Burn coal? The supply-side case for carbon capture and storage

Terrence Iverson*† Maria Elisa Belfiori*

Abstract

The paper studies the optimal environmental policy of a coalition of countries for which renewable energy and fossil energy with carbon capture and storage (CCS) comprise alternative forms of carbon abatement. Countries outside the coalition use dirty fossil energy and – for unmodeled political reasons – the coalition cannot permanently retire valuable fossil fuel deposits. In this setting, the coalition still uses the energy source with the lowest marginal social cost, but a novel margin enters the calculation. Specifically, when renewable energy is used, the coalition foregoes fossil deposits that eventually get burned by nonparticipants. In contrast, CCS effectively locks the carbon embedded in fossil fuel deposits underground, preventing future agents from burning the same deposit without CCS. Necessary conditions for the optimal policy show that renewable abatement gives rise to an extra environmental liability that accompanies the risk of future generations burning the foregone deposit. The liability comes due at the end of the fossil era. We calibrate the model to quantify the importance of the identified mechanism. The role of CCS is most important when the coalition is intermediate in size, roughly as big as the set of Annex I countries in the Kyoto Protocol.

Keywords: Supply-side environmental policy, carbon capture and storage, sub-global climate policy. JEL Codes: E27, Q35, Q43, Q54.

*Department of Economics, Colorado State University. 1771 Campus Delivery, Fort Collins, CO., 80523-1771 USA. We thank Daniel Kaffine, Larry Karp, Christian Traeger, Sammy Zahran and participants at the 2015 Front Range Energy Economics Workshop at CU Boulder and the SURED 2016 conference for helpful comments. Remaining errors are our own.

†Corresponding author. Terry.Iverson@colostate.edu
1 Introduction

As long as fossil energy remains cheaper than low-carbon alternatives, there will be a powerful incentive to use it. If the whole world were committed to a global climate agreement, a positive price on carbon or a permanent subsidy on renewables would be sufficient to keep profitable fossil fuel deposits locked in the ground over the long run. But with incomplete participation, demand-side policies alone cannot ensure this since there is nothing to keep deposits from being sold to non-coalition countries. It follows that as long as climate policy remains sub-global, constraints on fossil fuel supply are critical (Hoel 1994, Harstad 2012). Unfortunately, the act of permanently retiring valuable deposits will be difficult to enforce: through the rise and fall of political cycles and economic downturns, it only takes one “corrupt” (or locally-self-interested) political regime to negate a previously successful ban on extraction.

The paper takes this obstacle as a starting point. It then builds on the following under-appreciated observation: carbon capture and storage (CCS)—if feasible—would accomplish the same aim as supply-side policy. By burning fossil fuel deposits while storing carbon emissions underground, CCS eliminates the risk that non-coalition countries will burn them without CCS. This suggests a dual role for CCS that contrasts with conventional intuition about the role of CCS in climate policy. Specifically, CCS not only has the potential to provide clean energy, but it also avoids the future environmental damages that would arise from nonparticipants burning the foregone fossil deposits without CCS. The additional role becomes evident in a model in which the stock of fossil fuels is finite, climate policy sub-global, and efforts to permanently retire valuable deposits infeasible.

To illustrate the dual role of CCS in the simplest possible way, we consider a two-region, dynamic model with a climate externality that arises from burning fossil fuels. Each set of countries seeks to meet a given energy demand at the lowest possible cost. The coalition does this in a way that internalizes climate damages that accrue within its borders, while the non-coalition region ignores climate damages. In the model, available energy technologies include fossil, fossil with CCS, and renewables. Importantly, the paper assumes that conventional supply-side policies of the sort proposed by Hoel (1994) and Harstad (2012) are feasible. In this setting, a novel margin affects the choice between renewables and fossil energy with CCS. Along the optimal path, the correctly accounted social costs for the coalition take account of the “environmental liability” that accompanies use of renewable energy to abate carbon
emissions. The liability is evident when renewable abatement is compared to the alternative energy technologies, both of which use up fossil fuels in the process of producing energy. By foregoing the use of fossil fuels, renewables leave the deposit in the ground, available for future generation in the non-coalition region to burn.

Our first result shows that the environmental liability is trivial in a global economy where all countries abide by the climate treaty because the liability never materializes. This occurs because the global coalition optimally switches to 100 percent abatement with renewable energy before the full stock of fossil fuels is fully exploited. The setting with a global coalition thus supports the “conventional” intuition that CCS has value only to the extent that it provides a cost-effective means to abate carbon emissions.

The second result shows that the environmental liability component of social costs becomes important when participation in the climate treaty is incomplete—thus, when the coalition is sub-global. In this case, optimal policy for the coalition must take into account the present value of the environmental liability discounted back from the date at which fossil fuels are finally exhausted. Because the present value of the liability becomes more important as the end of the fossil fuel becomes more proximate, the coalition is prone to rely on renewable abatement early on and switch to CCS later as the end of the fossil era approaches.

The paper contributes to the literature on supply-side environmental policy. In a seminal paper, Hoel (1994) shows that policies affecting the supply of carbon are necessary to control supply-side leakage that arises when participation in a climate agreement is incomplete. Harstad (2012) extends this work in a fascinating contribution which shows that a sub-global climate coalition could achieve the first-best outcome by purchasing and retiring the mineral rights for fossil fuel deposits, provided it has access to a well-functioning global market for fossil fuel deposits. Our paper shows that absent such a market, carbon capture and storage (CCS) may be the coalition’s best alternative, since it provides a potentially more credible means to achieve the same end.

The paper also complements two prior contributions that consider ways in which renewable energy and CCS differ in their impact on the supply of fossil fuel. Hoel and Jensen (2012) use a two-period model to compare the Green-Paradox-like effects of cost reductions in renewables versus cost reductions in CCS. They find that while cost reductions for renewables always cause fossil fuel extraction to speed up, cost reductions for CCS does not. Indeed, it potentially has the opposite effect. Kalkuhl et al. (2015) use a dynamic general equilibrium model to consider the relative merits of policies that support CCS versus those
that support renewables. Their model allows for the general equilibrium impact of policies on the time path of fossil fuel extraction, thus accommodating Green-Paradox-like effects. Both papers consider renewable energy and CCS implemented at a global scale. In contrast, we study the relative benefit of using renewables and CCS in a two-region model with incomplete participation in a climate treaty. As a consequence, the extended role of CCS as a supply-side policy that we identify in our model does not arise in either Hoel and Jensen (2012) or Kalkuhl et al. (2015).

Finally, the paper contributes to the economic literature that considers the role of CCS to mitigate climate change. Important contributions include van der Zwaan and Gerlagh (2009), van der Zwaan and Smekens (2009), van der Zwaan and Tavoni (2011), and especially Moreaux and Withagen (2015). Like Moreaux and Withagen (2015), we study the optimal level of carbon emissions abatement. Unlike this paper, we also study the interaction between CCS and renewable energy and we consider sub-global policies only pursued by a coalition of countries.

Section 2 describes the model. Section 3 sets up the policy problem for a potentially sub-global coalition of abating countries. Section 4 studies the importance of the environmental liability within a global and sub-global climate agreement, and it presents the main theoretical results of the paper. Section 5 explores the quantitative significance of the ideas, and section 6 concludes.

2 Model

We employ a relatively simple dynamic model that captures the essential features of the situation. Time is continuous and runs from 0 to infinity. There is no uncertainty. The world consists of two regions: a coalition of abating countries, $A$, and a group of non-abating countries, $N$. Each region is characterized by a constant size parameter $\psi \in (0,1]$ that determines the region’s portion of global energy demand and its portion of global climate damages. Specifically, with global (flow) energy demand $E$, energy demand in $A$ is $\psi E$ and energy demand in $N$ is $(1 - \psi)E$. Moreover, given global climate damages $D(\cdot)$, climate damages in $A$ are $\psi D(\cdot)$. While energy policy in $A$ takes account of climate damages that accumulate within its borders, climate damages in $N$ are not internalized by decision-makers in either region.

