Abstract. Substantial evidence suggests that candidates may benefit from being listed early on voting ballots. I study such ballot order effects both theoretically and empirically. I consider three different voting models. First, I construct a pure rational model with strategic and sincere voters, where strategic voters may use ballot order to coordinate. This gives rise to ballot order effects despite order-independent preferences. Second, I derive a pure boundedly-rational voting model comprising sincere and donkey voters (i.e., sincere voters with order-dependent preferences), in which ballot order effects are caused by donkey voters alone. Third, I develop a general, non-standard model that consists of all three types of voters. Here, ballot order effects stem directly from both donkey voting and strategic voting and also indirectly from the added strategic response to donkey voters. I derive tests to evaluate all three models.

Turning to an empirical analysis, I provide empirical evidence to refute the rational and the boundedly-rational models individually, whereas the peculiar testable implications of the richer combined model are supported empirically. Using the richer model, I can directly estimate the share of donkey voters while accounting for the possible interplay between strategic and donkey voting. I estimate the share of donkey voters to be at least 1.71% in local Californian elections. I find that ballot order effects due to strategic voting double total ballot order effects (i.e., vote share percentage advantage). Preliminary analyses suggest that higher education levels and higher median age of the population lower the share of donkey voters, but increase the size of strategic ballot order effects. (JEL D03, D72)

1. Introduction

Elections are held to aggregate individual preferences and select the best possible outcome. However, it has long been observed that seemingly innocuous factors, like the order in which candidates are listed on voting ballots, affect voters. A quote from a 1940 U.S. court case reads, “It is a commonly known and accepted fact that in an election [...] those whose names appear at the head of the list have a distinct advantage.”¹ All 50 US states address such ballot order effects (BOE) in their assignment of candidates’ ballot positions, 30 states directly implement some

¹Elliot v. Secretary of State, 294 N.W. 171, 173 (Mich. 1940), as cited in [14].
randomization of ballot order to minimize these (undesirable) influences [14]. Such order effects are not confined to elections alone. For example, [9] argues that the alphabetical position of the first letter of a company’s name influences its trading activity in stock markets.

In the present paper, I study the sources of ballot order effects and quantify the relative impacts. I focus particularly on the role of strategic voters in creating ballot order effects. Strategic voters, sometimes called tactical voters, vote while taking into account the probability of casting a decisive vote in the election. They differ from other types of voters whose choices are governed solely by their preferences, so-called sincere voters. The prior literature on ballot order effects mostly focuses on sincere voters who prefer candidates listed early on the ballot, which I will call donkey voters, as a cause for ballot order effects. By contrast, I posit two potential channels of how strategic voters may create ballot order effects despite their order-independent preferences. I then derive and test various implications of this theory.

I start by constructing a general voting model with the three different voter types (sincere, strategic, and donkey) and illustrate their interaction in causing ballot order effects. I consider versions of the general model that differ in what types of voters create ballot order effects and test these models for their consistency with the observed empirical patterns of ballot order effects:

1. First, I consider a standard voting model which consists of strategic and sincere voters only, similar to [10, 17]. Ballot order effects in this model may be due to strategic voters using ballot order to facilitate coordination among multiple equilibria, which is a general phenomenon in voting games.²

2. Next, I consider voting models with donkey voters, but in which strategic voters do not react strategically to ballot order. Ballot order effects can therefore only come from the order-dependent rationales of donkey voters. To test these models empirically, I specify certain boundedly-rational motivations for donkey voting: Utility bump and Tie-breaking. In a nutshell, donkey voters may simply prefer the first candidate due to a “utility bump”, for example through a lack of knowledge about or interest in the election. Tie-breaking donkey voters use ballot order to break ties in their preferences, favoring earlier-listed candidates. Lastly, Satisficing assumes that voters incur a search cost when deciding on a vote, leading to a bias for candidates listed early on the ballot.

3. Finally, I develop a general model, in which ballot order effects are caused by both strategic voting and utility bump donkey voting, but also from an added strategic response to donkey voting. Strategic voters may perceive the first candidate to be more likely to win due to donkey voters and thereby change their voting behavior in response.

I show that there exists a class of elections, which we call type A elections, where strategic voters always vote for their most-preferred candidate. In particular, in two-candidate elections they vote sincerely, but more generally in type A elections with more than vote per voter, they have weaker incentives to vote strategically as they have more votes and thereby less need to abandon candidates. Define type B elections as elections that are not type A. Using this, I am able to show that one can empirically separate the above models along the following two dimensions:

²Sincere voters do not create ballot order effects with order-independent preferences.
³This class of elections is defined by elections where the number of votes per voter is equal to the number of candidates minus one.
(1) Do ballot order effects exist in type A elections with only two candidates?

(2) For $k \in \{A, B\}$, let $f_k$ express ballot order effects as a function of the first candidate’s “performance” in elections of type $k$. What is the shape of $f_B$ compared to $f_A$? That is, how do ballot order effects change when strategic voters have more incentive to vote strategically, and how do they depend on the winning chances of a candidate (i.e., performance)?

The pure rational voting model cannot generate ballot order effects in two-candidate elections, as strategic voters always vote for their most-preferred candidates in these elections. However, the data on dimension 1 show that ballot order effects indeed exist in type A elections with two candidates. This rejects the pure rational model, while simultaneously necessitating donkey voters to explain ballot order effects.

The data on dimension 2 show that the difference $f_B - f_A$ is inversely U-shaped, which is a prediction of the general voting model with sufficiently many strategic voters. To see this, observe that a defining characteristic of strategic voting is that candidates who are perceived to have low chances of winning, or are considered sure winners in elections with multiple winners, are abandoned to not have wasted votes on them. However, as there are relatively fewer votes in type B elections than in type A elections (by definition after fixing the number of candidates), this weakens the need to abandon such candidates, which creates an inverse U-shape of $f_B - f_A$, as it predicts such an inverse U-shape only for $f_B$, but relatively flatter $f_A$. Note that this prediction comes from an intuitive property of strategic voting rather than specific model properties.

To illustrate the intuition of the utility bump and the tie-breaking model, fix the number of candidates to three, $m = 3$. Now consider first an election with two votes per voter, $w = 2$. A utility bump then benefits candidates if they were the third-most preferred candidate without utility bump and the second most preferred candidate with the utility bump. This is because they would get no vote in the first case, but a vote in the latter one. However, if the number of voters per voter $w$ decreases, candidates will have to tie for the most-preferred candidate instead. Taking to the data, it shows that candidates performing well enjoy larger advantages than worse-performing candidates in type B elections, but in type A elections, ballot order effects do not change with performance, which refutes a tie-breaking and a special case of the utility bump donkey voting model. In a coming draft, I address a more general utility bump model.

Motivated by such evidence for the general model, I then use it to estimate the share of donkey voters, which is, as reasoned above, identical to the size of ballot order effects in two-candidate elections. Subsequently, I estimate strategic ballot order effects off of the difference in the shape of ballot order effects as a function of performance, utilizing the peculiar inverse U-shapedness of strategic ballot order effects.

The (publicly available) data that I use to empirically implement the theoretical results above include Californian local elections for city council, school board offices,

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4Formally, performance is going to be defined as the percentiles of the vote share percentage of a candidate compared to all other candidates in the same ballot position across a given election type.

5This qualification is necessary, as the general model includes both a donkey voting component and a strategic voting component. It may be that donkey voters create such a strong U-shape of ballot order effects to outweigh any inverse U-shape of strategic voting.
and county offices. The data consist of 10,073 elections from 1995 to 2012, which differ by their number of candidates running for office and potential winners. The unique characteristic of these elections is that the ballot order of the candidates is determined by a random alphabet by which candidates are listed according to their names. A new random alphabet is drawn after the entrance of candidates into the elections approximately 80 days before the election day. For example, if the new alphabet were ACB..., candidate Charlie would be listed before Brown. This randomly determined ordering is then identical on all ballots in a given election.

Using this data set and the general model, I estimate the share of donkey voters to be at least 1.71%. In elections with one voter per voter, I estimate that strategic voters constitute around 75% of the total first-listed candidate advantage (around 2% vote share percentage). These findings illustrate the potential empirical importance of strategic ballot order effects.

I run preliminary estimations, which I will include in a later draft of this paper, of interactions of demographics with the share of donkey voters and find that education at both the high school level and university level decreases the share of donkey voters. The effect of a high school education seems to be relatively stronger than a college education, which could be because high schools are usually local, whereas many students attend colleges away from home and thereby lose connection to their local communities. Moreover, higher age decreases the share of donkey voters, while simultaneously increasing strategic ballot order effects. This suggests stronger local community ties in older voters, which raises interest in local politics and enlarges the knowledge necessary to vote strategically.

