Dynamic Technology Subsidies: 
Paying People Not to Wait*

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We analyze how a policymaker should structure subsidies over time in order to induce agents to adopt a new durable good technology. We theoretically demonstrate that, if agents are myopic, a policymaker can price discriminate by using a subsidy that increases over time but can take advantage of technological change by using a subsidy that decreases over time. However, if agents plan their adoption around future subsidies, then the policymaker commits to low future subsidies in order to induce adoption in early periods. In order to measure the relative importance of these different channels, we estimate a dynamic model of household preferences in the California residential market for rooftop solar photovoltaics. We show that households’ expectations of future subsidies and technological progress increase the cost of the efficient policy by 50% and limit the regulator’s ability to reduce total spending by taking advantage of technological progress. Using the efficient subsidy could have allowed the state to spend 31% ($46 million) less to achieve the same level of adoption.

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Policymakers commonly subsidize adoption of new durable good technologies. The U.S. government pays hospitals to adopt electronic medical record systems and car buyers to choose electric vehicles. The U.S. and other countries have paid farmers to install more efficient irrigation systems, firms to build renewable energy projects, and households to replace their aging appliances. And many U.S. states pay homeowners to install solar panels. Many of these subsidies evolve over time according to an announced schedule. Policymakers often employ these subsidies to increase adoption, but these subsidies also interact with other goals such as limiting public spending. Two subsidy trajectories that eventually achieve the same level of adoption could have very different implications for the public purse. Yet the design of an efficient subsidy schedule has thus far remained an open policy question.

We formally analyze the dynamically efficient subsidy schedule. The regulator commits to a subsidy policy designed to achieve a target level of adoption by a certain date. The regulator values cumulative adoption and dislikes spending money. Consumers have heterogeneous, private values for the technology. We show that the regulator’s efficient subsidy schedule depends critically on three factors: the distribution of consumers’ private values for the technology, the anticipated pace at which technical change will reduce the private cost of adopting the technology, and whether consumers consider future changes in the cost of the technology and the level of the subsidy when they decide whether to adopt the technology.

To build intuition, first imagine that consumers are myopic and do not anticipate future changes in either the technology’s cost or the level of the subsidy when they decide whether to adopt the technology. In this case, the regulator can offer a low initial subsidy to induce adoption by consumers with a particularly high willingness-to-pay for the technology and can then increase the subsidy over time so as to obtain adoption from consumers with a lower willingness-to-pay. Within a period, the regulator cannot discriminate between consumers because all adopters receive the same subsidy, but the dynamic nature of the subsidy enables intertemporal price discrimination when consumers are myopic. By using a low subsidy early and a high subsidy later, the regulator avoids “over-subsidizing” consumers who would be willing to adopt the technology at a low subsidy level and thereby reduces the total cost of achieving a given level of adoption. The strength of this price discrimination channel is strong if there are a lot of “inframarginal” consumers who would adopt even at low subsidy levels and is weak when most consumers are on the margin.\(^1\)

The story changes if consumers anticipate future subsidies. In that case, if the regulator commits to offering a higher subsidy tomorrow, then some consumers will simply wait to take advantage of the higher subsidy.\(^2\) Consumers’ expectations constrain the regulator’s ability

\(^1\)Boomhower and Davis (2014) estimate the fraction of inframarginal adopters in an energy efficiency program in Mexico. They find that more than 65% of households are inframarginal and that about half of adopters would have adopted in the absence of any subsidy. Gowrisankaran and Rysman (2012) find that consumers who buy digital camcorders in later periods have lower values for the good than did consumers who purchased in earlier periods.

\(^2\)In the theory and simulations, our consumers have rational expectations about the evolution of technol-
to intertemporally price discriminate. In order to stimulate adoption today, the regulator commits to offering a relatively low subsidy in the future, even though the regulator would prefer to offer a higher subsidy once tomorrow actually arrives. The combination of consumer expectations and the ability to commit to a subsidy schedule thus favors using lower subsidies in later periods.

Finally, consider how improving technology (i.e., declining private costs of adoption) affects the efficient subsidy schedule. If the regulator anticipates that the technology will improve over time, then the efficient policy delays adoption until later periods. The prospect of technological change can therefore lead the regulator to offer a smaller subsidy early on. As technology improves, more people want to adopt the technology for a given subsidy. The regulator’s spending thus tends to increase over time. However, a regulator with a convex cost of public funds prefers to smooth spending over time. The regulator achieves this smoothing by using a smaller subsidy in later periods. Technological progress therefore generally favors a subsidy that decreases over time.

In order to understand how these competing forces affect the level and effectiveness of subsidies in a real-world setting, we quantitatively assess a prominent example of a dynamic technology subsidy. The California Solar Initiative (CSI) was a substantial subsidy for residential photovoltaic (rooftop solar) adoption. This program had a budget of over $2 billion and lasted from 2007–2014. The subsidy declined step-wise over time from $2.50/Watt to $0.20/Watt, with pre-subsidy installation costs over this period declining from around $9/Watt to around $4/Watt. We combine data on household-level installations with a dynamic discrete choice model to estimate the distribution of households’ benefit of installing solar conditional on household demographics. The estimation assumes that households know the full time-path of subsidies but allows solar system prices to evolve stochastically. The results show substantial heterogeneity in the private benefit of residential solar systems.

Once we have estimated the distribution of households’ private benefits of adopting solar, we compare the actual subsidy schedule to the efficient subsidy trajectory for myopic and forward-looking households facing either constant or improving technology. We find that the efficient subsidy would have started at a lower level and declined at a slower rate than did technology costs and subsidies and attempt to time their adoption of the technology. However, we remain agnostic about whether consumers accurately internalize the present discounted value of the stream of benefits from the technology. This potential undervaluation of a stream of future technology benefits has been discussed at length in the literature on the “energy efficiency paradox” (e.g., Busse et al., 2013; De Groote and Verboven, 2016).

Similarly, Conlon (2010) emphasizes that firms want to lower the price of a durable good over time in order to intertemporally price discriminate, but consumers’ willingness to wait for lower prices limits the rate at which firms can reduce prices.

Because consumers are forward-looking, we cannot use a static approach to estimate the implications of a change in the subsidy for adoption as, for instance, in Hughes and Fodolefsky (2015). Instead, we must recover consumers’ structural preferences that will remain stable even when the regulator changes expectations of future subsidies (Lucas Jr, 1976).
the actual subsidy. We find that using the efficient subsidy policy to achieve the actual level of adoption would have reduced state spending by 31% in expectation, without substantially reducing the consumer surplus that households would obtain from the policy.

Our results highlight how households’ rational expectations of future subsidies are critical for the cost of the efficient policy and the value of technological progress. The present cost of the policy is 50% greater and the efficient initial subsidy is 40% larger when households have rational expectations. In this case, households who adopt the technology today lose the option to adopt the technology tomorrow. In order to induce adoption today, the regulator must compensate households for giving up that option by offering them a higher subsidy. Households’ rational expectations also reduce the degree to which the regulator can take advantage of technological progress to reduce the total cost of the policy. When households do not anticipate that technology might improve over time, then the regulator can take advantage of its understanding of technological progress to reduce spending by 60%. However, when households are aware of the possibility of technological progress, the regulator can only reduce its spending by 40%. Expectations of technological progress and uncertainty about technological progress both increase households’ incentives to wait until later periods to adopt the technology. Technological progress therefore increases the opportunity cost of adopting the technology today, which requires the regulator to offer households a larger subsidy than would be necessary in a world in which households were ignorant of technological progress. Our results show how that household expectations can be critical to the cost and design of subsidy policies in environments with rapidly changing technology.

We also show that technological progress is critical for the shape of the efficient subsidy schedule. In the absence of technological progress, the efficient subsidy would increase over time. This type of policy is rarely enacted or discussed. The increasing subsidy is driven by the regulator’s preference to delay spending and, when households are myopic, by the potential for intertemporal price discrimination. In contrast, when the regulator anticipates the possibility of technological progress, the efficient subsidy declines over time as the regulator attempts to smooth its spending.

Finally, we estimate that much of the variation in preferences for solar across households comes from idiosyncratic stochastic shocks to their willingness-to-pay in each period. This

\[5\] Previous analyses have shown that the possibility of induced technical change (whereby a larger subsidy causes technology to improve faster) can raise the efficient subsidy early in order to actively induce the better technology and can adjust the subsidy in later periods in order to take advantage of the improved technology (Kalish and Lilien, 1983; Goulder and Mathai, 2000). These analyses implicitly assume myopic consumers. Our analysis implies a new effect: agents’ rational expectations about technical change can increase the subsidy required to convince them to adopt the technology, which reduces the benefit of inducing technical change. The regulator would like to use a large early subsidy to induce technical change, but without letting households realize that the large early subsidy will have this effect.

\[6\] The fact that the stochastic error terms explain a lot of the variation in willingness-to-pay makes sense in the solar context where consumers may only have access to credit in some periods (for instance, when they are already taking out a home-improvement loan or refinancing their mortgage) or when the cost of
means that in each time period, some households who did not previously adopt solar end up with high willingness-to-pay. Thus, there are always inframarginal households (even when the subsidy is constant over time), which dampens the regulator’s incentive to price discriminate through the use of an increasing subsidy. The possibility that low willingness-to-pay households will eventually become high willingness-to-pay households reduces the cost of the efficient subsidy policy by over 60%.

Our primary contribution is to ground the design of dynamic subsidy instruments in economic principles. Despite the prevalence of subsidies for durable investments, there has been little formal analysis of these instruments. Kalish and Lilien (1983) study the efficient subsidy trajectory in the presence of learning and of word-of-mouth diffusion. They argue that both channels call for a subsidy that declines over time. In their conclusion, they mention that a desire to avoid subsidizing high-value consumers could argue for an increasing subsidy schedule. Meyer et al. (1993) discuss how to design investment tax credits in order to obtain the “biggest bang for the buck.” They note that the investment incentive is determined by the credit offered to the marginal investor, whereas the regulator’s spending depends on the average credit offered to investors. Policymakers should aim to combine a high marginal credit with a low average credit. These papers’ informal observations illustrate the logic underpinning our intertemporal price discrimination channel. We formally demonstrate this channel, show how it depends on private actors’ expectations, and introduce new channels.

A larger literature has analyzed how monopolists should set prices for durable goods over time. In particular, several papers have explored the conditions under which a monopolist finds intertemporal price discrimination to be optimal. When production is costless, a monopolist should commit to offering a constant subsidy over time as long as all customers use the same discount rate and the monopolist is at least as impatient as its customers (Stokey, 1979; Landsberger and Meilijson, 1985). In that case, all sales happen in the first instant. However, intertemporal price discrimination can be optimal when production costs are convex (Salant, 1989) or declining over time (Stokey, 1979). Our setting follows these in assuming that the regulator can commit to a subsidy schedule and in analyzing the implications of rational expectations. We avoid a corner solution (i.e., a constant subsidy) because our regulator has a concave benefit function and a convex distaste for spending in each instant. Further, we reserve the label of “price discrimination” for forces that arise only because of adopters’ equilibrium decisions, so that we disentangle other dynamic forces from intertemporal price discrimination motives.

