What We Talk About
When We Talk About a Mutual Fund’s Reputation

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Abstract

Few studies explored the nature of the mutual fund’s reputation and how the reputation forms and evolves, though all funds and investors pay close attentions to it. We model the fund’s reputation as the market’s belief about whether the fund has the information to make profitable investments. We then propose an infinite-horizon model in which the fund’s reputation can be invested by costly information acquisition but may depreciate exogenously. Reputation has significant effects on the fund’s information acquisition decision, which affects the fund’s performance and flow-performance relationship. The model explains several stylized facts and offers new empirical implications.

Keywords: Mutual Fund, Reputation, Information Acquisition, Obsolete Information

JEL Classification Codes: C73, D83, G23

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1 Introduction

In the mutual fund industry, one of the central criteria the investors use to make a purchase decision is the reputation of the fund. In their survey, Capon, Fitzsimons, and Prince (1996) find that the fund manager’s reputation is the second most important selection criterion when retail investors make their purchasing decisions, with the mean of importance 4.00 and the standard deviation 0.77.\(^1\) The Investment Company Institute Survey (1997) confirms the importance of reputation in investors’ funds picking decisions, showing that reputation is the third highest consideration, after risk level and total return. Not only retail investors, but financial advisers also emphasize the importance of the fund manager’s reputation (Jones, Lesseig, and Smythe, 2005). Given the fact that financial advisers are the most important information channel for investors to make investment decisions, the fund’s reputation indeed stands out when investors consider the purchase.

More important, mutual fund managers are always compensated by a percentage of the total assets under management. Given the size of the fund, such a compensation scheme is indeed a flat fee, which seems not able to incentivize the fund manager to manage investors’ money well at a cost of her own disutility. However, fund managers in fact work hard for their investors because they care about their future revenues as a fraction of future sizes (Berk, 2005), which are usually thought to be increasing in their reputation.

However, the concept of mutual fund’s reputation is so familiar as to be taken for granted; thus, little attention has been paid to understanding the nature of the reputation in such an industry. In particular, what exactly do we talk about when we discuss a mutual fund’s reputation? In addition, it is widely agreed that the reputation of a mutual fund sets up a bridge between its past performance and expected performance in the future. Then does a fund’s reputation just reflect some persistent characteristics of the fund so that the past performance is positively correlated with the future performance? Or can the fund’s reputation affect the fund manager’s behaviors, thereby affecting the fund’s performance?\(^2\)

In this paper, we address these questions in a discrete-time infinite-horizon model and provide explanations for several empirical observations in the mutual fund industry. We model the reputation of a fund as the market belief about whether the fund has superior information.\(^3\) In-

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\(^1\) The importance has a 1 to 5 scale, with 5 meaning “most important.”

\(^2\) Warren Buffet once pointed out: “... If you think about that [your reputation can be ruined easily], you [fund managers] will do things differently.” This famous quote suggests that since fund managers care about their reputations, they may want to do something to promote their reputations, which may deviate from their myopic actions.

\(^3\) We do not consider the agency problem between the fund and the fund manager, which is an optimal contract
formation plays the central role in financial markets. The Efficient Market hypothesis asserts that stock prices have incorporated all available information in the market; therefore, no trader can earn an abnormal return unless he/she possesses information that has not been reflected by the prices. Consequently, if a mutual fund constantly outperforms the market, it must persistently have superior information. Moreover, information superiority has been shown as the reason why mutual funds are endogenously built (García and Vanden, 2009). Therefore, if a fund is known to have information, investors will purchase its shares. However, whether a fund is informed is unobservable to investors. So investors make decisions based only on their belief about whether a fund has information: When the market belief that the fund has information is high, the fund has a higher reputation, and more investors will purchase the fund.

Our definition of a mutual fund’s reputation differs substantively from, but complements that in Berk and Green (2004), who model the mutual fund’s reputation as the market belief over the fund manager’s ability. Such an ability is persistent and can be learned through the fund’s past performance and sizes. As a result, given the fund’s size, if the fund’s performance is better than investors’ predictions, the fund will have a better reputation, and more cash will flow into the fund. While the fund manager’s ability determines how efficiently the manager uses available information, our definition of mutual fund’s reputation differs in reflecting whether the fund manager has information to use.

There are two special features of information that differentiate our definition of a mutual fund’s reputation from that in Berk and Green (2004), as well as those in numerous type reputation models, for example, a borrower’s reputation in Diamond (1989). First, since information can be acquired, an uninformed mutual fund can become informed by acquiring information. So a mutual fund’s reputation in every period contains the market belief about the fund’s information acquisition behavior in that period. Second, information has short-run persistence but will become obsolete in the long run. As a result, conditional on that the fund is informed in the current period, it is possible that the fund is still informed in the next period even without the next period’s information acquisition; but conditional on that an uninformed fund never acquires information, the fund’s reputation will deteriorate over time.

Our model captures these critical properties of a mutual fund’s reputation. Given a fund’s prior problem. So in this paper, the fund’s reputation and the fund manager’s reputation refer to the same thing. We also treat a fund family’s reputation and its fund’s reputation the same.

4There is a huge literature in economics in which an agent’s reputation is modeled as the market belief over his/her types. Kreps and Wilson (1982), Milgrom and Roberts (1982), and Fudenberg and Levine (1989) define an agent’s reputation as the market belief whether the agent is of a commitment type; Mailath and Samuelson (2001) regard an agent’s reputation as the market belief that the agent is not an inept person. Mailath and Samuelson (2013) survey recent works on reputations in repeated games of incomplete information.
reputation, which is defined as the market belief about whether the fund is currently informed, the uninformed manager makes the information acquisition decision. If she acquires information, she becomes informed; otherwise, she remains uninformed. Investors do not observe the manager’s choice but form a belief over her information acquisition behavior. Such a belief, together with the fund’s prior reputation, constitutes the fund’s interim reputation. Investors will then make their purchase decisions based on the interim reputation. The fund’s period payoff depends on the number of investors who purchase. Investors will reevaluate whether the fund is informed in the current period after observing the investment outcomes and form a posterior reputation: Because an informed fund manager can generate a good outcome with a higher probability than an uninformed manager does, good outcomes will always promote the fund’s posterior reputation. Information may become obsolete, so the fund’s posterior reputation in the current period, discounted by the information depreciation, turns into the prior reputation in the next period.

Because the fund’s information superiority is privately known to the fund, and the fund’s information acquisition behavior is unobservable to investors, moral hazard is inevitably present. Hence, a mutual fund’s the reputation-building process is an incentive mechanism in our paper. Such an incentive mechanism, together with information’s special features, leads to a “work-work-shirk” stationary equilibrium of the model. In the equilibrium, in every period, an uninformed fund acquires information if and only if the reputation premium is greater than the information acquisition cost. If an uninformed fund is believed to possess information (high prior reputation), it “shirks” and does not acquire information. The reputation premium in this case is low, both because a good investment outcome and a bad investment outcome do not affect the fund’s posterior reputation significantly when the prior reputation is high, and because the effect of the information depreciation dominates the next period’s prior reputation. When an uninformed fund has a medium or low prior reputation, the fund “works” to acquire information with different probabilities. As a result, a low reputation fund has a lower interim reputation than a medium reputation fund. The reputation premium in these two scenarios are equal to the information acquisition cost.

Our model has different long-run implications about the flow-performance relationship from those in Bayesian learning models with permanent types, for example, Holmstrom (1999) and Berk and Green (2004). In models in which a fund has a permanent type, the fund’s flows are barely affected by the recent performance when the fund is sufficiently old. This is due to the convergence result in Bayesian learning models with persistent types: Because investors will ultimately learn the fund manager’s true ability, the fund’s size will not change much because of the fund’s recent performance when the fund is old. However, it has been documented in empirical studies that the most recent performance is the best predictor of the fund’s flow. Chevalier and
Ellison (1997) document that though flows into older funds are less sensitive to recent performance than those into younger funds, flows are significantly correlated to recent performance across all funds, including those older than 11 years. Relatedly, Coval and Stafford (2007) find that the most recent performance significantly affects the fund’s current flows, even after controlling for a long history of past performance. These empirical facts, however, appear in the “work-work-shirk” equilibrium of our model: A fund’s flows always positively correlated with the fund’s recent performance. Because the information superiority has short-run persistence only and the fund’s information acquisition decision is unobservable to investors, recent performance is a better indicator of whether the fund is currently informed.\(^5\)

This paper sheds light on some empirical findings about bad performing funds. Berk and Tonks (2007) divide bad performers into two groups: seasoned ones who are the worst performers in two consecutive periods and unseasoned ones who are the worst in the previous period but have had better performance before. Berk and Tonks find that seasoned funds in the bottom decile persistently perform badly, and seasoned funds in the bottom decile have a weak flow-performance relationship. This empirical finding echoes Carhart’s (1997) rules of thumb (I): Investors should avoid funds with persistently bad performance. Berk and Tonks attribute such an empirical finding to investors’ unwillingness to withdraw from bad performing funds. This difference between seasoned and unseasoned funds appears in the “work-work-shirk” equilibrium in our model, where investors have homogeneous willingness of withdrawing money. In the equilibrium, bad performing funds are believed to be uninformed, and they acquire information with a small probability. Therefore, their interim reputations are low. This explains why bad performers are expected to perform badly. Seasoned funds have similar interim reputations in two consecutive periods, which determine sizes in our model. Hence, the flow is not significant.

Complementing to Berk and Green (2004), we point out another reason why past performance is a very weak predictor of future performance (Carhart, 1997). A necessary condition for a fund to persistently outperform the market is that the fund be persistently informed. However, a previously informed fund may become uninformed in the current period. In the equilibrium, an uninformed fund acquires information with a probability less than the information depreciation probability; that is, the information acquisition behavior of the uninformed fund is not sufficient to compensate for the information depreciation. Cross-sectionally, since the previous period’s star funds do not acquire information if they become uninformed, and the previous period’s medium

\(^5\)Dangl, Wu, and Zechner (2008) also derive this implication. They extend the model of Berk and Green (2004) by assuming time-varying managerial abilities: as a result, recent performance is most informative about the fund manager’s current ability, so flow-performance sensitivity never converges to zero. We endogenize the change of the fund’s type in this paper, which enables us to analyze the difference between seasoned and unseasoned funds.
funds acquire information so that they have a high interim reputation, on average, the performance difference between star funds and medium funds shrinks significantly, which is consistent with the “no performance persistence” documented in empirical studies.

