Loss Aversion around a Fixed Reference Point in Highly Experienced Agents*

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Abstract

We study how reference dependence and loss aversion motivate highly experienced agents, professional basketball players. We find a very large “losing motivates” effect, an average team scores like a league leader when trailing by ten points and a bottom dweller leading by ten. Detailed data on players’ actions shows this effect comes through differential exertion of effort. Using betting spreads and lagged score margin, we test if expectations influence the reference point; they do not. The reference point appears remarkably stable around zero, far less malleable than previously found in experimental work studying less experienced agents.

JEL Codes: D03, D84, C9
Keywords: reference dependence, loss aversion, expertise, behavioral economics

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1 Introduction

Employees in every line of work face tradeoffs between the exertion of costly effort and the rewards of better performance. There is considerable evidence that people adjust the effort they exert depending on whether they feel they are currently falling short of or exceeding some internally meaningful standard. This internal standard—referred to as a “reference point”—splits the space of outcomes into “losses” and “gains.” An agent that responds to a reference point is said to have reference dependent preferences, combining this with an increased sensitivity to losses results in loss aversion.

The predictive power of a reference-dependence depends critically on the ability to model how an agent forms and changes her reference point. If it merely affords an additional free parameter in estimation, then it will beat a neoclassical model with uninteresting algebraic certainty. Accordingly, recent economic research, spearheaded by the theoretical work in Koszegi and Rabin (2006), has focused on pinning down reference points. Initial experimental findings indicate reference points are quite malleable. Chinese factory workers, public school teachers, and laboratory subjects all have been shown to adjust their reference point—in these cases over expected earnings—upward in response to re-framing their compensation as being docked for poor performance as opposed to rewarded for good (Hannan et al., 2005; Hossain and List, 2012; Fryer Jr et al., 2012). Relatedly, laboratory subjects immediately lowered their reference point in response to the good performance of an opponent, exhibiting what the authors referred to as a “discouragement effect” (Gill and Prowse, 2012).

But does this malleability extend beyond experimental settings and into situations where agents have substantial experience with their effort/performance tradeoff? It is typically difficult to study reference points in the wild. A series of papers on cab drivers reveals substantial disagreement among prominent researchers (Camerer et al., 1997; Farber, 2008; Crawford and Meng, 2011). Most relevant to our paper, Crawford and Meng (2011) argue that cab drivers’ effort provision is consistent with an income reference point fixed at the expected earnings at the start of the day and a static daily hours target. The disagreement surrounding this conclusion stems from the challenge in estimating the driver’s expected returns to effort (and the serial correlation therein) at any given time of day. And if a reference point cannot be pinned down, then it is difficult to make a reliable inference about loss aversion.

1 Abeler et al. (2011) present what could be interpreted as a related finding. Post et al. (2008) find that reference points adjust quite fluidly for “Deal or No Deal” game show contestants.

2 This has led some researchers to assert that the tight control of a field experiment, despite concerns about external validity, offers more reliable evidence on reference dependence (Goette et al., 2004; Fehr and Goette, 2007).
In this paper, we respond to this challenge with a unique observational environment in which nearly all payoff relevant factors can be precisely measured: the in-game performance of National Basketball Association (NBA) players. Importantly, the study of professional basketball requires few modeling assumptions to identify how reference points adjust over the course of a game. The neoclassical model (as presented in Section 3) has the strong prediction that, controlling for the quality of teams in a match-up, the marginal return (in terms of expected impact on the game outcome) of scoring one point should be a sufficient statistic for a team’s chosen level of effort. Since the game is zero sum, for an even match-up the team trailing faces the exact same marginal returns to scoring a point as their opponent, who is leading by the same margin. This feature is particularly useful to study loss aversion since the neoclassical model will predict symmetry around zero. To study reference point adjustment, we make use of score fluctuation throughout the game. Throughout, we control directly for the quality of players on the court and use the betting line to capture time-varying quality differences.

Static reference points typically make different behavioral predictions than their dynamically adjusting counterparts; our setting is no exception. If within-game fluctuation of the score margin is immediately incorporated into expectations, then we would expect loss aversion to push the currently leading team to play better and the losing team to get discouraged. Conversely, if the reference point is static, for example around zero (a tie game), then we’d expect the losing team to play better and the winning team to play worse.

We find very strong evidence that NBA players are loss averse around a fixed reference point of a score margin equal to zero. Specifically, they play with progressively greater intensity and effectiveness as their team falls further behind. Controlling for the quality of all players on the court, a NBA team that falls 10 points behind will score roughly three points more per 100 possessions. The estimate is robustly demonstrated in all four quarters of the game and with flexible methods of controlling for lineup quality. As a useful benchmark, in the last five seasons only 15 points per 100 possessions separated the absolute best offense from the absolute worst over the entire time frame. This means that an average NBA team plays like a playoff contender when 10 points behind and a bottom of the standings also-ran when 10 points ahead.

One might be concerned that this effect is driven by good teams letting bad teams mount a comeback, only to win the (now closer) game with a late push. This is not the case. The effect is robustly demonstrated to be very significant in subsamples of playoff, currently close and ex-ante evenly matched games. In most specifications, the magnitude is statistically indistinguishable in the playoffs and is only slightly smaller in competitive match-ups. For an interesting counterfactual, suppose a team could trick themselves into
believing the game was tied when they were ahead (so they would now view a close win as a “loss”) but keep the reference point fixed at zero when losing. We estimate they would win 5% more of their regular season games and 10% more of their playoff series. An NBA team seeking a similar improvement would typically have add a player costing $7–10 million per year to the roster, which is typically very difficult given the “luxury tax” penalties most teams would have to pay to make such an addition.