Gathering information on the appropriateness of solar for their particular home is high.

7Kalish and Lilien (1983) implicitly assume myopic consumers. Our analysis of technological change suggests that both the cost and the trajectory of the efficient subsidy will be sensitive to households’ expectations about their future preferences for solar.

8In simultaneous work, Kremer and Willis (2016) study the efficient subsidy trajectory in the presence of spillovers. They assume homogeneous private values for the technology and constant costs, whereas we emphasize the implications of heterogeneous private values and of declining costs.

9Many authors have also explored how a monopolist should price durable goods when it cannot commit
The next section contains the theoretical analysis. Section 2 introduces the data to be used in the empirical estimation. Sections 3 describes the dynamic structural model for estimating the distribution of household value of solar photovoltaics, and Section 4 reports the estimation results. Section 5 combines the empirically estimated distribution of private values with the theoretical analysis in order to explore the determinants of the efficient subsidy trajectory for rooftop solar. The final section concludes. The appendix provides evidence that California households were forward-looking, describes the numerical calibration, and extends the theoretical analysis to the case of a fixed budget.

1 Theoretical Analysis

We begin by theoretically analyzing the subsidy trajectory that efficiently incentivizes actors to adopt a new technology. The first subsection describes the setting, and subsequent subsections analyze the cases in which households are myopic and are forward-looking.

1.1 Setting

Actor $i$ values a technology at $v_i$ and decides on a single time at which to adopt the technology. We normalize the measure of potential adopters to 1. The twice-differentiable cumulative distribution function $F(v_i) \in [0, 1]$ gives the number of actors who are willing to pay no more than $v_i$ for the technology. Define $f(v_i) \geq 0$ as the density function (i.e., as $F'(v_i)$). We will be especially interested in the empirically relevant case where $f'(v_i) < 0$, meaning that the number of actors on the margin of adopting increases as the technology’s cost falls.

The private cost of the technology is $C(t)$. The technology’s cost declines exogenously: $\dot{C}(t) \leq 0$, where a dot indicates a derivative with respect to time. An actor who adopts the technology at time $t$ receives a subsidy of $s(t)$. Actor $i$’s net benefit of adopting is $v_i - C(t) + s(t)$. Let $Q(t)$ give the number of actors who adopted the technology prior to time $t$, so that $\dot{Q}(t)$ is adoption at time $t$. When potential adopters are myopic, they do not later periods’ prices (e.g., Coase, 1972; Stokey, 1981; Conlisk et al., 1984; Gul et al., 1986; Kahn, 1986; Besanko and Winston, 1996; Sobel, 1991). In considering this literature’s implications for actual markets, Waldman (2003) criticizes the assumption that commitments are not possible. He notes that firms often do appear to commit to policies in practice. Similarly, it is easy to provide examples in which policymakers appear to successfully commit to a subsidy schedule. Our theoretical analysis focuses on this environment with commitment. Much macroeconomics literature follows Kydland and Prescott (1977), Calvo (1978), and Fischer (1980) in analyzing the implications for monetary policy of a regulator’s incentives to surprise economic agents. Our environment instead relates more closely to the optimal taxation literature, which analyzes the regulator’s choice of, for instance, the trajectory of capital taxes under commitment (e.g., Judd, 1985; Chamley, 1986; Chari and Kehoe, 1999). Note that we are considering policies over relatively short timescales and that the regulator indeed followed through on its commitments in our empirical application.

We assume that the regulator is small relative to the world market, as in the later empirical application.
not anticipate future changes in the technology’s cost or in the subsidy. In that case, they compare the net benefit of adopting at time \( t \) to zero. When potential adopters are forward-looking, they anticipate all future changes in costs and in the subsidy, so that they compare the net benefit of adopting at time \( t \) to the net benefit of adopting at a later time, discounted at rate \( \delta > 0 \).

The regulator commits at time 0 to offer a subsidy \( s(t) \) to all \( \dot{Q}(t) \) actors who adopt the technology at time \( t \). The regulator aims to achieve total adoption \( Q_T \) by some given time \( T > 0 \). She knows the distribution of potential adopters’ values but does not know any particular actor’s value. The regulator dislikes spending money. Her distaste for spending money is \( G(s(t) \dot{Q}(t)) > 0 \), with \( G(\cdot) \) strictly increasing and strictly convex. The regulator also receives instantaneous benefit \( B(Q(t)) > 0 \) from cumulative adoption, with \( B(\cdot) \) strictly increasing and strictly concave. In our application to adoption of solar photovoltaics, the benefit function will capture the regulator’s value from production of solar electricity.

When selecting the subsidy trajectory, the regulator correctly anticipates how the technology’s cost will evolve and how actors will respond to the offered subsidy. At time 0, the regulator chooses the subsidy trajectory to maximize

\[
\int_0^T e^{-rt} \left[ B(Q(t)) - G(s(t) \dot{Q}(t)) \right] \, dt,
\]

for given discount rate \( r > 0 \), given initial adoption \( Q_0 \in [0, 1] \), and given terminal adoption \( Q_T \in (Q_0, 1] \). Potential adopters’ decisions determine how the announced subsidy trajectory affects \( \dot{Q}(t) \) and thus \( Q(t) \). We now specialize to the settings with myopic and forward-looking adopters.

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11 In order to focus on other effects, we ignore heterogeneity in potential adopters’ discount rates. The implications of such heterogeneity depend on whether actors with high discount rates tend to have high or low private values for the technology. The case with a positive correlation between discount rates and private values corresponds to Stokey (1979). The case with a negative correlation arises when adopting the technology provides a stream of benefits that potential adopters discount to a present value. The assumption of a common discount rate corresponds to a well-known aspect of the empirical methodology, in which the econometrician must assume a common discount rate because the discount rate is not well identified by the data.

12 The assumptions of a fixed terminal time \( T \) and of a fixed adoption target \( Q_T \) are not critical to our analysis. These assumptions affect the transversality conditions for the regulator’s problem, but they do not affect the necessary conditions that are the focus of our theoretical analysis.

13 The appendix shows that our theoretical analysis is robust to giving the regulator a fixed budget instead of a fixed adoption target. Intuitively, if the budget constraint does not bind, then that setting is equivalent to altering the present setting to allow \( Q_T \) free, which would affect the transversality condition but not the other necessary conditions. If the budget constraint does bind, then the problems are identical if we here fix \( Q_T \) at the value that results from solving the problem with a budget constraint.
1.2 Myopic Adopters

When potential adopters are myopic, they adopt the technology as soon as their net benefit of adoption is positive. Therefore, at time \( t \), all actors with \( v_i \geq C(t) - s(t) \) should adopt (or should already have adopted) the technology. Define \( V(t) \) as the value at which actors are just indifferent to adopting or not: \( V(t) = C(t) - s(t) \). The number of actors who have adopted the technology by \( t \) is \( Q(t) = 1 - F(V(t)) \), which implies \( \dot{Q}(t) = -f(V(t)) \dot{V}(t) \).

Note that \( \dot{V}(t) \leq 0 \) along any optimal path: there is no reason for the regulator to adopt a subsidy that makes net costs strictly increase.

Rather than imagining the regulator selecting the subsidy at each instant, imagine the regulator selecting the quantity of adoption via \( V(t) \), with the subsidy determined by this choice and by actors’ equilibrium conditions. Relabel the regulator’s control \( V(t) \) as \( y(t) \) and substitute for \( s(t) \). The regulator’s problem becomes:

\[
\max_{y(t)} \int_0^T e^{-rt} \left[ B\left(1 - F(V(t))\right) - G\left(-[C(t) - V(t)] f(V(t)) y(t)\right)\right] \, dt \\
\text{s.t.} \quad \dot{V}(t) = y(t) \\
V(0) = F^{-1}(1 - Q_0), \quad V(T) = F^{-1}(1 - Q_T).
\]

The Hamiltonian is:

\[
H(t, y(t), V(t), \lambda(t)) = e^{-rt} \left[ B\left(1 - F(V(t))\right) - G\left(-[C(t) - V(t)] f(V(t)) y(t)\right)\right] + e^{-rt} \lambda(t) y(t).
\]

\( \lambda(t) \) gives the (current) shadow value of \( V(t) \). The necessary conditions for a maximum are:

\[
\lambda(t) = -[C(t) - V(t)] f(V(t)) G\left(-[C(t) - V(t)] f(V(t)) y(t)\right),
\]

\[
-\dot{\lambda}(t) + r \lambda(t) = -B\left(1 - F(V(t))\right) f(V(t)) \]

\[
- G\left(-[C(t) - V(t)] f(V(t)) y(t)\right) \left[f(V(t)) y(t) - [C(t) - V(t)] f'(V(t)) y(t)\right],
\]

along with the transition equation, the initial condition, and the terminal condition. The first equation follows from the Maximum Principle and the second equation is the costate (or adjoint) equation for the state variable \( V(t) \). The first equation implies that \( \lambda(t) \leq 0 \): because lower \( \dot{V}(t) \) corresponds to greater adoption \( Q(t) \), this negative sign means that the shadow benefit of adoption is positive.

Differentiate equation (1) with respect to time and suppress the argument of \( G(\cdot) \):

\[
\dot{\lambda}(t) = -\left[\dot{C}(t) - y(t)\right] f(V(t)) G' - [C(t) - V(t)] f'(V(t)) y(t) G' \\
- [C(t) - V(t)] f(V(t)) G" \\
\left[-\left[\dot{C}(t) - y(t)\right] f(V(t)) y(t) - [C(t) - V(t)] f'(V(t)) y(t)^2 - [C(t) - V(t)] f(V(t)) \dot{y}(t)\right].
\]

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Substitute for \( \dot{\lambda}(t) \) from the costate equation and rearrange to obtain:

\[
\dot{s}(t) = \frac{-r \lambda(t)/f(V(t)) - B' - G' y(t) + [s(t)]^2 G'' \left[ f(V(t)) y(t) + f'(V(t)) [y(t)]^2 \right]}{G' - s(t) f(V(t)) G'' y(t)},
\]

where we use \( \dot{s}(t) = \dot{C}(t) - y(t) \) from the definition of \( V(t) \) and suppress the argument of \( B(\cdot) \). This differential equation tells us how the subsidy changes along an efficient trajectory.

The shape of the subsidy’s trajectory is determined by the four terms in the numerator, with the denominator always positive. First, the \(-r \lambda(t)/f(V(t)) \geq 0\) ensures that the regulator is indifferent to small deviations in the subsidy trajectory. Equation (1) requires that the social cost of each moment’s spending on marginal adopters equals the shadow benefit of adoption. For now, ignore complications introduced by other channels. In order for the regulator to be indifferent to small deviations in her policy trajectory, the shadow benefit of adoption must grow at the discount rate \( r \), which keeps its present value constant over time.\(^{14}\) The efficient subsidy schedule therefore tends to increase because the impatient regulator will tolerate a greater social cost of spending in later periods. As the regulator becomes more impatient (i.e., as \( r \) increases), the regulator more strongly dislikes spending money now to obtain later benefits and so uses a more sharply increasing subsidy schedule. We call this first force for an increasing subsidy a Hotelling channel, due to its similarity to the Hotelling (1931) analysis of exhaustible resource extraction.