In recent empirical studies, several proxies are used to measure a mutual fund’s reputation, such as past performance and size (Gerken, Starks, and Yates, 2014).\(^6\) Our model provides sufficient structures to analyze these variables’ roles. First, past performance of a fund contributes to its prior reputation, which determines the fund’s equilibrium information acquisition decisions. As a result, past performance will have causal effects on the fund’s performance. However, past performance cannot predict the fund’s current performance. For one thing, past performance of a fund is not equivalent to its prior reputation: Given the same performance in the last period, two funds with different interim reputations in the last period may have different prior reputations today. For another, the prior reputation does not contain the probability that an uniformed fund acquires information, which is an important factor in determining the fund’s ex-ante current performance. Second, a fund’s size is perfectly correlated to the fund’s interim reputation; therefore, it can predict the fund’s ex-ante current performance well. However, the size reflects the market belief about the fund’s information acquisition behavior and is endogenously determined in the model, so it does not have the “causal” effect on the fund’s performance, and investors cannot use the information of the current size to make the current purchasing decision.

While we focus on a mutual fund’s reputation in this paper, our model could be extended to analyze reputation in other industries in which information plays the most important role. Similar to mutual funds, hedge funds can persistently obtain abnormal investment returns only if they constantly have superior information. Recently, attention has been directed at hedge funds’ online reputation because potential investors use Google search as their first step in researching funds or managers. Since the search engine makes hedge funds much more transparent, investors can link hedge funds’ past performance to expected performance in the future when making investment decisions, which makes reputation a bigger concern in the hedge fund industry than before. Although hedge funds have many characteristics that are different from those of mutual funds, the insight of reputation and the tractable approach we provide in this paper can be used to analyze a hedge fund’s reputation.\(^7\) Our model could also be used to analyze the reputations

\(^6\)A fund’s age is also used to measure the fund’s reputation in Gerken, Starks, and Yates (2014). However, we cannot discuss the fund’s age in our model. First, an old fund has a better reputation because it survives for a long time. To analyze the survival of a fund, we need a model with fixed operating costs and the fund’s decision to exit. Such a model, though potentially interesting, is beyond the scope of this paper. Second, we focus on stationary equilibria, in which age will not play a role.

\(^7\)In the hedge fund industry, the incentive fee and the high-water mark are important parts of the fund’s compensation scheme. These two characteristics offer hedge fund managers option values, which require a different
of financial analysts, consultants, credit rating agencies, and auditors, where information is the product in these industries.

This paper enriches the reputation literature by studying a discrete-time infinite-horizon model in which reputation is defined as the market belief over an endogenous variable. In a recent paper, Board and Meyer-ter-Vehn (2013) study a model in which a firm’s reputation is the consumers’ belief about the quality of its product. Their model is different from ours in the following aspects. First, in their setting, a shock periodically arrives and resets the quality of the product; upon the arrival of the shock, the firm’s current effort determines the new quality. Hence, in their model, the firm controls the quality stochastically so he plays pure strategy in the equilibrium, while in our model, the uninformed fund can directly choose whether to be informed, which naturally leads mixing in the equilibrium.\(^8\) In addition, in their model, the firm’s reputation always responds to good/bad signals. However, in our “work-work-shirk”, the fund’s reputation changes only if the current signal is different from the previous signal.\(^9\)

The remainder of this paper is organized as follows. In Section 2, we present an infinite-horizon reputation model. Section 3 solves a “work-work-shirk” stationary equilibrium. Section 4 describes the equilibrium properties, which provide theoretical explanations to salient empirical facts. Section 5 concludes.

2 The Model

**Fund Manager.** A fund manager may or may not have superior information for arbitrage. In each period, the uninformed fund decides whether to acquire information. Denote by \(\sigma_t\) the probability that the uninformed manager acquires information. If the uninformed manager chooses not to acquire information, the manager remains uninformed; if the manager acquires information, the manager will become informed in period \(t\), but she needs to pay an effort cost \(c > 0\). We assume that the manager knows whether she knows, but this is her private information. The fund manager’s information acquisition behavior is unobservable to investors.

**Investment Outcomes.** In every period, the fund’s investment has two possible outcomes:

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\(^8\)Halac and Prat(2014) adopts a similar setting to study a dynamic inspection game between a worker and a manager.

\(^9\)Dilme (2012) proposed an alternative model of firm reputation where actions have lasting effects because of switching costs. Bohren (2013) also considers a reputation model in which the reputation of a firm is the persistent quality of its product. However, in her model, there is no asymmetric information. A firm’s reputation is an observable quality stock, which can be enhanced through the firm’s costly investment.
After fees, it either outperforms or underperforms the market. We call the former a “good outcome” and the latter a “bad outcome.” The fund manager’s information superiority determines the ex-ante distributions of the investment outcomes. If the manager is informed, she will generate a good investment outcome with probability $q \in (1/2, 1)$; if the manager is uninformed, the fund outperforms the market with probability $1 - q$.

**Investors.** There is a continuum of investors with measure 1. In each period, investors form a public belief about whether the fund is informed or uninformed. We assume that in period $t$, investors have all records of the fund’s past performance, which is denoted by $h_{t-1}$; that is, $h_{t-1} = \{s_\tau\}_{\tau=1}^{t-1}$, where $s_\tau \in \{B, G\}$ for each $\tau = 1, 2, \ldots, t-1$.

**Reputation.** We interpret the public belief that the manager being informed as the manager’s reputation. Formally, the fund’s prior reputation in period $t$ is

$$x_t = \Pr(\text{the fund is informed at the beginning of period } t| h_{t-1}).$$

The law of motion of the reputation within period $t$ is described as follows. At the beginning of each period $t$, the manager’s prior reputation is $x_t \in (0, 1)$, and $x_1$ is exogenously given. Recall that the manager’s information acquisition choice is not observed by the investors, but the market believes that she acquires information with probability $\tilde{\sigma}_t$. Then the manager’s interim reputation in period $t$ is

$$z_t = x_t + (1 - x_t)\tilde{\sigma}_t, \quad (1)$$

where $\tilde{\sigma}_t$ is the market belief that the manager acquires information in period $t$. Once the investment outcome is realized, the market will update the belief about whether the manager is informed in period $t$ again:

$$
\begin{cases}
  z_t^g = \frac{qz_t}{qz_t + (1 - z_t)(1 - q)}, & \text{if the investment outcome is good;} \\
  z_t^b = \frac{(1 - q)z_t}{(1 - q)z_t + q(1 - z_t)}, & \text{if the investment outcome is bad.} 
\end{cases} \quad (2)
$$

**Information Depreciation.** An informed manager in period $t$, however, will become uninformed in period $t + 1$ with probability $\lambda \in (0, 1)$. Given an interim reputation $z_t$ in period $t$, the prior reputation in period $t + 1$ is:

$$x_{t+1} = (1 - \lambda)z_t^s, \quad (3)$$

where $s \in \{b, g\}$. 

7
**Payoff.** For simplicity, we assume that the manager’s flow revenue in period \( t \) is \( z_t \), so the manager’s discounted continuation payoff at time \( t \) is

\[
\sum_{\tau=t}^{\infty} \delta^{t-1} \left[ z_\tau - 1\{\text{the manager acquires information at time } \tau\}c \right],
\]

where \( \delta \in (0,1) \) is the discount factor, and \( 1\{\cdot\} \) is the indicator function. We summarize the timing in each period \( t \) in Figure 1.

![Figure 1: Timing](image)

We are interested in a **Markov perfect equilibrium**, in which

1. the manager’s strategy depends only on the history only through her prior reputation \( x \). Namely, there exists a function \( \sigma : [0,1] \to [0,1] \) such that \( \sigma_t = \sigma(x_t) \);

2. the equilibrium strategy \( \sigma(x_t) \) maximizes the fund’s expected continuation payoff, (4);

3. the manager’s strategy is consistent with the market belief: \( \hat{\sigma}_t = \sigma(x_t) \); and

4. over time, the reputation evolves according to Bayes’ rule: equation (1), (2), and (3).

Because we focus on (stationary) Markov strategies, we can ignore the subscript \( t \) in the rest of this paper, and we denote \( x \) (and \( z \)) as the current prior (and interim) reputation and denote \( x' \) (and \( z' \)) as the next period prior (and interim) reputation.

### 2.1 Discussion of Assumptions

Before analyzing the model, we discuss its special features. In our model, information superiority has short-run persistence only and will disappear in the long run; that is, \( \lambda \in (0,1) \). As we will argue in Subsection 3.1, information’s short-run persistence is necessary for investors to pay attention to the fund’s past performance. However, information superiority may disappear quickly in the financial market. This is mainly due to the financial market’s efficiency: Any piece
of information will be learned quickly by hundreds of fund managers, so mispricing discovered by a fund manager with information superiority will be corrected soon. Consequently, when the arbitrage opportunity disappears, the associated information of the fund manager becomes useless; that is, the fund manager loses her information superiority.

In our model, an uninformed manager can endogenously become informed by acquiring information. Therefore, the mutual fund’s reputation in our model is the market belief over a variable that is changing endogenously and exogenously. Empirical studies have shown that funds can gain information superiority by acquiring information in various ways. Kacperczyk, Sialm, and Zheng (2007) demonstrate that mutual funds can obtain important information by concentrating on some industries. Cohen, Frazzini, and Malloy (2008) show the importance for mutual fund managers to build social networks, which may generate critical information for their investments. However, information acquisition is costly. We assume the information acquisition cost is the fund manager’s private cost, such as time and effort, and such a cost is not counted into the fund’s investment outcome.

The fund’s interim reputation contains the probability of the uninformed fund’s information acquisition. Because the fund’s information acquisition behavior is not observable to investors, the equilibrium is self-confirming. That is, in the equilibrium, an uninformed fund acquires information with probability $\sigma$ if and only if the market believes that the probability of information acquisition is $\sigma$.

In each period, the fund’s size is assumed to be its interim reputation. This reduced form assumption is just for simplicity. It is sufficient to capture the fact that the number of investors is increasing in the fund’s interim reputation because, by definition, investors think the fund is more likely to possess the information. However, this assumption can be justified in a number of ways. For example, we can imagine that each individual investor can access a private signal from private research about whether the fund manager is informed. Then given the interim reputation, there exist private signal structures that lead to the equation of the fund’s interim reputation and the number of investors purchasing the fund. In fact, our result is robust if we assume that in each period, the size of the fund is an increasing function of the fund’s interim reputation.

Also for simplicity, we assume the fund’s period revenue is equal to the size of the fund. By this assumption, we capture the fact that the management fee is a fixed percentage of the total money under management. Moreover, for any exogenous and constant management fee structure that depends on the fund’s prior reputation, our result is robust.

Finally, we assume the fund’s productivity has constant returns to scale. In our model, the total capital available in the market is finite, so the fund’s revenue is well defined with a constant returns to scale production function. If a fund is a price taker in the financial market, that is, the
fund’s investment does not affect the assets’ prices in the market, the fund’s short-run excess rate of return from investment, the alpha, is independent of the volume it trades. Therefore, if the fund has information, the marginal rate of return of investments is constant, which means the fund’s productivity has a constant return to scale in the short run. In the literature, the assumption of fund-level decreasing returns to scale is used to explain the fact that funds’ performance is not persistent (Berk and Green, 2004). By assuming constant returns to scale, we show that, in addition to decreasing returns to scale, the fund’s incentive to acquire information also contributes to the impersistence of a fund’s performance.