There are three alternative technologies to produce energy: fossil energy ($e_f$), renew-
able energy \((e_r)\) and fossil energy together with CCS \((e_c)\). Renewable energy and CCS are produced at constant marginal cost \(b\) and \(c\), respectively. The global supply of fossil fuels is characterized by a marginal extraction cost curve, denoted \(g(F)\), which is an increasing, linear function of cumulative extraction. Letting \(F(t)\) denote the stock of economically attractive fossil fuel remaining in the ground in \(t\), we have

\[
g(F(t)) = \phi_1 + \phi_2(F(0) - F(t)),
\]

where \(\phi_1 \geq 0\) and \(\phi_2 > 0\). \(F(0)\) is the aggregate stock of economically profitable fossil fuels at time 0. It is determined by the marginal cost of the backstop technology through the following condition:

\[
g(F(0)) = b. \tag{1}
\]

We further assume there are no fossil energy transportation costs and no resource rents. It follows that the user price of fossil energy is just the marginal extraction cost. The literature has shown that resource rents strengthen the case for using CCS relative to renewables (see Hoel and Jensen (2012)). By abstracting from them, we seek to isolate the contribution of the novel “supply-side value” of CCS for determining its role in sub-global climate policy.

The stock of economically attractive fossil fuels are exhausted at the (potentially infinite) future time “T”. This is the end of the fossil era, so

\[
F(T) = 0. \tag{2}
\]

The stock of fossil fuel evolves according to the law of motion

\[
\dot{F}(t) = -E + e_r(t), \tag{3}
\]

where \(e_r(t)\) is the time \(t\) flow of energy from renewables in the coalition. The initial and terminal conditions for \(F(t)\) are given by equations (1) and (2). Throughout, capital letters denote global variables, while small letters denote choice variables for the coalition.

A key assumption in the analysis is that a market for fossil fuel deposits does not exist. That is, we rule out supply-side policies of the sort proposed by Harstad (2012). As discussed earlier, leaving economically attractive fossil fuels unexploited is likely to be politically difficult—indeed, even if a given polity were committed to retiring local deposits in the present, the risk of future expropriation would be high. By ruling out conventional supply-side policy, the paper shows that CCS can accomplish the same aim. In this sense,
CCS can be viewed as a form of supply-side environmental policy and a credible mechanism for locking fossil carbon permanently underground.

Units are chosen so a unit of burned fossil fuel gives rise to a unit of atmospheric carbon in the long run. This is equivalent to a carbon cycle in which a fixed portion of emissions stays in the atmosphere permanently. It also abstracts from temperature inertia and other transitory carbon cycle dynamics. When fossil energy is coupled with CCS, atmospheric carbon increases by $\chi$ units, where $\chi \in [0, 1]$ denotes long run leakage of stored carbon. The stock of atmospheric carbon, $S$, evolves according to

$$\dot{S}(t) = E - e_r(t) - (1 - \chi)e_c(t).$$

Here, $e_c(t)$ is the time $t$ flow of fossil energy coupled with CCS in the coalition and $S(0) > 0$ is given. Notably, $e_c$ does not enter the state equation for $F$ because fossil fuels are used up when abating with CCS. In contrast, $e_r$ enters both state equations.

Climate damages are given by the function $D(S)$. The function $D$ is strictly increasing and strictly convex. In addition, marginal damages are assumed to be bounded. The strict convexity requirement is somewhat strong. For example, in the DICE model (Nordhaus (2013)), economic damages are a convex function of temperature, while temperature is a concave function of atmospheric carbon. Taking the composition of these two functions implies a version of our damage function in which $D''(S)$ is roughly constant. Nevertheless, Weitzman (2012) argues that the damage function in the DICE model implies implausibly low damages at high temperature and that a more plausible functional form would be significantly more convex. Moreover, the case with $D''(S)$ constant is approximately covered by assuming $D''(S) > 0$ with small magnitude.

Energy supply in $N$ is always met by the cheapest available option. Given the assumptions in the model, countries in $N$ use fossil energy until the end of the fossil era when they switch to renewables. The interesting economic choice entails the mix of energy supply in $A$. We study this problem in the next section.

3 The Coalition Policy Problem

The problem for countries in region $A$ (“the coalition”) is to choose the time path of fossil energy ($e_f$), renewable energy ($e_r$) and fossil energy with CCS ($e_c$) to minimize the present discounted social cost of energy, as viewed by the coalition. The social cost of energy includes
coalition energy costs and coalition climate damages. It is given by

\[ r(e_f, e_r, e_c; F, S, t) = g(F(t))e_f(t) + be_r(t) + [g(F(t)) + c]e_c(t) + \psi D(S(t)). \]  

(5)

Total energy supply in the coalition must satisfy the coalition’s (exogenous) energy demand. Thus,

\[ e_f(t) + e_r(t) + e_c(t) = \psi E \]  

for every \( t \). Both the stock of fossil fuel and the stock of atmospheric carbon are necessary state variables since cumulative CCS disrupts the otherwise fixed relationship between them.

The coalition’s optimal energy policy is the path \([e_f(t), e_r(t), e_c(t)]_{t=0}^{\infty}\) that minimizes

\[ \int_{0}^{\infty} e^{-\rho t} r(e_f, e_r, e_c; F, S, t) dt \]  

subject to (6), the law of motion for fossil fuel (3) and atmospheric carbon (4), the initial and terminal condition on the stock of fossil fuel (1)-(2), the initial condition on atmospheric carbon and the non negativity constraints on the choice variables.

While the planning horizon is infinite, we study an equivalent finite horizon planning problem where the terminal time is free and the salvage value captures the continuation value in the steady state after the economy optimally switches to the renewable backstop. This formulation naturally captures the restriction that CCS is feasible only if fossil fuels have not been exhausted. A detailed characterization of the solution to this problem is provided in Appendix 7.1.

Given a discount rate \( \rho \), the salvage value is given by

\[ \phi(S(T), T) = e^{-\rho T} \left[ \frac{b\psi E}{\rho} + \frac{\psi D(S(T))}{\rho} \right]. \]  

(8)

It includes the cost of meeting future coalition energy demand with renewable energy plus the cost of future climate damages. Damages in the terminal period are determined by the stock of atmospheric carbon that the economy ends up with in \( T \) since, by assumption, atmospheric carbon doesn’t depreciate. Using this, the objective function in (7) is replaced with

\[ \int_{0}^{T} e^{-\rho t} r(e_f, e_r, e_c; F, S, t) dt + \phi(S(T), T). \]  

(9)

To characterize the solution in the next section, it will be useful to define \( \lambda(t) \) as the Social Cost of Carbon (SCC) and \( \mu(t) \) as the Social Cost of Fossil Fuel (SCFF). The first is the co-state variable for the carbon cycle law of motion (4), while the second is the co-state
variable for the fossil fuel equation (3). The SCC is calculated by integrating the co-state equation for the carbon stock. This implies

$$\lambda(t) = \int_t^T e^{-\rho(s-t)} \psi D'(S(s)) ds + e^{-\rho(T-t)} \lambda(T). \quad (10)$$

The last term in (10) represents the social cost of carbon in the continuation problem that begins at the end of the fossil era. Taking the derivative of the continuation value (8) with respect to the carbon stock gives the standard relationship

$$\lambda(t) = \int_t^\infty e^{-\rho(s-t)} \psi D'(S(s)) ds. \quad (11)$$

So the SCC is just the present value of future marginal damages due to an extra unit of carbon emissions in $t$.