Second, the \(-B' < 0\) reflects that raising today’s subsidy in exchange for lowering tomorrow’s subsidy not only shifts the shadow benefit of adoption forward in time but also provides benefits tomorrow by raising cumulative adoption.\(^{15}\) This effect of valuing the total stock of adoption is familiar from Heal (1976) models of resource extraction, in which extraction costs increase in the cumulative quantity extracted. The additional benefit of an earlier subsidy favors a decreasing subsidy schedule. We call this force an opportunity cost channel because it captures how waiting to spend money on the subsidy foregoes benefits in the interim.

Third, the \(-G' y(t) \geq 0\) recognizes that the regulator cannot price discriminate within a period. Recall that \( y(t) \triangleq \dot{V}(t) \leq 0 \) measures the maximum gap between between the private values of households adopting at time \( t \). If the regulator offers a marginally greater

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\(^{14}\)Imagine that the regulator deviates by reducing adoption by \( \epsilon \) at time \( t \) and increasing adoption by \( \epsilon \) at time \( t + \Delta t \). And assume for the moment that the regulator’s marginal cost of funds is unity (the convex cost of funds enters through other channels). The regulator’s savings today are \( \epsilon s(t) \), which by equation (1) equals \(-\epsilon \lambda(t)/f(V(t)) \). The regulator invests this money and earns interest at rate \( r \) before spending \(-\lambda(t + \Delta t)/f(V(t + \Delta t)) \) to obtain the later adoption. For the regulator to be indifferent to this deviation for \( \Delta t \) small, it must be true that \(-\lambda(t + \Delta t)/f(V(t + \Delta t)) + \lambda(t)/f(V(t)) = -r \lambda(t)/f(V(t)) \). Letting \( \Delta t \) go to zero and using the derivative of equation (1) with respect to time, we have \( \dot{s}(t) = -r \lambda(t)/f(V(t)) \).

\(^{15}\)In footnote 14, the cost of delaying adoption should include \( B' \Delta t \). The logic of the footnote would then imply that \( \dot{s}(t) = -r \lambda(t)/f(V(t)) - B' \).
subsidy to some adopter at time \( t \), then it must offer that marginally greater subsidy to all adopters, including those who would have adopted at a lower subsidy. But if the regulator waits to offer the marginally greater subsidy in the next instant, then it avoids paying the extra money to the \(-y(t) f(V(t))\) inframarginal adopters at time \( t \). The more inframarginal adopters there are at time \( t \), the stronger the incentive to wait to offer the higher subsidy. This price discrimination channel thus favors an increasing subsidy.

The final term captures how the regulator dislikes spending lots of money in a single instant. The regulator would prefer to smooth spending over time. The terms in brackets determine how instantaneous adoption changes as time advances. First, if \( \dot{y}(t) < 0 \), then the measure of private values for which adoption is optimal is increasing over time, either because the regulator is increasing the subsidy or because installation costs are falling. This greater adoption works to increase subsidy spending, which favors using a declining subsidy in order to smooth spending over time. Second, if \( f'(V(t)) < 0 \), then the distribution of private values becomes thicker as more people find adoption to be optimal. In this case, subsidy spending again tends to increase over time, which favors using a declining subsidy schedule. Putting these pieces together, this cost convexity channel favors a decreasing subsidy schedule in the plausible case with \( \dot{y}(t) \leq 0 \) and \( f'(V(t)) \leq 0 \).

Now consider how anticipated improvements in technology affect the efficient subsidy schedule. First, more strongly declining costs (i.e., more negative \( \dot{C}(t) \)) make \( y(t) \) more negative for a given subsidy. The more rapidly that costs are declining, the greater the number of inframarginal adopters at a given subsidy, and the greater the incentive to price discriminate by using an increasing subsidy schedule. Second, the cost convexity channel depends on both the first and second derivatives of the cost function, via \( y(t) \) and \( \dot{y}(t) \). Rapid cost declines exacerbate the effect of moving to a thicker part of the distribution of private values (assuming \( f'(V(t)) < 0 \)), which favors a decreasing subsidy schedule. If costs are declining at an accelerating rate (\( \ddot{C}(t) < 0 \)), then \( \dot{y}(t) \) tends to be negative, again favoring a decreasing subsidy schedule. Third, declining costs tend to reduce the shadow benefit \( \lambda(t) \) of adoption by making it easier to obtain adoption in later periods. This effect weakens the Hotelling channel and thus favors a decreasing subsidy schedule. Combining these pieces, we see that declining costs can strengthen the price discrimination channel that favors an increasing subsidy schedule but otherwise work to make the subsidy decrease over time.

### 1.3 Forward-Looking Adopters

We have thus far considered the efficient subsidy schedule when potential adopters are completely myopic. However, when potential adopters anticipate future subsidies, the regulator can no longer induce adoption just by offering a large subsidy today; instead, the regulator must offer both a high subsidy today and a sufficiently small subsidy in the future.

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\(^{16}\text{Recall that } \dot{Q}(t) = -y(t) f(V(t)).\)
Instead of adopting the technology as soon as instantaneous net benefits are greater than zero, each actor \( i \) now chooses the optimal time \( \Psi_i \) to adopt the technology, for given subsidy and cost trajectories:

\[
\max_{\Psi_i} \ e^{-\delta \Psi_i} [v_i - C(\Psi_i) + s(\Psi_i)]
\]

The first-order necessary condition is\(^{17}\)

\[
\delta [v_i - C(\Psi_i) + s(\Psi_i)] = \dot{s}(\Psi_i) - \dot{C}(\Psi_i). \tag{3}
\]

The left-hand side is the cost of waiting until the next instant: the actor delays receiving the instantaneous payoff \( v_i - C(t) + s(t) \). The right-hand side is the benefit of waiting: when costs net of the subsidy are decreasing (i.e., when \( \dot{C}(t) - \dot{s}(t) < 0 \)), then the actor can save money by adopting the technology later. The optimal time of adoption balances these costs and benefits. As the potential adopter becomes perfectly patient (\( \delta \to 0 \)), the cost of waiting disappears, so the agent delays adoption until net costs reach their minimum.

Potential adopters’ stopping problems generate the equilibrium conditions that constrain the regulator’s choice of subsidy trajectory. As before, let the regulator’s control be which actors are marginal in each period, with \( V(t) \) denoting the marginal actors’ private value for the technology. Then, rearranging equation (3), the subsidy must evolve as

\[
\dot{s}(t) = \delta [V(t) - C(t) + s(t)] + \dot{C}(t). \tag{4}
\]

The regulator’s time 0 choice of subsidy schedule will be dynamically inconsistent because the regulator commits to offering a given subsidy at time \( t \) in part to affect potential adopters at times \( w < t \), but once time \( t \) arrives, those adoption decisions are in the past and thus irrelevant to a decision problem that starts from time \( t \). However, we assume that the regulator is able to commit at time 0 to not revise its announced subsidy schedule. There is a long tradition in the optimal taxation literature of analyzing similar “dynamic Ramsey” problems under the assumption of full commitment (e.g., Judd, 1985; Chamley, 1986), and this assumption is particularly applicable in the case of subsidies for new technologies.\(^{18}\)

\(^{17}\)Define net costs as \( z(t) \equiv C(t) - s(t) \). These necessary conditions are sufficient if \( \ddot{z}(\Psi_i) > \delta^2 [v_i - z(\Psi_i)] + 2\delta \dot{z}(\Psi_i) \). Substituting both equation (4) (obtained below) and its derivative with respect to time, we find that the necessary conditions are sufficient if \( -V(\Psi_i) > \delta [v_i - V(\Psi_i)] \). The left-hand side is positive. The inequality is satisfied for any \( v_i \) that is not too much greater than \( V(\Psi_i) \). But by the definition of \( V(\Psi_i) \) as the private value of the marginal adopter at time \( \Psi_i \), it cannot be true that \( v_i \) exceeds \( V(\Psi_i) \) by more than \( -V(\Psi_i) \) for any actor \( i \) who had not adopted prior to time \( \Psi_i \). Thus, if \( \delta < 1 \), then constraining the subsidy’s evolution via equation (4) ensures that the necessary conditions are sufficient for any agent who chooses to adopt the technology. In the numerical simulations, we verify that the sufficient condition holds for adopters along the entire trajectory.

\(^{18}\)For instance, our empirical application will consider California’s subsidies for rooftop photovoltaic (solar) systems. Most observers took for granted that the regulator would follow its announced subsidy schedule.
Again using \(y(t)\) for \(\dot{V}(t)\), the regulator solves

\[
\max_{y(t),s(0)} \int_0^T e^{-rt} \left[ B \left( 1 - F(V(t)) \right) - G \left( -s(t) f(V(t)) y(t) \right) \right] \, dt
\]

s.t. \(\dot{V}(t) = y(t)\)

\[
\dot{s}(t) = \delta [V(t) - C(t) + s(t)] + \dot{C}(t)
\]

\[
V(0) = F^{-1}(1 - Q_0), \quad V(T) = F^{-1}(1 - Q_T)
\]

\[
s(T) = C(T) - V(T) + J(V(T), C(T)).
\]

In the case with myopic agents, we did not need a terminal condition on the subsidy because agents did not care how the subsidy changed at \(T\). However, in the present setting, time \(T\) adoption depends on how the subsidy is changing at \(T\). We assume that the subsidy drops to zero at \(T\), with \(J(v_i, C(T))\) the present value to household \(i\) of having the option to adopt the technology at time \(T\). This household adopts the technology at time \(T\) if and only if \(v_i - C(T) + s(T) \geq J(v_i, C(T))\). The Hamiltonian is

\[
H(t, y(t), V(t), s(t), \lambda(t), \mu(t)) = e^{-rt} \left[ B \left( 1 - F(V(t)) \right) - G \left( -s(t) f(V(t)) y(t) \right) \right] + e^{-rt}\lambda(t) y(t) + e^{-rt}\mu(t) \left[ \delta [V(t) - C(t) + s(t)] + \dot{C}(t) \right].
\]

The costate variable \(\lambda(t)\) is the same as in the myopic setting. The new costate variable \(\mu(t)\) measures the degree to which the regulator is constrained at each instant by private actors’ equilibrium behavior and rational expectations: it measures the cost of keeping promises made to those who adopted the technology in past periods.