A final concern in interpreting these results is raised in Ariely et al. (2009): better performance in perceptual-motor tasks may not actually signal a heightened desire to win. In their study, subjects actually perform worse on routine tasks when monetary rewards are increased and there is evidence this effect has also been found in professional athletes (Baumeister, 1984; Wallace et al., 2005; Dohmen, 2008). By studying outcomes, we might be making an unreliable inference about effort (the choice variable). To address this concern, we dig deeper into player behavior. The change in offensive output is driven by the losing team’s improved performance in the most effort-intensive aspects of basketball. Losing teams secure more rebounds, turn the ball over less and see their star players work to shoulder an even larger burden of the scoring load than usual. We do not however, observe significant changes in shooting accuracy (conditional on shot location) during game play. Interestingly, players shoot “free throws”—unguarded shots taken after a foul is called—with lower accuracy when trailing. We also observe lower free throw accuracy in the playoffs and when the game is nationally televised, both scenarios in which a player is presumably motivated to try harder. A consistent story thus emerges. When trailing, players perform better on effort-intensive tasks and worse on focus-intensive tasks (the opposite being true for leading teams), but in both cases the evidence supports loss averse preferences.3

Had we only studied free throws we might have been tempted to conclude that reference points adjust quickly and players exhibit disappointment aversion. In a related paper, Pope and Schweitzer (2011) study professional golfers and conclude, based on a lower accuracy on “birdie” putts (a gain) than similarly difficult “par” putts (avoiding a loss), that golfers exhibit loss aversion around the reference point of “par.” A skeptic, however, could argue that the critique in Ariely et al. (2009) has bite; the worse performance on birdie putts is actually evidence of trying harder, but this effort is counterproductive. We guard against this critique by studying outcome metrics which vary from focus- to effort-oriented. We thus view our findings as some of the strongest evidence to date for both loss aversion and

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3 The negative impact of effort on performance for focus tasks has been explained by what psychologists refer to as detrimental “self-focus” (thinking too much about how to perform the task rather than just doing it) (Neiss, 1988; Beilock et al., 2002; Wilson et al., 2007; Gucciardi and Dimmock, 2008). It is sometimes referred to as “performance pressure.”
fixed reference dependence in highly experienced and incentivized agents. Fixed reference points, combined with robust loss aversion, in turn substantially increase the predictive power of reference-dependent models.

2 Background and Preliminaries

An NBA basketball game lasts for forty-eight minutes, with ties being settled by five-minute overtime periods, repeated as necessary. Five players from each team, referred to as the “lineup,” share the court at any given time. Game play is a sequence of “possessions” in which one team has the ball and attempts to score a “basket,” which is worth two or three-points, depending on the distance the shot is taken from. If a player is fouled while attempting to score a basket, he is awarded “free throws,” unguarded 15-foot shots worth one point each, corresponding to the point value of the attempted shot. The large number of possessions and high frequency of scoring (roughly half of all possessions result in some points for the offense) are crucial features of basketball that allow for win probabilities and the implied incentives to be easily modeled—in the Appendix, we discuss exactly how this is done.

The magnitude of an action’s expected impact on the game’s outcome depends heavily on the score margin and time remaining. We’ll use the “win value of a point” (WVP)—the expected (per-point) increase in the probability of winning the game for making a two-point basket in a specified game state—to quantify the importance of any particular moment in the game.\(^4\) Formally, let

\[
\text{WVP} \equiv \frac{\text{PW}(M_p + 2, t) - \text{PW}(M_p, t)}{2},
\]

where \(\text{PW}\) is the win probability function estimated in the Appendix.

In Figure 1, we plot WVP for two evenly matched teams. There is considerable variation in the expected importance of a point throughout the course of a game. Since basketball is zero-sum, point importance is equal for both teams at any given moment; this can be seen graphically in the symmetry of the WVP function with a spine at a lead of zero.\(^5\) However, if the team’s were of different quality, then the spine of the WVP function would be shifted. If, for example, the home team was superior, then WVP would be maximized (for a given amount of time remaining) when the away team had a sufficient

\(^4\)Just as in voting, either all points or pivotal or none are. The expectation effectively captures the probability the game will be decided by a single shot, which is naturally higher when the game is close and not much time is remaining.

\(^5\)Since the win probability surface is non-linear, the leading and trailing team face opposing risk-return tradeoffs. See Goldman and Rao (2014) for more details.
lead to give them a 50% chance to win the game.

3 Theoretical Framework

Most of the outcomes metrics used in this paper are determined by the simultaneous exertion of effort by the home ($e_h$) and away ($e_a$) teams and these outcomes will have different interpretations depending on which team is on offense. As an example, our key performance measure is the average number of points scored by the offense on one possession. For a fixed matchup of players, define the performance mapping for this metric as:

$$\rho(e_o, e_d) \equiv E[Ptsp|e_o, e_d].$$

Throughout, it will be assumed that $\rho$ is concave. If the returns to effort are monotonic for both teams, then $\rho$ is increasing in the first argument and decreasing in the second.

3.1 “Standard” Model of a Win-Maximizing Team

Consider two competing basketball teams with preferences only for winning the game and facing a separable marginal cost of effort normalized to one ($c(e) = e$).\(^6\)

Take an offense and defense competing on possession $p$ where the offense is ahead by

\(^6\)This normalization is without loss of generality because we have not specified the functional form of $\rho$. 

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Figure 1: The marginal impact of a point on win probability, as a function of score margin and time remaining in the game for the first three quarters (left) and only the final quarter (right).
M_p with t minutes remaining in the game. Let PW and WVP be the win probability and win value of a point functions discussed in section 2. For notational convenience, let us assume that all NBA possessions end in zero or two-points. Recall that WVP is defined as half the win probability return of making a two-point shot. Then on a given possession, the offense is seen to behave according to

\[
\max_{e_o} U_{\text{win}_o} \cdot E[\text{PW}(M_{p+1}, t) | M_p, t, e_o, e_d] - e_o = U_{\text{win}_o} \cdot (\text{PW}(M_p, t) + \text{WVP} \cdot \rho(e_o, e_d)) - e_o,
\]

The first order condition is

\[
U_{\text{win}_o} \cdot \text{WVP}(m_p, t_p) \cdot \frac{\partial \rho(e_o, e_d)}{\partial e_o} = 1. \tag{1}
\]

The defense’s problem is exactly parallel and has solution

\[
U_{\text{win}_d} \cdot \text{WVP}(m_p, t_p) \cdot \frac{\partial \rho(e_o, e_d)}{\partial e_d} = 1. \tag{2}
\]

It is straightforward that the offense and defense will both be more motivated on possessions with high \( WVP \), meaning they always scale effort in the same direction. If \( \rho \) is symmetric in offensive and defensive effort, then this model necessarily predicts equal performance across game states. See Figure 2 as an example.

![Figure 2: Predicted of the win-maximizing model with symmetric performance function, \( \rho = 1 + .2(e_o^{1/2} - e_d^{1/2}) \), specified identically for both the home and away team. The two right most panels plot levels of effort selected by each team which are both skewed towards high WVP moments at the end of close games. However, these net out resulting in the flat pattern of offensive efficiency observed in the left panel.

Of course the offense and defense may have different returns to effort. For instance, if

\( ^7 \)This is only done for notational convenience in this section. All results could be easily generalized. See Appendix for further discussion.
the defense had a more efficient technology to reduce scoring probability in high importance moments, then we’d expect scoring rates to drop at the end of close games for both teams. Alternatively, if one team has a higher utility of winning, for instance because they are playing in front of their home crowd, then \( U_{\text{win}_h} > U_{\text{win}_a} \) and this will lead to overall higher levels of effort throughout the game (particularly in high leverage moments) and thus better performance. See Figure 3 for an example.