3 Equilibrium Characterization

3.1 Preliminary Analysis

One of the most important features of our model is that the information has short-run persistence. That is, if the fund is informed today, it will remain informed with positive probability, so \( \lambda \in (0, 1) \). It is important for a fund to have incentives to acquire information. Consider the case of \( \lambda = 1 \). Because the fund’s information acquisition is unobservable to the market, for any given interim reputation in the current period, the fund’s prior reputation in the next period is 0. This is because the information has no persistence, therefore, today’s information surely becomes useless in the next period. Since the fund will be uninformed at the beginning of the next period, the fund’s continuation value is independent of whether the fund is informed or not in the current period. This argument, together with the fact that the interim reputation is also independent of the fund’s information superiority, implies that the reputation premium is 0. Since the effort cost is positive, the unique equilibrium when \( \lambda = 1 \) is that the fund never acquires information. Therefore, to analyze the reputation effects, we should assume that \( \lambda \) is sufficiently small.

Similarly, for an uninformed fund to have incentives to acquire information, a good investment outcome must be significantly more probable if the fund acquires information. Therefore, we maintain the assumption below in the rest of the paper.

Assumption 1. \((2 - \lambda)q > 1\).

Even with Assumption 1, when the fund’s interim reputation is too high, the effect of a good investment outcome on the prior reputation in the next period is dominated by that of the fund manager’s information depreciation. The reason is that the fund’s information depreciates linearly with probability $1 - \lambda$, while the effect of a good outcome is concave. Specifically, define $\hat{\omega}$ as the interim reputation such that the next period prior reputation following a good investment outcome is just $\hat{\omega}$. Then

$$\hat{\omega} = (1 - \lambda) \hat{\omega}^g = (1 - \lambda) \frac{q \hat{\omega}}{q \hat{\omega} + (1 - q)(1 - \hat{\omega})}.$$ 

Simple algebra implies that $\hat{\omega} = 1 - \frac{\lambda q}{2q - 1} \in (0, 1)$ by Assumption 1. Therefore, conditional on a good investment outcome, when the manager’s interim reputation is smaller than $\hat{\omega}$, the manager’s prior reputation in the next period will be higher than the current interim reputation. Conversely, the fund’s prior reputation in the next period will be smaller than its current interim reputation, when its current interim reputation is greater than $\hat{\omega}$. This intuition can be seen from Figure 2 and is summarized in Lemma 1 below.

Figure 2: The determination of $\hat{\omega}$. The thick curve represents the posterior reputation following a good performance by considering the information depreciation.

**Lemma 1.** Under Assumption 1, following a good investment outcome, a fund’s prior reputation
in the next period is

\[ x' = (1 - \lambda)z^g \begin{cases} < z, & \text{if } z > \hat{\omega}; \\ = z, & \text{if } z = \hat{\omega}; \\ > z, & \text{if } z < \hat{\omega}. \end{cases} \]

Denote by \( V_I(x) \) the fund’s continuation value if it is currently informed, and denote by \( V_U(x) \) the fund’s continuation value if it is currently uninformed and chooses to remain uninformed. Then,

\[
V_I(x) = z + \delta q [(1 - \lambda) V_I((1 - \lambda)z^g) + \lambda \max \{ V_U((1 - \lambda)z^g), V_I((1 - \lambda)z^g) - c \}] \\
+ \delta (1 - q) [(1 - \lambda) V_I((1 - \lambda)z^b) + \lambda \max \{ V_U(z^b), V_I((1 - \lambda)z^b) - c \}],
\]

(5)

and

\[
V_U(x) = z + \delta(1 - q) \max \{ V_U((1 - \lambda)z^g), V_I((1 - \lambda)z^g) - c \} \\
+ \delta q \max \{ V_U((1 - \lambda)z^b), V_I((1 - \lambda)z^b) - c \}.
\]

(6)

The fund’s flow payoff equals the fund’s interim reputation \( z = x + (1 - x)\hat{\sigma} \). Note that the interim reputation is not only affected by the fund’s prior reputation but also by the market belief about the uninformed fund’s information acquisition. The interim reputation, however, does not depend on the fund’s current information acquisition choice or the fact that it is informed or not, since neither its knowledge nor its action is observed by the market. Though an uninformed fund cannot affect its current flow payoff, it can obtain a higher continuation value by acquiring information. There are two benefits from information acquisition. First, the fund can generate a good outcome with a higher probability, so the fund can increase its prior reputation in the next period. Second, if the fund is informed in the current period, with positive probability \( 1 - \lambda \), the fund is still informed in the next period.

Lemma 2 below shows that in an equilibrium, a fund will not acquire information for sure.

**Lemma 2.** *In any equilibrium, \( \sigma(x) < 1 \) for each \( x \in [0, 1] \).*

The conclusion drawn in Lemma 2 is due to the fact that the fund is forward-looking. Suppose there is an equilibrium in which the uninformed fund with prior reputation \( x \) acquires information for certain, i.e., \( \sigma(x) = 1 \). In such an equilibrium, the market can rationally anticipate the fund’s behavior and form the interim belief \( z = x + (1 - x) \cdot 1 = 1 \). Consequently, the posterior reputation of the fund is \( z^g = z^b = 1 \), and the next period prior reputation of the fund is \( 1 - \lambda \), regardless of the current period’s investment outcome. Then the benefit of information acquisition depends
only on the difference of continuation values between an informed fund and an uninformed fund at prior reputation $1 - \lambda$. Such a difference of continuation values must be strictly greater than $c$ to support $\sigma(x) = 1$, which implies that an uninformed fund will acquire information for sure if its prior reputation is $1 - \lambda$. Because the fund discounts future values by $\delta < 1$, and the probability that an informed fund becomes uninformed is strictly less than 1, the difference of continuation values between an informed fund and an uninformed fund at the prior reputation $1 - \lambda$ will be strictly less than $c$, which leads to the contradiction. With Lemma 2, in the rest of the paper, the statement that “the fund acquires information” means the fund acquires information with positive probability.

Note that $V_U(\cdot)$ in Equation (6) is not the value function of the uninformed fund but the continuation value by assuming that the fund chooses to remain uninformed. The value function of an uninformed fund is $\max\{V_U(x), V_I(x) - c\}$. However, because of Lemma 2, in any equilibrium, we have

$$\max\{V_U(x), V_I(x) - c\} = V_U(x)$$

for any $x \in [0, 1]$. Consequently, on any equilibrium, Equations (5) and (6) can be simplified as

$$V_I(x) = z + \delta q\{[(1 - \lambda)V_I((1 - \lambda)z^b) + \lambda V_U((1 - \lambda)z^b)]$$

$$+ \delta(1 - q)[(1 - \lambda)V_I((1 - \lambda)z^b) + \lambda V_U(z^b)]\},$$

and

$$V_U(x) = z + \delta(1 - q)V_U((1 - \lambda)z^b) + \delta qV_U((1 - \lambda)z^b).$$

Given the prior reputation $x$, we denote $V_I(x) - V_U(x)$ as the reputation premium of the fund. Then, the fund’s optimal information acquisition rule is

$$\sigma(x) \begin{cases} 
= 0, & \text{if } V_I(x) - V_U(x) < c; \\
\in [0, 1], & \text{if } V_I(x) - V_U(x) = c; \\
= 1, & \text{if } V_I(x) - V_U(x) > c.
\end{cases}$$

That is, an uninformed fund acquires information if and only if the reputation premium is greater than the information acquisition cost. Notice that the reputation premium depends on the market belief $\hat{\sigma}$ via the interim reputation $z$. Put differently, the fund’s incentive to acquire information depends on whether the market believes it will do so, which is the self-confirming property of our equilibrium. Because the reputation premium is bounded in any equilibrium, $\sigma(x) = 1$ for any $x \in [0, 1]$ when $c$ is large. To avoid a trivial case, in the rest of the paper, we assume that $c$ is small.
3.2 A “Work-Work-Shirk” Equilibrium

Consider the following “work-work-shirk” strategy.

\[ \sigma(x) = \begin{cases} 
  \frac{\omega_0 - x}{1 - x}, & \text{if } x \in [0, \omega_1]; \\
  \frac{\omega_0 - x}{1 - x}, & \text{if } x \in (\omega_1, \omega_0]; \\
  0, & \text{if } x \in (\omega_0, 1]. 
\end{cases} \]  

(9)

Here, \( \omega_0 \geq 1 - \lambda \), and

\[ \omega_1 = (1 - \lambda)\omega_0 \leq \hat{\omega}; \]

that is, fix the interim reputation \( x \) in the current period, \( \omega_1 \) is the fund’s next period prior reputation, if it generates a bad outcome in the current period. This proposed strategy could be represented by Figure 3 below.

Figure 3: The “work-work-shirk” strategy

In the strategy profile, when the fund’s prior reputation \( x \leq \omega_1 \), the fund will acquire information with probability \( (\omega_1 - x)/(1 - x) \), leading to the interim reputation \( z = x + (1 - x)\sigma(x) = \omega_1 \). In this case, the next period prior reputations following a good investment outcome and a bad investment outcome are \( (1 - \lambda)z^g \in (\omega_1, \omega_0] \) and \( (1 - \lambda)z^b < \omega_1 \), respectively. When the prior reputation is \( x \in (\omega_1, \omega_0] \), the fund will acquire information with probability \( (\omega_0 - x)/(1 - x) \) so that its interim reputation is \( z = \omega_0 \). Then, the next period prior reputations following a good investment outcome and a bad investment outcome are \( (1 - \lambda)z^g \in (\omega_1, \omega_0] \) and \( (1 - \lambda)z^b \in [0, \omega_1) \),
respectively. Finally, when the prior reputation is high such that \( x > \omega_0 \), the fund will certainly not acquire information. So its interim reputation is \( z = x \), and its posterior reputations are \((1 - \lambda)z^g, (1 - \lambda)z^b \in (\omega_1, \omega_0]\). Such a law of motion of the “work-work-shirk” strategy is represented by the automaton in Figure 4 below.