The necessary conditions for a maximum are:

\[
\lambda(t) = -s(t) f(V(t)) G' \left( -s(t) f(V(t)) y(t) \right), \quad (5)
\]

\[
-\dot{\lambda}(t) + r\lambda(t) = - f(V(t)) B' \left( 1 - F(V(t)) \right)
\]

\[
+ s(t) f'(V(t)) y(t) G' \left( -s(t) f(V(t)) y(t) \right) + \delta \mu(t),
\]

\[
-\dot{\mu}(t) + r\mu(t) = f(V(t)) y(t) G' \left( -s(t) f(V(t)) y(t) \right) + \delta \mu(t), \quad (6)
\]

\[
\mu(0) = 0,
\]

along with the transition equations and the initial and terminal conditions. The first equation follows from the Maximum Principle, the next two equations are the costate (or adjoint)
equations, and the final equation is the transversality condition corresponding to the choice of \( s(0) \).

Solving for \( \mu(t) \) in equation (6), we find

\[
\mu(t) = -\int_0^t e^{-(\delta-r)(t-i)} f(V(i)) y(i) G'\left( -s(i) f(V(i)) y(i) \right) di \geq 0. \tag{7}
\]

Whereas costate variables in standard optimal control problems are forward-looking, here the costate variable \( \mu(t) \) is backward-looking. The costate variable \( \mu(t) \) is the discounted value of all past adoption. In the first instant, the regulator is not bound by past commitments, but over time the regulator becomes more bound by the commitments it has made in order to obtain past adoption. The regulator’s promises accrue as past adoption accrues, and these promises decay at rate \( \delta - r \). The more impatient that potential adopters are (the higher is \( \delta \)), the faster that past commitments fade away and the smaller \( \mu(t) \) is. The more impatient that the regulator is (the higher is \( r \)), the more commitments tend to accumulate and the bigger \( \mu(t) \) is: the regulator obtained early benefits by making promises about later dates’ subsidies.

Differentiate equation (5) with respect to time to obtain

\[
\dot{\lambda}(t) = \dot{s}(t) f(V(t)) G' - s(t) f'(V(t)) y(t) G' + s(t) f'(V(t)) [y(t)]^2 - s(t) f(V(t)) \dot{y}(t) \cdot
\]

where we suppress the argument of \( G(\cdot) \). Substitute for \( \dot{\lambda}(t) \) from the costate equation on \( V(t) \) and rearrange:

\[
\dot{s}(t) = \frac{-r \lambda(t)/f(V(t)) - B' + \hat{\mu}(t)/f(V(t)) + [s(t)]^2 G'' f(V(t)) \dot{y}(t) + f'(V(t)) [y(t)]^2}{G' - s(t) f(V(t)) G'' y(t)},
\]

This equation defines the dynamics of the efficient subsidy when potential adopters correctly anticipate future subsidies and costs. We see the same four channels as in the myopic case. However, we also have a new term: \( -\hat{\mu}(t)/f(V(t)) \leq 0 \). When \( \hat{\mu}(t) > 0 \) (as must be true in early instants), the regulator’s promises are accruing over time. The new channel then favors a decreasing subsidy, because the regulator has promised low future subsidies. However, when \( \hat{\mu}(t) < 0 \), the regulator’s promises are decaying sufficiently fast that the regulator is becoming less constrained by past promises as time passes. The regulator then has more freedom to raise the subsidy towards its ex post preferred level.

Differentiating equation (7), we find that the new term cancels the price discrimination channel and leaves us with \( \delta \mu(t)/f(V(t)) \geq 0 \). We call this net effect of forward-looking
agents a *promise-keeping channel*. In the myopic case, the price discrimination channel reflected the regulator’s ability to intertemporally price discriminate by raising the subsidy once adopters with greater private values had already claimed their subsidy. But when adopters are forward-looking, they might wait for the higher subsidies, which limits the regulator’s ability to price discriminate. Instead, the new term \( \delta \mu(t)/f(V(t)) \) reflects the time \( t \) cost of keeping past promises, as described above. This promise-keeping cost is weakly positive, favoring an increasing subsidy schedule. All else equal, a regulator who obtained a lot of adoption prior to time \( t \) must have promised a low time \( t \) subsidy, which makes her want to raise the subsidy from that level as time passes. When adopters are perfectly patient (\( \delta = 0 \)), the regulator must offer that low subsidy forever, but when adopters are impatient, the regulator can offer a higher subsidy in later periods without strongly disincentivizing adoption in early periods.\(^{19}\) Near the initial time, the promise-keeping channel is approximately zero. Recognizing that adopters anticipate future subsidies thus eliminates the price discrimination channel in early periods, which works to make the efficient subsidy decrease over those early periods.

# 2 Data

The theoretical analysis shows that the efficient subsidy schedule for a durable technology can be sensitive to whether consumers are forward-looking and to the distribution of private values in the population. In order to quantitatively evaluate the determinants of the efficient subsidy schedule in a high-stakes setting, we focus on households’ decisions about whether to install solar systems under the California Solar Initiative. We will use our estimates to evaluate the efficient subsidy policy and to understand how households’ technology adoption decisions would have changed under counterfactual subsidy policies.

The California Solar Initiative (CSI) was a state subsidy for distributed solar installations. We focus on the residential component of the policy, which subsidized each Watt installed. In each of the three major California utilities (Pacific Gas and Electric (PG&E), San Diego Gas and Electric (SDG&E), and Southern California Edison (SCE), the subsidy started at $2.50/Watt installed and declined over time to $0. All utilities exhausted their subsidies after 83 months. See Burr (2014) for more details on the CSI program.

The CSI maintains data on all applications for residential solar subsidies under the program. We use the CSI data as the universe of households that installed solar systems between July 2007 and May 2014.\(^{20}\) The data include the application date, the household’s zip code

\[^{19}\]This desire to offer a higher subsidy in later periods can also be interpreted as a price discrimination channel that is constrained by consumers’ willingness to wait, as determined by their discount rate \( \delta \).

\[^{20}\]There are suggestions that some households installed solar without applying for the CSI subsidy near the end of the time-frame. We will investigate this possibility further using recently released data on the universe of solar system interconnections to the electricity grid.
and utility, and an extensive set of solar system characteristics, including system size, manufacturer, installer, and cost.

Figure 1 shows the evolution of pre-subsidy system costs and of subsidies in the PG&E service area over this 83-month interval for the average system installed in our sample (5.4 kW). The cost of an average system rises in the initial periods when silicon costs are increasing and then declines over the majority of our sample as technology advances and silicon costs fall. At the same time, CSI subsidies decrease in a step-wise pattern.\textsuperscript{21} Because the subsidy level was a function of the total installations within each utility’s service area, the drops in the subsidy occur at different times in the three different utilities, which will help us understand how households respond to changes in the subsidy.\textsuperscript{22}

Figure 1: Monthly Average System Cost and CSI Subsidy for Pacific Gas and Electric

In addition to the information on solar system installations and costs, we use demographic

\textsuperscript{21}There was also a 30\% federal tax subsidy available for residential solar installation during this period. Our empirical strategy accounts for this additional subsidy.

\textsuperscript{22}The step-wise declines in the subsidy were triggered once a utility achieved a predefined level of solar adoption. We assume that these changes were actually scheduled for specific dates. Since it was broadly announced when the quantity targets were approaching, this should not substantially affect our estimation results. And in the theoretical setting, the regulator understands how its subsidy will determine adoption over time, so it could just as easily announce its subsidy as a function of time or of adoption.
data at the block group level from the American Community Survey to allow preferences for solar to vary with consumer demographics. Under the assumption that all residential households that install solar systems are owner-occupied, we focus on the demographics of owner-occupied households in each zip code. Given the short panel of solar installation data, we do not allow demographics to vary over time. We supplement the demographic data with information from the California Secretary of State’s office on Barack Obama’s share of votes in the 2012 Presidential election at the precinct level. Finally, we use data from the National Renewable Energy Laboratory (NREL) on the median solar direct normal irradiation at the zip code level to account for variation in the solar generation potential in different areas of California.  

Table 1 summarizes the demographic data. The first column presents the average demographics for owner-occupied housing in the three California utilities in our sample. The second column presents average demographics for zip codes with households that install solar, weighted by the number of installations in each zip code. We see that households that install solar live, on average, in zip codes with slightly higher income and more expensive homes than owner-occupied households overall. Households that installed solar systems are in zip codes with greater median solar direct normal irradiance than households overall, which means that they are generally in areas with greater overall solar electricity generating potential. Interestingly, households that install solar systems are in precincts that voted for Barack Obama at slightly lower rates than owner-occupied households overall, perhaps because of their higher income and home values or because of how political preferences are correlated with solar radiation in California. Households that install solar live in zip codes with approximately the same education, household size, and number of mortgages as owner-occupied households overall.

3 Empirical Model

In order to understand how California households value residential solar and how different subsidy trajectories may change their installation decisions, we estimate a dynamic model of residential solar system adoption. Our approach is consistent with previous literature that has modeled residential solar installation decisions as a dynamic decision (e.g., Burr, 2014; Reddix II, 2014; De Groote and Verboven, 2016). In the appendix, we provide reduced-form evidence that households are indeed forward-looking when deciding whether to install solar systems.

23We remove from the sample any solar installations in zip codes served by more than one of the major utilities because we do not know what price and subsidy non-installers in those zip codes faced. For similar reasons, we also focus our analysis only on households in the territories of the three major utilities (PG&E, SCE, and SDG&E).
Table 1: Demographic Summary Statistics

<table>
<thead>
<tr>
<th></th>
<th>Average Owner-Occupied Household Demographics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Overall Install Solar</td>
</tr>
<tr>
<td>Household Income ($)</td>
<td>88,664</td>
</tr>
<tr>
<td>Home Value ($)</td>
<td>476,483</td>
</tr>
<tr>
<td>Median Solar Radiation (kWh/m²/day)</td>
<td>5.86</td>
</tr>
<tr>
<td>Democratic Vote Share</td>
<td>0.59</td>
</tr>
<tr>
<td>Years of Schooling</td>
<td>13.6</td>
</tr>
<tr>
<td>Number of Mortgages (0-2)</td>
<td>0.94</td>
</tr>
<tr>
<td>Number of Household Members</td>
<td>2.6</td>
</tr>
<tr>
<td>Count</td>
<td>4,104,377</td>
</tr>
</tbody>
</table>
| Data is at the block group level except for installations and solar radiation, which are at the zip code level and Democratic vote share, which is at the precinct level. Owner-occupied households are assumed to have the average demographics of their zip code, weighted across block groups. Solar radiation is Direct Normal Irradiance.

3.1 Model

Our dynamic choice model is based on Hotz and Miller (1993) and is similar to those of Burr (2014) and Reddix II (2014), who also consider household adoption of solar in California. The major differences with Burr (2014) are that we estimate the correlation between consumer demographics and solar preferences and we allow the evolution of solar prices to be stochastic rather than fully known by consumers. Allowing preferences to be correlated with demographics is critical for recovering a realistic distribution of the benefit of installing solar that we can use in the subsidy simulations. Allowing the price path of solar systems to be stochastic is critical for estimating households’ private values for solar, as ignoring uncertainty would bias our results in the same way as using a static model would: if households are uncertain about future costs but we model them as being certain, then it will look like they are postponing adoption because they dislike solar rather than because they are maintaining the option of waiting to adopt until prices are particularly low. Reddix II (2014) does allow for stochastic prices, but he focuses on only a limited set of household demographics in favor of a more detailed model of the size of the system chosen by the household.