![Home Offense](image)

Figure 3: Predictions of the win-maximizing model with symmetric performance function, \( \rho = 1 + .2(e^{1/2} - e^{-1/2}) \) and a stronger home preference for victory \( U_{\text{win}_h} = 1.5 \cdot U_{\text{win}_a} \).

Finally, one might imagine that certain teams tend to have greater marginal returns to effort. These teams will tend to differentially outperform their opponents in high WVP moments. If teams with high marginal returns to effort are also better teams, then WVP will generally be higher when these teams are behind. This is because when a good team is trailing, the game is more likely to end close (so that WVP is high) because they are expected to come back. When a bad team is trailing, the likelihood of a comeback is lower and thus WVP is lower as well. If good teams do indeed have higher marginal returns to effort, then a rational win-maximizing basketball team could appear to perform differentially better when behind. However, such a model must predict a larger performance boost for the superior team in high leverage moments. Figure 4 provides an example. As will be shown in our empirical section, such a “clutch” performance boost is not supported in the data.

More generally, refer back to (1) and (2) and notice that the score margin and time...
remaining \((m_p, t_p)\) enter the first order condition for effort only through their impact on WVP. This is the key restriction of the “standard” model—for a given matchup, selected effort, and thus performance, can only be a function of incentives.

The next two subsections discuss extensions to the standard model that incorporate reference-dependent preferences. These models predict different patterns of team motivation and performance across game state, but on their own cannot predict patterns of choking observed on free-throws. Such discussion is reserved for the final subsection.

### 3.2 Reference-Dependent Preferences over Outcomes

We now maintain that teams get utility only from the outcome of the game, but allow that those outcomes are experienced relative to a reference point. It is immediate that loss aversion around a fixed reference point is not a meaningful construct since winning is a binary variable, hating to lose just means loving to win (and visa versa). Formally, it would rescale the value of \(U_{\text{win}}\) but would not effect the pattern of effort exertion across game states. Alternatively, the reference point could be endogenous to current game state and selected level of effort in order to represent the foreword looking probability of a win. Since losses around this reference point would be more painful than gains, both team’s
would be motivated to limit their exposure to variance (relative to the standard model). This would lead to a greater level of effort chosen by the leading team (increasing PW towards 1 reduces variance) and a lesser effort by the trailing team (increasing PW moves it toward 0.5, increasing variance). This is exactly as in the *discouragement effect* found in Gil and Prowse (2012) and we direct an interested reader to the theory section of that paper.

### 3.3 Reference dependent preferences over game state

Now we allow that players are not motivated just by the final outcome of the game, but also get a discounted flow of utility based on the current score margin. The idea is that each team has a reference point in mind for how they expect to play against a given opponent and gain utility from their performance with respect to that benchmark. Let \( v(\cdot) \) denote a standard loss averse utility function \( (x > 0 \implies v'(x) < v'(-x)) \) over the difference between the current score margin and the prescribed reference point. For example,

\[
v(x) = 1\{x \geq 0\}x^{\gamma_A} - \lambda 1\{x < 0\}|x|^{\gamma_B},
\]

where \( \gamma_A \) and \( \gamma_B \) denote separate utility exponents for teams that are *ahead* and *behind* their reference point. The idea is that players get more psychic utility from moving out of a losing state than moving into a winning state. For now, we assume the reference point \((R)\) is fixed at a constant level and does not update in response to game-play. Then utility for the home and away, with \( T \) possessions remaining in the game, is respectively given by

\[
V_{o,p} = E \left( \sum_{t=0}^{T} \beta^t [v(M_{p+t} - R) - e_{o,p+t}] \right)
\]

\[
V_{d,p} = E \left( \sum_{t=0}^{T} \beta^t [v(R - M_{p+t}) - e_{o,p+t}] \right),
\]

each of which can be represented recursively as (for example),

\[
V_{o,p}(M_p) = v(M_p - R) + \beta E[V_{o,p+1}(M_{p+1})].
\]

This recursive representation allows the model to be easily solved for specified parameter values (as is done in Figure 5). This parameterization implies that losing teams will outperform winning teams and that the amount by which they do so will grow with the score margin.

Of all the models considered, this is the only one capable of rationalizing the following
Figure 5: Predicted performance premium of a ref-dependent model over the score margin. $\gamma_A=.8$, $\gamma_B=.9$, $\lambda = 1$, $\beta = .8$. $\rho = 1 + .2(e_t^{1/2} - e_d^{1/2})$ is specified identically for each team.

factors of the data. (1) improved performance by the trailing team by an amount that is roughly independent of the amount of time left in the game (2) a performance premium function that is consistently steepest around a tie game (3) a relatively small performance boost for superior teams in high WVP moments.

3.4 Summary of Predictions

Our “standard” model is quite flexible in that it allows opposing teams to experience any possible relation between effort and performance, but requires that they have preferences only to win the game. As demonstrated, this allows for effort and performance to vary substantially over the course of the game, but only in such a way that WVP is a sufficient statistics for both. Thus, if we see that one team is systematically outplaying another when they are behind, we must conclude that WVP is actually high in these moments and that this team is superior. We must then also expect to see this team have a significant performance boost in other high WVP moments—specifically, the final moments of close games. As discussed in the following section, the losing motivates effect is found quite broadly and we find no evidence of any kind of clutch performance boost for superior teams. Thus, we must look for alternative explanations for team behavior.

We consider two forms of loss averse preferences to help us rationalize the data. Loss aversion over game outcomes, with a reference point that immediately updates and is endogenous to chosen effort, predicts disappointment aversion as in (Gill and Prowse, 2012). This factor, induces both leading and trailing teams should seek to avoid variance in the final outcome. In particular, NBA teams that find themselves behind, should reduce
effort in order to avoid simultaneously raising their reference point and exposing themselves to disappointment. Such a model unambiguously predicts the opposite of the “losing motivates” model we have observed.

Alternatively, loss aversion over the score margin is considerably more flexible. If the reference point is fixed over the course of the game, we have the unambiguous prediction that losing teams will outplay winning teams. If the reference point updates with a lag, then we may also find that lagged levels of a team’s lead may be positive predictors of a teams performance.

