also other possible relevant extensions of the model. An extension that breaks aggregation may allow to study the cross-sectional responses of banks depending their liquidity and leverage ratios. Introducing aggregate shocks would also allow us to investigate the role of liquidity requirements and capital requirements as macroprudential tools. We hope that the model we propose can be useful for policy analysis.
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A Proofs

A.1 Proof of Propositions 1, 2 and 3

This section provides a proof of the optimal policies described in Section 3.4. The proof of Proposition 1 is straightforward by noticing that once $E$ is determined, the banker does not care how he came up with those resources. The proof of Propositions 2 and 3 is presented jointly and the strategy is guess and verify. Let $X$ be the aggregate state. We guess the following. $V(E; X) = v(X)E^{1-\gamma}$ where $v(X)$ is the slope of the value function, a function of the aggregate state that will be solved for implicitly. Policy functions are given by:

$$DIV(E; X) = \text{div}(X)E, \quad \tilde{B}(E; X) = \tilde{b}(X)E, \quad \tilde{D}(E; X) = \tilde{d}(X)E \quad \text{and} \quad \tilde{C}(X) = \tilde{c}(X)E,$$

for $\text{div}(X), \tilde{b}(X), \tilde{d}(X)$ and $\tilde{c}(X)$ policy functions that are independent of $E$.

A.1.1 Proof of Proposition 2

Given the conjecture for functional form of the value function, the value function satisfies:

$$V(E; X) = \max \{DIV, \tilde{C}, \tilde{B}, \tilde{D}\} \in \mathbb{R} \cup \{\text{DIV}\} + \beta E \left[ v(X')(E'^{1-\gamma}) | X \right]$$

Budget Constraint: $E = q\tilde{B} + \tilde{C}p + DIV - \frac{\tilde{D}}{R^D}$

Evolution of Equity: $E' = (q'\delta + (1 - \delta))\tilde{B} + \tilde{C}p' - \tilde{D} - \chi((\rho + \omega'(1 - \rho))\frac{\tilde{D}}{R^D} - p\tilde{C})$

Capital Requirement: $\frac{\tilde{D}}{R^D} \leq \kappa \left( \tilde{B}q + \tilde{C}p - \frac{\tilde{D}}{R^D} \right)$

where the form of the continuation value follows from our guess. We can express all of the constraints in the problem as linear constraints in the ratios of $E$. Dividing all of the constraints by $E$, we obtain:

$$1 = \text{div} + q\tilde{b} + p\tilde{c} - \frac{\tilde{d}}{R^d}$$

$$\frac{E'}{E} = (q'\delta + 1 - \delta)\tilde{b} + \tilde{c}p' - \tilde{d} - \chi((\rho + \omega'(1 - \rho))\frac{\tilde{D}}{R^D} - p\tilde{C})$$

$$\frac{\tilde{d}}{R^D} \leq \kappa(\tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D})$$

where $\text{div} = DIV/E, \tilde{b} = \tilde{B}/E, \tilde{c} = \tilde{C}/E$ and $\tilde{d} = \tilde{D}/E$. Since, $E$ is given at the time of the decisions of $B, C, D$ and $DIV$, we can express the value function in terms of choice of these ratios.
Substituting the evolution of $E'$ into the objective function, we obtain:

$$
V(E; X) = \max_{\{w_b, w_c, w_d, \text{div}\} \in U(\text{div})} U(\text{div}E) + \beta \mathbb{E} \left[ v(X') (R(\omega, X, X') E^{1-\gamma}) | X \right]
$$

$$
1 = \text{div} + q\tilde{b} + p\tilde{c} - \tilde{d}
$$

$$
\frac{\tilde{d}}{R^D} \leq \kappa \left( \tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D} \right)
$$

where we use the fact that $E'$ can be written as:

$$
E' = R(\omega', X, X') E
$$

where $R(\omega', X, X')$ is the realized return to the bank’s equity and defined by:

$$
R(\omega', X, X') \equiv (q(X')\delta + (1-\delta))\tilde{b} + p(X')\tilde{c} - \tilde{d} - \chi((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - p(X)\tilde{c}).
$$

We can do this factorization for $E$ because the evolution of equity on hand is linear in all the terms where prices appear. Moreover, it is also linear in the penalty $\chi$ also. To see this, observe that

$$
\chi((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - p(X)\tilde{c}) = \chi((\rho + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c})
$$

by definition of $\{d, \tilde{c}\}$. Since, $E \geq 0$ always, we have that

$$
(r + \omega'(1-\rho))\frac{\tilde{D}}{R^D} - \tilde{C} \leq 0 \leftrightarrow \left( (r + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c} \right) \leq 0.
$$

Thus, by definition of $\chi$,

$$
\chi((r + \omega'(1-\rho))\frac{\tilde{D}}{R^D} - \tilde{C}) = \begin{cases} 
E \chi \left( (r + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c} \right) & \text{if } \left( (r + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c} \right) \leq 0 \\
E \chi \left( (r + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c} \right) & \text{if } \left( (r + \omega'(1-\rho))\frac{\tilde{d}}{R^D} - \tilde{c} \right) > 0
\end{cases}
$$

Hence, the evolution of $R(\omega', X, X')$ is a function of the portfolio ratios $b, c$ and $d$ but not of the level of $E$. With these properties, we can factor out, $E^{1-\gamma}$ from the objective because it is a constant when decisions are made. Thus, the value function may be written as:

$$
V(E; X) = E^{1-\gamma} \left[ \max_{\{w_b, w_c, w_d, \text{div}\} \in U(\text{div})} U(\text{div}E) + \beta \mathbb{E} \left[ v(X') R(\omega, X, X')^{1-\gamma} | X \right] \right]
$$

$$
1 = \text{div} + q\tilde{b} + p\tilde{c} - \tilde{d}
$$

$$
\tilde{d} \leq \kappa(\tilde{B}q + \tilde{C}p - \tilde{d})
$$
Then, let an arbitrary $\tilde{v}(X)$ be the solution to:

$$\tilde{v}(X) = \max_{\{w_b, w_c, w_d, \text{div}\}} U(\text{div}) + \beta \mathbb{E} \left[ \tilde{v}(X') R(\omega, X, X')^{1-\gamma} \right] |X|$$

$$1 = \text{div} + \tilde{b} + \tilde{c} - \frac{\tilde{d}}{R^D}$$

$$\frac{\tilde{d}}{R^D} \leq \kappa (\tilde{b}q + \tilde{c}p - \frac{\tilde{d}}{R^D})$$