By similar logic, the SCFF is

$$\mu(t) = -\phi_2 \int_t^T e^{-\rho(s-t)} (\psi E - e_r(s)) ds + e^{-\rho(T-t)} \mu(T). \quad (12)$$

The first term in (12) captures the reduction in future fossil-based energy production costs as a result of having an extra unit of fossil fuel in the ground. The extra unit shifts the marginal extraction cost curve down by one unit for each future period because the marginal extraction cost is increasing in cumulative extraction. For convenience later, we denote this term $\mu_1(t)$.

The second term in (12) captures the contribution to the SCFF in $t$ due to there being (all else equal) an extra unit of fossil fuel in the ground in $T$. By assumption, the marginal extraction cost in $T$ equals the backstop price $b$, so the production value of the extra unit in $T$ is zero. Thus, the social value of the extra unit in $T$ is the environmental liability due to the carbon emissions that result when it is burned. Because this interpretation is useful when interpreting the optimal policy below, we will often refer to $\mu(T)$ simply as the “environmental liability”.

Since $T$ is defined as the endogenous time at which the fossil era ends, we know that the last unit of fossil fuel is used up in that instant. Nevertheless, the magnitude of $\mu(T)$ still depends (endogenously) on what coalition policy turns out to be optimal in $T$. The interpretation of $\mu(T)$ as an environmental liability is easiest to see when the coalition abates fully with renewables in $T$—i.e., $e_r(T) = \psi E$. In that case, it is easy to show that

$$\mu(T) = \lambda(T). \quad (13)$$
The intuition is simple: with one unit of fossil fuel remaining in the ground and the coalition abating with renewables, the last unit gets burned in the non-coalition region. Thus, the social value of the extra unit of fossil fuel is just the social cost of carbon (from the perspective of the coalition).

More generally, we show in the appendix that

\[ \mu(T) = \frac{E - e_r(T) - (1 - \chi)e_c(T)}{E - e_r(T)}\lambda(T) + \frac{ce_c(T)}{E - e_r(T)} \]  

(14)

The expression simplifies in the two instances that turn out to be most common at the optimum. First, as already noted, if \( e_r(T) = \psi E \) and \( e_c(T) = 0 \), then \( \mu(T) = \lambda(T) \).

Alternatively, if the coalition abates fully with CCS—\( e_c(T) = \psi E \) and \( e_r(T) = 0 \)—then

\[ \mu(T) = (1 - \psi)\lambda(T) + \psi[c + \chi\lambda(T)] \]  

(15)

In this case, fraction \( \psi \) of the last unit is burned in the coalition with CCS, while fraction \( 1 - \psi \) is burned in region N. The portion burned in A increases coalition climate damages by \( \chi \), while increasing coalition energy cost by \( c \). Meanwhile, the portion burned in N leads to a unit-for-unit increase in climate damages. Overall, the social cost of fossil fuel at T reduces to the expression in (15).

When the coalition chooses to abate with renewable energy instead of CCS, it leaves more fossil in the ground for non-coalition countries to burn without CCS. This creates an environmental liability, which the coalition optimally takes into account by discounting the liability back from the end of the fossil era when the last unit of fossil fuel is finally exhausted. We turn to the optimal policy decision next.

4 Optimal Energy Policy

The policy that solves the coalition energy problem consists of using the technology at each instant with the lowest marginal social cost. While this statement is standard, the novelty in the analysis stems from the need to account for the environmental liability that accompanies a policy like renewable abatement that—relative to the alternatives—leaves an extra unit of fossil fuel in the ground. In our setting, the marginal social cost of energy is comprised of three pieces: the marginal production cost, the marginal externality cost, and the environmental liability. One could alternatively subsume the environmental liability term into the definition
of the marginal externality cost, but it is useful to keep them separate since they arise through different channels.

Table 1 decomposes the marginal social cost of energy into its three components for each energy source.\(^1\) For fossil energy, the marginal social cost is the marginal extraction cost together with the rise in future extraction costs that result from the accompanying increase in cumulative extraction plus the climate damages due to the increase in emissions. When fossil energy is used together with CCS, the unit cost of CCS is added to the production cost. At the same time, climate damages only arise when carbon leaks from storage. Finally, the marginal social cost of renewable energy equals the marginal production cost, \(b\), plus the environmental liability. While conventional intuition holds that renewables have no social cost beyond the cost of production, in the setting considered here, a sub-global planner must account for the present value of the environmental liability that arises from the fact that the unused fossil fuel deposits will eventually get burned in the non-coalition region.

In fact, the “conventional intuition” concerning the role of CCS and renewables for climate policy does arise in the model, though only as a special case: mainly, when the coalition is global \((\psi = 1)\). In this case, the noted environmental liability (and thus the supply-side value of CCS) does not arise. It then follows that CCS and renewables are substitutes and that CCS has value only to the extent that it can abate carbon emissions in a more cost-effective way. Given its importance, we state this result as a proposition. Proofs for all propositions are provided in the appendix.

**Proposition 1** Suppose that \(\psi = 1\). Then \(T \to \infty\) and \(\lim_{T \to \infty} e^{-\rho(T-t)} \mu(T) = 0\).

To understand the result, we need to consider some properties of the optimal policy. As time proceeds in the model, cumulative extraction of fossil fuels increases. This has three effects. First, rising extraction causes marginal extraction costs to rise. Second, it causes the impact on future extraction costs from using up an extra unit of fossil fuel to decrease. Finally, it causes the social cost of carbon to rise since carbon emissions are irreversible and damages are convex in atmospheric carbon. Lemma 1 in Appendix 7.2 shows that the net outcome of these three effects is to make abatement increasingly attractive in time. Indeed, both the relative value of renewables compared to fossil and the relative value of CCS compared to fossil increase with calendar time along the optimal path. Moreover, once it becomes optimal to abate, it is optimal to abate fully.

\(^1\) A formal derivation of the necessary conditions for an optimum is included in Appendix 7.1.
<table>
<thead>
<tr>
<th></th>
<th>Marginal Production Cost</th>
<th>Marginal Externality Cost</th>
<th>Environmental Liability Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fossil</td>
<td>$g(F(t)) + \mu_1(t)$</td>
<td>$\lambda(t)$</td>
<td></td>
</tr>
<tr>
<td>Fossil with CCS</td>
<td>$c + g(F(t)) + \mu_1(t)$</td>
<td>$\chi\lambda(t)$</td>
<td></td>
</tr>
<tr>
<td>Renewables</td>
<td>$b$</td>
<td>$e^{-\rho(T-t)}\mu(T)$</td>
<td></td>
</tr>
</tbody>
</table>

The result in the proposition arises because (as we show in the appendix) it necessarily becomes optimal to abate fully with renewables before the full stock of economically attractive fossil carbon is extracted. Once this happens, both state variables ($F$ and $S$) remain unchanged since the coalition comprises the entire world. Thus, it remains optimal to abate fully with renewable for all time. As a result, the terminal horizon $T$ is infinite, so the environmental liability never materializes—and its present value is zero. The stark feature of the model whereby the coalition abruptly switches from zero abatement (100 percent fossil energy) to 100 percent abatement follows from the assumption that alternative energy sources are perfect substitutes. Relaxing this assumption would imply an optimal path in which different energy sources are used simultaneously.