This paper’s main contribution are:

- to combine two strands of the voting literature, namely strategic voting and ballot order effects,
- to identify a new sources of ballot order effects arising from strategic voters,
- to help explain peculiar empirical patterns of ballot order effects,
- to provide evidence for the empirical significance of strategic voting for ballot order effects,
- to estimate the share of donkey voters while accounting for interactions with strategic voting,
- and to provide a rational framework to study order effects in other settings, e.g. financial markets.

Section 2 reviews related literature, section 3 introduces the general voting model. I provide examples and define ballot order effects in section 4. Section 5 derives theoretical results, section 6 introduces the data. Section 7 tests the individual voting models and section 8 estimates the share of donkey voters and strategic ballot order effects. Section 9 concludes. The appendix collects proofs and additional discussions.

2. Literature Review

This study contributes, in particular, to two strands of the literature.

There is a range of both theoretical and empirical studies on strategic voting. Theoretical strategic voting papers include [15, 17, 1, 16]. My theoretical model is closest to [17], which derives voting equilibria as the set of solutions in which

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6The number of votes allowed for each voter equals the number of possible winners in the data.
voters believe that only the top two contenders have a reasonable chance of winning. Contrarily, [16, 15] specify a complete structural model of voting games, in which the uncertainty about the election outcome comes from uncertainty about the number of voters in the election. This allows to solve the whole game, which turns out to have similar properties as the model I derive from [17].

Empirical studies aim to show the existence of strategic voting and compute the extent of strategic voting. [7] uses a regression discontinuity in the vote system assignment in Brazil to show that third place candidates are more often deserted in plurality elections than in runoff-elections, as hypothesized by strategic voting models. [3] considers Japanese elections, in which voters have one vote, but candidates run for more than one seat simultaneously. He derives testable implications of the equilibrium results for these multi-winner elections, namely that trailing and leading candidates will be deserted, and finds that the data indeed supports this strategic voting prediction. [10] estimate the share of strategic voters in Japanese General Elections and find a high share of voters to be strategic (around three fourths). They use variation of preferences and election outcomes both within and across counties, which provides the necessary identification of strategic voting in their paper.

[5] considers the possibility of identifying non-sincere voting in individual-level data, which requires multiple observations of voters. With regard to my model, I observe a given voter only once in my data (or at least I can not assign them to more than one election). I can still identify non-sincere voting if voting exhibits a correlation between vote share percentages and ballot order, which cannot be explained by sincere voting alone.

The other strand of literature to which my study is related concerns ballot order effects. These studies investigate the empirical extent of ballot order effects and the psychological motivation behind it. My study uses data from [12], which shows the existence of ballot order effects in Californian local elections and tests implications by the most prominent theory to explain ballot order effects, Satisficing. They show, using similar testable implications of the Satisficing model as mine, that it fails to explain some characteristics in the data, concluding that Satisficing can’t alone explain the empirical pattern - a result that my estimations provide as well. [8] examine show Californian state elections and find that ballot order effects are not significant in general elections, but are highly influential in primaries, changing the winner of the election in around 12% of primary elections. They consider a cognitive model similar to Satisficing. [11] studies elections in Ohio and finds ballot order effects averaging around 2.5%.

An experimental paper related to mine is [6], which allows for the possibility that strategic voter use ballot order as a coordination device, but concluded that the relationship is not strong enough to affect election outcomes in an experimental setting. One important difference, however, is the existence of polls in their study, which are missing in the local elections considered herein.

3. Model

I start by introducing a voting model and define sincere voting, donkey voting, and strategic voting. I then define the notion of a voting equilibrium.

7At least not in sincere elections. See section 4 for a more detailed discussion.
3.1. **Voter types.** I will refer to an election as having an *election type* \((w,m)\) if it consists of \(m\) candidates and \(w\) possible winners, where \(m > w \geq 1\). Note that \(w > 1\) is possible, for example in school board elections, when two seats on the board are up for taking. Voters can cast up to \(w \geq 0\) votes (including 0, i.e. abstaining), but no more than one vote for any candidate. The \(w\) candidates with the most number of votes win the election, while ties are broken with equal probability. All \(N\) voters vote simultaneously without prior communication.\(^9\) I further assume that voters incur no voting cost, which implies that turnout is exogenous. I discuss this assumption later in an empirical context.

There are two ingredients that govern a voter’s behavior: their utility \(u\) for candidates and their voter type \(\theta\). Note that I frequently refer to the set of \((u, \theta)\) as a type of a voter. I assume that these two are independently drawn from their respective distributions. The type of a voter \((u, \theta)\) is private information, but the distributions of types are publicly known. A given election is then a random draw of \(N\) voters from the pool of potential types.

A utility function \(u_i : C \to \mathbb{R}^m\) of a voter \(i\) is defined on the set of candidates and abstention \(C = \{c_1, \ldots, c_m, 0\}\), where 0 denotes abstention. Let \(\mathcal{U}\) denote the set of all utility vectors, and let \(g(u)\) denote the fraction of voters with utility \(u\).

I distinguish between three types of voters \(\theta\): sincere voters \(\theta = s\), donkey voters \(\theta = d\), and strategic voters \(\theta = t\). They differ in how their preferences are translated into votes and how ballot order affects them. I refer to the distribution of such voter types as \(\lambda = (\lambda_s, \lambda_t, \lambda_d)\), i.e., \(\text{Prob}(\theta = i) = \lambda_i\).

A sincere voter \(i\) simply votes to maximize his order-independent utility function \(u_i\).

In election types \((w, m)\), sincere voters choose a \(w\)-subset of candidates \(C_w \subseteq C\) that maximizes

\[
\sum_{c \in C_w} u_i(c),
\]

where \(\#C_w \leq w\), i.e. sincere voters may abstain.

To reflect that some voters give an advantage to candidates listed early on the ballot, I define *donkey voters* to be sincere voters with an order-dependent utility function \(u^y_i : C \times Y \to \mathbb{R}^m\), where \(y : \{1, \ldots, m\} \to C\) is an ordering in the set of all orderings \(Y\).

Assume order-dependence only with respect to the identity of the first-listed candidate:

\[
u^y_i = u_i + f(y),\]

where \(f(y) = f > 0\) if \(y = 1\), \(f(y) = 0\) otherwise.

However, when I test individual donkey voting models later on, I also consider other theories of donkey voting such as Tie-breaking\(^{10}\).

A strategic voter \(i\) votes to maximize an expected utility function that combines their order-independent utility \(u_i\) for candidates with beliefs about how likely their
vote affects the election outcome. They may affect the election outcome only if their vote either breaks or generates a tie between several candidates for the last winning spot in the election.

Denote by \( p_i(c_k, c_j) \) the perceived probability of the voter \( i \) that his vote ties or decides the election in favor of \( c_k \) over \( c_j \), or vice versa.\(^{11}\) This pivot probability \( p_i(c_k, c_j) \) is computed by the individual voter using the knowledge about the distribution of voter types and their strategies in equilibrium, while the random draws of the \( N \) voters prevent certainty.

The literature mostly deals with pivotal voting in single-vote elections, i.e., \( w = 1 \). In those, strategic voters choose their vote \( v_1 \) to maximize the compound utility \( \tilde{u}_i(c_k|p) \) that combines utilities for candidates with the pivot probability of casting the decisive vote for a candidate, summing across all candidates \( k \)

\[
\tilde{u}_i(c_k|p) = \sum_{c_j \in C} p_i(c_k, c_j) [u_i(c_k) - u_i(c_j)].
\]

This generalizes to elections with more than one vote \( w > 1 \), in which strategic voters vote for a \( w \)-subset of candidates \( C_w \subseteq C \) that maximizes

\[
\tilde{u}_i(C_w|p) = \sum_{c_k \in C_w} \sum_{c_j} p_i(c_k, c_j) [u_i(c_k) - u_i(c_j)],
\]

where \( \#C_w \leq w \) denotes the set of candidates for which strategic voters vote. This encompasses strategic abstention if the number of candidates voted for is less than \( w \).

3.2. Voting equilibrium. My model extends the model from [17] to elections with multiple winners and voters per voter and augments it with donkey voters. It assumes that voters perceive the ratios of pivot probabilities \( p_i(c_j, c_k)/p_i(c_l, c_n) \) sufficiently large that utility considerations will only play a role in determining the preferred candidate among the candidate pairs with sufficiently high pivot probabilities. While this may seem restrictive, it is intended to capture the intuitive idea that voters are unlikely to go through combined pivot probability and utility computations, but rather use the expected ranking to determine which candidates have chances to win and choose among them. Indeed, most results derived here depend on typical strategic voting characteristics such as the abandoning of weak and overly strong candidates rather than idiosyncrasies of the model. This suggests that the qualitative results are robust to the choice of the specific voting model.