We now show that if $\tilde{v}(X)$ exists, $v(X) = \tilde{v}(X)$ verifies the guess to our Bellman equation. Substituting $v(X)$ for the particular choice of $\tilde{v}(X)$ in (14) allows us to write $V(E; X) = \tilde{v}(X) E^{1-\gamma}$. Note this is true because maximizing over $\text{div}, \tilde{c}, \tilde{b}, \tilde{d}$ yields a value of $\tilde{v}(X)$. Since, this also shows that $\text{div}, \tilde{c}, \tilde{b}, \tilde{d}$ and independent of $E$, and $DIV = \text{div}E, \tilde{B} = \tilde{b}E, \tilde{C} = \tilde{c}E$ and $\tilde{D} = \tilde{d}E$.

### A.1.2 Proof of Proposition 3

We have from Proposition 2 that

$$v(X) = \max_{\{w_b, w_c, w_d, \text{div}\}} U(\text{div}) + \beta \mathbb{E} [v(X') |X|]$$

$$\mathbb{E}_{\omega'} \left( (q' \delta + (1 - \delta))\tilde{b} + p'\tilde{c} - \tilde{d} - \chi((\rho + \omega' (1 - \rho)) \tilde{d}/R^D - p\tilde{c}) \right)^{1-\gamma}$$

subject to

$$1 = q\tilde{b} + p\tilde{c} + \text{div} - \frac{\tilde{d}}{R^D}$$

$$\frac{\tilde{d}}{R^D} \leq \kappa (q\tilde{b} + p\tilde{c} - \frac{\tilde{d}}{R^D})$$

Now define:

$$w_b \equiv \frac{\tilde{b}q}{(1 - \text{div})}, \quad w_c \equiv \frac{\tilde{c}p}{(1 - \text{div})} \quad \text{and} \quad w_d \equiv \frac{\tilde{d}}{R^D(1 - \text{div})}.$$ 

and collecting terms on $1 = q\tilde{b} + p\tilde{c} + \text{div} - \frac{\tilde{d}}{R^D}$, we obtain:

$$\text{div} + (1 - \text{div})(w_b + w_c + w_d) = 1 \Leftrightarrow w_b + w_c - w_d = 1$$
Then using the definition of \( w_b, w_c, w_d \) have that \( v(X) \)

\[
v(X) = \max_{\{w_b, w_c, w_d, \text{div}\} \in \mathbb{R}^4_+} \left\{ U(\text{div}) + \beta \mathbb{E}[v(X') | X](1 - \text{div})^{1-\gamma} \right\}
\]

\[
\mathbb{E}_\omega^\prime \left\{ \frac{q \delta + (1 - \delta)}{q} w_b + p' w_c - w_d (R^D) - \chi((\rho + \omega' (1 - \rho)) w_d - w_c) \right\}^{1-\gamma}
\]

s.t.

\[
w_b + w_c - w_d = 1
\]

\[
w_d \leq \kappa (w_b + w_c - w_d)
\]

Using the definition of returns, we can define portfolio value as:

\[
\Omega^*(X) = \max_{\{w_b, w_c, w_d, \text{div}\} \in \mathbb{R}^4_+} \left\{ \mathbb{E}_\omega^\prime \left( R^B w_b + R^C w_c - w_d (R^D) - R^X(w_d, w_c) \right)^{1-\gamma} \right\}^{1/\gamma}
\]

s.t.

\[
w_b + w_c - w_d = 1
\]

\[
w_d \leq \kappa (w_b + w_c - w_d)
\]

Since, the solution to \( \Omega(X) \) is the same for any \( \text{div} \) and using the fact that \( X \) is deterministic,

\[
v(x) = \max_{\{w_b, w_c, w_d, \text{div}\} \in \mathbb{R}^4_+} \left\{ U(\text{div}) + \beta \mathbb{E}[v(X') | X](1 - \text{div})^{1-\gamma} \Omega^*(X)^{1-\gamma} \right\}
\]

which is the formulation in Proposition 3.

For \( \gamma \to 1 \), the objective becomes:

\[
\Omega(X) = \exp\{\mathbb{E}_\omega[\log(R(\omega, X, X'))]\}
\]

and for \( \gamma \to 0 \),

\[
\Omega(X) = \mathbb{E}_\omega[R(\omega, X, X')].
\]

A.2 Proof of Proposition 4

Taking first-order conditions on (3) and using the CRRA functional form for \( U(\cdot) \), we obtain:

\[
\text{div} = (\beta \mathbb{E}[v(X') | X])^{-1/\gamma} \Omega^*(X)^{-1/(1-\gamma)/(1-\gamma)} (1 - \text{div}) (1 - \gamma)
\]

and therefore one obtains:

\[
\text{div} = \frac{1}{1 + [\beta \mathbb{E}[v(X') | X](1 - \gamma) \Omega^*(X)]^{1-\gamma}}^{1/\gamma}.
\]
Substituting this expression for dividends, one obtains a functional equation for the value function:

\[
 v(X) = \frac{1}{1-\gamma} \left( 1 + \left[ \beta \mathbb{E}[v(X')|X] (\Omega^*(X))^{1-\gamma} \right]^{\frac{1}{\gamma}} \right)^{1-\gamma}
 + \beta \mathbb{E}[v(X')|X] (\Omega^*(X))^{1-\gamma} \left[ \left( 1 + \left[ \beta \mathbb{E}[v(X')|X] (\Omega^*(X))^{1-\gamma} \right]^{\frac{1}{\gamma}} \right)^{1-\gamma} \right]
 .
\]