The above intuition does not hold when the coalition is sub-global ($\psi < 1$). In this case, the extra unit of fossil fuel foregone under renewable abatement gets burned in the non-coalition region if it is not used within the coalition. Indeed, the next proposition shows that the end of the fossil era necessarily arrives in the sub-global model and, as a consequence, the environmental liability must be taken into account when computing optimal policy.

**Proposition 2** Suppose $\psi < 1$. Then $T < \infty$ and $e^{-\rho(T-t)}\mu(T) > 0$.

In the sub-global setting, an expanded role for CCS arises because the coalition has to worry that unused fossil fuel deposits will eventually get used in the non-coalition region. This implies a key difference between abatement with CCS and abatement with renewables: while fossil with CCS uses up fossil fuels, renewable energy does not. The unexploited fossil fuel deposits give rise to an environmental liability that comes due at the end of the fossil era. This is a key insight of the paper.

Finally, because the relevant environmental liability term in the last column of Table 1 entails the present value of the SCFF in $T$, it follows that the importance of this term...
for impacting optimal policy increases as the terminal horizon gets closer. This implies in turn that the relative value of using CCS increases as $T$ approaches. Indeed, if the environmental liability is large enough, the coalition necessarily uses CCS as the end of the fossil era approaches. This feature of the sub-global model differs qualitatively from the global coalition case, and we state it as a proposition.

**Proposition 3** Suppose that $\psi < 1$ and $c \leq (1 - \chi)\psi D'(F(0))$. Then $e_c(t) > 0$ for at least some $t$.

The proposition shows that CCS is always optimal for some period provided that the environmental damages are sufficiently large. Specifically, CCS is always used provided the “worst case climate damages,” which arise when the entire stock of economically profitable fossil fuels are burned, exceeds the leakage-adjusted marginal cost of CCS. In contrast, CCS is only used in the global model if it provides a cheaper means to abate carbon than renewables. Moreover, even if this condition holds initially in the global model, renewables still always overtake CCS before the end of the fossil era, and CCS is then never used again.

While our emphasis in the paper is on clarifying the novel reason for valuing CCS, it is worth emphasizing that renewable energy—as the backstop technology that constrains the total amount of fossil carbon that turns out to be economically accessible over the long run—still plays a critical role in the model.

To quantify the economic importance of CCS as a source of energy and as way of avoiding a potentially large environmental liability at the end of the fossil era, the next section presents a calibrated version of the model.

5 A Numerical Example

5.1 Calibration

The calibration of the model builds heavily on the energy and climate calibration assumptions in van der Ploeg (2015). Both the stock of fossil fuels and the stock of atmospheric carbon are expressed in teratons carbon (TtC). According to International Energy Agency (2012), proven international fossil fuel reserves are 0.781 TtC, but potential reserves are much larger. GEA (2012) estimate that total plausibly accessible carbon energy deposits in the earth’s crust are on the order of 10 TtC or more, though fortunately much of this will almost certainly remain economically unattractive to extract. van der Ploeg (2015) calibrates the model to
generate current fossil fuel prices, which implies total economical reserves of 1.72 TtC. In the baseline calibration, I adopt the somewhat higher value of 2.0 TtC.

We allow for a carbon cycle in which half of current emissions get taken up immediately by the biosphere and upper oceans, while the other half stays in the atmosphere permanently. Thus,

\[ S(t) = S(0) + 0.5[F(0) - F(t) - (1 - \chi)M(t)], \]

where\n
\[ M(t) = \int_0^t e_c(\tau) d\tau \tag{16} \]

is cumulative CCS. This specification roughly captures the expected relationship between emissions and future atmospheric stocks, though a more complete model would allow for a further portion of current emissions that decays geometrically (see Golosov et al. (2014)). The initial stock of atmospheric carbon is calibrated to \( S(0) = 0.85 \) TtC, which is equivalent to 400 ppm of CO2.

As in van der Ploeg (2015), higher atmospheric carbon stocks cause global mean temperature to rise through the following logarithmic relationship:

\[ T = \alpha + \beta \ln(S). \tag{17} \]

The parameters \( \alpha \) and \( \beta \) are calibrated to get a temperature sensitivity of 3.0 degrees Celsius. This implies \( \beta = \frac{3}{\ln(2)} = 4.238 \). In addition, the temperature change before the industrial revolution (when the stock of atmospheric carbon was 0.581 TtC) is assumed to be 0 degrees Celsius. This implies \( \alpha = -\beta \ln(0.581) = 2.350 \).

For simplicity, the model abstracts from any delay between rising atmospheric carbon and eventual temperature change. As noted in van der Ploeg (2015), this causes damages to occur immediately when plausibly damages follow emissions with a delay of approximately 70 years. Abstracting from temperature inertia biases the present value of damages upwards.

To model the relationship between temperature change and economic damages, I consider two possible damage functions. The first is taken from Nordhaus (2013):

\[ D^N(T) = \left[ 1 - \frac{1}{1 + (T/18.8)^2} \right] Y_t, \tag{18} \]

where \( Y_t \) is global GDP in the period. Taking the composite of the temperature equation, (17), and this damage function implies a relationship between the stock of atmospheric
carbon and damages that is roughly linear, so marginal damages—viewed as a function of atmospheric carbon—are roughly constant.

We also consider a damage function suggested by arguments in Weitzman (2012). Ackerman and Stanton (2012) suggest a calibrated damage function that is consistent with Nordhaus (2008) damage estimates at low temperature change and with Weitzman estimates at high temperature change. It is

$$D_{NW}(T) = \left[1 - \frac{1}{1 + (T/20.2)^2 + (T/6.08)^{6.76}}\right]Y_t. \quad (19)$$

Combining this damage function with the temperature equation (17), implies a damage function that is convex in the stock of atmospheric carbon. It can be viewed as accounting for so-called “fat-tailed” risks that may accompany large increases in atmospheric carbon.

To calibrate marginal extraction costs, we again follow van der Ploeg (2015). The current market price of (generalized) fossil fuels, $p(0)$, is assumed to be 470 USD/etC, where etC stands for equivalent ton carbon. Since there is no scarcity rent, we set $\phi_1 = 470$ USD/etC. In addition, the the long run renewable backstop price (with storage) is set to 1.5 times the current price of fossil fuels. Thus,

$$b = 1.5p(0) = 705 \text{ USD/etC.}$$

It follows that $\phi_2 = \frac{b - \phi_1}{F(0)} = 117.5 \text{ (TUSD/TtC^2)}$.

Global energy demand in the model is assumed to be constant. van der Ploeg (2015) calibrates current energy demand at 9.3 GtC, which corresponds to emissions with a Gross World Product of approximately 70 trillion USD. Emission growth over the last decade averaged 2.7 percent per year, and typical forecasts have it declining over time along a business as usual path. To account for this, we assume constant global energy demand that is double the value in that paper, thus $E = 18.6$ GtC. This implies cumulative energy demand over the next 85 years that is the same as would obtain if global energy demand were to grow at 1.5% per year (followed by no emission growth after 85 years).

The consumption discount rate is set to 3 percent per year, though this assumption is relaxed in the robustness exercise later on.

### 5.2 Optimal policy

The optimal policy is highly sensitive to two key features of the model: the discount rate and the damage function. The coalition’s willingness to employ CCS requires, on the margin,
that the present value of the environmental liability accompanying renewables is sufficiently large to justify the added cost of CCS. But because this liability doesn’t materialize until the end of the fossil era, the decision maker waits until the terminal horizon is “sufficiently proximate”.