![Automaton Diagram](image-url)

**Figure 4:** The automaton representation of the “work-work-shirk” equilibrium

Given the proposed strategy profile, there are only two relevant interim reputations, \( \omega_0 \) and \( \omega_1 \), in our analysis. To save notations, we denote by \( V_K^i = V_K(\omega_i) \) the fund’s continuation value when the fund’s interim reputation is \( \omega_i \), and it is in the status \( K = I \) (informed) or \( K = U \) (uninformed). Let’s first consider the fund’s continuation value when its prior reputation is \( x > \omega_0 \). Since investors believe the fund will not acquire information, the fund’s interim reputation is \( z = x \).

\[
V_I^i(x) = x + \delta [(1 - \lambda)V_I^0 + \lambda V_U^0],
\]

\[
V_U^i(x) = x + \delta V_U^0.
\]

Regardless of whether the fund is informed or not, its flow payoff will be its interim reputation \( z = x \). Because \( z > \omega_0 \), when the fund gets a good investment outcome, the fund’s prior reputation in the next period will be less than \( 1 - \lambda \), so it will be less than \( \omega_0 \). This is because the fund’s information superiority disappears with probability \( 1 - \lambda \); as a result, no matter how likely the fund was informed in the previous period, the fund’s prior reputation in the next period cannot be greater than \( 1 - \lambda \). When the fund gets a bad investment outcome, the fund’s prior reputation in the next period will be less than \( 1 - \lambda \) but larger than \( \omega_1 \), since the fund’s current interim reputation is greater than \( \omega_0 \). Therefore, when the fund’s interim reputation is \( z > \omega_0 \), the fund’s prior reputation in the next period is \( \omega_0 \), which is independent of the fund’s performance in the current period. Then, when the fund is informed, the fund will still be informed in the next period with probability \( 1 - \lambda \); but with probability \( \lambda \), an informed fund will become uninformed. For an
uninformed fund, it will still be uninformed at the beginning of the next period. By Lemma 2, in an equilibrium, keeping uninformed is at least as good as acquiring information to the uninformed fund, so its continuation value is just $V^0_U$.

When the fund’s prior reputation is $x \in (\omega_1, \omega_0]$, the fund’s interim reputation will jump to $\omega_0$. So the fund with the prior reputation $x \in (\omega_1, \omega_0]$ has continuation values:

$$
V^I_I(x) = \omega_0 + \delta \left[ q(1 - \lambda)V^0_I + q\lambda V^0_U + (1 - q)(1 - \lambda)V^1_I + (1 - q)\lambda V^1_U \right],
V^U_I(x) = \omega_0 + \delta \left[ (1 - q)V^0_I + qV^1_I \right].
$$

When the fund’s prior reputation is $x \in [0, \omega_1]$, the fund’s interim reputation will jump to $\omega_1$. In this case, the fund’s continuation value is:

$$
V^I_I(x) = \omega_1 + \delta \left[ q(1 - \lambda)V^0_I + q\lambda V^0_U + (1 - q)(1 - \lambda)V^1_I + (1 - q)\lambda V^1_U \right],
V^U_I(x) = \omega_1 + \delta \left[ (1 - q)V^0_I + qV^1_I \right].
$$

If the strategy profile described in Equation (9) is an equilibrium, we must have $\omega_1 < \hat{\omega}$. Suppose $\omega_1 \geq \hat{\omega}$, then when the fund’s interim reputation is $\omega_1$, the fund’s next period prior reputation will be lower than $\omega_1$, regardless of whether the investment outcome is good or bad, by the definition of $\hat{\omega}$. Then, according to the proposed strategy profile, the fund’s next period interim reputation is $\omega_1$, independent of the current period investment outcome. This will break the equilibrium because the fund with a prior reputation less than $\omega_1$ will certainly not acquire information. Therefore, to support the existence of $\omega_0$ and $\omega_1$ and thus the equilibrium, we require that the distance between $1 - \lambda$ and $\hat{\omega}$ is small enough, such that when the fund with the current interim reputation $1 - \lambda$ gets a bad outcome, the fund’s next period prior reputation is smaller than $\hat{\omega}$. This requirement can be rewritten as $1 < \lambda + q(1 + q)$. By Assumption 1, $q > 1/(2 - \lambda)$. So a sufficient condition is $\frac{1}{2 - \lambda} \geq 1 - \lambda$, which is equivalent to $f(\lambda) = 1 - 7\lambda + 5\lambda^2 - \lambda^3 \leq 0$.

We can show that $f(\lambda)$ is strictly decreasing for all $\lambda \in (0, 1)$, $f(0) > 0$, and $f(1) < 0$, so there is a unique $\hat{\lambda} \approx 0.16$, such that $f(\hat{\lambda}) = 0$. Therefore, we will maintain Assumption 2 below.

**Assumption 2.** $\lambda \geq \hat{\lambda}$.

The requirement that $\omega_1 < \hat{\omega}$ also sets an upper bound for us to choose $\omega_0$. That is, the supremum of $\omega_0$ to support the equilibrium will be the interim reputation, with which the fund reaches the next period prior reputation $\hat{\omega}$ after getting a bad outcome. This supremum is the solution to the equation $(1 - \lambda)\frac{(1 - q)x}{(1 - q)x + q(1 - x)} = \hat{\omega}$, which is denoted by

$$
\omega = \frac{q(2 - \lambda) - 1}{(1 - \lambda)(1 - q)(2q - 1) + 2q[q(2 - \lambda) - 1]}.
$$
Therefore, we can only choose \( \omega_0 \in [1 - \lambda, \bar{\omega}) \).

In the strategy profile described in Equation (9), an uninformed fund will be indifferent between acquiring information and remaining uninformed when its interim reputation is \( \omega_0 \) or \( \omega_1 \). Hence, if this strategy profile is an equilibrium, we must have

\[
V_I^0 - V_U^0 = V_I^1 - V_U^1 = c. \tag{10}
\]

In addition, since the fund with the prior reputation greater than \( \omega_0 \) will certainly not acquire information,

\[
V_I(x) - V_U(x) < c. \tag{11}
\]

Figure 5 illustrates the fund’s continuation value for any prior reputation.

Figure 5: The Equilibrium Continuation Value of the Manager

Note that an informed (uninformed) fund’s payoff depends on its interim reputation \( z \) instead of its prior reputation \( x \). For an informed (uninformed) fund whose prior reputation is \( x \in (0, \omega_1] \), its interim reputation is \( \omega_1 \) regardless of its prior reputation, so its continuation value is constant in such an interval. The same argument applies in the interval \((\omega_1, \omega_0] \). For a fund whose prior reputation is \( x \in (\omega_0, 1), \) the market belief is \( \hat{\sigma} = 0 \), so its interim reputation is equal to its prior reputation, and its continuation value is a linear function of its prior reputation.

Equation (10) will help to pin down \( \omega_0 \), which will determine the whole equilibrium construction. Specifically, Equation (10) implies that

\[
\frac{\delta(2q - 1)}{1 - \delta(1 - \lambda)} \left[ \omega_0 - (1 - \lambda) \frac{(1 - q)\omega_0}{(1 - q)\omega_0 + q(1 - \omega_0)} \right] = c. \tag{12}
\]
The left-hand side of Equation (12) is a continuous function over \([1 - \lambda, \bar{\omega}]\); therefore, the left-hand side of Equation (12) has its maximum and minimum over \([1 - \lambda, \bar{\omega}]\), which are denoted by \(\bar{y}\) and \(\underline{y}\), respectively. We characterize the “work-work-shirk” equilibrium in the proposition below.

**Proposition 1.** Suppose the parameter vector \((\delta, q, \lambda)\) satisfies Assumption 1 and Assumption 2. There exists a pair \((\underline{y}, \bar{y})\) with \(0 < \underline{y} < \bar{y}\), such that for each \(c \in (\underline{y}, \bar{y})\), there exists a “work-work-shirk” stationary equilibrium described in Equation (9), where \(\omega_0\) is pinned down by Equation (12).

Some remarks about Proposition 1 are in order. First of all, the key equilibrium condition is Equation (10). Because the flow payoffs to the informed fund and the uninformed fund are the same, the difference in values at any prior reputation depends only on continuation values. More important, given the proposed strategy profile, regardless of whether the prior reputation is \(\omega_0\) or \(\omega_1\), if the fund receives a good outcome, its interim reputation in the next period is \(\omega_0\); if the fund receives a bad outcome, its interim reputation in the next period is \(\omega_1\). Since the conditional probability of a good outcome is independent of the fund’s reputation, the difference between an informed fund’s continuation value and an uninformed fund’s continuation value is independent of its reputation.

Second, the “work-work-shirk” equilibrium requires medium levels of the effort cost; that is, \(c \in (\underline{y}, \bar{y})\). When the effort cost is too large, we cannot have an equilibrium in which the fund acquires information. Because the reputation premium is bounded, the reputation premium will be dominated by the effort cost when the effort cost is too large. On the other hand, when the effort cost is too small, the “work-work-shirk” strategy is not an equilibrium. Because \(\omega_0 \geq 1 - \lambda\) and \(\omega_1 < \hat{\omega}\), their difference \(\omega_0 - \omega_1\) is bounded below by \((1 - \lambda) - \hat{\omega}\). From Equation (10), the reputation premium at interim reputations \(\omega_0\) and \(\omega_1\) is determined by \(\omega_0 - \omega_1\), so if the effort cost is too small, the fund will acquire information for sure at interim reputations \(\omega_0\) and \(\omega_1\), which contradicts Lemma 2.

### 3.3 Other Equilibria

In section 3.2, we characterize a “work-work-shirk” equilibrium in which there are two “work” regimes and one “shirk” regime. In reputation models, there is a “shirk” regime when the agent’s reputation is very high so that his incentive is too small to work. However, one may wonder if it is reasonable to focus on the equilibrium with two “work” regimes. In this subsection, we argue that the model has neither a “work-shirk” equilibrium nor an equilibrium with more than two “work” regimes. More details are presented in Appendix B.

In an equilibrium with \(N\) “work” regimes \((N \neq 2)\),
• there is a decreasing sequence \( \{\omega_k\}_{k=0}^N \) such that \( \omega_N = 0 \), and \( \omega_0 \leq 1 \).

• when the uninformed fund’s reputation \( x \) is in the \( k \)th “work” regime \( (\omega_k, \omega_{k-1}] \), it acquires information with probability \( \sigma(x) = \frac{\omega_{k-1} - x}{1 - x} \) so that its interim reputation is \( \omega_{k-1} \), for \( k = 1, 2, ... N - 1 \), and a good performance pushes the fund’s reputation up to regime \((k - 1)\)th regime, and a bad performance leads it down to regime \((k + 1)\)th regime.

• when the uninformed fund’s reputation is \( x \in (\omega_0, 1] \), it does not acquire information.

When \( N = 1 \), it is a “work-shirk” equilibrium in which the fund’s interim reputation will be \( \omega_0 \) after a period in which its posterior reputation is lower than or equal \( \omega_0 \). When \( N > 2 \), the fund’s reputation jumps up step-by-step as long as its investment performance is good, but its reputation is always lower than \( \omega_0 \).