Therefore, we obtain the following functional equation:

\[
 v(X) = \frac{1}{1-\gamma} \left[ 1 + (\beta(1-\gamma)\Omega^*(X))^{1-\gamma} \mathbb{E}[v(X')|X] \right]^{\frac{1}{\gamma}}
 .
\]

We can treat the right-hand side of this functional equation as an operator. This operator will be a contraction depending on the values of \( \beta(1-\gamma)\Omega^*(X) \). Theorems in Alvarez and Stokey (1998) guarantee that this operator satisfies the dynamic programming arguments.

In a non-stochastic steady state we obtain:

\[
 v^{ss} = \frac{1}{1-\gamma} \left( \frac{1}{1 - (\beta \Omega^*1-\gamma)^{\frac{1}{\gamma}}} \right)^{\gamma}
\]

and

\[
 \text{div}^{ss} = 1 - \beta^{\frac{1}{\gamma}} \Omega^{*1/\gamma}-1.
\]

### A.3 Proof of Lemma 1

Define the threshold \( \bar{\omega} \) shock that determines whether the bank has a reserve deficit or surplus, i.e., the shock that solves \( \rho + (1-\rho) \bar{\omega} \) \( w_d = w_c \). This shock is:

\[
 \bar{\omega}(1, L) = \bar{\omega}(w_d, w_c) \equiv \frac{w_c}{w_d} \rho \left( \frac{L - \rho}{1 - \rho} \right)
\]

where \( L \) is the reserve ratio. We can express the expected liquidity cost in terms of \( \bar{\omega} \):

\[
 \mathbb{E}_{\omega'} [R^x (w_d, w_c)] = \chi_d \left[ \int_{\bar{\omega}(w_d, w_c)}^{1} ((\rho + (1-\rho) \omega') w_d - w_c) f (\omega') d\omega' \right]
 + \chi_b \left[ \int_{-\infty}^{\bar{\omega}(w_d, w_c)} ((\rho + (1-\rho) \omega') w_d - w_c) f (\omega') d\omega' \right]
 .
\]

We separate the integral into terms that depend on \( \omega' \) and the independent terms. We obtain that the expected liquidity cost:

53
\[
\begin{align*}
\rho w_d - w_c \chi_l (1 - F(\bar{\omega}(w_d, w_c))) + \\
+ (1 - \rho) w_d \chi_l (1 - F(\bar{\omega}(w_d, w_c))) \mathbb{E}_{\omega'} \left[ \omega' > \bar{\omega}(w_d, w_c) \right] \\
+ (1 - \rho) w_d \chi_b (F(\bar{\omega}(w_d, w_c))) \mathbb{E}_{\omega'} \left[ \omega' \leq \bar{\omega}(w_d, w_c) \right].
\end{align*}
\]

From here, we can factor \( \omega^d \) from all of the terms in the expression above:

\[
\begin{align*}
\mathbb{E}_{\omega'} [R^x (w_d, w_c)] &= w_d ((\rho - L) [\chi_l (1 - F(\bar{\omega}(1, L))) + \chi_b F(\bar{\omega}(1, L))] \\
+ (1 - \rho) (\chi_l (1 - F(\bar{\omega}(1, L))) \mathbb{E}_{\omega'} [\omega' > \bar{\omega}(1, L)] \\
+ (1 - \rho) (\chi_b (F(\bar{\omega}(1, L))) \mathbb{E}_{\omega'} [\omega' \leq \bar{\omega}(1, L)]) \\
&= w_d \mathbb{E}_{\omega'} \left[ \bar{R}^x (1, L) \right].
\end{align*}
\]

From the expression above, we find that if we multiply \( \{w_d, w_c\} \) by any constant, the expected liquidity cost increases by that same constant. Thus, \( \mathbb{E}_{\omega'} [R^x (w_d, w_c)] \) is homogeneous of degree 1 in \( \{w_d, w_c\} \).

### A.4 Proof of Lemma 2

The closed form expression for \( \mathbb{E}_{\omega'} [R^x_c (1, L)] \) is obtained as follows. Given an \( \omega' \), the reserve surplus per unit of deposit is \( (\rho - L + (1 - \rho) \omega) \):

\[
\begin{align*}
\mathbb{E}_{\omega'} [R^x_c (1, L)] &= \chi_l \int_{\frac{L - \rho}{1 - \rho}}^{1} (\rho - L + (1 - \rho) \omega') f(\omega) d\omega + \\
&\quad \chi_b \int_{-\infty}^{\frac{L - \rho}{1 - \rho}} (\rho - L + (1 - \rho) \omega') f(\omega) d\omega.
\end{align*}
\]

Taking the derivative with respect to \( L \) yields:

\[
\begin{align*}
\mathbb{E}_{\omega'} [R^x_c (1, L)] &= (\chi_b - \chi_l) \{ (\rho - L + (1 - \rho) \omega') f(\omega) \frac{L - \rho}{1 - \rho} \} \\
&- \left( \chi_b F\left( \frac{L - \rho}{1 - \rho} \right) + \chi_l \left( 1 - F\left( \frac{L - \rho}{1 - \rho} \right) \right) \right).
\end{align*}
\]
A.5 Proof of Proposition 6

Since the objective is linear, the solution to the leverage decision is:

\[ w^* = \begin{cases} 
0 & \text{if } R^L < 0 \\
[0, \kappa] & \text{if } R^L = 0 \\
\kappa & \text{if } R^L > 0 
\end{cases} \]

Substituting this result implies that the return to the bank’s equity is \( R^E = R^b + \max \{ \kappa R^L, 0 \} \).