In each period \( t \), each household \( i \) who has not yet installed solar has a choice of investing in solar and receiving benefit \( U_{i1t} \), where

\[
U_{i1t} = \alpha_i X + \alpha_i P ((1 - \phi) P_t - s_{it}) + \varepsilon_{i1t} + E \left[ \sum_{\tau=1}^{\infty} \beta^{t+\tau} (\psi_{iX} + \varepsilon_{i\tau}) \right].
\]  

(9)

The value of solar in the current period is \( \alpha_i X + \alpha_i P ((1 - f) P_t - s_{it}) + \varepsilon_{i1t} \), which includes
the upfront cost of solar installation $P_t$, the federal subsidy rate $\phi$, the household’s current CSI subsidy $s_{it}$, and a logit error $\varepsilon_{it}$. The coefficients $\alpha_{iX}$, $\psi_{iX}$, and $\alpha_{iP}$ are themselves functions of household demographics: $\alpha_{iX} = \alpha_0 + \sum_{d=1}^{D} \alpha_d Z_{di}$, $\psi_{iX} = \psi_0 + \sum_{d=1}^{D} \psi_d Z_{di}$, and $\alpha_{iP} = \alpha_0 + \sum_{d=1}^{D} \alpha_d Z_{di}$ where $Z_{di}$ is household $i$’s demographic characteristic $d$. The upfront benefit of installing solar is the current social, aesthetic, or reputational benefits to the household net of any nonmonetary fixed costs of researching solar systems or providers and the disruption of the solar installation process. For many households, the upfront nonmonetary cost of installing solar is likely to be large, which would make the upfront net benefit negative.

A household that installs solar never makes another decision in our model, but receives a stream of future benefits which is assumed to continue indefinitely into the future: $\sum_{\tau=1}^{\infty} \beta^{t+\tau}(\psi_{iX} + \varepsilon_{i\tau})$. This stream of benefits includes any electricity bill savings from solar generation as well as continuing social or aesthetic benefits (or costs) of having a solar system, net of any long-run costs of maintaining a solar system. We will not separately estimate the upfront benefits and the continuing benefits of solar but rather will estimate the present discounted value of the stream of net household benefits of installing a solar system: $b_i = \alpha_{iX} + \mathbb{E} \left[ \sum_{\tau=1}^{\infty} \beta^{t+\tau}(\psi_{iX} + \varepsilon_{i\tau}) \right]$.

If the household chooses not to install solar, then it receives:

$$U_{i0t} = \varepsilon_{i0t} + \beta \mathbb{E} V(\Omega_{t+1}|\Omega_t), \quad (10)$$

where $\varepsilon_{i0t}$ is a logit draw and $\beta \mathbb{E} V(\Omega_{t+1}|\Omega_t)$ is the discounted continuation value. $\Omega_t = \{P_t, t\}$ is the vector of time $t$ state variables, which includes the cost of a solar system ($P_t$) and, via the index of the current period, the current state of the preannounced subsidy policy. Consistent with policymakers committing to a subsidy schedule, we assume that the entire subsidy schedule is known to households and that households expect the subsidy to be 0 after period $T$.

However, neither households nor the regulators know the trajectory of solar system prices. Instead, they know only that solar costs evolve according to an AR1 process:

$$P_{t+1} = \gamma_0 + \gamma_1 P_t + \mu_t, \quad (11)$$

where $\mu_t$ is a normally distributed residual and we estimate the coefficients $\gamma_0$ and $\gamma_1$ from the observed price path.\textsuperscript{24}

Finally, many households do not actually face the choice of installing residential solar. This may be because they live in a condominium or other multi-family dwelling and therefore do not have the right to install solar on their roof, or it may be because their roof’s slope,

\textsuperscript{24}We use the average per-Watt installed system price in each of the three utilities in each month to estimate the parameters of the AR1 model. When expressed in tens of thousands of dollars for the average, 5.4kW, system, prices are estimated to evolve according to the equation: $P_{t+1} = 0.0006468 + 0.9925089 P_t + \mu_t$, where $\mu_t$ has a standard error of 0.16111.
orientation, or shading are not conducive to solar. We therefore also include a variable $\Phi$ that indicates whether a household is able to consider residential solar. This variable is fixed over time and can be thought of as a permanent, random shock to the household’s preference for residential solar.

We estimate the model via maximum likelihood, where the log-likelihood function in each period takes the form:

$$ LL_t = \sum_{i=1}^{N_t} \left\{ \Phi \left[ \frac{\exp(b_i X + \alpha_i P_{it})}{\exp(b_i X + \alpha_i P_{it}) + \exp(\beta \mathbb{E}V(\Omega_{t+1}|\Omega_t))} \right] 1\{i \text{ adopts in } t\} + \Phi \left[ \frac{\exp(\beta \mathbb{E}V(\Omega_{t+1}|\Omega_t))}{\exp(b_i X + \alpha_i P_{it}) + \exp(\beta \mathbb{E}V(\Omega_{t+1}|\Omega_t))} \right] 1\{i \text{ does not adopt in } t\} \right\} $$

where the tildes represent the net-of-subsidy price of installing solar ($\tilde{P}_{it} = (1 - \phi) P_{it} - s_{it}$). The log-likelihood function is summed over the number of households that have not yet installed solar and therefore changes in each period with $N_t$. While we would ideally like to estimate the percentage of households who consider installing solar, for now we instead assume that 7.5% of the households who have not yet installed solar at the start of our timeframe actually do have the ability to consider installing solar, and we conduct sensitivity checks when this rate is alternately 10% and 5%.\(^{25}\)

At each step of the likelihood maximization, we solve the value function recursively from the final period for each demographic group. In the final period, the CSI subsidy is zero and we assume that households expect the subsidy to remain zero in all periods after the end of our data. Households do, however, expect solar system costs to continue evolving after the end of our data. Households make an installation decision every month, using a monthly discount factor of 0.99 (for an annual discount rate of approximately 12%). Standard errors are calculated as the square root of the inverse of the outer product of the Jacobian.

### 3.2 Implementation

In order to implement our estimator using the CSI data, we need to limit the time-frame of our analysis and make some assumptions about the distribution of household demographics. First, there was a 30% federal tax credit available for residential solar installation during our time-frame, represented in the preceding model by $\phi$. However, the federal tax credit was initially capped at $2,000 (which was strictly binding for almost all systems), and this cap was only lifted on January 1, 2009. Because we do not know whether households anticipated this

\(^{25}\)We ran one version of our estimation where we did allow the estimation to optimize over the probability that $\Phi = 1$ in addition to the $b_i X$ and $\alpha_i P$ coefficients and the resulting estimate was 0.075. Unfortunately, estimating this probability also inflated the standard errors on our other coefficients substantially, so instead we take the approach of fixing the probability that $\Phi = 1$ and conducting sensitivity checks. We find that changing this probability does not affect the point estimates of our other coefficients substantially.
federal policy change, we limit our analysis to installations that occurred after the cap was lifted on January 1, 2009. Because the CSI data only includes information on installations that applied for CSI subsidy funding, we also end our estimation period in May 2012, when the first utility’s (PG&E’s) subsidy ended. These restrictions leave us with a 41-month time-frame for estimating households’ preferences.26

The CSI provides data on the applications to install solar, which includes information on the price paid for the solar system and the household’s zip code but not the household’s demographics. In order to estimate how preferences vary with demographics, we simulate over the distribution of demographics within each zip code as given by the American Community Survey (Nevo, 2001; Berry et al., 2004).27 There are over 2,500 zip codes in California, so identification of the demographic preference coefficients comes from the fact that, for instance, particularly high-income zip codes are more likely to have higher rates of solar installation conditional on prices than are low-income zip codes. We assume that only owner-occupied households have the potential option to install solar. It is important to emphasize how identification works in this model. Price changes at the utility level are coming largely from changes in subsidies and changes in panel costs via technological advancement, input costs, and exchange rates.28 This means that unobserved shocks to local adoption are likely to be uncorrelated with average utility-level solar system prices and we are therefore estimating the causal effect of changes in system prices on adoption. However, we are not arguing that our estimates of the relationship between household demographics and preferences are causal. For example, while we are estimating how changes in the price of a solar system will affect adoption, and how this effect will vary over households with different home values, our estimates should not be interpreted as suggesting how a change in housing values will affect solar installation rates. This is because households in high-home-value zip codes will differ from households in low-home-value zip codes in unobservable ways that we are not capturing. Critically, we need causal estimates only for price because we will study policies that determine net-of-subsidy prices, not demographics.

26We are in the process of merging our data with the recently-released dataset of all applications for connection to the California electricity grid, which will allow us to extend our estimation window somewhat for households in SCE and SDG&E.

27Unfortunately, the joint distribution of demographics is not available at the zip code level in the ACS. To capture some of the correlation between demographic characteristics, we draw from the unconditional distribution of each demographic characteristic in each block group within a zip code. Thus, if one block group has, on average, higher income and education while another block group has, on average, lower income and education, this tendency for income and education to be correlated will be captured in our simulations even though we do not know the actual covariance between income and education within a block group.

28If there is imperfect competition among local solar installers, then local prices could be correlated with local demand. We mitigate this concern by using monthly average prices by utility.
4 Empirical Results

The dynamic empirical model generates estimates of households’ valuation of residential solar systems, which we will use to quantitatively evaluate the efficient subsidy trajectory. We first describe the estimated preferences and their implications for the consumer surplus generated by the actual CSI subsidy program.

Table 2 presents the estimated coefficients from our dynamic demand model. The first set of estimates measures the present value of the benefit of installing a solar system of the median size. The average household that considers installing solar has a negative valuation of solar, even after controlling for the fact that the overwhelming majority of households are not able to even consider installing solar systems. This result makes sense, because the decision to adopt solar comes with substantial nonmonetary costs of researching whether solar is a good option for the household, of finding an installer to evaluate the home, and of understanding whether financing is available to help with the substantial upfront cost of solar. Many households will likely perceive this cost of considering solar substantial enough that they would require a substantial upfront payment or other “push” to even evaluate whether solar is a reasonable option for them.

The benefit of installing solar is increasing in the solar radiation in the household’s zip code, which makes sense given that the quantity of radiation directly determines the generation potential of a given system and therefore the stream of electricity bill savings that can be expected from a solar system. We also allow the benefit of installing solar to evolve over time, which accounts for technological improvements that increase the efficiency and overall quality of solar systems. We find that the benefit of installing solar is increasing over time, but that this rate of increase is declining. Finally, we find that households with more schooling perceive a somewhat higher value of installing solar and that the value of solar varies with utility, where households in the Southern California Edison (SCE) territory have a slightly lower value of solar and households in the San Diego Gas and Electric (SDG&E) territory have a slightly higher value of solar.