The second key feature is the damage function. Because CCS (compared to abatement with renewables) has the effect of avoiding damages that materialize relatively far out in the future, these damages have to be sufficiently large for the present value of damages viewed from an earlier period to be large enough to justify the added cost of CCS. Indeed, when damages follow the Nordhaus damage function, in our baseline calibration with \( \psi = 0.5 \), the coalition never uses CCS unless \( \frac{c}{b-p_0} \leq 0.44 \). Thus, the marginal cost of capturing and storing carbon would need to be significantly less than half the markup of renewables (the “long run” price with storage) over the current price of fossil energy for CCS to play an important role in climate policy. While this is conceivable, the economic case for CCS is much stronger when damages are increasing in the stock of carbon. One interpretation of increasing marginal damages, advocated in Weitzman (2012), is that they account for the possibility that damages could become catastrophic at very high levels of atmospheric carbon. I adopt the Weitzman-Nordhaus damage function for the remainder of this section.

With Weitzman-Nordhaus damages, (again with \( \psi = 0.5 \)) the threshold value for CCS to be used is \( \frac{c}{b-p_0} \leq 2.53 \). For the analysis below, the baseline value for the marginal CCS cost is \( c = 0.7 \ast (b - p_0) \). While this implies that the “long run” CCS cost is less than the markup of renewables over the current price of fossil energy, it is worth noting that the markup of renewables over fossil energy declines in time as the marginal extraction cost rises; in contrast, the markup of CCS over fossil is always the constant cost \( c \). Thus, by the time CCS is actually used, it is more costly as a means to abate carbon than the renewable alternative.

Figure 1 reports the temperature change for three alternative coalition sizes (\( \psi \)). The middle case, \( \psi = 0.5 \), assumes the coalition comprises 50 percent of global energy demand. In addition, the figure shows a high participation case with \( \psi = 0.75 \) and a low participation case with \( \psi = 0.25 \). For each value of \( \psi \), the figure shows the optimal trajectory of temperature under two scenarios: one in which the coalition has access to renewables and CCS, and one in which the coalition only has access to renewables (CCS is unavailable). Colors indicate the value of \( \psi \). Circles indicate the scenario with renewables only, while boxes indicate the scenario with CCS. In addition, dotted vertical lines show, for the scenario with CCS, when
Figure 1: Colors indicate the coalition fraction, $\psi$. Dotted vertical lines (for each $\psi$ value) indicate when renewable abatement optimally begins; dashed lines indicate when the coalition transitions to CCS. Circles and squares distinguish the renewable-only case from the case with renewables and CCS.

The coalition optimally transitions from no abatement to full abatement with renewables ($\tilde{t}$). The dashed vertical lines show when the coalition optimally transitions from full abatement with renewables to full abatement with CCS ($\hat{t}$).

A number of effects are evident in the figure. First, the coalition begins renewable abatement earlier, the higher $\psi$. This effect is reinforced by two factors. First, with higher $\psi$, the coalition internalizes a greater fraction of global climate damages and hence values abatement more. Second, higher $\psi$ makes renewable abatement more impactful, since more effectively delays the onset of severe climate damages. In addition, the time at which the coalition transitions to full CCS is increasing in $\psi$. This again reflects the fact that renewables are more effective at delaying the onset of severe damages, so the threshold for triggering the transition to CCS is also postponed.

A further effect is that the maximum long run temperature increase is decreasing in $\psi$, though the impact of CCS on long run temperature change responds more dramatically to increases in $\psi$ when $\psi$ is relatively low. Moving from $\psi = 0.25$ to $\psi = 0.5$ has the effect of lowering the long run temperature increase by an additional 0.30 degrees C. In contrast,
Figure 2: The dashed grey line is the horizon of the fossil era when only renewables are available. The top line of the shaded region is the horizon of the fossil era with renewables and CCS, and the bottom line of the shaded region indicates when CCS is first used. The shaded region shows the duration of the CCS era. The horizontal axis denotes $\psi$.

Figure 3: The outcome variable is the period CCS is used (the shaded region in figure 3) over the horizon of the fossil era. This is plotted as a function of $\psi$.
moving from $\psi = 0.5$ to $\psi = 0.75$ lowers the long run temperature increase by an additional 0.03 degrees C. Nevertheless, the timing at which the eventual temperature change is realized differs considerably across scenarios. With $\psi = 0.75$, it is delayed by over a century relative to the case with $\psi = 0.5$.

Figure 1 suggests that the role of CCS in optimal climate policy depends on the size of the coalition. To clarify this relationship, Figure 2 plots the horizon of the fossil era (the terminal time $T$) and the transition from renewables to CCS ($\hat{t}$), both as functions of $\psi$. In addition, Figure 3 plots $\frac{T - \hat{t}}{T}$ as a function of $\psi$. This captures how meaningful the time at which CCS is used is a fraction of the total fossil horizon.

According to the latter measure, the role of CCS in optimal climate policy is highest when the coalition comprises roughly 40 percent of global energy demand. Intriguingly, this is similar to the size of the Annex I countries in the Kyoto Protocol. The Annex I countries are roughly the same as the OECD countries, and the OECD comprised roughly 50% of global carbon dioxide emissions in 1990 and roughly a third of global emissions in 2013 (Olivier et al. 2014).
5.3 The value of CCS

To quantify the value of CCS, we compare the maximized value of the objective function with the maximized value under the constraint that only renewables are available. Let \([e_r^*(\tau), e_c^*(\tau), F^*(\tau), S^*(\tau)]\) solve the general coalition’s problem. Then define

\[
J(t) = \int_{T-t}^T e^{-\rho \tau} \left[ g(F^*(t+\tau))\psi_E + (b - g(F^*(t+\tau)))e_r^*(t+\tau) + ce_c^*(t+\tau) + \psi_D(S^*(t+\tau)) \right] dt + e^{-\rho(T-t)} \left[ \frac{b\psi_E}{\rho} + \frac{\psi_D(S^*(T))}{\rho} \right].
\]

This is the present value of the coalition’s energy costs plus the coalition’s climate damages viewed from the perspective of \(t\) under the optimal policy. Similarly, define \(H(t)\) as the analogous forward-looking period-\(t\) continuation value under the constrained optimal policy for the analogous problem in which CCS does not exist—abatement is chosen optimally with only renewables available. Then the time-\(t\) value of CCS is defined as

\[
V(t) = J(t) - H(t).
\]

Figure 4 plots \(V(t)\) for the high, medium and low values of \(\psi\) reported in Figure 2. For each \(\psi\), \(V(t)\) is plotted from period zero (today) until \(\hat{t}\), the time at which CCS is first used. To account for the fact that a larger coalition internalizes a larger portion of global climate damages, \(V(t)\) is drawn as a percent of the coalition’s GDP, which is proportional to \(\psi\). As one might expect, the value grows at the discount rate over the time interval in which CCS is not used. The value trajectory lines up almost exactly for the \(\psi = .25\) and \(\psi = .5\) cases. Though CCS is adopted later for the \(\psi = 0.5\) case, and its value at the time it is first adopted is significantly higher. In contrast, for the \(\psi = .75\) case, the value of CCS is near zero for the first hundred years or so. It then rises rapidly, reaching over 15 percent of coalition GDP by year 250. It is worth noting that the absolute value of CCS is strictly increasing in \(\psi\), though this partly reflects the fact that the magnitude of coalition climate damages is increasing in \(\psi\).