Thus, the bank’s dividend decision is:

\[ \text{div} = \begin{cases} 
0 & \text{if } \beta R^E > 1 \\
[0, 1] & \text{if } \beta R^E = 1 \\
1 & \text{if } \beta R^E < 1 
\end{cases} \]

In any steady state, it must be that \( \beta R^E = 1 \) and \( \text{div} = R^E - 1 \), because otherwise equity is not constant. If the leverage constraint is non-binding in the steady state, then by the condition above \( R^L = 0 \), and therefore \( R^B = 1/\beta \). Otherwise, there is a positive spread. The statement in the Proposition follows.

A.6 Proof of Proposition 7

Since the objective of the liquidity management subproblem is linear, we have that its value is:

\[
\max \left( \sum_{L=0}^{\rho} \left( -\chi_b \rho - \left( R^B - R^C \right) \rho - \left( \left( R^B - R^C \right) - \chi_l \right) \frac{1 + \omega d}{\omega d} - \chi_l \rho \right) \right).
\]

Here we study the equilibrium in the interbank market. An equilibrium is studied as the Nash equilibrium of a game, that is we study the choice of \( L \) of a given bank, given a choice \( \tilde{L} \) by other banks.

**Case 1** (\( \tilde{L} = 0 \)). Assume all banks choose \( \tilde{L} = 0 \). If an individual bank chooses \( L \leq \rho \), the cost of reserve deficits equals the discount window rate \( \chi_b = r^{DW} \) because there are no other banks to borrow reserves from. Therefore, we have that \( (R^B - R^C) < r^{DW} \) is necessary and sufficient to guarantee that \( L = 0 \) is not an equilibrium —because we require positive reserves in a monetary equilibrium.

**Case 2** (\( \tilde{L} = \rho \)). If all banks chooses \( \tilde{L} = \rho \), a bank deviating to \( L = 0 \) would pay \( r^{DW} > (R^B - R^C) \), because, again, no banks would lend reserves to that bank. Thus, \( L = \rho \) dominates
$L = 0$ when other banks choose $L = \rho$ and $r^{DW} > (R^B - R^C)$. This shows that $(R^B - R^C) < r^{DW}$ is necessary and sufficient to guarantee that $L = 0$ not an equilibrium when $\tilde{L} = \rho$.

So far, $(R^B - R^C) < r^{DW}$ is enough to argue that $\tilde{L} \geq \rho$ in a symmetric Nash equilibrium. Assume now that also $(R^B - R^C) > r^{ER}$ holds.

**Case 3** ($\tilde{L} = \frac{1 + \omega d}{\omega d}$). If all banks set $\tilde{L} = \frac{1 + \omega d}{\omega d} > \rho$, no bank will be short of reserves. Thus, $\chi_l = r^{ER}$ since $\gamma^+ = 0$. Thus, an individual banks is better off deviating by reducing $L$ to $\rho$.

**Case 4** ($\tilde{L} = \rho$). Instead, if all banks set $\tilde{L} = \rho$, then, $\chi_l = r^{ER}$ since again $\gamma^+ = 0$. Thus, $L = \rho$ is an optimal choice because deviating to $\frac{1 + \omega d}{\omega d}$ is not profitable.

Hence, $r^{ER}_t < R^B - R^C < r^{DW}_t$ will hold in any equilibrium with positive reserves and this implies $L^* = \rho$. 
B Evolution of Bank Equity Distribution

Because the economy displays equity growth, equity is unbounded and thus, the support of this measure is the positive real line. Let $\mathcal{B}$ be the Borel $\sigma$-algebra on the positive real line. Then, define $Q_t(e, E)$ as the probability that an individual bank with current equity $e$ transits to the set $E$ next period. Formally $Q_t : \mathbb{R}_+ \times \mathcal{B} \rightarrow [0, 1]$, and

$$Q(e, E) = \int_{-1}^{1} \mathbb{1}\{e_t(\omega) e \in E\} F(d\omega)$$

where $\mathbb{1}$ is the indicator function of the event in brackets. Then $Q$ is a transition function and the associated $T^*$ operator for the evolution of bank equity is given by:

$$\Gamma_{t+1}(E) = \int_{0}^{1} Q(e, E) \Gamma_{t+1}(e) \, de.$$ 

The distribution of equity is fanning out and the operator is unbounded. Gibrat’s law shows that for $t$ large enough $\Gamma_{t+1}$ is approximated by a log-normal distribution. Moreover, by introducing more structure into the problem, we could easily obtain a Power law distribution for $\Gamma_{t+1}(E)$. We will use this properties in the calibrated version of the model.
C  Algorithm

C.1  Steady State

1. Guess prices for loans $q$ and for the probability of a match in the interbank market $\gamma^-, \gamma^+$. 


3. Compute associated average equity growth and average surplus in the interbank market.

4. If equity growth equals zero and the conjectured probability of a match in the interbank market is consistent with the average surplus, stop. Otherwise, adjust and continue iterating.

Algorithm to solve transition dynamics in baseline model

C.2  Transitional Dynamics

1. Guess a sequence of loan prices $q_t$ and for the probability of a match in the interbank market $\gamma^-_t, \gamma^+_t$.

2. Solve by backward induction banks’s dynamic programming problem using 3 for banks’ portfolio and 4 for value function and dividend rates.

3. Compute growth rate of equity and average surplus in interbank markets.

4. Compute price implied by aggregate sequence of loans resulting from (2) and (3), and the probability of a match according to average surpluses computed in (3).