We show that such equilibria do not exist if (1) \( N = 1 \) under some mild conditions\(^{11}\), or (2) \( N > 2 \). The reason is as follows. For the equilibrium in which \( N = 1 \), \( x = \omega_0 \) is an absorbing state so that the fund’s reputation is constant regardless of its investment performance. That implies that the uninformed fund has no incentive to acquire information with positive probability, which violates the equilibrium requirement. For equilibria in which \( N > 2 \), one has to ensure that the uninformed fund is indifferent between acquiring information or not in \( N \) states, which requires \( N \) equations holding. We show that it is impossible as long as \( N > 2 \).

Notice that we do not claim that the “work-work-shirk” equilibrium is the unique Markov perfect equilibrium. The family of equilibrium we study is featured with multiple “blocked work regimes”: In each state “work” regime, \( x \in (\omega_{k+1}, \omega_k] \), the fund’s interim reputation is \( z = \omega_k \). We focus on such a family of equilibrium because it is intuitive and technically tractable. There may exist equilibrium in which the fund’s interim reputation does not equal \( \omega_k \). We leave this issue to future research.

4 Reputation Effects

The “work-work-shirk” equilibrium characterized in Proposition 1 shows that a fund’s initial reputation has great effects on an uninformed fund’s incentives to acquire information. Because a fund’s information acquisition decision determines its ex-ante investment outcome, we should expect that the fund’s reputation has significant causal effects on the fund’s performance, size, and cash flows. In this section, we first analyze how to explain empirical measures of funds’ reputations

\(^{11}\)Without further restrictions, we cannot prove or disprove the existence of a “work-shirk” equilibrium.
in existing studies. Then we analyze the reputation effects in the “work-work-shirk” equilibrium and provide potential explanations for some empirical observations.

4.1 Reconcile Existing Measures of a Fund’s Reputation

A fund’s past performance is usually employed to measure the fund’s reputation. Ippolito (1992) and Gerken, Starks, and Yates (2014), among others, treat past performance as one proxy of a fund’s reputation when empirically analyzing investors’ fund purchasing choices. Berk and Green (2004) assume that more capital will flow into the fund when the fund has a good investment performance. All these studies implicitly assume that past investment performance reflects the quality of the fund’s money management and that such a quality is persistent. As a result, if a fund performed well before, it should perform well in the future. These studies, however, overlook the fund’s incentives. Hence, the fund’s reputation will not have causal effects on the fund’s future performance in these frameworks.

Past performance is an important factor in determining the fund’s prior reputation in our model. This is due to the information’s short-run persistence. In our model, if a fund’s investment outcome is good today, investors will believe that the fund is more likely to have superior information today. If the fund is informed today, the fund will be informed tomorrow with probability $1 - \lambda$, without considering the probability that an uninformed fund manager acquires information.

In the “work-work-shirk” equilibrium, the fund’s prior reputation determines the fund’s information acquisition decision, so the fund’s past performance has causal effects on the fund’s investment performance. However, a fund’s past performance is not the best predictor of the fund’s future performance because we should take into account the probability that an uninformed fund acquires information. Let’s consider two funds with prior reputations $x_1, x_2 \in (\omega_1, \omega_0)$, where $x_1 > x_2$. Assume that the prior reputation is uniquely determined by past performance. Just from these two funds’ prior reputations, the expected probabilities that the funds will generate good investment outcomes are $x_1q + (1 - x_1)(1 - q)$ and $x_2q + (1 - x_2)(1 - q)$, respectively. That is, if we ignore the fund’s information acquisition behavior, we should predict that the fund with prior reputation $x_1$ should perform better than the fund with prior reputation $x_2$. However, in the equilibrium, the fund with prior reputation $x_1$ acquires information with probability $(\omega_0 - x_1)/(1 - x_1)$, while the fund whose prior reputation is $x_2$ acquires information with a higher probability, $(\omega_0 - x_2)/(1 - x_2)$. Consequently, these two funds have the same interim reputation $\omega_0$, which means these two funds have the same ex-ante performance.

Another common proxy of a fund’s reputation is the fund’s size (for example, see Gerken, Starks, and Yates, 2014). In the “work-work-shirk” equilibrium, the fund’s interim reputation is the best predictor of the fund’s performance. Since the fund’s interim reputation equals the fund’s
size, it follows that the fund’s current size is the best predictor of its performance in the current period. That is, a fund with a larger size is expected to perform better. This is consistent with empirical findings in Elton, Gruber, and Blake (2011). However, the fund’s size, as the interim reputation, has no causal effect on the fund’s performance because the fund’s size is endogenously determined in the equilibrium. Also note that investors cannot use the information of a fund’s size to smartly make purchasing decisions, because when an investor receives information about the fund’s size, the investor has lost the current period’s purchasing opportunity. This is similar to the argument in Zheng (1999) that investors cannot use Gruber’s “smart money” (1996) information to select funds.

### 4.2 Flow-Performance Relationship

A fund’s net flow in any period $t$ is the difference between the fund’s size in period $t$ and that in period $t - 1$ (ignoring the fund’s natural growth). In our model, the fund’s size in any period $t$ is just the fund’s interim reputation $z_t$. Therefore, the fund’s net flow $f_t$ in period $t$ is:

$$f_t = z_t - z_{t-1}.$$  

In this subsection, we discuss how the fund’s past performance affects the fund’s current flow. Since we assume the fund’s investment outcome is either good or bad, we are not trying to explain the cross-sectional convexity of flow-performance relationship that is first documented by Sirri and Tufano (1998). Instead, we focus on the time series flow-performance relationship for a specific fund.

In the “work-work-shirk” equilibrium characterized by Equation (9), a fund’s interim reputation $z_t$ in any period $t$ is determined by the fund’s prior reputation in period $t$. Since the fund’s size in period $t$ is assumed to be equal to the fund’s interim reputation in period $t$, the fund’s size in period $t$ is also determined by the fund’s prior reputation in period $t$. In the “work-work-shirk” equilibrium, the fund’s prior reputation can be pinned down by the fund’s performance in the previous period, so we have Lemma 3 below.

**Lemma 3.** In the “work-work-shirk” equilibrium described by Equation (9), in any period $t \geq 3$, the fund’s size is

$$z_t = \begin{cases} 
\omega_0, & \text{if } s_{t-1} = G; \\
\omega_1, & \text{if } s_{t-1} = B. 
\end{cases}$$  

(13)

The conclusion in Lemma 3 that a fund’s size is uniquely determined by the fund’s performance in the previous period is due to the information’s two special features. On one hand, the information superiority disappears with probability $\lambda$, so the prior reputation in any period is below $\omega_0$.
(since $\omega_0$ by construction is greater than $1 - \lambda$). On the other hand, a bad investment outcome leads to a low prior reputation in the next period that is at most $\omega_1$, while a good investment outcome leads to a prior reputation in the next period that is strictly greater than $\omega_1$. Then in the equilibrium, investors will take the fund’s likelihood to acquire information into account, so that the interim reputation is $\omega_0$ for any prior reputation between $\omega_1$ and $\omega_0$, and the interim reputation is $\omega_1$ for any prior reputation less than $\omega_1$. Therefore, after initial periods, a fund’s size depends only on the fund’s performance in the previous period.

The fund’s flow in any period $t$ is the net increment of the fund’s size from period $t - 1$ to period $t$. By Lemma 3, the fund’s size in period $t$ is determined by the fund’s performance in period $t - 1$ only, so the fund’s flow in period $t$ will depend on the fund’s performance in period $t - 1$ and that in period $t - 2$. Proposition 2 below summarizes the flow-performance relationship in the “work-work-shirk” equilibrium.

**Proposition 2.** In the “work-work-shirk” equilibrium described by Equation (9), in any period $t \geq 4$, the fund’s flow depends on the fund’s performance in the two most recent periods. In particular,

$$f_t = \begin{cases} 
\omega_1 - \omega_0 < 0, & \text{if } s_{t-1} = B \text{ and } s_{t-2} = G; \\
0, & \text{if } s_{t-1} = s_{t-2} = G \text{ or } s_{t-1} = s_{t-2} = B; \\
\omega_0 - \omega_1 > 0, & \text{if } s_{t-1} = G \text{ and } s_{t-2} = B.
\end{cases}$$

Proposition 2 has important qualitative empirical implications. First of all, after the initial periods, the fund’s flow in period $t$ always depends on the fund’s performance in recent periods. As a result, no matter how long the fund’s record is, the fund’s reputation will not converge. The driving forces of this implication are the information depreciation and the unobservable fund’s information acquisition. Because the information has only short-run persistence, the fund’s recent performance is most important when investors make inferences as to whether the fund is informed at the beginning of the current period. The investors also care about whether the fund, if it is uninformed, acquires information in the current period. If the information acquisition behavior is observable to investors, the fund’s performance in a period in which the fund acquires information will be irrelevant to the fund’s next period size and thus flow. Corollary 1 below summarizes this argument.

**Corollary 1.** Independent of the fund’s age, the fund’s performance in the recent periods are relevant to predict the fund’s flow.

Corollary 1 is supported by some empirical findings. Chevalier and Ellison (1997) document that even for funds older than 11 years, their current flows are significantly correlated to their
performance in recent periods. Coval and Stafford (2007) find that the most recent performance is significantly correlated to the current flow, even after controlling for a long history of performance.

The conclusion drawn in Corollary 1 differs from the convergence result in Bayesian learning models with persistent types, such as Holmstrom (1999) and Berk and Green (2004). In such models, the fund’s optimal size depends on the fund’s ability, which is the fund’s persistent type. Investors can learn the fund’s ability by observing the fund’s past sizes and performance. Then with a long record, the investors’ posterior belief about the fund’s true ability will converge to 1. This convergence result implies that after controlling for a long history of performance, the fund’s size should not change much by the most recent fund’s performance. In other words, in the Bayesian learning models with persistent types, the fund’s performance in recent periods can hardly affect the fund’s flow.

Proposition 2 also implies that the fund’s most recent performance predicts the fund’s current flow best, which has been widely supported by empirical evidence. Sirri and Tufano (1998) conclude that investors respond most strongly to the most recent fund history, based on the observation that investors’ reactions to performance are not markedly stronger for extreme performance measured after five years than after one year. Coval and Stafford (2007) also document that the fund’s investment return in period \( t-1 \) has the most important impact on the fund’s flow in period \( t \), followed by the fund’s investment returns in period \( t-2 \) and period \( t-3 \); investment returns before period \( t-4 \), however, have much smaller economic effects on the current flow. Corollary 2 below summarizes this empirical implication of the model.