I do not

Now, before defining the notion of an equilibrium, I want to provide conditions that are intended to reflect such intuitive properties of voting behavior.

First some definitions. Consider a voting game \((C, U, g, \lambda, y, w, m, S)\) with notations as above, i.e., \( C \) the set of candidates, \( U \) the set of utility types, \( g \) the distribution of utilities, \( \lambda \) the distribution of types, \( y \) the ordering, \( w \) the number of voters per voter and possible winners, \( m \) the number of candidates, and \( S \) the space of uncertainty.

Let \( a \in \{0, 1\}^m = A \) be an action such that \( \sum_m a_j \leq w \), where \( a_j = 1 \) if the voter gives a vote for candidate \( c_j \), \( a_j = 0 \) if not. Consider a voter of type \((u, \theta)\). A strategy is then an action \( a(u, \theta) \in A \) and the vector of pivot probabilities \( p_i(u, \theta) \) captures the perceived probability of deciding the election.

\(^{11}\)This assumes symmetric pivot probabilities, which is not necessarily true.
Let $t_j(s)$ be the vote share percentage of candidate $j$ in equilibrium given the random draw of utilities $g(u,s)$ and voter type $\lambda(\theta, s)$:

$$t_j(s) = \frac{\int_{u,\theta} a_j^i(u, \theta) g(u, s) \lambda(\theta, s) d(u, \theta)}{\sum_{k=1}^{m} \int_{u,\theta} a_j^i(u, \theta) g(u, s) \lambda(\theta, s) d(u, \theta) \cdot w}.$$  

Note that this induces an election to be a distribution over election outcomes $t_j(s)$, one for each such random draw $s$. Therefore, let the expected vote share percentage be $\tau(j) = E_S[t_j(s)]$, taken over the probability distribution of uncertainty. Note that the upcoming consistency conditions between pivot probabilities and the equilibrium outcome are defined with respect to the expected vote share percentage $\tau$.

Define $r : C \times \tau \rightarrow \{1, \ldots, m\}$ to be the (expected) vote share rank of a candidate (i.e. the ranking of candidates in terms of vote share percentages) given the expected election outcome $\tau$. If $r(c_j)$ is less than $r(c_k)$ it means that $c_j$ has a higher expected vote share percentage than $c_k$.

Define for any candidate $c_k$ the competing candidate $opp(c_k)$ by

$$opp(c_k) = c_j \text{ s.t. } \left\{ \begin{array}{ll}
    r(c_j) = w, & \text{if } r(c_k) > w, \\
    r(c_j) = w + 1, & \text{if } r(c_k) \leq w.
\end{array} \right.$$  

This means that if a candidate is ranked below the $w$-th position (i.e., loses), his competing candidate will be the candidate in the $w$-th position (last winning candidate). Contrarily, if a candidate is placed among the top $w$-positions (i.e., wins), his competing candidate will be the candidate ranking in the $w+1$-th position (first losing candidate). The competing candidate will become an important ingredient for a strategic voter’s choice in that the competing candidate is perceived to be most likely to overtake FORMULATE

The figure 3.2 presents an example of competing candidates in an election of type $(w=2, m=5)$, i.e., two voters per voter and winning candidates and five candidates.

The first condition states that voters perceive the the probability of a tie between any two candidates to be non-zero. This condition later implies that strategic voters will not vote for their least-preferred candidate in elections with only one vote. It

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$^{12}$Note that vote share percentages may exceed one if sufficiently many voters do not give out all votes.
strategic voting and ballot order effects

posits that there exists some uncertainty in the election and that it is not pre-decided.

Uncertainty condition:
There exist \( c_j, c_k \in C \) such that \( p(c_j, c_k) > 0 \).

The next assumption states that voters perceive only ties between two candidates at most as possible.

Assumption No Higher-order Ties
Assume that \( p_i(c_k, C) = 0 \) for any non-singleton set \( C \subseteq C \) (three-way, four-way ties, etc.).

I now present two conditions that further restrict the beliefs of strategic voters in equilibrium. The first condition restricts comparisons of pivot probabilities for a given candidate.

M-pivot probability condition:
For all candidates \( c_j \in C \), it holds that
\[
p(c_j, \text{opp}(c_j)) \geq M \cdot p(c_j, c_k), \quad \forall c_k \neq \text{opp}(c_j).
\]

This condition states that voters regard the probability of a candidate to tie with his competing candidate for the last winning spot to be at least \( M \)-times larger than the probability of him tying with any other candidate for the last winning spot. It becomes more restrictive the larger \( M \) is. For \( M \) large enough, the differences in pivot probabilities are larger than any differences in utilities. The importance of this is that strategic voters then only vote for a candidate \( c_j \) if they prefer him over their respective competing candidate \( \text{opp}(c_j) \). This can be thought of as a limiting case when pivot probabilities converge to zero (e.g. through an increase in the number of voters) and large pivot probabilities converge more slowly. I provide an example of this choice process later on.

Different models in the literature satisfy this assumption for different values of \( M \). [17] considers arbitrarily large \( M \), similar to [15] if the number of voters is large enough. [10] assume a condition with \( M = 1 \).

Next, I assume that voters can compare pivot probabilities of candidates on the “same side”, i.e. if both candidates are winning or losing. Specifically, the candidate who is closer in vote share percentages to their competing candidate is perceived to more likely tie for the last winning spot than the candidate who is further away in vote share percentage terms.

M-sorting condition:
If two candidates \( c_j, c_k \) are both losing, i.e., \( r(c_j), r(c_k) > w \), then
\[
r(c_j) < r(c_k) \Rightarrow p(c_j, \text{opp}(c_j)) \geq M \cdot p(c_k, \text{opp}(c_k)).
\]

Similarly, for two winning candidates \( c_j, c_k, r(c_j), r(c_k) \leq w \), then
\[
r(c_j) > r(c_k) \Rightarrow p(c_j, \text{opp}(c_j)) \geq M \cdot p(c_k, \text{opp}(c_k)).
\]

Because in elections with only one vote per voter, strategic voters only vote for the top two candidates, only the previous M-pivot probability condition is needed for single-vote elections. However, in elections with multiple votes per voter, strategic

\[\text{This is a common assumption in the literature [10]}\]

\[\text{A sufficient threshold would be } M \geq \left( \left( \binom{m}{2} - 1 \right) \cdot 2 \cdot \max_{i,j}|u_i(c_j)| \right).\]
voters may vote for candidates other than the two candidates around the winning threshold. The M-sorting condition therefore provides the required structure on how pivot probabilities compare beyond the first vote. I generally assume that one value of $M$ holds for both conditions.

First define a general equilibrium notion. The set $(a,p)$ is a symmetric Bayesian Nash pure-strategy equilibrium if it satisfies for each voter type $(u,\theta)$

- Voters choose $a_i$ to maximize their expected utility given their pivot probabilities $p_i$.
- $p_i(c_k,c_j) = p(c_k,c_j)$, i.e., all voters have identical pivot probabilities.

We are now ready to define a voting equilibrium.

**Voting equilibrium:**

A set of strategies and pivot probabilities $(a,p)$ is a voting equilibrium if $(a,p)$ is a symmetric Bayesian Nash pure-strategy equilibrium and it satisfies the no higher-order condition, the uncertainty condition, the M-pivot probability condition, and the M-sorting condition for all sufficiently large $M$.

The existence of such voting equilibria is proven in the appendix.

The figure 3.2 illustrates a voting equilibrium. Voters compute pivot probabilities as a response to their uncertainty about what other types of voters participate in the election. They use these pivot probabilities to select their equilibrium actions. Note that this equilibrium action is non-stochastic and independent of the particular random draw $s$ of voters (as voters do not know the outcome of this draw, they cannot possibly condition their decision on it). These actions induce a distribution over possible election outcomes and therefore give rise to a unique expected election outcome. A voting equilibrium requires that this expected election outcome be related with pivot probabilities via the consistency conditions defined above.

I now present a simple numerical example of such a voting equilibrium. Let there be an election with three candidates, A, B, and C, and one possible winning candidate. Candidates are ordered alphabetically on the ballot, i.e., candidate A is listed first. There shall be 4 voters, each one can give out one vote to any candidate. Let voter 1 and 2 be always sincere and voter 3 always strategic. The last voter, voter 4, may be sincere with 50% probability and a donkey voter with 50% probability. Voter 1, 2, and 4 shall have identical utility functions when sincere, preferring $B \succ A \succ C$. However, if voter 4 is a donkey voter, he will vote for candidate A. The strategic voter’s preferences are $C \succ A \succ B$.