5. If the conjectured price equal effective price from (4) and the average surplus computed in (4) are consistent with the guessed sequences, stop. Otherwise, continue iterating until convergence.
D Microfoundation for Loan Demand and Deposit Supply

There are multiple ways to introduce a demand for loans and a supply of deposits. Here, the demand for loans emerges from firms who borrow working capital from banks and the supply of deposits from the households’s savings decision. With working capital constraints, a low price for loans, \( q_t \), translates immediately into labor market distortions and, therefore, has real output effects. This formulation is borrowed from the classic setup of Christiano and Eichenbaum (1992). To keep the model simple, we deliberately model the real sector so that the loans demand is static—in the sense that it does not depend on future outcomes—and the supply of loans is perfectly elastic.

D.1 Households

Households’s Problem. Households obtain utility by consuming and disutility from providing labor. They work during the lending stage and consume during the balancing stage. This distinction is irrelevant for households but matters for the sequence of events that we describe later. Households have quasi-linear utility in consumption and have a convex cost of providing labor given by \( h_t^{1+\nu} \). The only savings instruments available to households are bank deposits and their holding of shares of firms. Households solve the following recursive problem:

\[
W (s_t, d_t; X_t) = \max_{\{c_t \geq 0, h_t, d_{t+1} \geq 0\}} c_t - \frac{h_t^{1+\nu}}{1+\nu} + \beta^D E \left[ W (s_{t+1}, d_{t+1}; X_{t+1}) | X_t \right]
\]

subject to the budget constraint:

\[
d_{t+1} + c_t + p_t^s s_{t+1} = s_t (z_t + p_t^s) + w_t h_t + R^D_t d_t + T_t.
\]

Here, \( \beta^D \) is the households’s discount factor and \( \nu \) the inverse of the Frisch elasticity. In the budget constraint, \( d_t \) are deposits in banks that earn a real rate of \( R^D \), \( h_t \) are hours worked that earn a wage of \( w_t \), and \( s_t \) are shares of productive firms. The price of shares is \( p_t^s \) and these pay \( z_t \) dividends per share. Finally, \( c_t \) is consumption and \( T \) are lump sum transfers from the government.

The first-order conditions for the households’s problem yield the following labor supply:

\[
\frac{1}{w_t^{\nu}} = h_t.
\]

This supply schedule is static and only a function of real wages. Hence, the total wage income for the household is \( w_t^{1+\nu} \). In turn, substituting the optimality condition in this problem and using the fact that in equilibrium \( s_{t+1} = s_t \), we can solve for the optimal policy decisions, \( \{c, d\} \).
independently from the labor choice. The solution is immediate and given by,

\[
\{c, d\} = \begin{cases} 
    c_t = w_t^{1+\nu} + R^D d_t + T_t; d_{t+1} = 0 & \text{if } R^D < 1/\beta^D, \\
    c_t \in [0, y_t], d_{t+1} = y_t - c_t & \text{if } R^D = 1/\beta^D \\
    c_t = 0; d' = w_t^{1+\nu} + R^D d_t + T_t & \text{if } R^D > 1/\beta^D
\end{cases}
\]

These two results imply that households consume all cash on hand in the period they receive it if the interest is very low and they do not save, or carry real balances. If \(R^D = 1/\beta^D\), they are indifferent between consuming or savings. Otherwise, they either do not consume or save all of their resources. We will consider parameterizations where in equilibrium \(R^D = 1/\beta^D\).

D.2 Firm’s Problem

**Firms.** Firms maximize \(E[\sum_{t=0}^{\infty} m_t z_t]\) where \(z_t\) are dividend payouts from the firms and \(\mu_t\) is the stochastic discount factor of the representative household. Given the linearity of the household’s objective, the discount factor is equivalent to \(m_t = (\beta^D)^t\).

**Timing.** A continuum of firms of measure one is created at the lending stage of every period. Firms choose a production scale together with a loan size during their arrival period. In periods after this scale choice is decided, firms produce, pay back loans to banks; the residual is paid in dividends.

**Production Technology.** A firm created in period \(t\) uses labor \(h_t\), to produce output according to \(f_t(h_t) \equiv A_t h_t^{1-\alpha}\). The scale of production is decided during the lending stage of the period when the firm is created. Although the scale of production is determined immediately at the time of creation, output takes time to be realized. In particular, the firm produces \(\delta^s (1-\delta) f_t(h_t)\) of its output during the \(s\)-th balancing stage after its scale was decided.

Labor is also employed when the firm is created, and workers require to be paid at that moment.\(^{42}\) Since firms do not possess the cash flow to pay their workers —no equity injections are possible—firms need to borrow from banks to finance the payroll. Firms issue liabilities to the banking sector —loans— by, \(l_t\), in exchange for deposits —bank liabilities—, \(q_t l_t\), that firms can use immediately to pay workers. The repayment of those loans occurs over time. In particular, firm repay \(\delta^s (1-\delta) l_t\) during the \(s\)-th lending stage after the loan was made. Notice that the repayment rate \(\delta\) coincides exactly with the \(\delta\) rate of sales. This delivers a problem for firms similar to the one in Christiano and Eichenbaum (1992) with the difference in the maturity. Taking as given wages a labor tax \(\tau^l\), and the loan prices \(q_t\), the problem of the firm created during the period \(t\)

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\(^{42}\)This constraint emerges if it is possible that the firm defaults on this promise and defaults on its payroll Bigio (2011). The implicit assumption is that banks have a special advantage of monitoring loans compared to households.
is:

\[
\max_{\{h_t, l_t\}} \sum_{s=1}^{\infty} (\beta^D)^{s-1} z_{t+s-1}
\]

subject to:

\[z_{t+s-1} = \delta^s (1 - \delta) A_t f_t (h_t) - \delta^s (1 - \delta) l_t\]

and

\[(1 + \tau^f_t) w_t h_t = q_t l_t.\]

Substituting \(z_{t+s-1}\) into the objective function, and substituting the working capita loan, yields a static maximization problem for firms:

\[
\max_{\{h_t\}} A_t h_t^{1-\alpha} \left(1 + \frac{\tau^f_t}{\alpha} - \frac{\tau^f_t h_t}{q_t}\right)
\]