4 Main Results

We first estimate the aggregate impact of game state on team performance. We are mindful of two possible confounds. First, good teams are more likely to be ahead. If no attempt is made to control for player or team quality is made, then mechanically, the offensive team’s score margin and scoring efficiency should be positively correlated. Second, losing teams may be more likely to keep their better players in the game. Such behavior is also not rationalized by a standard model and may reflect the loss averse preferences of a team’s coaching staff. We refer to this as the effect of coach loss aversion in order to distinguish it from the effect of player loss aversion which we define as the impact of a change in the score margin on the performance of a fixed set of players.\footnote{Note that insofar as coaches call better plays when trailing, for a fixed set of players, we’ll attribute this to the players.}

4.1 Quantifying loss aversion

Our first specification does not control for lineup composition, it is designed to measure the combined effect of player and coach loss aversion on team performance. We use the closing Las Vegas betting spread for each NBA game as an additional regressor to control for differences in team quality.\footnote{Obtained from covers.com.} The spread is the score margin that a team must win by in order for an even odds bet on them to be declared a winner. Empirical research demonstrates it represents the expected median score differential for that game and is a strong (if not sufficient) control for differential team quality in each game (Paul et al., 2004). Specifically, we estimate:

\[
E[\text{Pts}_p] = \alpha + \delta_1\text{Home}_p + \delta_2\text{Spread}_p + \gamma\text{Playoffs}_p \\
+ \beta_1\text{Lead}_p \times (1 - \text{Playoffs}_p) + \beta_2\text{Lead}_p \times \text{Playoffs}_p, 
\]  

(4)

\footnote{Note that insofar as coaches call better plays when trailing, for a fixed set of players, we’ll attribute this to the players.}

\footnote{Obtained from covers.com.}
where $\beta_1$ and $\beta_2$ represent the gross impact of loss aversion on team performance in the regular season and playoffs respectively. We estimate (4) separately for each quarter of the game and use fixed effects to control for the way each possession originates.\(^\text{10}\)

Table 1: Total impact of lead on offensive performance (Points/100 possessions)

<table>
<thead>
<tr>
<th></th>
<th>1st Quarter</th>
<th>2nd Quarter</th>
<th>3rd Quarter</th>
<th>4th Quarter</th>
<th>Average Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>All Data $\hat{\beta}_1$</td>
<td>-.3205</td>
<td>-.4263</td>
<td>-.3683</td>
<td>-.6930</td>
<td>-.4520</td>
</tr>
<tr>
<td></td>
<td>(.0396)***</td>
<td>(.0255)**</td>
<td>(.0250)**</td>
<td>(.0344)**</td>
<td>(.0159)***</td>
</tr>
<tr>
<td></td>
<td>$\hat{\beta}_2$</td>
<td>-.1378</td>
<td>-.3060</td>
<td>-.1645</td>
<td>-.6447</td>
</tr>
<tr>
<td></td>
<td>(.1826)</td>
<td>(.0960)***</td>
<td>(.1157)</td>
<td>(.1634)***</td>
<td>(.0719)***</td>
</tr>
<tr>
<td>Evenly Matched Teams $\hat{\beta}_1$</td>
<td>-.2481</td>
<td>-.4310</td>
<td>-.3580</td>
<td>-.6435</td>
<td>-.4202</td>
</tr>
<tr>
<td></td>
<td>(.0559)***</td>
<td>(.0327)***</td>
<td>(.0315)***</td>
<td>(.0440)***</td>
<td>(.0211)***</td>
</tr>
<tr>
<td></td>
<td>$\hat{\beta}_2$</td>
<td>-.0965</td>
<td>-.1608</td>
<td>-.0531</td>
<td>-.5699</td>
</tr>
<tr>
<td></td>
<td>(.2655)</td>
<td>(.1190)</td>
<td>(.1440)</td>
<td>(.2079)***</td>
<td>(.0964)***</td>
</tr>
<tr>
<td>Close Games $\hat{\beta}_1$</td>
<td>-.4101</td>
<td>-.4049</td>
<td>-.7426</td>
<td>-.5192</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(.0332)***</td>
<td>(.0385)***</td>
<td>(.0517)***</td>
<td>(.0242)***</td>
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<td></td>
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<td></td>
<td>(.1431)**</td>
<td>(.1962)</td>
<td>(.2516)***</td>
<td>(.1166)***</td>
<td></td>
</tr>
</tbody>
</table>

Estimates of $\beta_1$ and $\beta_2$ presented in Table 1. The impact of lead is strongly negative and easily statistically significant in all specifications and all four quarters of the regular season data. The overall average for regular season games indicates that trailing by 10 points increases output per 100 possessions by 4.5 points. As a frame of reference, 10 points per 100 possessions typically separates very good offenses from very bad offenses. The observed magnitudes are indeed very large. A playoff contender morphs into merely a below average team up 10 points and one of the best teams in the league when down by 10. Estimates of the impact in playoff games, which are economically more important, naturally have larger standard errors due to the smaller sample size, but are significantly negative in the second and fourth quarters and only slightly smaller in aggregate than the estimates obtained from regular season data. The global average for playoffs is a strongly significant 3.13 points per 100 possessions, or about 70% of the regular season effect. Interestingly, even though the magnitude of the effect is smaller in the playoffs it is actually twice as likely to impact the game outcome given how much closer playoff games are on average.

A natural concern is that the effect is driven by good team vs. bad team match ups in which the good team plays lackadaisically with a lead, only to “turn it on” late in the game.\(^\text{10}\)Possessions originating off steals or defensive rebounds tend to be more valuable than those originating off of dead balls or made baskets by the opposing team. Controlling for this soaks up some residual variability but does not significantly effect our results.
to ensure the win. Uneven match-ups are less likely to occur in the playoffs, so the fact that we see significant loss aversion there as well is our first clue this concern does not have bite. Panels 2 and 3 restrict the estimation to ex-ante evenly matched teams (games where the spread was less than 6 in magnitude) and close games (games which were within 10 points at the start of that quarter). Neither restriction significantly impacts the estimates and all specifications show a larger gross impact of losing on performance in the fourth quarter. As will be shown in future specifications, this is driven in part by an increased impact of coach loss aversion.