The voting equilibrium is now as follows. The sincere voters always votes for candidate B, the donkey voter may or may not. If candidate B has three votes, the strategic voter’s vote will not decide the election, but it may do so when the third voter is a donkey voter, as his vote will then go to candidate A. If this is the case, the strategic voter may then generate a tie for his preferred candidate, A. Therefore, his pivot probability is $p(A,B) = 0.5$ and the pivot probabilities for the other candidate pairs are zero. Using the utility functions from above, he maximizes $p(A,B)(u(A) - u(B))$, so he votes for candidate A.

The equilibrium outcomes are then with 50% probability $(A = 2, B = 2, C = 0)$ and with 50% probability $(A = 1, B = 3, C = 0)$. The expected election outcome is $(A = 1.5, B = 2.5, C = 0)$, so that the requirements on pivot probabilities are $p(A,B) \geq M \cdot p(A,C)$, $p(A,C) \geq M \cdot p(B,C)$, $p(A,B,C) = 0$ and $p(A,B) \geq 0$ with
**Figure 3.2.** Graphical representation of an equilibrium

\[ M \] sufficiently large. These are satisfied if the voter believes to have rational pivot probabilities because \( p(A, B) = 0.5 \) and \( p(A, C) = p(B, C) = p(A, B, C) = 0 \).

4. **Sources of ballot order effects**

In this section, I formally define ballot order effects, outline potential sources and give examples.

Assume that we observe the election outcome for each possible ordering \( y \) of an election, i.e., the identical election just with different ballot orderings. This allows us to compute the average vote share percentage of a ballot position \( k \) across all ballot orderings, which we denote by \( E(\tau_k) \).

Define a vote share percentage advantage \( b_k \) of position \( k \) to be the difference between the expected vote share percentage \( E(\tau_k) \) of a candidate in position \( k \) from the average expected vote share percentages of all other positions, formally\(^{15}\)

\[
b_k = E(\tau_k) - \frac{1}{m-1} \sum_{i \neq k} E(\tau_i).
\]

Note that without any ballot order effects in general, it holds that \( E(\tau_k) = \frac{w}{m} \) for all \( k \).

When I talk about ballot order effects, I usually refer to the vote share percentage advantage of the first position, \( b_1 \). I will say that ballot order effects exist if the first ballot position enjoys an advantage, \( b_1 > 0 \).

4.1. **Sources of ballot order effects: Donkey voters.** Donkey voters directly create ballot order effects. Here, I consider two theories of donkey voting, Satisficing and Tie-breaking, and I review other theories in the appendix. Therein, I also provide a more formal treatment of Satisficing and Tie-breaking. The reader be

\(^{15}\)An alternative, equivalent definition would be to define \( b_k \) as \( b_k = (E(\tau_k) - \frac{w}{m}) \frac{m}{m-1} \).

reminded that donkey voters are sincere voters, so their order dependence in voting is created through order-dependent preferences.

In a utility bump model, donkey voters may simply prefer the first candidate due to a “utility bump”, for example through a lack of knowledge about or interest in the election.

In a tie-breaking model, donkey voters may use ballot order to break ties in their preferences for voters. They then choose the earlier-listed candidate when in doubt for whom to vote, which would imply preference for earlier-listed candidates, in particular for the first-listed candidate.

The most commonly used theory to explain ballot order effects is called Satisficing, [12]. It states that voters incur a (psychological) search cost if they consider an additional candidate on the ballot. This leads to an optimal reservation utility strategy, in which voters search until the expected marginal benefit from searching no longer exceeds marginal costs. I call this reservation utility the aspiration level of the voter. This generates an advantage for candidates listed first on the ballot, as they will always be considered, which can only increase the chances of receiving a vote.

4.2. Sources of ballot order effects: Strategic voters. I now present two ways of how strategic voters may react to ballot order despite order-independent utilities. First, candidates listed first may benefit if strategic voters use ballot order to coordinate; second, an initial advantage due to donkey voters may be further reinforced by strategic voting.\(^\text{16}\)

4.2.1. Coordination using ballot order. Strategic voters are faced with a fundamental coordination problem in elections, which generally results in multiple equilibria. This induces a need for coordination for strategic voters, who may use ballot order for this purpose. To illustrate how such coordination may then benefit the candidates early on the ballot, I assume that strategic voters vote for candidates listed first on the ballot when indifferent between several candidates (and one of them is listed first).

**Coordination assumption:**

Strategic voters vote for the first-listed candidate when deciding on a vote if they are i) indifferent between two or more candidates, ii) one of which is listed first, and iii) the first candidate is not the least-preferred candidate.

As an example, consider an election with four candidates and one possible winner, \((w = 1, m = 4)\).

Let there be 5 voters: 2 sincere and 3 strategic voters. The 4 candidates are (A,B,C,D) and are ordered alphabetically on the ballot \(y=(A,B,C,D)\). Let the preferences of the strategic voter also be alphabetic

\[ A \sim B \succ C \succ D, \]

\(^{16}\) Notice that these examples are constructed to be simple and to transmit the intuition behind the channels of strategic ballot order effects. However, the examples do not exhibit any uncertainty and are, therefore, slightly different from the model introduced above.
where strategic voters are indifferent between candidates A and B. Let the vote distribution of the sincere voters be

\((A = 0, B = 0, C = 2, D = 0)\).

Table 4.1 lists the set of pure-strategy equilibria, where I only show one equilibrium of each kind (eq. 4-7 show one of three possibilities each). For simplicity, assume that every equilibrium is selected with equal probability.

<table>
<thead>
<tr>
<th>Equilibria</th>
<th>Eq 1</th>
<th>Eq 2</th>
<th>Eq 3</th>
<th>Eq 4</th>
<th>Eq 5</th>
<th>Eq 6</th>
<th>Eq 7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voter 1's (\sigma_1)</td>
<td>(1,0,0,0)</td>
<td>(0,1,0,0)</td>
<td>(0,0,1,0)</td>
<td>(0,0,1,0)</td>
<td>(0,0,0,1)</td>
<td>(0,1,0,0)</td>
<td>(1,0,0,0)</td>
</tr>
<tr>
<td>Voter 2's (\sigma_2)</td>
<td>(1,0,0,0)</td>
<td>(0,1,0,0)</td>
<td>(0,0,1,0)</td>
<td>(0,0,0,1)</td>
<td>(0,0,0,1)</td>
<td>(0,0,1,0)</td>
<td>(0,0,1,0)</td>
</tr>
<tr>
<td>Voter 3's (\sigma_3)</td>
<td>(1,0,0,0)</td>
<td>(0,1,0,0)</td>
<td>(0,0,1,0)</td>
<td>(0,0,1,0)</td>
<td>(0,0,1,0)</td>
<td>(0,0,1,0)</td>
<td>(0,0,1,0)</td>
</tr>
</tbody>
</table>

**Figure 4.1.** Pure-strategy equilibria where \(\sigma_1=(1,0,0,0)\) denotes a vote profile, where candidate 1 gets voter 1’s vote.

Equilibria 1 and 2 are equilibria in which the candidates successfully coordinate on candidates A or B, while equilibria 3 to 7 exhibit failed coordination in that candidate C is selected as the sole winner. However, notice that in equilibria 3 to 7, at least one voter is indifferent between voting for candidates A or another candidate, but votes for one of the other candidates. The coordination assumption therefore rules out equilibria 3 to 7 and leaves only equilibria 1 or 2.

The average vote share percentage for the first candidate given ordering \(y\), candidate A, under the coordination assumption is then \(E(\tau_1|y) = \frac{3}{10}\). Changing the order, one gets \(E(\tau_1|y(B) = 1) = \frac{3}{10}\) (eq. 1 & 2), \(E(\tau_1|y(C) = 1) = \frac{3}{10}\) (eq. 1, 2 & 3), and \(E(\tau_1|y(D) = 1) = \frac{3}{25}\) (eq. 1-7). The average vote share percentage of the first candidate is therefore \(E(\tau_1) = 0.33\). With an expected vote share percentage of \(\frac{3}{10} = \frac{1}{4}\), ballot order effects \(b_1\) are then \(b_1 = (0.33 - 0.25)\frac{\frac{1}{4}}{\frac{3}{25}} = \frac{8}{75}\) (using the alternative and equivalent definition of ballot order effects mentioned in footnote 13).

4.2.2. Strategic response to ballot order effects. Strategic voters may change their voting behavior in response to the increase in winning chances of the first candidate due to donkey voting. The example is simply a presentation of how donkey voting can have multiplicative effects when voters are strategic.