Taking first-order conditions from this problem and the household's first-order condition, \(w_t h_t = h_t^{1+\nu}\), yields an allocation for labor

\[h_t = \left[\frac{q_t A_t (1 - \alpha)}{(1 + \tau^f_t)}\right]^{\frac{1}{\alpha + \nu}}.\]

Now, using the working capital constraint, \((1 + \tau^f_t) w_t h_t = l_t q_t\), and the expression above:

\[(1 + \tau^f_t) \left[\frac{q_t A_t (1 - \alpha)}{(1 + \tau^f_t)}\right]^{\frac{1}{\alpha + \nu}} = l_t q_t.\]

Clearing \(q_t\) from this expression yields:

\[l_t = (A_t (1 - \alpha))^{\frac{1+\nu}{\alpha + \nu}} \left((1 + \tau^f_t) q_t\right)^{\frac{1-\alpha}{\alpha + \nu}}.\]

This is the expression in equation (7) and proves the following proposition:

**Proposition 9** The demand for loans takes the form:

\[q_t = \Theta_t I^C_t\]

where

\[\Theta_t = (1 + \tau^f_t) \left(A_t (1 - \alpha)\right)^{-\frac{1+\nu}{\alpha + \nu}} \quad \text{and} \quad \epsilon = (\alpha + \nu) \frac{\alpha}{(1 - \alpha)}.\]

Standard calibrations assume \(\alpha = \frac{1}{3}\) and some \(\nu \in \left[\frac{1}{3}, 2\right]\). This provides the bounds \(\epsilon \in [1.0, 3.5]\).
E Data Analysis (not for publication)

E.1 Aggregate Monetary Data

All the aggregate monetary time series are obtained from the Federal Reserve Bank of St. Louis Economic Research Database, FRED© available at:

http://research.stlouisfed.org/fred2/.

These series are used in the construction of Figure 12.

Daily Series. The series for interest rates in the upper-left panel of Figure 12 are daily. We use the following data for the construction of policy rates:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source Acronym</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily Fed Funds Rate</td>
<td>DFF</td>
<td>FRED</td>
</tr>
<tr>
<td>Daily Fed Funds Target Rate</td>
<td>DFEDTAR</td>
<td>FRED</td>
</tr>
<tr>
<td>Daily Fed Funds Target Rate Upper Limit</td>
<td>DFEDTARU</td>
<td>FRED</td>
</tr>
<tr>
<td>Daily Fed Funds Target Rate Lower Limit</td>
<td>DFEDTARL</td>
<td>FRED</td>
</tr>
<tr>
<td>Primary Credit Rate (Discount Window Rate)</td>
<td>DPCREDIT</td>
<td>FRED</td>
</tr>
</tbody>
</table>

To reconstruct a series for the fed funds target rate, we use the Daily Fed Funds Target Rate when this series is available. Otherwise, we take the average of the Daily Fed Funds Target Rate Upper Limit and Daily Fed Funds Target Rate Lower Limit.

Weekly Series. The data used to reconstruct the balance sheet components of the Fed is weekly. These series are used in the upper-middle panel of Figure 12. We use the following weekly data:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source Acronym</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Weekly Fed Total Assets (Less of Consolidation)</td>
<td>WALCL</td>
<td>FRED</td>
</tr>
<tr>
<td>Securities Held Outright</td>
<td>WSHOL</td>
<td>FRED</td>
</tr>
<tr>
<td>Securities, Unamortized Premiums and Discounts, and Loans</td>
<td>WSRLL</td>
<td>FRED</td>
</tr>
<tr>
<td>Treasury Securities</td>
<td>WSHOTS</td>
<td>FRED</td>
</tr>
<tr>
<td>Federal Agency Debt</td>
<td>WSHOFDSL</td>
<td>FRED</td>
</tr>
<tr>
<td>Mortgage-Backed Securities</td>
<td>WSHOMCB</td>
<td>FRED</td>
</tr>
<tr>
<td>Bank Credit of All Commercial Banks</td>
<td>TOTBKCR</td>
<td>FRED</td>
</tr>
</tbody>
</table>

We directly plot the series for Treasury Securities. The series that corresponds to Mortgage-Backed Securities plus Agency Debt (MBS+Agency) is the difference between Securities Held Outright and Treasury Securities. We call liquidity facilities, the series that includes Securities, Unamortized Premiums and Discounts, Repurchase Agreements, and Loans. All other assets
correspond to the Weekly Fed Total Assets (Less of Consolidation) minus the sum of Securities Held Outright and Securities, Unamortized Premiums and Discounts, Repurchase Agreements, and Loans. The upper-right panel is constructed by dividing the Fed’s Weekly Fed Total Assets by the series for Bank Credit of All Commercial Banks.

**Monthly Series.** Finally, we use monthly data to report excess and required reserves and the money multiplier. These series appear in the bottom-left and -right panels of Figure 12. The series correspond to:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source Acronym</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Excess Reserves</td>
<td>EXCRESNS</td>
<td>FRED</td>
</tr>
<tr>
<td>Required Reserves</td>
<td>REQRESNS</td>
<td>FRED</td>
</tr>
<tr>
<td>M1 Money Multiplier</td>
<td>MULT</td>
<td>FRED</td>
</tr>
</tbody>
</table>

### E.2 Individual Bank Data

We use information on FDIC Call Reports for data on commercial banks in the construction of all the time series that are based on individual bank data. There was a considerable amount of mergers and acquisitions in the industry. Moreover, many US chartered banks report very small amounts of lending activities during certain periods relative to their assets, something that may underrepresent many of the ratios we discuss. To present a consistent view of bank ratios, commercial lending and the growth rates of different accounts, we follow Bigio and Majerovitz (2013) in the construction of the data we report in the paper and in this appendix. Bigio and Majerovitz (2013) use data filters based on those used by Kashyap and Stein (2000), and on den Haan et al. (2002). This filter gets rid of abnormal outliers and adjusts the data for mergers.