6 Conclusion

In a world in which participation in a global climate treaty is incomplete and where it is politically infeasible to permanently retire valuable fossil fuel deposits, renewables suffer
from 100 percent supply side leakage over the long run. As a result, abating with renewables does not change the long run climate outcome relative to the business as usual case. In contrast, CCS permanently retires the carbon embedded in fossil fuel deposits, and thus does change the long run climate outcome. Despite this difference, renewables have the potentially substantial merit of delaying the onset of severe climate damages. If participation is high, this delaying effect could be large, on the order of centuries. As a result, the relative attractiveness of renewables increases with the size of the climate coalition provided the size of the coalition is sufficiently large. In contrast, when participation is low, the delaying effect of renewables is smaller, and the relative case for CCS increases. In the quantitative analysis, I find that the role of CCS for climate policy is most important when the coalition comprises about 40 percent of global energy demand. Interestingly, this happens to coincide approximately with the size of the Annex I group of countries as defined in the Kyoto Protocol.

The analysis understates the relative attractiveness of CCS versus renewables for at least two reasons. First, by abstracting from resource rents, the model does not account for the insight, studied in Hoel and Jensen (2012) and Kalkuhl et al. (2015), that renewables suffer from Green-Paradox-like price effects, while CCS does not. Incorporating this general equilibrium effect into the current model would make CCS even more attractive. A further advantage of CCS not considered is that CCS avoids the so-called Stranded Asset problem—the fact that demand side climate policies reduce the asset value of fossil fuel reserves, which creates a political economy problem as losers have an incentive to obstruct the policy process. For these reasons, the analysis might be viewed as conservative.

An interesting extension to consider in future research would be to model the process through which each technology develops—for example, through learning by doing. This would attenuate the rather stark feature of the current model whereby abatement is initially zero, and CCS is used only at the last possible moment.

References


GEA (2012): Global Energy Assessment - Toward a Sustainable Future, Cambridge Univer-


7 Appendix

7.1 Characterization of the Solution to the Policy Problem

In Section 3, we described the coalition’s optimal policy problem and in Table 1 in Section 4, we summarized the margins that the coalition takes into account when deciding its optimal policy. We now provide a formal characterization of the solution to the coalition’s problem. While the text writes the problem as one of minimizing costs, we solve the problem as maximizing the negative cost. Thus, the co-state variables reported in the text \((\lambda, \mu)\) are the negative of the co-state variables presented here \((\bar{\lambda}, \bar{\mu})\).

The Hamiltonian is

\[
H(t, e_r, e_c, \bar{\lambda}, \bar{\mu}) = e^{-\rho t} \left[ -g(F(t))\psi E - (b - g(F(t)))e_r - ce_c \
- \psi D(S(t)) \right] + \bar{\lambda}(t)[E - e_r - (1 - \chi)e_c] - \bar{\mu}(t)[E - e_r].
\]

The optimal policy each period solves

\[
\max_{e_r, e_c} H(t, e_r, e_c, \bar{\lambda}, \bar{\mu})
\]

s.t.

\[
e_r(t) \geq 0, \quad e_c(t) \geq 0,
\]

\[
e_c(t) + e_r(t) \leq \psi E.
\]
with $S(0)$ and $F(0)$ given. Let $\theta_1(t)$, $\theta_2(t)$ and $\theta_3(t)$ be, respectively, the multipliers on the non-negativity constraint for $e_r(t)$, the non-negativity constraint for $e_c(t)$ and the upper bound constraint. The first-order conditions for $e_r(t)$ and $e_c(t)$ can be written

$$- e^{-\rho t}(b - g(F(t))) - \tilde{\lambda}(t) + \tilde{\mu}(t) = \theta_3(t) - \theta_1(t)$$

and

$$- e^{-\rho t}c - (1 - \chi)\tilde{\lambda}(t) = \theta_3(t) - \theta_2(t).$$

In addition, complementary slackness requires

$$\theta_1(t)e_r(t) = 0, \quad \theta_2(t)e_c(t) = 0, \quad \theta_3(t)(\psi E - e_r(t) - e_c(t)) = 0$$

with

$$\theta_1(t) \geq 0, \quad \theta_2(t) \geq 0, \quad \theta_3(t) \geq 0, \quad \theta_4(t) \geq 0.$$

And the co-state equations are

$$\dot{\tilde{\lambda}}(t) = -\frac{\partial H}{\partial S} = e^{-\rho t}\psi D'(S(t))$$

and

$$\dot{\tilde{\mu}}(t) = -\frac{\partial H}{\partial F} = -e^{-\rho t}(\psi E - e_r(t))\phi_2.$$

Because $S(T)$ is free, we must have that

$$\tilde{\lambda}(T) = \phi_S(S(T), T) = -e^{-\rho T}\psi D'(S(T))\frac{1}{\rho}$$

Moreover, the free terminal time constraint must satisfy

$$H(T, S(T), F(T), \tilde{\lambda}(T), \tilde{\mu}(T)) + \phi_t = 0$$

Noting that $\phi_t = -e^{-\rho T}[b\psi E + \psi D(S(T))]$, this implies

$$\tilde{\mu}(T) = \frac{E - e_r(T) - (1 - \chi)e_c(T)}{E - e_r(T)}\tilde{\lambda}(T) + \frac{ce_c(T)}{E - e_r(T)}.$$  \hspace{1cm} (22)

Integrating the co-state equation for $F$ gives

$$\tilde{\mu}(t) = \phi_2 \int_t^T [e^{-\rho s}(\psi E - e_r(s))] ds + e^{-\rho T}\tilde{\mu}(T).$$
Note that if we are speaking to the corresponding minimization problem in the text, and if we wrote the multiplier in current value terms, we would write:

\[ \mu(t) = -\phi_2 \int_0^{T-t} \left[ e^{-\rho s} (\psi E - e_r(t + s)) \right] ds + e^{-\rho(T-t)} \mu(T). \]

Integrating the co-state equation for \( S \) gives

\[ \tilde{\lambda}(t) = -\int_t^T e^{-\rho s} \psi D'(S(s)) ds + \tilde{\lambda}(T) \]

\[ = -\int_t^\infty e^{-\rho s} \psi D'(S(s)) ds. \]

Translating this into the multiplier for the cost minimization problem stated in the text, expressed in current value terms, we would similarly have

\[ \lambda(t) = \int_0^\infty e^{-\rho s} \psi D'(S(t + s)) ds \]

Because it will be useful to prove the main results of the paper, we show next that the coalition’s optimal policy displays a switching property: the coalition starts off using fossil energy and switches to carbon abatement (either fossil energy with CCS or renewables energy) at some time \( \hat{t} \). We proceed by parts. Lemma 1 shows that the marginal social cost of using fossil energy, relative to the social cost of using renewable energy, increases monotonically in time. Thus, once it becomes optimal to switch to renewables, it is never optimal to switch back to no abatement. Lemma 2 shows that the marginal social cost of using fossil energy, relative to the social cost of using fossil energy coupled with CCS, increases monotonically in time. Thus, once it becomes optimal to switch to CCS, it is never optimal to switch back to no abatement. Finally, Lemma 3 shows that the marginal social cost of using renewables, relative to the social cost of using fossil energy with CCS, depends on the size of the environmental liability. If the environmental liability is large enough, then the marginal social cost of using renewables increases with time and the coalition’s best policy is to switch to fossil energy with CCS at some time \( \hat{t} \).

**Lemma 1** Let \( f(t) \equiv g(F(t)) - e^{-\rho \lambda}(t) + e^{-\rho t} \tilde{\mu}(t) - b \). Suppose \( \psi = 1 \). Then \( f'(t) \geq 0 \). Alternatively, suppose \( \psi < 1 \). Then \( f'(t) > 0 \).

**Proof of Lemma 1.** From (20), this Lemma states that, ignoring the option to alternatively use CCS, the marginal social cost of using fossil energy, relative to the social cost of using
renewable energy, is increasing in time. Moreover, absent the option of using CCS, once \( f(t) \geq 0 \) we have that \( e_r(t) = \psi E \) for all remaining periods.