Consider an election with one voter per voter and four candidates \((w = 1, m = 4)\) and 5 voters: 2 sincere voters, 1 strategic voter, and 2 voters for whom I will consider different scenarios. The 4 candidates are (A,B,C,D) and are ordered alphabetically on the ballot \(y=(A,B,C,D)\).

Let the preferences of the strategic voter also be alphabetic

\(A \sim B \succ C \succ D\).

In the first scenario, let the 2 voters be sincere. Thus all non-strategic voters are sincere. Let the vote distribution of the sincere voters be such that

\((A = 0, B = 0, C = 2, D = 2)\)
(they vote independently of the ordering). The strategic voter would then vote for candidate C, as he prefers $C > D$, and this ensures that C wins instead of D. Candidate C is not listed first.

In the next scenario, if the 2 voters vote for the first-listed candidate (if they were donkey voters, for example), the vote distribution of the non-strategic votes is then

$$(A = 2, B = 0, C = 0, D = 2).$$

The strategic voter might now vote for candidate A and therefore changes his vote towards the first-listed candidate as a reaction to donkey voting. The computation of ballot order effects can be done similarly to above.

5. Theoretical results

In this section, I derive results of the general voting model that allow me to identify strategic ballot order effects. I use these results to derive testable implications of the pure rational voting model and to contrast the general model with the pure donkey voting models later on. These results also provide empirical strategies to estimate the share of strategic ballot order effects and donkey voters in the general model. Note that the following results do not precondition on the behavior of donkey voters nor sincere voters, but are rather statements solely about strategic voters\(^{17}\).

Proofs and detailed descriptions can be found in the appendix.

I first define three groups of elections which differ by what incentives they provide for strategic voters to react to ballot order. I show how strategic ballot order effects differ between these groups and define their empirical counterparts, which allows the empirical identification of strategic ballot order effects. Note that these differ from the ones in the introduction, as I split type A elections into group 1 and group 2 elections.

The first group shall include elections in which strategic voters always vote sincerely. Examples for this group are elections with only two candidates.

To define the second group, recall that strategic voters vote for a candidate if they i) prefer this candidate over their competing candidate and ii) consider the probability to cast a decisive vote for this candidate to be large enough (relative to the probabilities for other candidates). The second group shall then include all elections in which strategic voters vote according to i), but independently of ii). I will show shortly that examples would be elections in which voters have as many votes as there are candidates but one, $w = m - 1$. There, they can vote for all candidates with a potential benefit, even for the least-likely candidate to tie for a winning spot.

Formally, as noted before, strategic voters maximize the expected sum consisting of pivot probabilities $p(c_j, c_k)$ that a vote decides the election between any candidate pair $(c_j, c_k)$ and the consequential payoff of candidate $c_j$ winning instead of $c_k$, $(u_{c_j} - u_{c_k})$, i.e., the votes $v^i$ of any strategic voter $i$ are such that

$$c_k \in v^i \iff c_k \in \arg \max_{c_k \in C_w} \sum_{c_k \in C_w} p_i(c_k, \text{opp}(c_k)) \left[u_{c_k} - u_{\text{opp}(c_k)}\right].$$

\(^{17}\)They do require that there is a sufficiently large share of strategic voters, which indirectly restricts the shares of sincere voters and donkey voters.
An election belongs to the second group if the choice of any strategic voter $i$ can be reduced to
\[ c_k \in v^i \iff p(c_k, \text{opp}(c_k))[u_{c_k} - u_{\text{opp}(c_k)}] \geq 0. \]
Note that this voting behavior is not sincere voting: a sincere voter is oblivious of pivot probabilities, while strategic voters in the second group use pivot probabilities to determine competing candidates.

Finally, the third group shall include all remaining elections that do not fall into any of the other groups. Examples here are elections, in which strategic voters may not vote candidates that they believe to have very small chances of winning, e.g., an election with three candidates but only one vote per voter ($w = 1, m = 3$).

The first proposition states that all elections with two candidates belong to group 1, as they exhibit no strategic voting at all. In our model above, I assume that some pivot probability in any election must be non-negative. This implies that voting for the less preferred candidate in a two-candidate election is strictly dominated.

**Proposition 1.** Elections with two candidates belong to group 1.

Proposition 1 helps us evaluate the pure strategic voting model as it directly provides the testable implication that there are no ballot order effects in elections with only two candidates.

The next theorem addresses elections in which voters have $m - 1$ votes, where they can vote for all but one candidate.

**Proposition 2.** Elections of type $(w, m)$ with $w = m - 1$ belong to group 2.

Due to the limited amount of strategic voting in elections of group 2, strategic ballot order effects may be smaller in elections of group 2 than group 3. The next remark provides some justification for this claim. In group 2, every voter will always vote for their most preferred candidate regardless of pivot probability. This is not necessarily true for elections of other types.

**Remark.** In elections of group 2, strategic voters always vote for their most-preferred candidate.

The next proposition strengthens this claim by showing that with sufficiently many strategic voters and no correlation between the utilities for candidates\(^{18}\), there will be a unique equilibrium outcome $\tau$ in elections with $w = m - 1$. This limits the effect of strategic coordination using ballot order in these elections, reducing the potential for strategic ballot order effects.

**Proposition 3.** With sufficiently many strategic voters and if the utility of voters for different candidates is uncorrelated, there will be a unique equilibrium vote share percentage $\tau$ for any voting equilibrium in elections with $m = w + 1$.

The next proposition formalizes this claim by stating that strategic ballot order effects in group 3 are indeed stronger than in group 2 if voters perceive the magnitude of ballot order effects in group 2 elections to be similar to or less than the magnitude of ballot order effects in group 3 elections and if the number of close elections between the groups is comparable.

\(^{18}\)The assumption of uncorrelated individual utilities implies that there always exists at least one Condorcet loser (two or more candidates may tie for last place).
Proposition 4. Strategic ballot order effects imply a larger difference in vote shares in elections of group 3 than in group 2, everything else equal.

Proposition 4 is similar to findings in other studies that strategic voting is more prevalent in elections with higher magnitudes (ratio of candidates \( m \) to seats \( w \))[4].

So far, the propositions related to how strategic ballot order effects vary across election types, but we can say more. An important characteristics of strategic voters is that they vote less often for candidates for which they believe to have little chances of casting a decisive vote. Due to the M-sorting condition for voting equilibria, pivot probabilities are lower for candidates with very low vote share percentages, and very high vote share percentages in multi-winner elections (\( w > 1 \)). This suggests that strategic voters abandon these extreme candidates, which may translate into “inversely U-shaped” strategic ballot order effects as a function of vote share percentages. However, as our identification strategy also makes use of the previous propositions, we need to make vote share percentages comparable across different election types\(^1\). To do this, we instead state how strategic ballot order effects vary as a function of the percentile of a candidate’s vote share percentage within an election type.

The next theorem states this formally, before some assumptions:

1. Candidate entry is independent in an election.
2. Every equilibrium has equal probability.\(^2\)
3. The share of strategic voters is sufficiently high to ensure a positive correlation between vote share percentile and mean utility, see \(7\).
4. The share of donkey votes is either uncorrelated with quality or increasing or inversely U-shaped with respect to quality.

**Theorem 5.** Given assumptions 1 through 4, strategic ballot order effects as a function of vote share percentiles an inverse U-shape.

Figure ?? provides a numerical example of how the shape of strategic ballot order effects may compare between single-vote elections and multi-vote elections. It shows that ballot order effects, for some parameters to create ballot order effects roughly comparable to what we see in the data, may indeed be more inversely U-shaped as a function of vote share rank (vote share rank and percentiles are interchangeable in the example) in group 3 elections relative to group 2 elections.

6. Data

I use data from the *California Elections Data Archive* (CEDA), a publicly available collection of local Californian elections outcomes. These elections determine county supervisors, city council members, etc., in California’s 58 counties and more than 1,100 school and community college districts.\(^3\) I supplement this data set with registration data at the block level from the *Statewide Database* for a subset of the data set and with demographic data from the census.

\(^1\) An election result of 50% is average in an election with two candidates, but above average with three candidates.
\(^2\) In effect, we only need a weaker version of this assumption.
\(^3\) The data used here include replication data prepared by [12]. I merge the two data sets, but drop all elections that are only in their data set and not mine (or that cannot be merged due to missing/inaccurate data).
The cleaned CEDA data include a total of 10,073 elections from the years 1995 to 2012. They include elections with different combinations of number of candidates \( m \) and possible winners and vote per voter \( w \). Elections are non-partisan, meaning that candidates are not allowed to state party affiliation. These local elections are usually held together with general and primary elections in each even year. General elections are held in November, primaries in June, and special local elections in the beginning of the year. In odd years, local elections are usually held at around the same dates, but without accompanying general and primary elections.