**Filters.** The details of the filters we use are provided in Bigio and Majerovitz (2013).⁴³ In a nutshell, the first and last quarters when a bank is in the sample are dropped. All observations for which total loans, assets, or liabilities are zero are dropped. Those observations which are more than five —cross-sectional— standard deviations away from the —cross-sectional— mean for the quarter, in any of the aforementioned variables for which growth rates are calculated, are dropped. If a bank underwent a merger or acquisition—or a split, transfer of assets, etc.—it is dropped from the panel data but not from the aggregate time series.

**Seasonal Adjustments.** Most series feature strong seasonal components. Moreover, we find seasonal components at the bank level. We use standard seasonal adjustment procedures to correct for seasonality at the bank level.

**Series.** The bottom-middle panel of Figure 12 reports two time series for commercial and industrial loans (C&I loans). These series are constructed using the filters explained above and reported as percent deviations from the value of the series during 2007Q4. The first series is

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⁴³This guide is available at Bigio’s website.
the time series for commercial and industrial loans. The other series adjusts the original series for increases in lending that have to do with prior commitments. The series adjusted for prior commitments and is constructed the following way. First, we construct an upper bound for the use of loan commitments, subtracting the value of the stock of loan commitments at a given quarter from the stock during 2007Q4. Then, the adjusted series for C&I loans is the original series minus the series for the use of loan commitments.

The following section of this appendix describes some statistics for several bank balance-sheet accounts. That analysis guides our judgements of using total deposits to calibrate the withdrawal distribution, $F_t$, in our model. We narrow the analysis to the statistics of total deposits (TD), demand deposits (DD), total liabilities (TL), tangible equity (TE), equity (E), and loans net of unearned income (LNUI).

**Bank Ratios.** The bank ratios reported in Figure 13 are the following: Tangible leverage is the value of total liabilities minus intangibles over the value of equity minus intangibles. The liquidity ratio is constructed as the sum of reserves (cash) plus treasury securities over total assets. The dividend rate is the value of dividends relative to equity. The series for the return on equity is income over the value of equity. We report the cross-sectional average for every bank and every quarter in the cross section. We report two averages, simple averages and averages weighted by asset size.

**Summary of Individual Variables.** The summary of the series we use is found here:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source Acronym</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>total deposits (TD)</td>
<td>rcfd2200</td>
<td>Call Reports</td>
</tr>
<tr>
<td>demand deposits (DD)</td>
<td>rcfd2948</td>
<td>Call Reports</td>
</tr>
<tr>
<td>total liabilities (TL)</td>
<td>rcfd2948</td>
<td>Call Reports</td>
</tr>
<tr>
<td>intangible</td>
<td>rcfd2143</td>
<td>Call Reports</td>
</tr>
<tr>
<td>cash</td>
<td>rcfd0010</td>
<td>Call Reports</td>
</tr>
<tr>
<td>treasury holdings</td>
<td>rcfd0400+rcfd8634</td>
<td>Call Reports</td>
</tr>
<tr>
<td>tangible equity (TE)</td>
<td>Equity Intangible</td>
<td>Call Reports</td>
</tr>
<tr>
<td>equity (E)</td>
<td>TotalAssets-TotalLiability</td>
<td>Call Reports</td>
</tr>
<tr>
<td>total loans</td>
<td>rcfd2122</td>
<td>Call Reports</td>
</tr>
<tr>
<td>net of unearned income (LNUI)</td>
<td>rcfd2122</td>
<td>Call Reports</td>
</tr>
<tr>
<td>commercial and industrial loans</td>
<td>rcfd1766</td>
<td>Call Reports</td>
</tr>
<tr>
<td>commercial and industrial loans (commitments)</td>
<td>rcfd3816+rcfd6550</td>
<td>Call Reports</td>
</tr>
<tr>
<td>total assets</td>
<td>rcfd2170</td>
<td>Call Reports</td>
</tr>
<tr>
<td>income</td>
<td>riad4000</td>
<td>Call Reports</td>
</tr>
<tr>
<td>dividends</td>
<td>riad4460+riad4470</td>
<td>Call Reports</td>
</tr>
</tbody>
</table>
E.3 Data Analysis

1990-2010 Sample Averages. The summary statistics for the quarterly growth rate of the aggregate time series is presented in Table 3.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD</td>
<td>1.018</td>
<td>0.064</td>
<td>536074</td>
</tr>
<tr>
<td>DD</td>
<td>1.027</td>
<td>0.810</td>
<td>536074</td>
</tr>
<tr>
<td>TL</td>
<td>1.018</td>
<td>0.063</td>
<td>536074</td>
</tr>
<tr>
<td>TE</td>
<td>1.018</td>
<td>0.058</td>
<td>536074</td>
</tr>
<tr>
<td>E</td>
<td>1.019</td>
<td>0.067</td>
<td>536074</td>
</tr>
<tr>
<td>LNUI</td>
<td>1.022</td>
<td>0.061</td>
<td>536074</td>
</tr>
</tbody>
</table>

The data exhibits very similar patterns when we compare the average growth and standard deviation of the growth rate of total deposits and total liabilities. Demand deposits, on the contrary, are almost ten times as volatile as total deposits. This is one reason to use total deposits as our data counterpart for calibrate $F_t$. Although less volatile than demand deposits, total deposits still feature substantial volatility. The standard deviation of this series is 6.4 percent per quarter, and it is close to the volatility of total liabilities, 6.3 percent. Total deposits are also more correlated with equity growth —both for tangible and total equity. The correlation matrix of the variables in the analysis is reported in Table 4.