**Corollary 2.** A fund’s performance in most recent periods has the most important impact on its current flow.

Finally, as noted by Berk and Tonks (2007), in the “work-work-shirk” equilibrium, the correlation of the fund’s performance in recent periods also has significant effects on the fund’s current flow. As in Berk and Tonks (2007), we define funds with \( s_{t-2} = s_{t-1} = B \) as *seasoned* bad funds and those with \( s_{t-2} = G \) and \( s_{t-1} = B \) as *unseasoned* bad funds. Similarly, funds with \( s_{t-2} = s_{t-1} = G \) are called *seasoned* good funds, and funds with \( s_{t-2} = B \) and \( s_{t-1} = G \) are called *unseasoned* good funds. Proposition 2 then implies that unseasoned funds have more significant flows than seasoned funds do. This implication is mainly due to the self-confirming property of the equilibrium. For example, the seasoned bad funds have a low interim reputation in period \( t-1 \), so the size is \( \omega_1 \) in period \( t-1 \). Then another bad performance \( s_{t-1} = B \) makes investors think the fund is more likely to be uninformed, which, together with the information depreciation, leads to a very low prior reputation in period \( t \). That is, \((1 - \lambda)\omega_1^b < \omega_1\). At this point, it seems that there should be significant flows out from the fund. However, given that the prior
reputation in period \( t \) is less than \( \omega_1 \) in this case, investors will believe that the fund acquires information with probability \( (\omega_1 - (1 - \lambda)\omega_b^1)/(1 - (1 - \lambda)\omega_b^1) \) so that the fund’s interim reputation in period \( t \) goes back to \( \omega_1 \) in period \( t \). Therefore, there are no significant cash outflows from seasoned bad funds. The self-confirming property, however, increases the flows into unseasoned good funds. A fund with a bad performance in period \( t - 2 \) has an interim reputation \( \omega_1 \) in period \( t - 1 \). The fund then generates a good investment outcome in period \( t - 1 \), which promotes the fund’s prior reputation to \( (1 - \lambda)\omega_g^1 \). By Lemma 1, because \( \omega_1 < \hat{\omega} \), \( (1 - \lambda)\omega_g^1 \) > \( \omega_1 \). Without considering the probability of the fund’s information acquisition, the fund’s flow in period \( t \) would be \((1 - \lambda)\omega_g^1 - \omega_1\). However, investors rationally believe that the fund acquires information with the probability \([\omega_0 - (1 - \lambda)\omega_g^1]/[1 - (1 - \lambda)\omega_g^1]\) so that the interim reputation of the fund in period \( t \) is \( \omega_0 \). Since \( \omega_0 \) is strictly greater than \((1 - \lambda)\omega_g^1\), the cash inflow is greater. These arguments lead to Corollary 3 below.

**Corollary 3.** In the “work-work-shirk” equilibrium, seasoned funds have insignificant flows, while unseasoned funds’ flows are more significant. Moreover, whether a fund has outflows or inflows depends on the fund’s most recent performance.

### 4.3 Reputation Effects on Performance

In this subsection, we discuss some cross-sectional implications of the model about the fund’s performance. To better explain documented empirical observations, we sort all funds into three groups according to their prior reputations: Bad funds, with prior reputations smaller than or equal to \( \omega_1 \); medium funds, with prior reputations between \( \omega_1 \) and \( \omega_0 \); and star funds, with prior reputations strictly greater than \( \omega_0 \). To provide a cross-sectional prediction, we also assume that there is a continuum of funds with measure 1 that are identical except for their past performance. Assume funds’ past performance are represented by their prior reputations, which are uniformly distributed over \([0, 1]\).

Let’s first analyze bad funds’ performance. In the “work-work-shirk” equilibrium, an uninformed fund with a prior reputation \( x \leq \omega_1 \) will acquire information with probability \((\omega_1 - x)/(1 - x)\), so its interim reputation is \( \omega_1 \). On the other hand, all medium funds will have an interim reputation \( \omega_0 \) in the “work-work-shirk” equilibrium, and all star funds’ interim reputations are equal to their prior reputations, which are greater than \( \omega_0 \). Therefore, the bad funds will perform worst. Two reasons lead to this prediction. First, a bad fund with bad past performance is less likely to have information at the beginning of the current period. More important, an uninformed bad fund acquires information with a low probability because investors do not believe it is likely to acquire information. In the “work-work-shirk” equilibrium, bad funds’ information
acquisition decisions are necessary to support the behaviors of medium funds and star funds. This rationalizes Carhart’s rules-of-thumb I (1997): Investors should avoid funds with persistently bad performance.

However, bad funds can be still alive because some investors are still purchasing them. Such investors are rational because an uninformed bad fund will acquire information with a positive probability. Therefore, the ex-ante average performance of bad funds in the current period, $\omega_1 q + (1 - \omega_1)(1 - q)$, is greater than the average past performance of bad funds, $\omega_1 / 2$.

Now, let’s compare the performance of medium funds and star funds. A medium fund with a prior reputation $x \in (\omega_1, \omega_0]$ acquires information with a probability $(\omega_0 - x)/(1 - x)$. Therefore, all medium funds will have the same interim reputation in the current period. Conversely, a star fund with a prior reputation $x > \omega_0$ will not acquire information, so its interim reputation is the same as its prior reputation. Denote by $D_0$ the difference between average past performance of star funds and those of medium funds, then

$$D_0 = \frac{\omega_0 + 1}{2} - \frac{\omega_1 + \omega_0}{2} = \frac{1 - \omega_1}{2}.$$ 

Also denote by $D_1$ the difference between the average expected current performance of star funds and those of medium funds, then

$$D_1 = \left[\frac{1 + \omega_0}{2} q + \left(1 - \frac{1 + \omega_0}{2}\right)(1 - q)\right] - \left[\omega_0 q + (1 - \omega_0)(1 - q)\right] = (2q - 1)\frac{1 - \omega_0}{2}.$$ 

Because $\omega_0 > \omega_1$ and $q \in (1/2, 1)$, we have Proposition 3 below, which shows that star funds are performing better than medium funds, but the difference between average star funds’ performance and average medium funds’ performance shrinks.

**Proposition 3.** In the “work-work-shirk” equilibrium,

$$D_1 > 0 \quad \text{and} \quad D_1 - D_0 < 0.$$ 

The mechanism leading to the unpredictability of funds’ past performance on their current performance in our model differs substantially from that in Berk and Green (2004). In Berk and Green (2004), star funds receive a huge amount of cash inflows, which drive down star funds’ performance because of the decreasing returns to scale production function. This will equalize the ex-ante current performance of star funds and those of medium funds instantly. In the “work-work-shirk” equilibrium of our model, the difference between star funds’ performance and medium funds’ performance shrinks because an uninformed star fund will not acquire information while an uninformed medium fund will acquire information with a positive probability. Star funds still perform better than medium funds in the short run because star funds are more likely to be informed at the beginning of the current period: Since the information has some persistence, star funds, which perform better in the past, are more likely to be informed.
5 Conclusion

In this paper, we model a mutual fund’s reputation as the market belief about whether the fund is informed about the current best investment strategy. Information has two important features: First, it has short-run persistence but will become outdated in the long run; second, an uninformed mutual fund can become informed by acquiring information. Therefore, our definition of a mutual fund reputation is the market belief over an endogenous variable. We then propose a new discrete time infinite-horizon model to analyze the reputation effects on the fund’s performance, size, and cash flows.

The model predictions provide potential explanations for several empirical observations in the mutual fund industry. First, the information features and the moral hazard affect the fund’s flow-performance relationship. A fund’s recent performance has significant effects on the fund’s flow, and such effects are independent of the fund’s age. The fund’s most recent performance is most relevant to the fund’s current flow. We also show that seasoned funds have insignificant flows, while unseasoned funds’ flows are more significant. Second, two documented empirical facts present in the equilibrium of our model: Bad funds perform worse, and there is no performance persistence for medium funds and star funds.

Our model provides sufficient structures to understand the empirical measures of a mutual fund’s reputation. In particular, a fund’s past performance contributes to the fund’s prior reputation, thereby causally affecting its current performance, size, and cash flows. However, a fund’s past performance neither equals the fund’s prior reputation nor contains the probability that an uninformed fund acquires information; therefore, past performance cannot provide a good prediction of the fund’s current performance. A fund’s size can best predict the fund’s current performance. However, the fund’s size is endogenously determined, so it does not have causal effects. In addition, investors cannot use the information of the fund’s size to select funds.

This paper has both applied and theoretical contributions. From an applied aspect, our model provides a framework for future studies of reputations in industries, where information or knowledge plays the most important role, for example, reputations of financial analysts, consultants, and investment banks. From a theoretical aspect, this paper enriches the reputation literature by studying an infinite-horizon discrete-time model in which reputation is defined as the market belief over an endogenous variable.
Appendix A  Omitted Proofs

Proof of Lemma 1. Under Assumption 1, \( \hat{\omega} \in (0, 1) \). Define function
\[
g(z) = (1 - \lambda) \frac{zq}{zq + (1 - z)(1 - q)} - z.
\]
By the definition of \( \hat{\omega} \), when \( z = \hat{\omega} \), \((1 - \lambda)z^g = z\). That is, \( g(\hat{\omega}) = 0 \). Then we just need to show that \( g(z) \) is strictly decreasing. Since \( (zq)/(zq + (1 - z)(1 - q)) \) is strictly concave, so \( g'(z) \) reaches the highest value when \( z = 0 \). Since \( q > 1/2 \),
\[
g'(0) = (1 - \lambda) \frac{q}{1 - q} - 1 < 0.
\]
Therefore, for all \( z \), \( g'(z) < 0 \). Therefore, \( g(z) \) is strictly decreasing. Hence, when \( z < \hat{\omega} \), \( g(z) > 0 \), implying that \((1 - \lambda)z^g > z\); when \( z > \hat{\omega} \), \( g(z) < 0 \), implying that \((1 - \lambda)z^g < z\).

Proof of Lemma 2. Suppose \( \sigma(x) = 1 \) for some \( x \). Then the manager with the prior reputation \( x \) will have the interim reputation
\[
z = x + (1 - x)\sigma(x) = 1.
\]
Then the manager’s reputation in the next period will be \( 1 - \lambda \), independent of the investment outcome. To support the equilibrium, we must have \( V_I(x) - V_U(x) \geq c \), which implies that
\[
V_I(1 - \lambda) - \max \{V_I(1 - \lambda) - c, V_U(1 - \lambda)\} \geq \frac{c}{\delta(1 - \lambda)}. \tag{15}
\]
If \( V_I(1 - \lambda) - c > V_U(1 - \lambda) \), Equation (15) above becomes
\[
V_I(1 - \lambda) - [V_I(1 - \lambda) - c] = c \geq \frac{c}{\delta(1 - \lambda)}.
\]
Since \( \delta < 1 \) and \( 1 - \lambda < 1 \), this equation obviously does not hold.