Roughly half of the elections are school board elections (5,843), a third are city council elections (3,377), and the remaining elections (853) are other for county or city offices such as mayor, CSD/CSA director, etc.

I exclude elections for which I do not have the ordering of candidates (4,102),\(^{22}\) incomplete election data (46), for which the number of potential winners does not coincide with the number of votes per voter (228) or if the number of votes per voter is equal or higher than the number of candidates (188). I exclude runoff elections as they provide potentially different incentives for voters (1,034)\(^{23}\), as well as elections that cross county borders (249).

In order to double-check incumbency status of candidates, as it is separately collected in the data set, I reconstruct incumbency status based upon the last

\(^{22}\)The numbers refer to the amount of elections dropped after the previous step was executed.

\(^{23}\)My data set does not explicitly denote which elections are runoff elections, but rather indicates in the first round whether a candidate proceeded to the second round or potentially could have. This allows me to easily distinguish the first rounds of runoff elections from plurality elections. The second round of runoff elections can then be deduced by comparing the set of candidates in two-candidate elections in the next election cycle to elections of the previous election cycle.
election cycle’s outcome. Sporadic online-searches show that this new variable may improve accuracy\textsuperscript{24}, so I use it in my baseline estimations.

A special characteristic of the data is that after the deadline to enter as a candidate, the California Secretary of State office randomizes the ordering of candidates on the ballot. It does so by randomly drawing a new alphabet by which candidates are ordered on all ballots in an election. As an example, if a new alphabet would be drawn to be A C B instead of A B C, “Charlie” would be listed before “Brown”. This random determination ensures that ballot order is uncorrelated with any characteristic of the candidates.

The data initially do not include the ordering of candidates, which I infer from publicly available random alphabets. My data for ballot ordering for the years 1995 to 2008 comes from \cite{12} where applicable and I collect additional alphabets for the years 2008-2012.\textsuperscript{25} I assign ballot ordering manually according to these publicized alphabets. \cite{12} show that manual assignment of ballot order by the econometrician matches the actual ballot order 97\% of the time (for the San Bernardino County), where the discrepancies stem from confusion about what constitutes the last name of the candidate.

I assume turnout to be exogenous in my theoretical model. I justify this with that in each election, there are multiple electoral races on each ballot, which decreases the influence of any single election on whether voters turn out. I argue that turnout is then orthogonal to the outcome of any specific election, with the remaining differences in turnout captured by time or county fixed effects\textsuperscript{26}.

My voting model requires that differences in pivot probabilities exceed any differences in utilities. I did not introduce an underlying rationale for the uncertainty present in the model, but one important determinant is the number of voters. With an increasingly large number of voters, the relative differences in pivot probabilities become sufficiently large to outweigh differences in utilities. This begs the question for what number of voters the above model is (approximately) true. \cite{2} tests a pivotal voter model and presents evidence against it for elections with fewer than 900 voters. In light of this, I run my baseline regressions for elections with at least 900 voters, which excludes 9\% of the elections (912). Robustness checks suggest that alternative restrictions provide qualitatively similar results\textsuperscript{27}.

Below I present a few summary statistics, table 6.1. I then illustrate in table 6.2 a few comparative statistics of ballot order effects when sorting elections by number of candidates, number votes per voter or magnitudes, the ratio between number of candidates and number of winners.

7. Testing individual voting models

In this section, I empirically test the individual models, the pure-donkey and pure-rational voting model, in which only one type is responsible for ballot order effects, and then comment on how the general model predicts the observed patterns

\textsuperscript{24}The new incumbency status differs from the one in the data set in 116 elections. Note that it is not possible to construct this variable for all elections.

\textsuperscript{25}My assignment coincides for all but one election with \cite{12} for the overlapping year 2008.

\textsuperscript{26}County or time fixed effects will actually turn out to have rather negligible impacts on ballot order effects.

\textsuperscript{27}I provide a table with restrictions to fewer number of voters in the appendix, still TBD.
### Variable Obs Mean Std. Dev. Min Max

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. Dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>Candidates</td>
<td>43,444</td>
<td>5.58</td>
<td>3.42</td>
<td>2</td>
<td>54</td>
</tr>
<tr>
<td>No. of Votes/Winners</td>
<td>43,444</td>
<td>2.346</td>
<td>1.015</td>
<td>1</td>
<td>15</td>
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<tr>
<td>Vote Share Percentage</td>
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<td>0.468</td>
<td>0.215</td>
<td>0.00</td>
<td>1.59</td>
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<tr>
<td>County</td>
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<td>30.11</td>
<td>14.98</td>
<td>1</td>
<td>58</td>
</tr>
<tr>
<td>Total Votes</td>
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<td>29,765</td>
<td>65,635</td>
<td>14</td>
<td>872,335</td>
</tr>
<tr>
<td>Votes</td>
<td>43,444</td>
<td>5,819</td>
<td>15,841</td>
<td>2</td>
<td>613,376</td>
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<tr>
<td>Incumbent</td>
<td>43,444</td>
<td>0.321</td>
<td>0.467</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

#### Figure 6.1. Summary statistics

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
</tr>
</thead>
<tbody>
<tr>
<td>First</td>
<td>3.53***</td>
<td>4.43***</td>
<td>2.99***</td>
<td>8.31***</td>
<td>3.70</td>
<td>-4.58**</td>
<td>7.35***</td>
<td>9.34***</td>
<td>9.73***</td>
</tr>
<tr>
<td></td>
<td>(0.396)</td>
<td>(0.558)</td>
<td>(0.913)</td>
<td>(1.84)</td>
<td>(3.13)</td>
<td>(1.76)</td>
<td>(0.520)</td>
<td>(0.371)</td>
<td>(0.372)</td>
</tr>
<tr>
<td>Scenario</td>
<td>28</td>
<td>29</td>
<td>30</td>
<td>31</td>
<td>32</td>
<td>33</td>
<td>34</td>
<td>35</td>
<td>36</td>
</tr>
<tr>
<td>Observations</td>
<td>10,343</td>
<td>7,262</td>
<td>2,934</td>
<td>862</td>
<td>279</td>
<td>136</td>
<td>6,814</td>
<td>15,285</td>
<td>16,484</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.009</td>
<td>0.010</td>
<td>0.004</td>
<td>0.031</td>
<td>0.006</td>
<td>0.011</td>
<td>0.037</td>
<td>0.033</td>
<td>0.028</td>
</tr>
</tbody>
</table>

Robust standard errors in parentheses

*** p < 0.01, ** p < 0.05, * p < 0.1

#### Figure 6.2. Ballot order effects by magnitude (\( \frac{m}{w} \)) and number of winners (w).

of ballot order effects. I derive properties of these models and evaluate them along three dimensions of the data:

First, the data reveal ballot order effects in two-candidate elections. Second, the first candidate enjoys an advantage over second-listed candidates, disproportionally more than the second candidate’s advantage over the third candidate. Third, vote share advantages of the first candidate are more “inversely U-shaped” as a function of the first candidate’s performance in elections with higher magnitudes (higher ratio of candidates to votes per voter/number of winning candidates).

#### 7.1. Pure rational voting model.

I first consider the pure rational voting model, a voting model without donkey voters. Proposition 1 states that in elections with two candidates there is no strategic voting and, consequently, no strategic ballot order effects. But in a pure rational model, ballot order effects must be strategic, which implies proposition 6.

**Proposition 6.** The pure rational voting model implies no ballot order effects in elections with two candidates.

To check this testable implication, I run a regression of vote share percentages \( \tau_{ij} \) on an indicator of whether candidate \( i \) in election \( j \) is listed first \( F_{ij} \), restricting my sample to two-candidate elections.\(^{28}\) I include several control variables \( d_{ij} \) and run the regression with different restrictions on the total number of votes.\(^{29}\)

---

\(^{28}\) This regression is equivalent to estimating ballot order effects.

\(^{29}\) I will soon include turnout data, I am in the process of collecting it.
The regression equation is then
\[ \tau_{ij} = \alpha + \beta F_{ij} + \gamma d_{ij} + \varepsilon. \]

I cluster standard errors by election.

As can be seen in table 7.1, I still find significant ballot order effects in two candidate elections. The first candidate enjoys a 2.09% (.639) advantage in a simple regression with no controls, which decreases to 1.71% (.562) in the main regression specification (4) where I exclude elections with fewer than 900 votes and include all controls. The existence of such ballot order effects provides evidence against the pure rational voting model without donkey voters, the canonical voting model.

![Figure 7.1. Ballot order effect in two candidate elections.](image-url)

### 7.2. Pure donkey voting models and general model

I now evaluate the general model against two pure donkey voting models, models with sophisticated donkey voters, but no strategic ballot order effects. I consider tie-breaking donkey voters and sincere voters with a “utility bump” for the first candidate. I derive some implications of these models that allow testing them empirically.