4.1.1 Player loss aversion

Our next specification allows us to isolate only the impact of player loss aversion on team performance. In order to do this, we non-parametrically condition on the entire five-man lineup employed by both teams at any given moment of game play. Specifically, we estimate:

\[
E[Pts_p] = \alpha_{Off_p,Def_p} + \delta_1 Home_p + \delta_2 Playoffs_p \\
+ \beta_1 Lead_p^* \times (1 - Playoffs_p) + \beta_2 Lead_p^* \times Playoffs_p, \tag{5}
\]

where \( Off_p \) and \( Def_p \) denote the unique five man offensive and defensive players employed on possession \( p \) and \( \alpha_{Off_p,Def_p} \) is a unique fixed effect for each combination. Thus specification (5) is identified only by comparing performance of exactly equivalent match-ups across given different score margins, truly holding all else equal (unique 10 player combinations). Specification (6) is slightly less general. It achieves much more power by allowing for additively separable fixed effects for the offensive and defensive lineup, but may be confounded if match-up specific effects interact strongly with the score margin.

\[
E[Pts_p] = \alpha_{Off_p} + \nu_{Def_p} + \delta_1 Home_p + \delta_2 Playoffs_p \\
+ \beta_1 Lead_p^* \times (1 - Playoffs_p) + \beta_2 Lead_p^* \times Playoffs_p, \tag{6}
\]

By incorporating so many fixed effects, specifications (5) and (6) reduce our data to a short, wide panel. Since the current score margin is endogenous to lagged performance, this can create substantial dynamic panel bias. Our solution is to again preform the entire analysis separately for each quarter and to use the score margin at the beginning of that quarter (denoted \( Lead^*_p \) in (5) and (6)) as a proxy for the current score.

\[11\]We discuss this later, this could also include play-calling by the coach.
Table 2: Player (only) impact of lead on offensive performance (Points/100 possessions)

<table>
<thead>
<tr>
<th></th>
<th>1st Quarter</th>
<th>2nd Quarter</th>
<th>3rd Quarter</th>
<th>4th Quarter</th>
<th>Average Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Single Fixed Effect</td>
<td>.0181</td>
<td>-.4283</td>
<td>-.2396</td>
<td>-.4954</td>
<td>-.2863</td>
</tr>
<tr>
<td>(5)</td>
<td>(.2443)</td>
<td>(.2375)*</td>
<td>(.0733)**</td>
<td>(.1902)**</td>
<td>(.0993)**</td>
</tr>
<tr>
<td>Double Fixed Effect</td>
<td>-.5224</td>
<td>-.1222</td>
<td>-.1326</td>
<td>-.4567</td>
<td>-.3085</td>
</tr>
<tr>
<td>(6)</td>
<td>(.2783)*</td>
<td>(.2293)</td>
<td>(.0964)</td>
<td>(.1771)**</td>
<td>(.1033)**</td>
</tr>
<tr>
<td>Penalized Regression</td>
<td>-.4413</td>
<td>-.3152</td>
<td>-.2481</td>
<td>-.3064</td>
<td>-.3277</td>
</tr>
<tr>
<td>(7)</td>
<td>(.0615)**</td>
<td>(.0343)**</td>
<td>(.0218)**</td>
<td>(.0316)**</td>
<td>(.0201)**</td>
</tr>
</tbody>
</table>

† All standard errors clustered at the game level.

Estimates from our single fixed effect model (5) rely on a very limited set of comparisons and are thus noisy but nonetheless demonstrate a negative, statistically significant ($p < .01$) impact of lead on score margin for both regular season and playoff data. Our additively separable fixed effect model has much higher precision and produces similar point estimates. Here we find strongly significant loss aversion behavior in every quarter of the regular season and all but the third quarter of our playoff data. The aggregated estimates are shown in the final column; they are roughly two-thirds the magnitude of the overall loss aversion estimates shown in Table 1 indicating that substitution decisions—preferences of the coaching staff—contribute to the overall effect, particularly in the fourth quarter.

Our final specification further generalizes (6) to allow for each player to have a unique additively separable impact on his team’s performance. This type of regression model—often referred to as an adjusted plus/minus model—is a popular way of measuring the value of professional basketball players\(^{12}\). We use a ridge penalization (chosen by cross-validation) to control the coefficient estimates for each player and then add additional, unpenalized regressors to estimate the impact of the score margin. Formally, we estimate

\[
E[Ptsp] = \sum_{i \in Offp} \alpha_i - \sum_{i \in Defp} \nu_i + \delta_1 Home_p + \delta_2 Playoffs_p \\
+ \beta_1 Lead_p \times (1 - Playoffs_p) + \beta_2 Lead_p \times Playoffs_p, \tag{7}
\]

\(^{12}\)Regularized regression has been used to evaluate NBA player performance since (Sill, 2010). Jeremias Englemann has worked to improve and popularize the technique and it is now the basis for the ‘real plus minus’ stat posted on ESPN.
where the penalization factor ($\lambda = 2,400$) is applied to the $L^1$ norm of $\alpha$ and $\nu$. This method imposes strong separability in the contribution of each player, but in returns provides a dramatic increase in power. The penalization also biases all coefficients on player effects toward zero. We do not penalize the coefficients on our lead variables, but some bias could be transmitted through to these.

Taking advantage of the increased power, we replace the estimates of $\beta$ in (7) with a non-parametric function of the game state. Figure 6 demonstrates the results.

![Figure 6: Offensive efficiency achieved by “replacement players” across game states as estimated according to (7). Holding player quality constant, both home and away teams improve dramatically as they fall behind and the effect is only slightly smaller in the playoffs.](image)

4.2 Fixed vs. quickly adjusting reference point

Recall that a quickly adjusting and fixed reference points make entirely different predictions in this setting. If a loss-averse team that is leading quickly adjusts the reference point up to the current score margin, then a close victory will now be viewed as a “loss” from this
vantage point. We use in game fluctuations in score margin to get at this question. For example, if a team jumps out to a ten point lead at half time, will they adjust their reference point up? Will their opponent adjust down? Does that ten point differential now bifurcate the margin space into new gain and loss regions? If so, then we would expect an early lead to predict improved performance over the remainder of the game.

4.2.1 Reduced form analysis

We test for this by including additional lags of the score margin our specifications given in Table 1. If basketball dynamically update their reference point over the course of a game, we should expect coefficients on these regressors to be positive. If they maintain a fixed reference point throughout the game then the current score margin should be a sufficient statistic for the entire history of the game and the coefficients should be zero.

The results for all four of our specifications are printed in Table 3. Nearly all the coefficients on lagged lead lack statistical significance. Of the 24 estimates only 4 are statistically significant at the 10% level. In particular our most powerful specification, based on ridge regression, provides very tight estimates around zero for all coefficients, indicating that reference point updates over the course of a basketball game are neither statistically nor economically significant. Our double fixed effect specification shows an interesting pattern of significant positive coefficient on first quarter performance and a negative coefficient on second quarter performance. Identification here is dominated by the performance of common lineups and, in particular, each team’s starting lineup. Starting lineups typically play the most in the first and third quarter. Then this pattern of coefficients indicates that starters play the best in the third quarter when they had built a lead in the beginning of the first quarter, but saw their teammates give up a lead at the end of that quarter. Further investigation may be warranted.