In the case that \( V_I(1 - \lambda) - c \leq V_U(1 - \lambda) \), Equation (15) becomes
\[
V_I(1 - \lambda) - V_U(1 - \lambda) \geq \frac{c}{\delta(1 - \lambda)} > c.
\]
As a result, \( \sigma(1 - \lambda) = 1 \) in the equilibrium under consideration. Then
\[
V_I(1 - \lambda) - V_U(1 - \lambda) = \delta(1 - \lambda)[V_I(1 - \lambda) - V_U(1 - \lambda)]
\]
which is a contradiction!
Proof of Proposition 1. From the description of the “work-work-shirk” strategy, we can see that only values of $V^0_I$, $V^0_U$, $V^1_I$, and $V^1_U$ matter. That is, in order to verify whether the proposed strategy is an equilibrium, we first need to solve these values. Consider the manager with a prior reputation $\omega_0$ first. Since the uninformed manager is believed not to acquire information at this reputation, her interim reputation is the same as the prior reputation. Then her continuation value is

$$V^I_I(\omega_0) = \omega_0 + \delta \left[ q(1-\lambda)V^0_I + q\lambda V^0_U + (1-q)(1-\lambda)V^1_I + (1-q)\lambda V^1_U \right],$$

$$V^U_I(\omega_0) = \omega_0 + \delta \left[ (1-q)V^0_U + qV^1_U \right].$$

Take difference, we have

$$V^I_I(\omega_0) - V^I_U(\omega_0) = \delta \left[ q(1-\lambda)(V^0_I - V^0_U) + (1-q)(1-\lambda)(V^1_I - V^1_U) + (2q-1)(V^0_U - V^1_U) \right] \quad (16)$$

Similarly, if we calculate the difference between the informed manager’s value and the uninformed manager’s value at the prior reputation $\omega_1$, we have

$$V^I_I(\omega_1) - V^I_U(\omega_1) = \delta \left[ q(1-\lambda)(V^0_I - V^0_U) + (1-q)(1-\lambda)(V^1_I - V^1_U) + (2q-1)(V^0_U - V^1_U) \right] \quad (17)$$

We can see that

$$V^I_I(\omega_0) - V^I_U(\omega_0) = V^I_I(\omega_1) - V^I_U(\omega_1) = c,$$

then the equilibrium requirement Equation (10) is satisfied because the manager will randomize when the prior reputation $x \in [0, \omega_1) \cup (\omega_1, \omega_0)$. Given Equation (10), when the manager’s prior reputation $x > \omega_0$, the difference between values of an informed manager and an uninformed manager is

$$V^I_I(x) - V^I_U(x) = \delta(1-\lambda)(V^0_I - V^0_U) < c,$$

because $\delta(1-\lambda) < 1$. That is, the equilibrium condition Equation (11) is satisfied. As a result, the uninformed manager with the prior reputation $x > \omega_0$ will choose not to acquire information.

With the equilibrium condition (10), Equation (16) implies

$$c = V^I_I(\omega_0) - V^I_U(\omega_0) = \delta \left[ q(1-\lambda)(V^0_I - V^0_U) + (1-q)(1-\lambda)(V^1_I - V^1_U) + (2q-1)(V^0_U - V^1_U) \right]$$

$$= \delta \left[ (1-\lambda)c + (2q-1)(V^0_U - V^1_U) \right],$$

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which implies that

\[ V^0_U - V^1_U = \omega_0 - \omega_1 = \frac{1 - \delta(1 - \lambda)}{\delta(2q - 1)} c. \]

By definition,

\[ \omega_1 = (1 - \lambda) \frac{(1 - q)\omega_0}{(1 - q)\omega_0 + q(1 - \omega_0)}, \]

so we have Equation (12)

\[ \omega_0 - (1 - \lambda) \frac{(1 - q)\omega_0}{(1 - q)\omega_0 + q(1 - \omega_0)} = \frac{1 - \delta(1 - \lambda)}{\delta(2q - 1)} c. \]

That is, for a given set of parameters, we can identify \( \omega_0 \), which is the starting point of the equilibrium construction.

Given the constructed \( \omega_0 \) and \( \omega_1 \), we know

\[
\begin{align*}
V^0_U &= \omega_0 + \delta \left[ (1 - q)V^0_U + qV^1_U \right], \\
V^1_U &= \omega_1 + \delta \left[ (1 - q)V^0_U + qV^1_U \right].
\end{align*}
\]

Therefore,

\[ V^0_U - V^1_U = \omega_0 - \omega_1. \]

By arranging terms, we can solve

\[
\begin{align*}
V^0_U &= \frac{1}{1 - \delta} \left[ \omega_0 - \delta q \left( V^0_U - V^1_U \right) \right] \\
&= \frac{1}{1 - \delta} \left[ \omega_0 - \delta q \left( \omega_0 - \omega_1 \right) \right] \\
&= \frac{(1 - \delta q)\omega_0 + \delta q\omega_1}{1 - \delta}.
\end{align*}
\]

and

\[
\begin{align*}
V^1_U &= V^0_U - (\omega_0 - \omega_1) \\
&= \frac{\delta(1 - q)\omega_0 + (1 - \delta(1 - q))\omega_1}{1 - \delta}.
\end{align*}
\]

Then we can calculate that

\[
\begin{align*}
V^0_I &= c + \frac{(1 - \delta q)\omega_0 + \delta q\omega_1}{1 - \delta}, \\
V^1_I &= c + \frac{\delta(1 - q)\omega_0 + (1 - \delta(1 - q))\omega_1}{1 - \delta}.
\end{align*}
\]

Given the derived continuation values, an uninformed fund will be indifferent between acquiring information and remaining uninformed when its prior reputation is below \( \omega_0 \); the fund will
certainly not acquire information if its prior reputation is greater than \( \omega_0 \). By construction, the fund’s strategy is consistent with the market belief. Therefore, the proposed strategy profile is a stationary equilibrium.

Proof of Proposition 3. We assume that an informed fund will become uninformed with probability \( \lambda \), so the prior reputation

\[ x_t \leq (1 - \lambda)z_t^0 \leq 1 - \lambda \leq \omega_0, \forall t \geq 2. \]

Then in the equilibrium, for any \( t \geq 2 \),

\[ z_t = \begin{cases} 
\omega_0, & \text{if } x \in (\omega_1, \omega_0] \\
\omega_1, & \text{if } x \in (0, \omega_1]. 
\end{cases} \]

By construction, \( \omega_1 = (1 - \lambda)\omega_0^0 \) implies that if \( z_t = \omega_0 \),

\[ z_{t+1} = \begin{cases} 
\omega_0, & \text{if } s_t = G; \\
\omega_1, & \text{if } s_t = B. 
\end{cases} \]

Because \( \omega_1 < \omega_1 \), so \( (1 - \lambda)\omega_0^0 > \omega_1 \). That is, when the fund’s interim reputation is \( \omega_1 \) in period \( t \), the fund’s prior reputation in period \( t + 1 \) is strictly greater than \( \omega_1 \). Hence, \( z_t = \omega_1 \) and \( s_t = G \) lead to \( x_{t+1} \in (\omega_1, \omega_0) \). Then in the equilibrium, \( z_{t+1} = \omega_0 \). Conversely, \( z_t = \omega_1 \) and \( s_t = B \) will lead to \( x_t < \omega_1 \). Consequently, \( z_{t+1} = \omega_1 \) in this case in the equilibrium. Therefore, when \( z_t = \omega_1 \),

\[ z_{t+1} = \begin{cases} 
\omega_0, & \text{if } s_t = G; \\
\omega_1, & \text{if } s_t = B. 
\end{cases} \]

Then the interim reputation in period \( t + 1 \) depends on the performance in period \( t \) only. Since the argument above holds for any period \( t \geq 2 \), we have, for any period \( t \geq 3 \),

\[ z_t = \begin{cases} 
\omega_0, & \text{if } s_{t-1} = G; \\
\omega_1, & \text{if } s_{t-1} = B, 
\end{cases} \]

which is exactly Equation (13).

Proof of Proposition 2. By Lemma 3, if \( s_{t-1} = B \) and \( s_{t-2} = G \), we have \( f_t = z_t - z_{t-1} = \omega_1 - \omega_0 < 0 \). If \( s_{t-1} = s_{t-2} = B \) or \( s_{t-1} = s_{t-2} = G \), \( z_t = z_{t-1} \), which implies that \( f_t = 0 \). Finally, if \( s_{t-1} = G \) and \( s_{t-2} = B \), we have \( f_t = z_t - z_{t-1} = \omega_0 - \omega_1 > 0 \).
Proof of Proposition 3. $D_1 > 0$ directly follows from the assumptions $q \in (1/2, 1)$ and $\omega_0 < 1$. We also have
\[
\frac{1 - \omega_1}{2} > \frac{1 - \omega_0}{2},
\]
because $\omega_0 > \omega_1$. Then $2q - 1 < 1$ implies that $D_1 < D_0$. \qed

Appendix B Other Equilibria (Not for publication)

In this section, first we show that there exists no “work-shirk” equilibrium in which
\[
\sigma(x) = \begin{cases} \frac{\omega_0 - x}{1 - x}, & \text{if } x \leq \omega_0; \\ 0, & \text{if } x > \omega_0, \end{cases}
\]
for some $\omega_0 \in [0, 1]$ under a mild condition.

**Proposition 4.** There exists no “work-shirk” equilibrium for $\omega_0 \geq (1 - \lambda)z^b(\hat{w})$.

**Proof.** Suppose there is a “work-shirk” equilibrium s.t.
\[
\sigma(x) = \begin{cases} \frac{\omega_0 - x}{1 - x}, & \text{if } x \leq \omega_0; \\ 0, & \text{if } x > \omega_0, \end{cases}
\]
for some $\omega_0 \in [0, 1]$. We want to show that such an equilibrium does not exist. There are three cases: (1) $\omega_0 \geq 1 - \lambda$, (2) $\omega_0 \in [\hat{\omega}, 1 - \lambda)$, and (3) $(1 - \lambda)z^b(\hat{w}) \leq \omega_0 < \hat{\omega}$.