Substituting from above and rearranging,

\[
\begin{split}
    f(t) &= \left[ \phi_1 + \phi_2(F(0) - F(t)) + \phi_2 \int_0^{T-t} e^{-\rho s}(\psi E - e_r(t+s))ds \right] \\
    &\quad + \left[ e^{-\rho t} \tilde{\mu}(T) - e^{-\rho t} \tilde{\lambda}(t) \right] - b. \\
\end{split}
\]  

Define the first bracketed term as \( f_1(t) \) and the second bracketed term as \( f_2(t) \).

\[
\begin{split}
    f_1'(t) &= -\phi_2 \dot{F}(t) - \phi_2 \int_0^{T-t} e^{-\rho s} e'_r(t+s)ds + \phi_2 e^{-\rho(T-t)}(\psi E - e_r(T)) \\
    &\geq \phi_2 (E - e_r(t)) - \phi_2 \int_0^{T-t} e^{-\rho s} e'_r(t+s)ds. \\
\end{split}
\]

The inequality in the second line follows because the last term in the first line is positive since \( e_r(T) \leq \psi E \). Apply integration by parts to the second term in the second line to get

\[
\begin{split}
    f_1'(t) &\geq \phi_2 (E - e_r(t)) - \phi_2 \int_0^{T-t} (\rho) e^{-\rho s} e_r(t+s)ds \\
    &\geq \phi_2 E - \phi_2 \rho \int_0^{T-t} e^{-\rho s} e_r(t+s)ds. \\
\end{split}
\]

Suppose \( \psi < 1 \). The right-hand side of \( f_1'(t) \) is strictly positive since

\[
\int_0^{T-t} e^{-\rho s} e_r(t+s)ds < \int_0^{T-t} e^{-\rho s}(1)ds \leq \int_0^{\infty} e^{-\rho s}ds = \frac{1}{\rho}. 
\]

The first inequality is strict since \( e_r(t) \leq \psi E < E \). Hence, for \( \psi < 1 \), \( f_1'(t) > 0 \) for all \( t \leq T \).

Alternatively, suppose \( \psi = 1 \). Then the right-hand side of \( f_1'(t) \) is strictly positive unless \( e_r(t) = \psi E \) for all future periods. Therefore, \( f_1'(t) > 0 \) unless at some point it becomes optimal to switch to full renewable abatement for the rest of time—in which case \( f_1'(t) \geq 0 \).

Next, consider \( f_2 \).

\[
\begin{split}
    f_2 &= e^{\rho t} \tilde{\mu}(T) - e^{\rho t} \tilde{\lambda}(t) \\
    &= e^{-\rho(T-t)} e^{\rho T} \tilde{\mu}(T) - e^{\rho t} \tilde{\lambda}(t) \\
\end{split}
\]

First note that, if \( T = \infty \) then the present value of the first term is zero, and thus not consequential. Suppose then that \( T < \infty \). Because CCS is unavailable by hypothesis,

\[
\tilde{\mu}(T) = \tilde{\lambda}(T) 
\]
Thus,

\[ f_2(t) = e^{-\rho(T-t)}e^{\rho T} \tilde{\lambda}(T) - e^{\rho t} \tilde{\lambda}(t) = \int_0^{T-t} e^{-\rho s} \psi D'(S(t + s)) ds. \]

By Leibnitz’s Rule,

\[ f_2'(t) = e^{-\rho(T-t)} \psi D(S(T)) - \psi D(S(t)) - \int_0^{T-t} [(-\rho)e^{-\rho s} \psi D(S(t + s))] ds. \]

Suppose \( \psi < 1 \). Then \( D(S(t)) \) is strictly increasing in \( t \) for \( t < T \) since emissions are irreversible and always positive. Thus, for \( \psi < 1 \) we have that \( f_1'(t) > 0 \) and \( f_2'(t) > 0 \) and, therefore, \( f'(t) > 0 \).

Alternatively, suppose \( \psi = 1 \). Then \( D(S(t)) \) is strictly increasing in \( t \) unless abatement in all future periods is 100 percent, in which case it is constant. It follows, for all such \( t \), that

\[ f_2'(t) = e^{-\rho(T-t)} \psi D(S(T)) - \psi D(S(t)) + \rho \int_0^{T-t} [e^{-\rho s} \psi D(S(t + s))] ds \]

\[ = -\psi D(S(t))(1 - e^{-\rho(T-t)}) + \psi D(S(t))(1 - e^{-\rho(T-t)}) \]

\[ = 0. \]

Thus, for \( \psi = 1 \) we have that \( f_1'(t) \geq 0 \) and \( f_2'(t) > 0 \) and, therefore, \( f'(t) \geq 0 \). \( \square \)

**Lemma 2** Let \( g(t) = -(1 - \chi) \tilde{\lambda}(t) - c \). Suppose \( \psi = 1 \). Then \( g'(t) \geq 0 \). Alternatively, suppose \( \psi < 1 \). Then \( g'(t) > 0 \).

**Proof of Lemma 2.** From (21), the Lemma states that, ignoring the option to alternatively use renewable energy, the marginal social cost of using fossil energy, relative to the social cost of using fossil energy with CCS, is increasing in time. Moreover, absent the option to use renewable energy, once \( g(t) \geq 0 \) we have that \( e_s(t) = \psi E \) for all remaining periods.

Suppose \( \psi < 1 \). Then \( -\tilde{\lambda}(t) \) (the social cost of carbon) is strictly increasing in \( t \) for \( t < T \) since emissions are irreversible and always positive. Thus, for \( \psi < 1 \) we have that \( g'(t) > 0 \).

Alternatively, suppose \( \psi = 1 \). Then \( -\tilde{\lambda}(t) \) is strictly increasing in \( t \) unless abatement in all future periods is 100 percent, in which case it is constant. Thus, for \( \psi = 1 \) we have that \( g'(t) \geq 0 \). \( \square \)

26
Lemma 3 Let \( r(t) = f(t) - g(t) \), with \( f(t) \) and \( g(t) \) defined as in Lemmas 1 and 2. Suppose \( T = \infty \), then \( r'(t) > 0 \). Suppose \( T < \infty \), then \( r'(t) \) can be either positive or negative.

For convenience, define current value versions of the time dependent Lagrange multipliers as follows:

\[
\hat{\theta}_1(t) = e^{-\rho t} \theta_1(t); \quad \hat{\theta}_2(t) = e^{-\rho t} \theta_2(t); \quad \hat{\theta}_3(t) = e^{-\rho t} \theta_3(t).
\]

Maintaining the definitions of \( f \) and \( g \) from Lemmas 1 and 2, the first-order conditions for \( e_r(t) \) and \( e_c(t) \) can be written

\[
f(t) = \hat{\theta}_3(t) - \hat{\theta}_1(t)
\]

and

\[
g(t) = \hat{\theta}_3(t) - \hat{\theta}_2(t).
\]

In this case, \( f(t) < 0 \) implies \( e_r(t) = 0 \), while \( g(t) < 0 \) implies \( e_c(t) = 0 \). In contrast, either \( f(t) > 0 \) or \( g(t) > 0 \) imply \( e_r(t) + e_c(t) = \psi E \)—though which technology will be used depends on the relative magnitude of \( f \) and \( g \).

If \( f(t) > g(t) > 0 \), we must have that \( e_c(t) = 0 \). To see this, suppose not. Then \( e_c(t) > 0 \) implies

\[
\hat{\theta}_3(t) = g(t) < f(t).
\]

But \( f(t) - \hat{\theta}_3(t) = -\hat{\theta}_1(t) > 0 \) is a contradiction. The first equality follows from the first-order condition for \( e_r(t) \). Similarly, \( g(t) > f(t) > 0 \) implies \( e_r(t) = 0 \).