To further complement this analysis, we break down our nonparametric estimates from Figure 6 into separate functions of game state depending on performance in the first quarter. Specifically, we define a team that was ahead (behind) by more than 5 at the end of the first quarter to have been ‘Ahead Early’ (‘Behind Early’). This results in a roughly equal split of our data into thirds. We use our same ridge regression approach to control for player quality, results for each combination of first quarter performance and home/away are presented below. Estimates are presented in Figure 7. There is no evidence that first quarter performance has an impact on the level or slope of the performance function in later quarters.
Table 3: Impact of lagged score margins on offensive performance (Points/100 possessions)

<table>
<thead>
<tr>
<th>Spread Regression</th>
<th>Second Quarter</th>
<th>Lag Margin at 42 min remaining</th>
<th>Lag Margin at 36 min remaining</th>
<th>Lag Margin at 24 min remaining</th>
</tr>
</thead>
<tbody>
<tr>
<td>(4)</td>
<td>Third Quarter</td>
<td>-.0526 (.0378)</td>
<td>-.1025 (.0472)**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fourth Quarter</td>
<td>-.0527 (.0430)</td>
<td>.0365 (.0436)</td>
<td>.0321 (.0562)</td>
</tr>
<tr>
<td>Singe Fixed Effect</td>
<td>Second Quarter</td>
<td>.0847 (.3273)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(5)</td>
<td>Third Quarter</td>
<td>.0692 (.1443)</td>
<td>.0696 (.0989)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fourth Quarter</td>
<td>.1994 (.3238)</td>
<td>-.3295 (.2254)</td>
<td>-.2491 (.2111)</td>
</tr>
<tr>
<td>Double Fixed Effect</td>
<td>Second Quarter</td>
<td>-.0533 (.0627)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6)</td>
<td>Third Quarter</td>
<td>.1591 (.0516)**</td>
<td>-.1265 (.0378)**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fourth Quarter</td>
<td>-.1273 (.0685)*</td>
<td>-.0078 (.0513)</td>
<td>-.0118 (.0486)</td>
</tr>
<tr>
<td>Penalized Regression</td>
<td>Second Quarter</td>
<td>-.0291 (.0245)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(7)</td>
<td>Third Quarter</td>
<td>-.0163 (.0334)</td>
<td>-.0340 (.0240)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Fourth Quarter</td>
<td>-.0164 (.0321)</td>
<td>.0182 (.0293)</td>
<td>-.0025 (.0182)</td>
</tr>
</tbody>
</table>

† Standard errors clustered at the game level in all specifications.

4.2.2 Estimating the reference point as a latent variable

In the last subsection we looked at evidence of drift in the RP, and found none. In this section we’ll try to directly estimate it as a latent variable. The basic strategy is to estimate Figure 6 on various subsets of the data and look for areas of greatest steepness. Figure 8 does this for the aggregate data; it gives estimates the derivative in lead from the regression results presented in Figure 6. The first derivative is maximal at 0 in the aggregate data.

We now repeat this analysis splitting our sample into teams that are favorites (spread > 5), underdogs (spread < -5), and evenly matched (anywhere in between) and estimating a non-parametric game-state function for each group. Results are presented in Figure 9. Note that going from favorite to underdog does not display a significant level effect because that
Figure 7: Nonparametric estimates of the impact of game state over the last 36 minutes broken up by performance over the first 12.

Figure 8: Nonparametric derivative of the estimates in Figure 6. Both Home and Away teams appear to show the sharpest changes in relative performance around a score margin of zero.

has already been soaked up by our controls for player quality. The overall shape of the game function seems to be roughly identical in all six pictures (except that it is shifted up for the home team). More importantly, we do not observe a significant performance boost for a favored team in high WVP moments (at the end of close games). This was a prerequisite for the standard model to explain a pattern of apparent loss motivation in the data.
5 Additional Results and Robustness

In the introduction we discussed two potential critiques of our findings of strong, fixed reference point loss aversion in NBA players. We dispatched the first, that the effect is driven by behavior in meaningless games or from uneven matches, in the last section. The second is that effort/performance mapping is not monotonic. An additional concern is that it is in fact the referees driving the effect by favoring the trailing team (not this would presumably not apply to the coach loss aversion inferred through substitution patterns). We address these concerns here.

5.1 Breaking down loss aversion: Effort intensive tasks

In basketball, after a missed shot there is scramble for the ball known as rebounding. Securing a rebound involves physically “boxing out” and out jumping the opponent. We again apply our fixed effect estimation methodology, the results are given in Table 4.

A rebound is worth a possession and adds a little value to the resulting offensive possession. All together it is worth around 1.2 points. There is roughly one available rebound for every 2 possessions. +1 to rebounding rate is worth around +.6 to team efficiency. The LA effect on rebounding rate is worth around +.07. We now estimate the same model on turnover probability. The results are presented in Table 5.

A turnover costs a possession which is worth slightly more than 1 pt. As such +1 to turnover rate is worth slightly more than +1 to points scored. The LA effect on turnovers
Table 4: Impact of game state on Offensive Rebounding Rate (Offensive Rebounds /100 missed shots)

<table>
<thead>
<tr>
<th></th>
<th>Quarter 1</th>
<th>Quarter 2</th>
<th>Quarter 3</th>
<th>Quarter 4</th>
<th>Average Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread Regression</td>
<td>(\hat{\beta}_1)</td>
<td>(-.1649)</td>
<td>(-.1142)</td>
<td>(-.0983)</td>
<td>(-.0933)</td>
</tr>
<tr>
<td></td>
<td>(\hat{\beta}_2)</td>
<td>(.0558)</td>
<td>(-.1898)</td>
<td>(-.0401)</td>
<td>(-.0435)</td>
</tr>
<tr>
<td>Double Fixed Effect</td>
<td>(\hat{\beta}_1)</td>
<td>(-.2249)</td>
<td>(-.0977)</td>
<td>(-.1114)</td>
<td>(-.0854)</td>
</tr>
<tr>
<td></td>
<td>(\hat{\beta}_2)</td>
<td>(.0386)</td>
<td>(-.1779)</td>
<td>(-.0203)</td>
<td>(-.0151)</td>
</tr>
</tbody>
</table>

Table 5: Impact of game state on Turnover Rate (Turnovers /100 possessions)

<table>
<thead>
<tr>
<th></th>
<th>Quarter 1</th>
<th>Quarter 2</th>
<th>Quarter 3</th>
<th>Quarter 4</th>
<th>Average Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>Spread Regression</td>
<td>(\hat{\beta}_1)</td>
<td>(.1011)</td>
<td>(.1058)</td>
<td>(.0838)</td>
<td>(.1315)</td>
</tr>
<tr>
<td></td>
<td>(\hat{\beta}_2)</td>
<td>(.0732)</td>
<td>(.0775)</td>
<td>(.0662)</td>
<td>(.0640)</td>
</tr>
<tr>
<td>Double Fixed Effect</td>
<td>(\hat{\beta}_1)</td>
<td>(.1086)</td>
<td>(.1104)</td>
<td>(.0913)</td>
<td>(.1287)</td>
</tr>
<tr>
<td></td>
<td>(\hat{\beta}_2)</td>
<td>(.2306)</td>
<td>(.0863)</td>
<td>(.0704)</td>
<td>(.0630)</td>
</tr>
</tbody>
</table>

is worth around +1.1.