**Case I.** Suppose that $\omega_0 \geq 1 - \lambda$, then for any $x \leq \omega_0$, $\sigma(x) \geq 0$, and $V_I(x) - V_U(x) = c > 0$. However, for any $x \leq \omega_0$, the fund’s interim reputation is $z = \omega_0$. Because both $(1 - \lambda)z^g(\omega_0)$ and $(1 - \lambda)z^b(\omega_0)$ are strictly less than $1 - \lambda \leq \omega_0$, so that the fund’s next period interim reputation is $\omega_0$ again regardless of its current performance. Hence, for each $x \leq \omega_0$, we have
\[
V_U(x) = V_U^0 = \omega_0 + \delta V_U^0,
\]
and
\[
V_I(x) = V_I^0 = \omega_0 + \delta(1 - \lambda)V_I^0 + \delta\lambda V_U^0.
\]
Hence, we must have that, for each $x \leq \omega_0$,
\[
V_I(x) - V_U(x) = V_I^0 - V_U^0 = c = \delta(1 - \lambda)[V_I^0 - V_U^0] = \delta(1 - \lambda)c,
\]
which is impossible because $\delta(1 - \lambda) < 1$ and $c > 0$. 31
Case II. Suppose that $\omega_0 \in [\hat{\omega}, 1 - \lambda)$. Consider a fund with a prior reputation $\omega_0$. To support the equilibrium, $V_I^0 - V_U^0 = c > 0$. However, at $\omega_0$, the next period prior reputations are $(1 - \lambda)z^g$ following a good investment outcome or $(1 - \lambda)z^b$ following a bad investment outcome. Since $(1 - \lambda)z^b < (1 - \lambda)z^g \leq \omega_0$, we have

$$V_I^0 - V_U^0 = c = \delta(1 - \lambda)(V_I^0 - V_U^0) = \delta(1 - \lambda)c,$$

according to the proposed strategy profile. However, this cannot be true, since $\delta, \lambda \in (0, 1)$ and $c > 0$.

Case III. Suppose that $(1 - \lambda)z^b(\hat{\omega}) \leq \omega_0 < \hat{\omega}$. In the proposed strategy profile, for all $x \leq \omega_0$, the fund is randomizing, so that the interim reputation is just $\omega_0$. This implies that the fund’s reputation premium at $\omega_0$ must be equal to the information acquisition cost; otherwise, the fund with a prior reputation $x \leq \omega_0$ will not randomize. Therefore, we have

$$V_I^0 - V_U^0 = \delta q(1 - \lambda)(V_I((1 - \lambda)z^g(\omega_0)) - V_U((1 - \lambda)z^g(\omega_0))) + \delta(1 - q)(1 - \lambda)(V_I^0 - V_U^0) + \delta(2q - 1)(V_U((1 - \lambda)z^g(\omega_0)) - V_U^0) = c. \tag{18}$$

For $x \in [\omega_0, \hat{\omega}]$,

$$V_I(x) - V_U(x) = \delta q(1 - \lambda)(V_I((1 - \lambda)z^g(x)) - V_U((1 - \lambda)z^g(x))) + \delta(1 - q)(1 - \lambda)(V_I((1 - \lambda)z^b(x)) - V_U((1 - \lambda)z^b(x))) + \delta(2q - 1)(V_U((1 - \lambda)z^g(x)) - V_U^0) \tag{19}$$

By the strategy profile, we must have $V_I(x) - V_U(x) \leq c$. In the rest of the proof, we are going to show that $V_I(x) - V_U(x) > c$ for $x \in [\omega_0, \hat{\omega}]$, and thus the equilibrium does not exist.

Because $(1 - \lambda)z^b(\hat{\omega}) \leq \omega_0 < \hat{\omega}$, for any $\hat{\omega} \geq x \geq \omega_0$, $\hat{\omega} \geq (1 - \lambda)z^g(x) \geq x$ and $(1 - \lambda)z^b(x) \leq \omega_0$. Namely, the next period reputation is still lower than $\hat{\omega}$ after a good outcome, whereas it is lower than $\omega_0$ after a bad outcome, and thus $V_K((1 - \lambda)z^b(x)) = V_K^0$ for any $x \in [\omega_0, \hat{\omega}]$ and $K = U, I$. Thus Equation (18) becomes

$$V_I(x) - V_U(x) = \delta q(1 - \lambda)(V_I((1 - \lambda)z^g(x)) - V_U((1 - \lambda)z^g(x))) + \delta(1 - q)(1 - \lambda)(V_I^0 - V_U^0) + \delta(2q - 1)(V_U((1 - \lambda)z^g(x)) - V_U^0). \tag{19}$$

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By the strategy profile, for \( x \in [\omega_0, \hat{\omega}] \), \( \sigma(x) = 0 \), and thus
\[
V_U(x) = x + \delta(1 - q)V_U[(1 - \lambda)z^\theta(x)] + \delta q V_U[(1 - \lambda)z^b(x)]
\]
given the boundary condition that \( V_U(x) = V_U(\omega_0) \) for each \( x \leq \omega_0 \). By using the standard contraction mapping technique in Stokey, Lucas, and Prescott (1989), one can show that \( V_U(x) \) is continuous and strictly increasing for \( x \in [\omega_0, \hat{\omega}] \). As a result, the third term of the right-hand side of Equation (19), \( (V_U((1 - \lambda)z^\theta(x)) - V_U^0) \) is strictly increasing. Again, by applying the standard contraction mapping technique on the functional equation (19), one can show that \( V_I(x) - V_U(x) \) is continuous and strictly increasing for \( x \in [\omega_0, \hat{\omega}] \). Because \( V_I^0 - V_U^0 = c \), \( V_I(x) - V_U(x) > c \); thus, the equilibrium does not exist.

Next, consider the existence of a “work-shirk” equilibrium when \( \omega_0 \) is small. Unfortunately, we cannot prove or disprove the existence of a “work-shirk” equilibrium without further restriction. The following proposition shows that a “work-shirk” equilibrium does not exist for a large \( \lambda \).

**Proposition 5.** There exists no “work-shirk” equilibrium if \( \lambda \geq 1/2 \).

**Proof.** Suppose not. Following case III in the proof of 4, for \( x \in [\omega_0, \hat{\omega}] \), we have
\[
V_I(x) - V_U(x) = \delta q(1 - \lambda)(V_I((1 - \lambda)z^\theta(x)) - V_U((1 - \lambda)z^\theta(x)))
+ \delta(1 - q)(1 - \lambda)(V_I((1 - \lambda)z^b(x)) - V_U((1 - \lambda)z^b(x)))
+ \delta(2q - 1)(V_U((1 - \lambda)z^\theta(x)) - V_U((1 - \lambda)z^b(x))). \tag{20}
\]

We want to show that \( V_I(x) - V_U(x) > c \) to disprove the existence of the equilibrium. Unfortunately, because \( \omega_0 \) is too small, it is not true that \((1 - \lambda)z^b(x)\) for any \( x \geq \omega_0 \), so \( V_I(x) - V_U(x) \) may not be monotone in general.

However, for \( x \) small enough such that \((1 - \lambda)z^b(x) \leq \omega_0 \), we have
\[
V_I(x) - V_U(x) - [V_I^0 - V_U^0] = \delta q(1 - \lambda)[V_I((1 - \lambda)z^\theta(x)) - V_I((1 - \lambda)z^\theta(\omega_0))]
= \delta[(1 + \lambda)q - 1][V_U((1 - \lambda)z^\theta(x)) - V_U((1 - \lambda)z^\theta(\omega_0))]. \tag{21}
\]
The assumption \( \lambda \geq 1/2 \) and Assumption 1 together imply that \( \delta[(1 + \lambda)q - 1] \geq 0 \). Again, the standard contraction mapping technique implies that \( V_I(x), V_U(x) \) are strictly increasing in \( x \in [\omega_0, \hat{\omega}] \), so \( V_I(x) - V_U(x) - [V_I^0 - V_U^0] > c \) for such a \( x \); thus, the fund has the incentive to deviate.
When $\lambda < 1/2$, $\delta[(1+\lambda)q-1] < 0$, the argument above fails. We conjecture that a “work-shirk” equilibrium does not exist as well. However, we cannot prove or disprove its existence.

Last, we show that there exists no “work-work-...-shirk” equilibrium in which

1. there exists a decreasing sequence $\{\omega_i\}_{k=1}^N$, where $N > 2$, $\omega_N = 0$ and $\omega_0 \leq 1$,

2. $\sigma(x) = 0$ for $x \geq \omega_0$,

3. $\sigma(x) = \frac{\omega_k-x}{1-x}$ for $x \in (\omega_{k+1}, \omega_k]$ and $k \in \{0, 1, ..N-1\}$, and

4. for $x \in (\omega_{k+1}, \omega_k]$ and $k \in \{0, 1, ..N-1\}$ and $k < N-1$, the posterior belief after a good outcome $(1-\lambda)z^g \in (\omega_k, \omega_{k-1}]$; while the posterior belief after a bad outcome $(1-\lambda)z^b \in (\omega_{k+2}, \omega_{k+1}]$, where $\omega_{N+1} = \omega_N = 0$.

First, we consider the case in which $N = 3$. In Proposition 6, we show that a “work-work-work-shirk” equilibrium does not exist. Then we apply Proposition 6 to show that an equilibrium with $N > 3$ “work” regimes and one “shirk” regime does not exist either.

**Proposition 6.** There exists no “work-work-work-shirk” equilibrium.

*Proof.* Suppose there exists such an equilibrium s.t.

$$\sigma(x) = \begin{cases} \frac{\omega_k-x}{1-x}, & \text{if } x \leq \omega_2; \\ \frac{\omega_1-x}{1-x}, & \text{if } x \in (\omega_2, \omega_1]; \\ \frac{\omega_0-x}{1-x}, & \text{if } x \in (\omega_1, \omega_0]; \\ 0, & \text{if } x > \omega_0, \end{cases}$$

for some $1 \geq \omega_0 > \omega_1 > \omega_2 \geq 0$. Let $V_U^i = V_U(\omega_i)$ and $V_I^i = V_I(\omega_i)$ for $i = 0, 1, 2$. Then the manager’s indifference condition at $x = \omega_0, \omega_1, \omega_2$ implies that

$$V_I^0 - V_U^0 = c > 0,$$

$$V_I^1 - V_U^1 = \delta q(1-\lambda)(V_I^0 - V_U^0) + \delta(1-q)(1-\lambda)(V_I^2 - V_U^2) + \delta(2q-1)(V_U^0 - V_U^2) = c,$$

and

$$V_U^2 - V_U^2 = \delta q(1-\lambda)(V_I^1 - V_U^1) + \delta(1-q)(1-\lambda)(V_I^2 - V_U^2) + \delta(2q-1)(V_U^0 - V_U^2) = c.$$

Simple algebra implies that

$$(2q-1)\delta(V_U^0 - V_U^2) = c[1-\delta(1-\lambda)],$$

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\[(2q - 1)\delta(V'^1_U - V'^2_U) = c[1 - \delta(1 - \lambda)];\]

which further implies that \(V'^1_U = V'^0_U\), which is a contradiction!

By the same logic, we have the following corollary.

**Corollary 4.** There exists no “work-.....-work-shirk” Equilibrium with \(N\) “work” regimes for each \(N > 2\).

**Proof.** Suppose such an equilibrium exists, then we can apply the proof of Proposition 6 to show the contradiction for the first two “work” regimes: \((\omega_2, \omega_1), (\omega_1, \omega_0)\) to show that such an equilibrium cannot exist.

\[\square\]
References