Thus, \( r(t) = f(t) - g(t) \) represents the marginal social cost of renewables, relative to the social cost of fossil energy with CCS. \( r(t) \) can be interpreted as the “relative case for renewables”. Importantly, if \( f(t) > 0 \) and \( r(t) > 0 \) then \( e_r(t) = \psi E \) is optimal. In contrast, if \( g(t) > 0 \) and \( r(t) < 0 \) then \( e_c(t) = \psi E \) is optimal.

Substituting from above,

\[
r(t) = \left[ g(F(t)) + \phi_2 \int_0^{T-t} e^{-\rho s}(\psi E - e_r(t + s))ds + \chi \tilde{\lambda}(t) \right] + \left[ e^{-\rho(T-t)} e^{\rho T} \tilde{\mu}(T) \right] + (c - b).
\]

(25)

The first bracketed term is increasing in \( t \)—the first two terms comprise \( f_1(t) \) from Lemma 1, and the last term was already noted to be increasing in \( t \). In contrast, the second bracketed term is decreasing in \( t \). This implies competing effects that push in opposite directions over time.
Suppose $T < \infty$. Then, these competing effects always operate. Therefore, $r(t)$ can be either increasing or decreasing. Alternatively, suppose $T = \infty$. In this case, $\lim_{T \to \infty} e^{-\rho(T-t)} \hat{\mu}(T) = 0$ (since $D'$ is bounded by assumption). Therefore, $r(t)$ is increasing in time. □

7.2 Proofs of Main Results

Proof of Proposition 1. Suppose $T < \infty$. Then by the definition of $T$, it must be that for some sufficiently small $\epsilon > 0$, the optimal policy is to use either fossil or fossil plus CCS for the interval $[T - \epsilon, T]$. It is also possible that a combination of the two is optimal, but we will rule out this possibility later. For either policy to be optimal, it must have lower total social cost than any feasible alternative, including a policy that switches immediately to renewable abatement and uses renewables for all time.

First consider the policy that uses fossil energy for the indicated time interval, then optimally switches to the renewable backstop at $T$. Given this policy, let $[\hat{S}(t)]_{t=T-\epsilon}$ be the corresponding path of carbon and $[\hat{F}(t)]_{t=T-\epsilon}$ the corresponding path of fossil fuel. The total social cost for the policy starting in $T - \epsilon$ is

$$TSC_f = \int_0^\epsilon \exp(-\rho s) g(F(0) - \hat{F}(T - \epsilon + s)) ds + \exp(-\rho \epsilon) \int_0^\infty \exp(-\rho s) b ds + \int_0^\infty \exp(-\rho s) \psi D(\hat{S}(T - \epsilon + s)) ds. \quad (26)$$

By similar arguments, a policy that uses renewables for all time starting in $T - \epsilon$ has total social cost

$$TSC_r = \int_0^\epsilon \exp(-\rho s) b ds + \exp(-\rho \epsilon) \frac{b}{\rho} + \int_0^\infty \exp(-\rho s) \psi D(S(T - \epsilon)) ds. \quad (27)$$

Combining terms,

$$TSC_f - TSC_r = \left[ \int_0^\epsilon \exp(-\rho s) g(F(0) - \hat{F}(T - \epsilon + s)) ds - \int_0^\epsilon \exp(-\rho s) b ds \right]
+ \left[ \int_0^\infty \exp(-\rho s) \psi D(\hat{S}(T - \epsilon + s)) ds - \int_0^\infty \exp(-\rho s) \psi D(S(T - \epsilon)) ds \right].$$

Because $g(\cdot)$ is linear with slope $\phi_1$ and because $g(F(0)) = b$, the first term in brackets is easy to bound. In particular, we know that for all $s$ the discount factor $\exp(-\rho s) \leq 1$. Thus, if we fix this at 1, we get an upper bound on the magnitude of the value in brackets. In fact, since the value is negative, it gives a lower bound on the value. With the discount factor
fixed at 1, the resulting integral is simply the area of a triangle with base $\epsilon$ and height $\phi_1 \epsilon$. We thus have that the value of the first bracketed term is greater than or equal to

$$-\frac{1}{2} \phi_1 \epsilon^2.$$ 

Next, consider the second bracketed term. Since $\epsilon$ is arbitrarily small, the second term can be simplified by taking a first order Taylor Series approximation around the damage function when $S = S(T - \epsilon)$. In particular, we use the fact that for all $t > T - \epsilon$,

$$D(S'(t)) \approx D(S(T - \epsilon)) + D(S'(T - \epsilon)) \epsilon.$$ 

Making use of both facts, the difference to be computed can be bounded as follows:

$$TSC_f - TSC_r \geq \epsilon \left( -\frac{1}{2} \phi_1 \epsilon^2 + \frac{D'(S(T - \epsilon))}{\rho} \right).$$

For sufficiently small $\epsilon > 0$, this must be positive, which contradicts the conjecture that the proposed fossil policy is optimal.

Next, consider a policy that uses fossil plus CCS for the indicated time interval, then optimally switches to the renewable backstop at $T$. The comparison to a renewable only policy is very similar to that above. The only difference is that CCS has added marginal production cost $c$ over the interval $[T - \epsilon, T]$ and in addition climate damages are multiplied by the leakage rate, $\chi$. Thus, by similar arguments, the difference in total social is

$$TSC_c - TSC_r \geq \epsilon \left( -\frac{1}{2} \phi_1 \epsilon^2 + c + \frac{\chi \psi D'(S(T - \epsilon))}{\rho} \right).$$

For sufficiently small $\epsilon > 0$, this must be positive, which contradicts the conjecture that the proposed fossil plus CCS policy is optimal.

Since each fossil-based policy, on its own, is more expensive than the renewable only alternative, any combination of the two would also be more expensive. The contradiction implies that the initial conjecture that $T < \infty$ must be wrong. It follows that $T = \infty$ for the case in which $\psi = 1$. □

**Proof of Proposition 2.** Suppose that $\psi < 1$. Further, suppose that $e_r(t) = \psi E$ for all $t$. Note that these assumptions imply a lower bound on the path of fossil fuel extraction. In particular, given $F(0)$ that satisfies Eq.(1), the stock of fossil fuel evolves according to the following law of motion

$$\dot{F}(t) = -(1 - \psi) E$$
This implies that $T$ satisfies

$$T \leq \frac{F(0)}{(1 - \psi)E} < \infty$$

**Proof of Proposition 3.** By Lemma 2, $c \leq (1 - \chi)\psi D'(F(0))$ implies that $g(0) > 0$. Therefore $e_c(t) + e_r(t) = \psi E$ for all $t \geq 0$. Suppose for a contradiction that $e_c(t) = 0$ and $e_r(t) = \psi E$ for all $t \in [0, T]$. Using Lemma 3, we have that

$$r(t) = g(F(t)) + \phi_2 \int_0^{T-t} e^{-\rho s}(0) ds + \chi \tilde{\lambda}(t) - e^{-\rho(T-t)}\tilde{\mu}(T) + (c - b)$$

for all $t$. Using Eq.(1) and (13) and taking limits as time reaches the terminal period $T$, we have that

$$\lim_{t \to T} r(t) = c - (1 - \chi)\tilde{\mu}(T)$$

$$= c - (1 - \chi)\frac{\psi D'(F(0))}{\rho},$$

which is negative by assumption of the proposition. Therefore, using Lemma 3, we have that $\lim_{t \to T} e_c(t) > 0$, which is a contradiction. Hence, it must be that $e_c(t) > 0$ for at least some $t$. □