Overall, effort intensive tasks can account for 80% of the loss aversion effects we observe.

[Under construction]

5.2 Breaking down loss aversion: focus intensive tasks

The most focus intensive action in NBA basketball is the free throw. A free throw is an uncontested 15-foot shot taken while the play is stopped after the commission of a foul. We a simple fixed effect specification to capture the impact of the score margin on the probability of making a free throw. Results are presented in Table 6. As before, \(\hat{\beta}_1\) and \(\hat{\beta}_2\) represent the coefficient in regular season and playoff data respectively.

Contrary to all our findings so far, we find evidence that players actually have worse free throw accuracy when trailing (the statistical significance is not overwhelming, however). A look at playoff and nationally televised games—those with higher economic and psychological stakes—sheds some light on this potential inconsistency (of our sample of
Table 6: Impact of Game State on Free Throw Percentage (%)

<table>
<thead>
<tr>
<th>Quarter 1</th>
<th>Quarter 2</th>
<th>Quarter 3</th>
<th>Quarter 4</th>
<th>Average Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
<td>(5)</td>
</tr>
<tr>
<td>$\hat{\beta}_1$</td>
<td>.0610</td>
<td>.0244</td>
<td>.0291</td>
<td>.0187</td>
</tr>
<tr>
<td>(0.0290)**</td>
<td>(0.0167)</td>
<td>(0.0135)**</td>
<td>(0.0115)</td>
<td>(0.0095)**</td>
</tr>
<tr>
<td>$\hat{\beta}_2$</td>
<td>.0177</td>
<td>.0961</td>
<td>.0299</td>
<td>.0286</td>
</tr>
<tr>
<td>(0.1307)</td>
<td>(0.0718)</td>
<td>(0.0522)</td>
<td>(0.0416)</td>
<td>(0.0409)</td>
</tr>
</tbody>
</table>

† Player-season FE included. Standard errors clustered at game level.

7,410 games, 332 occur in the playoffs and 646 are nationally televised by a cable (ESPN, TNT) or broadcast (NBC) network. In Table 7 we use the same fixed-effect estimator and add dummy variables for playoff and national TV status.

Table 7: Impact of Game Type on Free Throw Percentage (%)

<table>
<thead>
<tr>
<th>Away Nat. TV</th>
<th>Home Nat. TV</th>
<th>Away Nat. TV</th>
<th>Playoffs</th>
<th>Home Playoffs</th>
<th>Playoffs</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) -1.3839</td>
<td>-.6270</td>
<td>-.8687</td>
<td>-1.8650</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(.5144)***</td>
<td>(.4820)</td>
<td>(.3712)**</td>
<td>(.3843)***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(2) -.9882</td>
<td></td>
<td>-.8687</td>
<td>-1.8650</td>
<td></td>
<td>-1.3791</td>
</tr>
<tr>
<td>(.3687)***</td>
<td></td>
<td>(.3712)**</td>
<td>(.3843)***</td>
<td></td>
<td>(.2796)***</td>
</tr>
</tbody>
</table>

† Lead and Home variables and player-season F.E. included in each specification. Standard errors clustered at game level.

The estimates indicate an across the board decline in free throw shooting percentage in playoff and nationally televised games. The difference between the effects on home and away shooters in specification (1) is statistically insignificant for both cases. As such, specification (2) aggregates over home and away to show that both playoffs (coeff. $-0.98, \ t = -2.92$) and national TV (coeff. $-1.4, \ t = -5.25$) induce significant drops in free throw accuracy. This provides overwhelming statistical evidence of a performance decline when the player presumably “cares” more about the outcome. This provides an explanation as to why players shoot free throws better while leading and worse while losing. When they try harder, they do worse. When they are losing, they try harder. For most aspects of basketball this helps, but for a focus intensive task like free throw shooting, it hurts. In a related paper, we also find they shoot free throws worse in other “high pressure” moments, such as at the end of close games.13 This nuance of the results, instead of revealing an inconsistency, actually strengthens our primary finding.

13Put citation. Psychologists have linked this to increased “self focus.”
5.3 Do Officials Favor The Losing Team

A final concern to address is that it is in fact the referees, not the players, driving loss aversion. NBA referees are not perfect, with past work documenting a bias for the home team and for players of the same race (Price and Wolfers, 2010). Perhaps referees are predisposed to “keep the game interesting.” We have already seen evidence that this sort of bias cannot account for all the loss aversion we observe as it is unlikely the it would inspire star players to shoot more or coaches to play their better players when losing, but not winning. Any analysis of referee bias is challenging because of the endogeneity of players’ actions to the purported bias. These concerns are present here as well, but what we will be able to show is that the magnitudes of foul rate differences are not of the same order as the point differentials we saw—it is simply not plausible they are driving the effect, but we cannot rule out a very small contribution.

Taking each whistled foul as a data point, we replicate specification ?? to predict whether or not it will be called on the offense. The baseline offensive foul rate is about 8% in our data. There is roughly one foul called for every four possessions of basketball and the value of the foul is typically to swing possession of the ball (worth around one point). Thus a one percentage point increase in the offensive foul rate is worth a quarter point of offensive efficiency. The results are presented in Table 8.

We are able to confirm the standard results of a lower foul rate for the home team and also show statistically significant apparent favoritism for the trailing team. Both effects seem to stronger in later quarters. Averaged across course of the entire game we estimate a -1.2% absolute bonus to a teams offensive foul rate can be achieved either by playing at home or seeing a twenty point decrease in their lead (e.g. +10 to -10). Each of these effects represents a 15% relative decline in offensive foul rate, but only represents a roughly 0.3 point/100 change swing in offensive efficiency. In contrast, the loss aversion impact of moving from +10 to -10 was 9 points (27x higher).