Note that a rejection of an unrestricted donkey voting model is ad-hoc not possible, as this would require a rejection of any kind of irrationality. Nevertheless, I interpret my findings as suggestive in that ballot order effects are created (to an extent) in a strategic way. I treat tie-breaking and utility bump models similarly, as they imply similar testable predictions. However, to do this, I need to assume that the utility bump for candidates is sufficiently small.

**Assumption Identical utility bump:**

The utility bump \( f \) is sufficiently small.

Donkey voting models are mute on the relationship between candidates’ performance and ballot order effects, but there may be a correlation between the **quality of a candidate** (e.g., mean utility of voters for a candidate) and their performance, which may cause possible indirect correlation. The next statement addresses this

---

\footnote{The distinction between strategic and sincere voters, and donkey voters, is artificially done in my model. I label strategic voters as those who vote in a certain way, but it is possible that those voters are not strategic, but are observationally equivalent to strategic voters. The question then is rather whether I am willing to do the same with donkey voters who vote “strategically.”}
in that with sufficiently many strategic voters, such correlation between the quality of a candidate and their performance is generally expected to be positive.

Let a voter’s $i$ utility for a candidate $j$ be $u_i^j = U_j + \varepsilon_i^j$, where $U_j$ denotes the mean utility of voters for candidate $j$ and $\varepsilon_i^j \sim F$ denotes the individual utility shock, which is distributed according to $F$.

**Proposition 7.** Given a ballot position. Let individual utilities for candidates be independent of each other $\varepsilon_i^j \perp \varepsilon_i^k$ and the distribution of utility shocks be independent of candidate’s mean utility $F \perp U$. Let every equilibrium be played with equal probability and candidate entry into the election be independent of each other. With sufficiently many strategic voters, it holds that expected vote share percentage $E\tau_j$ and mean utility $U_j$ are positively related, i.e., $E\tau_j > E\tau_k \Leftrightarrow U_j > U_k$.

Next, assume that the distribution of candidate quality across voters is symmetric, i.e., $F$ is symmetric. Further assume that distribution of candidate entry with respect to their mean utility $U$, which I denote by $F_E$ is also symmetric, i.e., as many high-utility candidates enter as low-utility candidates.

Tie-breaking donkey voters use ballot order to break ties in their preferences. In an election with one vote per voter, $w = 1$, donkey voters only then use ballot order if they are indifferent between their two most-preferred candidates. For multi-vote elections $w > 1$, the indifference to be broken needs to be between the $w$-th and $w+1$-th most-preferred candidate. This suggests that most donkey votes in a single-vote elections are in favor of first-listed high-quality candidates, as these are the ones who are more likely to tie for most-preferred candidate. Increasing the number of candidates, this “threshold” for tying for first place becomes higher, thereby moving the mass of donkey votes to the “right” (if performance is increasing on the x-axis). Contrarily, when increasing the number of voters per voter, this would move the mass of donkey voters to the “left”.

The next proposition states this formally, requiring assumptions about identical and symmetric distributions of candidates across election types:

**Proposition 8.** In a Tie-breaking model, given the assumptions for proposition 7 and if $F$ and $F_E$ are symmetric and identical across election types, ballot order effects for low-performing candidates (lower 50% percentiles) in elections with $(m = w + 1, w \geq 2)$ should be identical to ballot order effects for high-performing candidates (upper 50% percentiles) in elections with $(m, 1)$, and vice versa.

The general model produces ballot order effects as both a response to donkey voting and coordination by strategic voters. As shown before, there exist significant differences between elections of group 2 and group 3 in the general model, which therefore allows for significantly different shapes between these elections.

**7.2.1. Empirical implementation.** When comparing elections with different number of candidates or winners, vote share percentages are not directly comparable. A 50% vote share might be an average result with 2 candidates, but a good result for an election with 3 candidates. To account for this, I construct a measure of vote share percentages that ranks an election against elections of the same type (i.e., same $m$ and $w$), similar to sorting elections into percentiles. To do so, I separate candidates into $m$ groups for each ballot position. I then assign an aggregate rank to each candidate, determined by the relative size of this candidate’s vote share percentage relative to all other vote share percentages of the same election type.
For example, if I had data on three elections of type \((w, m)\), and the vote share percentages for the first candidates would be \((.3; 2.8)\) in the three elections, the relative ranks would be \((2, 3, 1)\).

I now want to compare the size of ballot order effects between the lower and the upper half of these aggregate ranks across elections with \((m = 3, w = 2)\) and \((m = 3, w = 1)\). Following proposition 8, a pure donkey voting model with tie-breaking or utility bump model with sufficiently low bump predicts no difference.

I run a regression of vote share percentage \(\tau_{ij}\) on an interaction between an indicator of whether a candidate is listed first \(F_{ij}\), the candidate’s aggregate rank is higher than 0.5 (upper half) \(h_{ij}\), and whether the election is a single-vote or multi-vote election \(g_j\). I again include controls \(d_{ij}\) and all lower interaction terms, but omit them for brevity:

\[
\tau_{ij} = \alpha + ... + \beta_3 F_{ij} h_{ij} g_j + \gamma d_{ij} + \varepsilon.
\]

Below I show the estimation result of \(\beta_3\) of this regression in specification (7), which is significantly different from zero and therefore rejects the pure donkey voting model as described above.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>% Vote Share</th>
<th>Vote Share</th>
<th>Vote Share</th>
<th>Vote Share</th>
<th>Vote Share</th>
<th>Vote Share</th>
<th>Vote Share</th>
</tr>
</thead>
<tbody>
<tr>
<td>First %</td>
<td>3.76***</td>
<td>1.49*</td>
<td>-</td>
<td>2.47***</td>
<td>2.12***</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>First h</td>
<td>-</td>
<td>-</td>
<td>2.28**</td>
<td>-</td>
<td>-</td>
<td>-0.35</td>
<td>-</td>
</tr>
<tr>
<td>(\beta_3) First h g</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>2.63**</td>
</tr>
<tr>
<td>Excluded percentiles</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
<td>1%</td>
</tr>
<tr>
<td>Half</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Election type</td>
<td>Upper</td>
<td>Lower</td>
<td>Both</td>
<td>Upper</td>
<td>Lower</td>
<td>Both</td>
<td>Both</td>
</tr>
<tr>
<td>Observations</td>
<td>822</td>
<td>822</td>
<td>1,644</td>
<td>1,662</td>
<td>1,662</td>
<td>3,327</td>
<td>4,971</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.1812</td>
<td>0.0478</td>
<td>0.7374</td>
<td>0.0441</td>
<td>0.1537</td>
<td>0.6681</td>
<td>0.8542</td>
</tr>
</tbody>
</table>

(Bootstrapped percentile 500 repetitions, clustered by election)

*** p<0.01, ** p<0.05, * p<0.1

**Figure 7.2.** Estimation of ballot order effects, top-half versus low-half, single-vote versus multi-vote.

8. Estimation of Donkey Voters and Strategic Effects

The last section provided evidence that neither a pure rational nor a pure donkey voting model is consistent with the empirical patterns of the data, which instead supports the general model. I therefore use the general model to estimate the share of donkey voters and the strategic share of ballot order effects in this section.

I estimate the share of donkey voters in two ways. First, it follows from proposition 5 that any ballot order effects in two candidate elections must be due to donkey voters, which directly provides an estimate of the share of donkey voters. Additionally, I implement a naive regression to estimate the share of donkey voters and the share of strategic effects by using propositions 5 and 4. This regression pools all election types and includes an indicator for whether an election belongs to group 1 or group 2. If the share of donkey voters does not change with the type of election, we can identify it off of the variation in ballot order effects across election types.
To allow for that the share of donkey voters may change by election type, I use a similar regression to before in the test of the pure donkey voting models to estimate the difference in slopes between elections of group 2 and 3. This allows me to compute the share of strategic ballot order effects, from which I can also reconstruct the share of donkey voters.

8.1. Share of donkey voters. Proposition 5 directly implies the following corollary.

**Corollary 9.** In the general model, donkey voters are the only voters to cause ballot order effects in elections with two candidates.

Using the same regression as above and referring back to table ??, I find that the share of donkey voters is 1.71% [0.61,2.81] for elections with at least a 900 votes. The point estimate increases to around 2.8% when restricting the sample to moderately large elections.