We find that households are sensitive to the net-of-subsidy cost of installing solar, although the estimate is only marginally statistically significant ($t=1.39$). There is, however, substantial variation in households’ price sensitivity, with households in high home value zip codes being less sensitive to prices than are households in zip codes with lower home values. All households are estimated to have a disutility for solar system price, but the price coefficient for households with home values over $1$ million is very close to zero.\footnote{We also explored interacting mean preferences and price sensitivity with income, number of mortgages (0,1,2+), democratic vote share, number of people in the household, and interactions between these variables, but none of these variables lead to statistically significant heterogeneity in the estimated coefficients.}

Our model simplifies actual decisions to install a solar system by omitting some factors that would affect a household’s adoption decision. In particular, we do not observe the household’s electricity use or the suitability of its particular roof for solar electricity gener-
Table 2: Dynamic Demand Estimates

<table>
<thead>
<tr>
<th>Benefit of Solar</th>
<th>-9.0078***</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(2.6959)</td>
</tr>
<tr>
<td>* Median Radiation (kWh/m²/day)</td>
<td>5.9447**</td>
</tr>
<tr>
<td></td>
<td>(2.6776)</td>
</tr>
<tr>
<td>* Years of Schooling</td>
<td>0.0458</td>
</tr>
<tr>
<td></td>
<td>(0.0609)</td>
</tr>
<tr>
<td>* Time Trend</td>
<td>0.1315***</td>
</tr>
<tr>
<td></td>
<td>(0.0121)</td>
</tr>
<tr>
<td>* Time Trend Squared</td>
<td>-0.0023***</td>
</tr>
<tr>
<td></td>
<td>(0.0005)</td>
</tr>
<tr>
<td>* SCE</td>
<td>-0.5508</td>
</tr>
<tr>
<td></td>
<td>(0.4743)</td>
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<tr>
<td>* SDG&amp;E</td>
<td>0.7817</td>
</tr>
<tr>
<td></td>
<td>(0.5197)</td>
</tr>
<tr>
<td>Price ($10,000s)</td>
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<tr>
<td></td>
<td>(0.9327)</td>
</tr>
<tr>
<td>* Home Value ($1,000,000s)</td>
<td>0.5731**</td>
</tr>
<tr>
<td></td>
<td>(0.2601)</td>
</tr>
</tbody>
</table>

Log-likelihood: 194,286
Months: 41

Standard errors in parentheses.

...
based on whether the household expects to receive a draw of the error terms that would make it worthwhile to install solar. Households who live in high radiation areas and consider installing solar expect an average consumer surplus of $5,152, whereas households who live in low radiation areas and consider installing solar expect a consumer surplus of only $3,227.\textsuperscript{30}

Similarly, households in expensive homes (over $1 million) expect to receive a consumer surplus of $4,809 from the CSI policy, whereas households in inexpensive homes (less than $200,000) expect to receive a consumer surplus of only $3,707. These differences are almost entirely due to differences in the likelihood that these households will install a solar system at all during the policy’s time-frame rather than from differences in the timing of adoption within the policy’s time-frame.

5 The efficient subsidy trajectory for rooftop solar

We now combine our structural estimates of household values for solar with the model of efficient policy developed in Section 1.\textsuperscript{31} We calibrate the regulator’s benefit to the value of solar energy (including emission displacement) from Baker et al. (2013), with the concavity of the regulator’s benefit reflecting how the intermittent nature of solar energy reduces its marginal value once there is a lot of solar on the electric grid (Gowrisankaran et al., 2016). The appendix details the calibration and solution method.

We begin by comparing the efficient subsidy to the actual subsidy. We then analyze how factors such as stochasticity, technological change, and households’ expectations affect the efficient subsidy.

5.1 The efficient subsidy versus the actual subsidy

The top panel of Figure 2 plots the efficient subsidy trajectory as well as the actual subsidies for each of the three major utilities during our estimation period. The efficient subsidy here yields the same expected cumulative adoption by the end of the 41-month horizon (0.68% of all households) as did the combination of the actual subsidies. As previously discussed, the actual subsidies decrease in steps, with PG&E’s subsidy declining from $1.55/Watt to $0.25/Watt over the period and SCE’s subsidy declining from $2.20/Watt to $0.35/Watt.\textsuperscript{32} In contrast, the efficient subsidy starts at $1.60/Watt and smoothly declines to a minimum of $0.58/Watt in period 31 before rising to $0.68/Watt in period 41.

\textsuperscript{30}Households who do not consider installing solar expect to receive no consumer surplus from the policy. These calculations abstract from how the CSI is funded and therefore do not take into account any changes in electricity bills that may arise from the need to fund the policy.

\textsuperscript{31}For cases with stochastic costs and/or stochastic preferences, we add a time-0 expectation operator to the regulator’s objective in Section 1 and require that $E_0[Q(T)] = Q_T$.

\textsuperscript{32}PG&E’s subsidy declines to $0 immediately after our estimation period, while SCE and SDG&E continue to have positive subsidies for a short period after our estimation window.
Figure 2: The trajectories of the subsidy level, per-period adoption, and per-period discounted spending under the efficient subsidy policy as well as the actual policies.
The efficient subsidy starts at a smaller value and declines less rapidly than did the actual subsidy. The middle panel of Figure 2 shows that the efficient subsidy delays adoption until later periods. This delayed adoption arises both because the regulator prefers spending money later and because the regulator anticipates that the technology will be cheaper in later periods. The bottom panel of Figure 2 shows that the efficient subsidy substantially reduces subsidy spending over the first half of the 41-month interval while only slightly increasing the present value of subsidy spending in later periods. The desire to delay spending leads the efficient subsidy to decline more slowly than did the actual policy. And the desire to smooth spending over time leads the efficient subsidy to decline smoothly.

The efficient subsidy is relatively flat over the middle periods, yet we see a high rate of adoption. This high rate of adoption is due not only to declines in the cost of installing solar. Instead, it arises because our preference estimates suggest a high amount of stochasticity in consumers’ willingness-to-pay for solar. Thus, households whose stochastic draws give them a low willingness-to-pay for solar in one period may have a high willingness-to-pay in a later period. By keeping the subsidy relatively constant over this interval, the regulator is able to continually encourage adoption by households that receive favorable preference draws in later periods.

The efficient policy is able to induce the same amount of total adoption at much lower total cost than was observed in practice. Our model suggests that regulators should have expected to spend $148 million (in present value terms) over the 41-month interval under the actual subsidy schedule, whereas the efficient subsidy would have cost only $102 million in expectation. Some of this difference in spending does result in a small reduction in households’ consumer surplus: the actual subsidy schedule increased expected consumer surplus by $3,988 per household (among those that consider installing solar), whereas the efficient subsidy schedule would increase expected consumer surplus over those same households by $3,911 per household.

5.2 Determinants of the efficient subsidy

We now combine the empirical results with the theoretical analysis to understand how the efficient subsidy trajectory depends on assumptions about consumer expectations and the rate of technological change.

Table 3 reports the present value of subsidy spending along the efficient trajectory for

33 These households may become able to access sources of credit that were not available in earlier periods (for instance, if they are refinancing their mortgage anyway or performing other home improvements) or may be approached by a solar installer with information about how solar would work for their particular home.

34 The efficient subsidy analyzed here does not optimize over the allocation of solar systems to high radiation areas (which would offer greater electricity generation), except insofar as the potential for electricity generation is reflected in households’ private values for solar. While it is possible to design a spatially differentiated subsidy (see Kokoza, 2017), we here focus on how to differentiate a subsidy over time.
Table 3: The efficient initial and terminal subsidies as well as the present value of spending along the efficient subsidy trajectory that achieves 3% adoption.

<table>
<thead>
<tr>
<th></th>
<th>Stochastic Model</th>
<th></th>
<th>Deterministic Model</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tech Progressay</td>
<td></td>
<td>Tech Constant</td>
<td></td>
</tr>
<tr>
<td>Myopic households</td>
<td>Tech Progress ay</td>
<td></td>
<td>Tech Constant</td>
<td></td>
</tr>
<tr>
<td>Present value of expected spending ($billion)</td>
<td>1.5</td>
<td>3.7</td>
<td>5.2</td>
<td>7.4</td>
</tr>
<tr>
<td>Present value of expected spending ($/W)</td>
<td>2.3</td>
<td>5.5</td>
<td>7.9</td>
<td>11.2</td>
</tr>
<tr>
<td>Initial subsidy ($/W)</td>
<td>4.4</td>
<td>6.1</td>
<td>6</td>
<td>6.1</td>
</tr>
<tr>
<td>Terminal subsidy ($/W)</td>
<td>3.2</td>
<td>7.6</td>
<td>11.2</td>
<td>16.1</td>
</tr>
<tr>
<td>Forward-looking households</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Present value of expected spending ($billion)</td>
<td>2.3</td>
<td>3.9</td>
<td>6.3</td>
<td>8.5</td>
</tr>
<tr>
<td>Present value of expected spending ($/W)</td>
<td>3.5</td>
<td>6</td>
<td>9.5</td>
<td>12.9</td>
</tr>
<tr>
<td>Initial subsidy ($/W)</td>
<td>6.1</td>
<td>7</td>
<td>15</td>
<td>15.1</td>
</tr>
<tr>
<td>Terminal subsidy ($/W)</td>
<td>4.2</td>
<td>7.7</td>
<td>11.2</td>
<td>16.1</td>
</tr>
</tbody>
</table>

achieving adoption of 3% in 41 months. It analyzes the efficient subsidy in the full empirical model as well as for three other cases: a case in which households receive new draws of each $\varepsilon_i$ in each period but the cost and quality of solar technology are held fixed at their initial values, a deterministic case in which households receive only a single draw of each $\varepsilon_i$ for all time and technology is known to improve according to its expected trajectory, and a deterministic case in which households receive only a single draw of each $\varepsilon_i$ for all time and the cost and quality of solar technology are fixed at their initial values. In each case, we report results when households are forward-looking (with rational expectations over future subsidies, over possible values of $\varepsilon_i$, and over technological progress) and when households are myopic (with no consideration of their option to adopt in a future period).

Begin by considering the cases in which households’ private values for solar PV can change over time (first two columns). Comparing the top panel to the bottom panel, we see that households’ anticipation that future periods could bring improved technology, a greater private value for solar, and/or a different subsidy increases spending along the efficient subsidy trajectory and raises both the initial and the terminal subsidy. When households anticipate that these variables may change over time, they have an incentive to wait to install solar at a later date. In order to incentivize installation in some given period, the regulator must offer households a subsidy that is high enough to not only give them positive net value from installing solar but also to compensate them for forsaking the option to adopt solar in some later period.