<table>
<thead>
<tr>
<th></th>
<th>One</th>
<th>Two</th>
<th>Three</th>
<th>Four</th>
<th>Simple Mean</th>
<th>Weighted Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home</td>
<td>-0.0107</td>
<td>-0.00774</td>
<td>-0.0158</td>
<td>-0.0165</td>
<td>-0.0127</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>t=-2.43</td>
<td>t=-2.30</td>
<td>t=-3.25</td>
<td>t=-4.14</td>
<td>t=-6.06</td>
<td>t=-5.95</td>
</tr>
<tr>
<td>Lead</td>
<td>0.000309</td>
<td>0.000604</td>
<td>-9.32e-05</td>
<td>0.00174</td>
<td>0.000639</td>
<td>0.000496</td>
</tr>
<tr>
<td></td>
<td>t=1.14</td>
<td>t=3.33</td>
<td>t=-0.26</td>
<td>t=3.09</td>
<td>t=3.45</td>
<td>t=3.68</td>
</tr>
</tbody>
</table>

Table 8: Lineup fixed effect estimates of the impact of lead on offensive foul rate.
6 Conclusion

Our study of highly experienced agents reveals reference dependent preferences resulting in a strong “losing motivates” effect. Moreover the reference point appears to be stable and exogenous—players respond to whether they are in the proverbial red or black, regardless of what would have been a reasonable expectation. Past experimental work has demonstrated that reference points can adjust quickly in inexperienced agents, leading to discouragement effects and opening up the possibility of using decision frames to motivate effort. Our results indicate that at least some of these findings do not extend to highly experienced agents. The stability of the reference point around the focal point of zero informs economic interactions beyond basketball. For instance, investors may respond to the returns of a single investment by ignoring the performance of the S&P 500 during the holding period. In our setting, there are huge incentives to eliminate the reference dependence for a good team (because they are leading frequently) but not only are coaches unable to overcome players' psychology, the exhibit they appear to exhibit the same dependence themselves, indicating just how deep reference dependent preferences run in this setting.

References


Appendix

Win Probability Models

We will now compute how important actions are at a given game state in terms of the magnitude of the expected impact on the game outcome. Consider two teams, home (h) and away (a). Let \( S_{h,N} \) and \( S_{a,N} \) denote the current scores for the home and away team with \( N \) offensive possessions (for each team) remaining in the game. Let \( P_{h,i} \) and \( P_{a,i} \) denote the number of points scored by the home/away team on the \( i \)th possession from the end of the game. The home team wins if:

\[
S_{h,0} > S_{a,0} \iff S_{h,N} + \sum_{i=1}^{N} P_{h,i} > S_{a,N} + \sum_{i=1}^{N} P_{a,i} \iff \sum_{i=1}^{N} P_{h,i} - P_{a,i} > S_{a,N} - S_{h,N}.
\]

To model how teams generate points, let \( \{\mu_h, \sigma_h^2\} \) and \( \{\mu_a, \sigma_a^2\} \) represent the mean and variance of points per possession that each team is able to achieve in the match-up. If the number of remaining possessions, \( N \), is large, the central limit theorem gives the probability of the home team winning as:

\[
P(\text{Home Win}) = P(S_{h,0} > S_{a,0}) = P\left(\sum_{i=1}^{N} (P_{h,i} - P_{a,i}) > S_{a,N} - S_{h,N}\right)
\xrightarrow{N} \Phi\left(\frac{S_{h,N} - S_{a,N} + N(\mu_h - \mu_a)}{\sqrt{N(\sigma_h^2 + \sigma_a^2)}}\right),
\]

where \( \Phi \) is the CDF of the standard normal distribution. The marginal impact of a point scored on winning the game is easily obtained by differentiating equation (8) to get

\[
\frac{dP(\text{Home Win})}{dS_{h,N}} = \phi\left(\frac{(S_{h,N} - S_{a,N}) + (N(\mu_h - \mu_a))}{\sqrt{N(\sigma_h^2 + \sigma_a^2)}}\right) \frac{1}{\sqrt{N(\sigma_h^2 + \sigma_a^2)}},
\]

where \( \phi \) is the standard normal PDF or by using the discrete analog given in the text. Given the normality, we estimate this equation with a Probit. We impute the number of remaining possessions using the team-specific paces-of-play in a given match-up and by adding one possession to the team currently holding the ball. The coefficient estimates of this model are given in the Appendix. The projections for the probability the home team wins again an evenly matched opponent are given are shown in Panel 1 of Figure 10.

One might be concerned that the parametric procedure we have employed relies too heavily on the normality afforded by the central limit theorem. Panel 2 depicts non-
parametric estimates of the win probability function constructed from applying a two-dimensional Gaussian kernel to the 2x1 game state vector (margin, time remaining) with bandwidths $h_{\text{margin}} = 1.5$ and $h_{\text{minutes}} = 1$. Values of the nonparametric estimator are censored for game states rare enough that the associated standard error exceeds 0.03.

The two approaches yield nearly identical win probability projections, even as the end of the game nears. Because of the well known difficulty of differentiating non-parametric estimators, we proceed with the values of WVP generated by the parametric model.

Using the parameter estimates given in Appendix Table 1, we apply equation (2) to get the marginal value of a point as a function of time remaining and score margin, which are plotted in Figure 1 for two evenly matched teams. Quarters 1–3 are shown in the left panel and fourth quarter in the right panel (note the big change in z-axis scale). The most important point in the first 3 quarters of the game is worth less than 0.05 wins in expectation. In the final minute of a close game, making two free throws can increase the chance the team will win by over thirty percentage points. The median WVP is 0.02, and Figure ?? gives the distribution for the shots in our sample.

**Appendix Table 1: Coefficient estimates for our probit model of win probability model.**

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Lead:</td>
<td>$S_{h,N} - S_{a,N}$</td>
<td>$1 { \text{Possession} }$</td>
<td>$\sqrt{N} \cdot 1 { \text{Home} }$</td>
<td>$\sqrt{N} \cdot \Delta \hat{\mu}_{\text{Off}}$</td>
<td>$\sqrt{N} \Delta \hat{\mu}_{\text{Def}}$</td>
</tr>
<tr>
<td>Coefficient</td>
<td>2.780</td>
<td>1.972</td>
<td>0.00623</td>
<td>0.00176</td>
<td>0.00180</td>
</tr>
<tr>
<td>$t$-value</td>
<td>43.61</td>
<td>26.69</td>
<td>11.72</td>
<td>15.79</td>
<td>15.23</td>
</tr>
</tbody>
</table>

Games=5,254, Possessions=902,803

Standard errors are clustered at the game level.