8.1.1. Pooled regression. To pool all election types, I need to account for that a given share of donkey voters will have different impacts on the first candidate advantage in different election types. Firstly, a donkey voter voting for the first candidate will take on average $\frac{1}{m-1}$ votes from every other candidate (as the probability that he would have voted for another candidate is $\frac{1}{m-1}$)\(^\text{31}\), but give one more vote to the first candidate. However, if voters have more than one vote, donkey voters may give out $w-1$ votes to other candidates. This implies that giving one vote to the first candidate will lead to a bonus vote of $1 - \frac{w-1}{m-1} = \frac{w}{m-1}$ over the average $\frac{w-1}{m-1}$ votes to other candidates. I include an additional table that allows donkey voters to only give out one vote.

Before running the regression, I assume that the share of donkey voters does not vary by election type. This assumption is identical to when donkey voters always vote for the first candidate on the ballot (i.e., the utility bump is sufficiently large).

**Assumption Fixed share of donkey voters:**
The share of donkey voters does not vary by election type.

I therefore run a regression for all election types of

$$
\tau = \alpha + \mu \frac{m-w}{m-1} F + \text{type2} * F + \text{type3} * F + d,
$$

where, again, $F$ denotes an indicator for whether a candidate is listed first, $\mu$ the share of donkey voters, type2 and type3 indicators for whether an election belongs to type 2 or type 3, and $d$ a set of control variables as described above. I include dummies for each election type and cluster standard errors by election.

Table 8.2 shows that the estimated share of donkey voters reduces to around 1.3% [0.26,2.34], whereas type 2 and type 3 strategic voting have substantial impacts on the first candidate advantage, with each constituting around 77% or 72% respectively of total ballot order effects. An estimation with only one vote per donkey voter better fits the share of donkey voters estimated before, and still estimates a roughly doubling effect from strategic ballot order effects in group 3 elections, where the difference between candidates and voters per voter is larger than 2 (46% of total ballot order effects). While relying on strong assumptions that the share

\(^{31}\)It is possible that donkey voters may vote only for the first candidate, which then reduces to simply estimating the first-listed candidate advantage without mechanical adjustments.
of donkey voters does not change with the number of candidates \( m \) or number of voters per voter \( w \), this regression nevertheless suggests that strategic voting has significant impacts on ballot order effects. Below, I estimate a the share of strategic ballot order effects with weaker assumptions for elections with three or four candidates.

**Discussion of results.** Most of the literature on ballot order effects is concerned with estimating the size of the first-listed candidate advantage or with investigating the...
causes of these effects [11, 13, 8, 12]. The only study known to me that considers ballot order effects due to voters with order-independent preferences is [6]. They investigate the coordination motive of strategic voters, but they do not find any empirical significance of the coordination motive in their experiment. However, they provide polls to candidates, which allow easier coordination and reduce the need for other coordination devices, such as ballot order.

To the best of my knowledge, my study is the first to estimate the share of donkey voters while accounting for strategic voting. The estimate of 1.71% (or 1.3%) is comparable to findings of ballot order effects in the literature. The estimates usually differ by whether the data include local elections, in which ballots usually contain more than one election, or more salient elections like presidential elections.

The existence of such donkey voters who simply prefer early-listed candidates calls for ballot-by-ballot randomization in elections to mitigate the disturbing effects.

8.1.2. Share of first-listed candidate advantage due to strategic voting, take 2. To weaken the above assumption and instead consider an alternative one, I follow a different approach.

Theorem 5 states that strategic ballot order effects in the general model are more inversely U-shaped as a function of the aggregate rank of a candidate in elections of group 3 than in group 2 elections. To account for that donkey voters may bias the shape of ballot order effects, I estimate the shape of ballot order effects as a function of the aggregate rank in elections of type \( (w = 1, m = 3) \) and \( (w = 1, m = 4) \) relative to \( (w = 2, m = 3) \) and \( (w = 3, m = 4) \), of which the former election types belong to group 3, while the latter ones belong to group 2. I attribute the estimated difference in shape to strategic voters, as previous evidence rejected a tie-breaking or utility bump donkey voting theory. However, I need to impose the following assumption:

**Assumption Fixed shaped of donkey ballot order effects:**

The relationship between ballot order effects and performance of a candidate *due to donkey voters* does not vary by election type or is more “U-shaped” from group 3 to group 2 elections.

The table ?? shows the estimation results for an OLS regression where I run a similar regression to before

\[
\tau_{ij} = \alpha + (F_{ij}, ar_{ij}, ar_{ij}^2, g_j) + \beta_1 F_{ij} \ast ar_{ij} \ast g_j + \beta_2 F_{ij} \ast ar_{ij}^2 \ast g_j + d_{ij},
\]

where notation is as before: \( F_{ij} \) is an indicator for when a candidate is listed first, \( ar_{ij} \) and \( ar_{ij}^2 \) stand for the aggregate ranks as defined above, \( g_j \) for an election belonging to group 3, and \( d \) are the control variables.

Table 8.3 presents the results for elections with at least 900 voters. I exclude elections that are located at the extremes to reduce the effect of outliers. Using the estimates of \( \beta_1 \) and \( \beta_2 \), I can then compute ballot order effects by integrating the quadratic function

\[
b = \int_0^1 \beta_1 x + \beta_2 x^2 dx.
\]

This yields the equation \( b = \beta_1 / 2 - \beta_2 / 3 \). However, accounting for the possible existence of strategic BOE in group 2 elections, I use 4 to derive a lower bound of strategic ballot order effects by dividing the estimated BOE by 2.
For three-candidate elections, I find that strategic ballot order effects are around 1.18%* in three-candidate single-vote elections, while they are around 1.40% in four-candidate, single-vote elections. The estimates of total ballot order effects are 2.28%** in three-candidate single-vote elections, which implies that strategic voters increase ballot order effects by 107% in these elections; similarly, for four-candidate single-vote elections, total ballot order effects are 3.08%**, so that strategic voters increase ballot order effects by 83% in these elections.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>β₁</td>
<td>8.25</td>
<td>6.37</td>
<td>3.21</td>
<td>17.45**</td>
<td>19.32**</td>
<td>22.51**</td>
</tr>
<tr>
<td>β₂</td>
<td>-5.29</td>
<td>-3.23</td>
<td>0.75</td>
<td>17.74**</td>
<td>-18.14**</td>
<td>-20.82*</td>
</tr>
<tr>
<td>Strategic BOE</td>
<td>1.18*</td>
<td>1.05</td>
<td>0.93</td>
<td>1.40</td>
<td>1.81*</td>
<td>2.16**</td>
</tr>
<tr>
<td># of candidates</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Excluded percentiles</td>
<td>1%</td>
<td>5%</td>
<td>10%</td>
<td>1%</td>
<td>5%</td>
<td>10%</td>
</tr>
<tr>
<td>Observations</td>
<td>4,971</td>
<td>4,569</td>
<td>4,059</td>
<td>3,664</td>
<td>3,368</td>
<td>2,992</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.991</td>
<td>0.996</td>
<td>0.998</td>
<td>0.987</td>
<td>0.994</td>
<td>0.996</td>
</tr>
</tbody>
</table>

Bootstrapped t-percentile 2,000 repetitions, clustered by election

** p < 0.01, *** p < 0.05, * p < 0.1

Figure 8.3. Estimation of strategic ballot order effects.

Discussion of results. The literature’s general recommendation is to randomize the ballot order by each individual ballot in a given election, as such one can eliminate any bias from a ballot order which is constant throughout an election.

While I support these recommendations, I stress the fact that ballot order randomization is crucial if strategic incentives in elections become important. Whether elections present strong strategic incentives depends on various factors. The political importance of an election (local versus national, presidential versus senate) magnifies the differences in benefits between candidates, thereby raising the payoff to strategic voting. The public awareness of an election influences the precision with which strategic voters can predict an election outcome, which also increases the benefits from strategic voting. The number of candidates generally allows for a finer distribution of candidates across the political spectrum, thus offering more viable candidates for strategic voters to switch to. If an election is predictably close, strategic voters will simply have more reasons to vote misaligned.

Currently, it is often the case that incumbent parties occupy the first place on election ballots, thereby preventing small parties from gaining significant shares.

8.2. Influences on ballot order effects. I want to investigate how various demographic characteristics affect the share of donkey voters and strategic effects on ballot order effects. For this, I will combine my data set about local elections in California with census data from 2000 and 2010, data from the American Community Survey from years that are available (typically 2007 and up) and voter statistics from the California Secretary of State.

However, I have not done this yet, but plan to do so. Preliminary results are available on request for the naive regression.

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32 As mentioned before, even in highly publicized elections donkey voting is still present.
Estimates suggest that educational attainment negatively influences the share of donkey voters, but more so on the high school level rather than on college level (due to a potential weakening of local ties).

Moreover, more absentee voting is associated with fewer donkey voters, also as expected.

9. Concluding Section

TBD

References