35We here study a case with 3% adoption because the case with 3% adoption allows us to consider how the regulator would subsidize myopic households, whereas the case with 0.68% adoption and myopic households would not require any subsidy. All other results are qualitatively similar for a target of 0.68% adoption.
Now consider how fixing the cost and quality of solar technology at their initial values changes the efficient policy. Comparing the first and second columns, we see that holding technology constant increases subsidy spending. Importantly, the increase in spending is greater when households are myopic ($2.2 billion as opposed to $1.6 billion) because households’ anticipation of technological progress increases their value of maintaining their option to adopt solar until some later period. The regulator must spend more to induce households to adopt solar in a given period when households anticipate the possibility of better technology in the future.36

Next, contrast the stochastic model with the deterministic model. Spending and subsidies are 2–3 times greater under the deterministic model. The reason is that a given household’s probability of having a relatively high value for solar at some point in the 41-month window is greater when its value for solar can change over time (i.e., when it receives a new draw of each $\varepsilon_i$ in each period). When each household’s value for solar is fixed at its initial level, the regulator will eventually have to convince lower-value households to adopt solar.

Figure 3 plots the efficient subsidy (top) and adoption (bottom) trajectories when households receive a new draw of each $\varepsilon_i$ in every period (left) and when households receive only a single draw of each $\varepsilon_i$ for all time (right). Consistent with the previous discussion, the efficient policy offers a larger subsidy when households receive only a single draw of each $\varepsilon_i$ for all time and/or when households are forward-looking. The efficient subsidy trajectory is sensitive to the potential for technological progress. When households receive a new draw of each $\varepsilon_i$ in each period, the efficient subsidy in a world with constant technology increases slightly, whereas the efficient subsidy in a world with technological progress declines over much of the horizon. When households receive a single draw of each $\varepsilon_i$ for all time, the efficient subsidy in a world with constant technology increases sharply over time, whereas the efficient subsidy in a world with technological progress either declines (if households are forward-looking) or follows a much flatter trajectory (if households are myopic).37

In all cases, the regulator delays adoption until later periods so as to substitute technological progress for subsidy spending.

Figure 4 uses the theoretical analysis to decompose the deterministic model’s efficient subsidy trajectory into the components analyzed in equations (2) and (8). The vertical sum of the components gives the instantaneous change in the subsidy $\dot{s}(t)$, labeled “Total”. This decomposition explains the diversity of subsidy trajectories. First, the top panels of Figure 4

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36 In the deterministic model, the reduction in spending due to technological progress is not sensitive to whether households are forward-looking or myopic. This result suggests that it is uncertainty about future technological progress, rather than expectations of progress, that is most responsible for increasing households’ option value.

37 The efficient subsidy increases most sharply when consumers are myopic and receive only a single draw of each $\varepsilon_i$ for all time. The reason is that making consumers forward-looking limits the regulator’s ability to intertemporally price discriminate through an increase subsidy, and allowing households to receive a new draw of each $\varepsilon_i$ in each period limits the gains from intertemporally price discriminating because it increases the number of inframarginal households in later periods.
Figure 3: The ex ante efficient subsidy trajectory (top) that achieves a target of 3% adoption when each household receives a new draw of $\varepsilon_i$ in each period (left) and when each household receives only a single draw of $\varepsilon_i$ for all time (right). Also, adoption along this efficient trajectory (bottom).
show why the efficient subsidy increases strongly over time when households are myopic: the price discrimination channel is large. By starting with a relatively small subsidy and raising it over time, the regulator avoids paying a large subsidy to households who would adopt even for a smaller subsidy.\footnote{The price discrimination channel is especially important in early periods because the marginal adopter is far in the right tail of the distribution of households. In this case, most adopters tend to be inframarginal.} In a world with technological progress (right panels), the price discrimination motive is eventually tempered by the regulator’s preference for spreading spending over time, as captured by her convex cost of public funds. The efficient subsidy trajectory flattens out so as to limit the surge in adoption induced by technological progress.

When households are forward-looking, their expectations and ability to time adoption constrain the regulator’s ability to price discriminate and to take advantage of technological progress. We have already discussed how the regulator must offer forward-looking households a larger subsidy in order to compensate them for forsaking their option to adopt solar at a

Figure 4: The change in the efficient subsidy at each instant (\( \dot{s}(t) \), labeled “Total”), as well as each component from equations (2) and (8). All panels use a target of 3% adoption. The opportunity cost component (not plotted) is negative but very small in magnitude.
later time. The bottom left panel of Figure 4 shows that the larger required subsidy amplifies the negative cost convexity channel. Further, the large, positive price discrimination channel is replaced by a promise-keeping channel that begins at zero and increases only slowly. Therefore, when technology is constant, the efficient subsidy increases much more slowly than in the case with myopic households, and it even declines strongly over time in the case with technological progress.\footnote{Using an increasing subsidy does not eliminate early adoption when households are forward-looking and technology is constant because households who are not perfectly patient will adopt the technology as long as the subsidy is not increasing too fast. The more patient that households are, the less freedom the regulator has to use an increasing subsidy.} The increasing promise-keeping channel reflects how the regulator would like to use a higher subsidy in later periods but is constrained by promises it made to convince households to adopt the technology in earlier periods.

6 Conclusions

We have demonstrated the forces that determine how to structure a subsidy designed to induce adoption of a new technology over time. In particular, we have shown that if consumers are myopic, then the regulator can reduce its overall spending by using an increasing subsidy schedule as a means of intertemporally price discriminating. However, if consumers have rational expectations over future subsidies, then their ability to wait for the higher subsidies constrains the regulator’s ability to price discriminate. Further, the regulator must offer consumers a relatively large subsidy in order to compensate them for forsaking their option to adopt the technology in a later period. Rational expectations thus increase the total cost of the policy program.

Quantitatively, we find that the California regulator could have reduced its spending on rooftop solar by 31\% if it had implemented the efficient subsidy trajectory. The regulator’s expectation of technological progress makes the subsidy decrease over time. Consumers’ rational expectations of future subsidies, technological change, and preferences increase the required spending by 50\% and limit the degree to which the regulator can take advantage of technological progress to reduce the cost of the policy.

Future work should explore the implications of rational expectations and technological dynamics in other policy environments. For instance, economists commonly recommend emission taxes that increase over time and also subsidies for research that would improve technology. Yet many economic models abstract from consumer and firm expectations of policy and of technology. Future work should consider when private decisions to wait are socially inefficient and how to design policy to manage private actors’ expectations.
References


Appendix

The first section of the appendix provides empirical evidence that consumers are forward-looking in the California market for rooftop solar, which requires us to estimate a dynamic choice model in order to study their adoption decisions. The second section describes the numerical calibration of the optimal subsidy conditional on the estimates of our empirical model. The third section demonstrates that analyzing a fixed budget does not substantially affect our theoretical analysis of the efficient subsidy trajectory.

A Evidence that Consumers are Forward-Looking

Before we estimate consumers’ private values for residential solar, we must understand whether consumers actually do think about future solar costs and subsidies when they make their investment decisions. There are a few reasons why we might think that the dynamic trajectory of prices and subsidies would matter for households’ solar installation decisions. First, there are examples in the literature on goods other than residential solar where consumers make decisions in expectation of future policy changes (e.g., Mian and Sufi, 2012). Second, it is not necessary that households themselves are informed about the future trajectory of solar subsides as long as solar installers are informed. If installers use the fact that subsidies will be changing to encourage households to install solar now, then households will act as if they were directly informed about the subsidy schedule.

To understand whether households in California behave as if they are forward-looking, we regress the weekly counts of residential solar subsidy applicants on measures of current and future solar system costs and controls. In particular, we focus on future drops in the CSI subsidy, changes in the prices of solar modules and of silicon, and changes in the US-China exchange rate. When the subsidy is about to decrease, more households will choose to install solar now if households are forward-looking.\footnote{This effect has been shown previously in Burr (2014).} Similarly, for a given current cost of solar modules, high input (silicon) costs suggest that the cost of solar modules is about to increase, so forward-looking households will want to install now. Since China produces a large number of solar panels, if the US-China exchange rate is increasing, then panels will be less expensive in the future and forward-looking households should wait to invest in solar.

In order to test whether household behavior responds to these measures of future costs, we combine the CSI application data with a solar module price index and a silicon price index from Bloomberg. We also include the 3-month change in the dollar-yuan exchange rate,\footnote{We construct this change as the difference between the exchange rate realized in 3 months and the current exchange rate. We will replace this variable with the 3-month exchange rate future minus the current exchange rate in a future draft.} and a set of controls including the VIX (a measure of the expected volatility of U.S. equities), the current dollar-yuan exchange rate, the current Euro-yuan exchange rate, the 6-month...
Treasury bill interest rate, and the DOW real estate investment trust index. These controls aim to capture macroeconomic conditions that affect the incentive to invest in solar.42

Table 4: Evidence that Consumers are Forward-Looking

<table>
<thead>
<tr>
<th>Dependent Variable: Log Weekly CSI Applications</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Within 2 months of subsidy drop</td>
<td>0.300***</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
</tr>
<tr>
<td>Solar Module Price Index</td>
<td>-2.701***</td>
</tr>
<tr>
<td></td>
<td>(0.619)</td>
</tr>
<tr>
<td>Silicon Price Index</td>
<td>0.012**</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
</tr>
<tr>
<td>3 month change in Dollar-Yuan Exchange Rate</td>
<td>-2.045***</td>
</tr>
<tr>
<td></td>
<td>(0.694)</td>
</tr>
<tr>
<td>VIX (Instrumented)</td>
<td>-0.032**</td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
</tr>
<tr>
<td>Number of utility-weeks</td>
<td>650</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.6779</td>
</tr>
</tbody>
</table>

Standard errors clustered by week. Controls include the Dollar-Yuan exchange rate, the Euro-Yuan exchange rate, the 6 month T-bill interest rate, the DOW REIT index, utility fixed effects and utility-specific time trends. Instruments are month-over-month changes in 10 year bond yields for Greece, Italy, Russia, and Spain. F=482.55 Results are not substantially different if the VIX is not instrumented.

The regression results presented in Table 4 suggest that consumers are forward-looking in their decision to install solar (or at least that solar installers are forward-looking). The coefficient estimates suggest that more households submit applications to install solar systems when subsidies are about to decline.43 Higher solar module prices reduce installation, but conditional on solar module prices, higher silicon prices (which suggest that module prices will be higher in the future) increase installation. Similarly, if the US-China exchange rate is increasing (conditional on the current level of the exchange rate) then solar panels

42We instrument for the VIX with month-over-month changes in 10-year bond yields for several economically-volatile developed countries (Greece, Italy, Russia, and Spain) in order to isolate sources of market uncertainty that should not directly affect California households through channels such as employment or income.

43Since the CSI subsidies were tied to cumulative installation in each utility, it is possible that higher adoption is causing subsidy declines. To test for this, we conducted a separate analysis on only media markets served by two utilities. If local conditions were leading households to install solar and thereby trigger declines in the subsidy, then we would see increased adoption in households in, for instance, Southern California Edison, when their neighboring utility, Pacific Gas and Electric, was about to have a subsidy decline. We found no evidence that this was true, leading us to conclude that reverse-causality is not a major problem for our analysis.
will likely be less expensive in the future. Indeed, our estimates suggest that an increasing
US-China exchange rate reduces installations today. Finally, it is interesting that economic
uncertainty, as captured by the instrumented VIX, tends to decrease solar installations, as
would be expected if households account for option value when timing their installation
decision (which again implies that households are not fully myopic).