Quarterly Cross-Sectional Deviations. Part of the variation in the bank-quarter statistics presented above follow from the influence of aggregate trends and seasonal components. To decompose the variation of these liabilities into their common trend, we present the summary statistics in terms of deviations of these variables from their quarterly cross-sectional averages. Table 5 presents the summary for cross-sectional deviations.

A comparison between tables 3 and 5 reveals that the series for deviations from the cross-sectional mean preserve much of the variation of the aggregated time series. This is evidence
### Table 4: Cross-sectional correlation for quarter-bank observations

<table>
<thead>
<tr>
<th>Variables</th>
<th>TD</th>
<th>DD</th>
<th>TL</th>
<th>TE</th>
<th>E</th>
<th>LNUI</th>
</tr>
</thead>
<tbody>
<tr>
<td>TD</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>DD</td>
<td>0.059</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TL</td>
<td>0.286</td>
<td>0.050</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TE</td>
<td>0.117</td>
<td>0.005</td>
<td>0.102</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>0.145</td>
<td>0.006</td>
<td>0.098</td>
<td>0.855</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>LNUI</td>
<td>0.527</td>
<td>0.024</td>
<td>0.198</td>
<td>0.153</td>
<td>0.155</td>
<td>1.000</td>
</tr>
</tbody>
</table>

### Table 5: Summary statistics for cross-sectional deviations from mean growth

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>devTD</td>
<td>0</td>
<td>0.043</td>
<td>536074</td>
</tr>
<tr>
<td>devDD</td>
<td>0</td>
<td>0.123</td>
<td>536074</td>
</tr>
<tr>
<td>devTL</td>
<td>0</td>
<td>0.042</td>
<td>536074</td>
</tr>
<tr>
<td>devTE</td>
<td>0</td>
<td>0.039</td>
<td>536074</td>
</tr>
<tr>
<td>devE</td>
<td>0</td>
<td>0.041</td>
<td>536074</td>
</tr>
<tr>
<td>devLNUI</td>
<td>0</td>
<td>0.045</td>
<td>536074</td>
</tr>
</tbody>
</table>
of a fair amount of idiosyncratic volatility in total deposit growth across banks. Table 6 shows the correlation in cross-sectional deviations from quarterly means across these variables. These correlations are almost identical to the correlations of historical growth rates. This implies that the idiosyncratic component is very important to explain the cross-correlations, more so than common aggregate trends.

Table 6: Correlation for Cross-Sectional Deviations from Means

<table>
<thead>
<tr>
<th>Variables</th>
<th>dev TD</th>
<th>dev DD</th>
<th>dev TL</th>
<th>dev TE</th>
<th>dev RTE</th>
<th>dev E</th>
<th>dev RE</th>
</tr>
</thead>
<tbody>
<tr>
<td>dev TD</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dev DD</td>
<td>0.389</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dev TL</td>
<td>0.844</td>
<td>0.345</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dev TE</td>
<td>0.082</td>
<td>0.027</td>
<td>0.135</td>
<td>1.000</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>dev RTE</td>
<td>0.050</td>
<td>0.016</td>
<td>0.097</td>
<td>0.854</td>
<td>1.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dev E</td>
<td>0.152</td>
<td>0.052</td>
<td>0.238</td>
<td>0.727</td>
<td>0.635</td>
<td>1.000</td>
<td></td>
</tr>
<tr>
<td>dev RE</td>
<td>0.118</td>
<td>0.040</td>
<td>0.187</td>
<td>0.635</td>
<td>0.769</td>
<td>0.881</td>
<td>1.000</td>
</tr>
</tbody>
</table>

The correlation between the cross-sectional deviations of tangible equity growth and the counterpart for total deposits 8.2 percent. In the model, this correlation is very high — though not 1 do to the kink in $\chi(\cdot)$ — because deposit volatility is the only source of risk for banks. In practice, banks face other sources of risks which include loan risk, duration risk, and trading risk. This figure, however, suggests that deposit withdrawal risks are non-negligible risks for banks. Figure 1, found in the body of the paper, reports the empirical histograms for every bank-quarter growth observation and decomposes the data into two samples, pre-crisis (1990Q1-2007Q4) and crisis (2008Q1-2010Q4). We use the empirical histogram of the quarterly deviations of total deposits to calibrate $F_t$, the process for withdrawal shocks.

**Tests for Growth Independence.** We have assumed that the withdrawal process is i.i.d. over time and across banks. This assumption is critical to solve the model without keeping track of distributions. This assumption implies that if we subtract the common growth rates of all the balance sheet variables in our model, the residual should be serially uncorrelated. We test the independence of the deviations-from-means quarterly growth rates using an OLS estimation procedure. We run the deviations in quarterly growth rates from the cross-sectional averages against their lags. The evidence from OLS autoregressions does not support the assumption that of time-independent growth because autocorrelations are significant. Table 7 reports the autocorrelation coefficients of all the variables in deviations. Though none are statistically equal...
to zero most of these autocorrelation coefficients are low. The low values of the autocorrelation coefficients are suggestive that assuming i.i.d. provide a good approximation to the actual process.

Table 7: Autocorrelation coefficients for cross-sectional deviations from mean growth

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>devTD</td>
<td>0.171</td>
<td>(0.001)***</td>
<td>526641</td>
</tr>
<tr>
<td>devDD</td>
<td>-0.262</td>
<td>(0.001)***</td>
<td>526641</td>
</tr>
<tr>
<td>devTL</td>
<td>0.196</td>
<td>(0.001)***</td>
<td>526641</td>
</tr>
<tr>
<td>devTE</td>
<td>0.204</td>
<td>(0.001)***</td>
<td>526641</td>
</tr>
<tr>
<td>devE</td>
<td>0.225</td>
<td>(0.001)***</td>
<td>526641</td>
</tr>
<tr>
<td>devLNUI</td>
<td>0.376</td>
<td>(0.001)***</td>
<td>526641</td>
</tr>
</tbody>
</table>