Given that households behave as if they are forward-looking, it is important to use a
dynamic model of residential solar adoption in order to estimate the benefits of installing
a solar system. If we used a static model of solar adoption, we would underestimate the
benefits to households of adopting: some households that value solar above the current
system cost will choose not to install for now as they wait for technology to advance and for
prices to drop. Correctly accounting for the value of waiting for lower prices is critical to
understanding the trade-offs regulators face in structuring solar subsidies.

B Numerical Calibration and Solution Method

We begin by describing the calibration. We then describe how we solve the stochastic and
deterministic models.

B.1 Calibration

We here describe the calibration of the regulator’s benefit from cumulative adoption and the
cost of public funds. Assume that the regulator’s benefit function is quadratic:

\[ B(Q(t)) = \gamma_1 Q(t) + \gamma_2 Q(t)^2. \]

We calibrate \( \gamma_1 \) as the marginal social benefit of solar from Baker et al. (2013). They
simulate a 5 kW array, whereas we use a 5.4 kW array. They report a south-facing array in
San Francisco as generating 7,220 kWh (ac) per year. Assume that this energy production
is evenly distributed across months. They report the value of solar to the electric grid
("weighted average \( \lambda \)") as $0.055/kWh. They report the emission displacement rate as 1.11
pounds of carbon dioxide (CO\(_2\)) per kWh. Using the U.S. government’s year 2015 social cost
of carbon (with a 3% discount rate) of $36/tCO\(_2\), we have the emission benefit of a 5 kW
array as $0.0181/kWh. Over the course of a month, the combined grid and emission benefit
of a 5.4 kW array is

\[ (5.4/5)(7220/12)(0.0181 + 0.055)/10^4 = 0.0048 \]

in tens of thousands of dollars, which we use as the marginal social benefit of the first array
to be adopted (i.e., the first 5 kW array to be adopted generates $48 per month of social
value). The parameter \( \gamma_1 \) applies to the fraction \( Q(t) \) of the population that has adopted:

\[ \gamma_1 = 0.0048 \sum_j N_j. \]
Assume that the marginal social value of an installed solar array falls by x% by the time we reach z% adoption (e.g., because of concerns about intermittency). Then

\[ \gamma_2 = -\frac{x}{2z} \gamma_1 \leq 0. \]

Gowrisankaran et al. (2016) estimate that the intermittency of solar electricity would impose costs of $46/MWh if solar photovoltaics provided 20% of electricity in Arizona.\(^{44}\) For our 5.4 kW array, this works out to a cost of

\[ (\frac{5.4}{5}) \times \frac{(7220/12)}{(46/1e3)}/1e4 = 0.0030 \]

in tens of thousands of dollars, which means that our marginal benefit of solar would decline by 62.5% (yielding \( x = 62.5 \)). Using California’s 2014 consumption of 296,843 GWh,\(^{45}\) providing 20% of electricity means providing 59,369 GWh. To generate this much electricity, we require \( 59369e6/[(5.4/5) \times 7220] \) arrays (or nearly 8 million arrays). We therefore have \( z = 100 \times 59369e6/[(5.4/5) \times 7220]/ \sum_j N_j \), or an adoption rate of 185%.

Finally, assume that the regulator’s cost of funds is quadratic:

\[ G(z) = g_1 z + g_2 z^2, \]

where \( z \) is spending per capita in tens of thousands of dollars (i.e., \( z = s(t) \hat{Q}(t) \)). Let the marginal cost of the first funds be unity. Then we have:

\[ g_1 = \sum_j N_j. \]

Let the marginal cost of funds double when spending reaches \( x \) dollars. So the cost of funds doubles when we have an \( s(t) \hat{Q}(t) \) such that \( z \sum_j N_j = x/10^4 \). Thus, we have

\[ g_2 = \frac{1}{2} \frac{g_1}{x} 10^4 \sum_j N_j. \]

We assume that \( x \) is $1 million.

The regulator’s horizon is 41 months. As in the empirical model, we assume that households use a monthly discount rate of 1% and we measure time in months. We assume that the regulator uses the same discount rate as its households. We begin with 0.02% of households having adopted solar (\( Q_0 = 0.0002 \)).

---

\(^{44}\)This scenario assumes that conventional sources of electricity are reoptimized around the 20% solar penetration rate. This number does not account for how the marginal value of electricity may decline in solar penetration.

\(^{45}\)http://energyalmanac.ca.gov/electricity/total_system_power.html
B.2 Solving the stochastic setting

We now describe how we solve the setting in which each household can receive a new draw of each \( \varepsilon_i \) in each period and in which installation costs may evolve stochastically. For any candidate subsidy trajectory, we simulate the empirical model for some particular draws of the random variables in order to obtain adoption in each period and in order to obtain the regulator’s present value over this simulation. Averaging over 200 such simulations gives expected adoption at the end of the horizon and gives the regulator’s expected value from committing to the candidate subsidy trajectory. We use the Matlab Knitro solver to search for the 41-element trajectory of per-month subsidies that maximizes the regulator’s expected value while matching expected adoption at the end of the horizon to \( Q_T \).

B.3 Solving the deterministic setting

We now describe how we solve the setting in which each household receives only a single draw of each \( \varepsilon_i \) for all time and in which the evolution of installation costs is deterministic.

To start, consider how the structural empirical model provides the desired distribution over private values \( v_i \) from the theoretical setting. The household’s private value from adopting solar is equal to the price at which the household would be indifferent between adopting solar and not adopting solar if adoption were a now-or-never decision. From equations (9) and (10), household \( i \) is indifferent to adopting solar at time \( t \) when

\[
\alpha_{iX} + \alpha_{iP} P_t + \varepsilon_{i1t} + \mathbb{E} \left[ \sum_{\tau=1}^{\infty} \beta^{t+\tau} (\psi_{iX} + \varepsilon_{i\tau}) \right] = \varepsilon_{i0t} + \beta \mathbb{E} V(\Omega' | \Omega),
\]

which we can write as

\[
b_i + \alpha_{iP} P_t + \varepsilon_{i1t} - \varepsilon_{i0t} = \beta \mathbb{E} V(\Omega' | \Omega), \tag{A-1}
\]

with

\[
b_i = \alpha_{iX} + \mathbb{E} \left[ \sum_{\tau=1}^{\infty} \beta^{t+\tau} (\psi_{iX} + \varepsilon_{i\tau}) \right].
\]

When adoption is a now-or-never decision, the expectation on the right-hand side of equation (A-1) is zero. Household \( i \)’s private value for solar is then

\[
v_i = -\frac{1}{\alpha_{iP}} (b_i + \varepsilon_{i1t} - \varepsilon_{i0t}),
\]

where each \( \epsilon \) is a draw from a type I generalized extreme value distribution with location parameter 0 and scale parameter 1 and where each estimated \( \alpha_{iP} \) is negative. Rewrite as

\[
\varepsilon_{i1t} - \varepsilon_{i0t} = -\alpha_{iP} v_i - b_i.
\]
The difference of two type I generalized extreme value random variables is itself a random variable following a logistic distribution with location parameter 0 and scale parameter 1. The cumulative distribution function $P(\cdot)$ of $v_i$ is:

$$P(v_i) = \frac{1}{1 + e^{\alpha_i P v_i + b_i}}.$$

The number of households with the same demographic characteristics as household $i$ is $N_i$. Aggregate across demographic groups to obtain the cumulative distribution function for private values $v$ across all demographic groups:

$$F(v) = \frac{\sum_i N_i \frac{1}{1 + e^{\alpha_i P v_i + b_i}}}{\sum_i N_i}.$$

Differentiating yields the density function of private values:

$$f(v) = -\frac{\sum_i N_i \alpha_i P e^{\alpha_i P v_i + b_i}}{(1 + e^{\alpha_i P v_i + b_i})^2}.$$

Differentiating a second time yields:

$$f'(v) = -\frac{\sum_i N_i \alpha_i^2 P e^{\alpha_i P v_i + b_i} (1 - e^{\alpha_i P v_i + b_i})}{(1 + e^{\alpha_i P v_i + b_i})^3}.$$

Now consider the evolution of the private cost of installing solar. The empirical results account for changes in both the monetary cost of installing solar panels and the preference for solar, which we interpret as changes in the quality of solar panels. We take $C(t) = \xi(t) - \chi(t)$ to measure the quality-adjusted cost, with $\xi(t)$ the direct monetary cost and $\chi(t)$ a discount to reflect quality improvements since time 0. As described above for $b_i$, the trend in quality will affect demographic groups differentially via $\alpha_i P$. We abstract from the time trend’s demographic dependence by using a population-weighted average of $\alpha_i P$ when calibrating $\chi(t)$ to the empirical estimates. The empirical estimates yield $\chi(t) = \chi_0 \cdot t + \chi_1 \cdot t^2$, with $\chi_0 > 0$ and $\chi_1 < 0$.

We regress the cost of installation (beginning with the elimination of the federal subsidy cap) against a constant and its lagged value:

$$\xi_{t+1} = \theta_0 + \theta_1 \xi_t,$$

where $t$ is measured with 0 as the start date and costs are measured in tens of thousands of dollars per 5.4 kW system. This regression yields $\theta_0 = 0.00065$ and $\theta_1 = 0.99$. The initial cost of installing is $C(0) = \xi(0) = 4.4$, with the initial period serving as the reference for quality ($\chi(0) = 0$).
Subtracting $\xi_t$ from each side and passing to the continuum, we have
\[ \dot{\xi}(t) = \theta_0 + [\theta_1 - 1]\xi(t). \]

Solve the differential equation:
\[ \dot{\xi}(t) + [1 - \theta_1]\xi(t) = \theta_0 \]
\[ \Leftrightarrow e^{[1-\theta_1]t}\left\{\dot{\xi}(t) + [1-\theta_1]\xi(t)\right\} = e^{[1-\theta_1]t}\theta_0 \]
\[ \Leftrightarrow \int_0^t e^{[1-\theta_1]s}\left\{\dot{\xi}(s) + [1-\theta_1]\xi(s)\right\} ds = \int_0^t e^{[1-\theta_1]s}\theta_0 ds \]
\[ \Leftrightarrow \xi(t) = \frac{\theta_0}{1-\theta_1}\left[ 1 - e^{-[1-\theta_1]t} \right] + e^{-[1-\theta_1]t}\xi(0). \quad (A-2) \]

Substituting into $\dot{\xi}(t)$, we have
\[ \dot{\xi}(t) = \theta_0 e^{-[1-\theta_1]t} - [1 - \theta_1]e^{-[1-\theta_1]t}C(0). \]

Differentiating with respect to time, we have:
\[ \ddot{\xi}(t) = -\theta_0(1 - \theta_1)e^{-[1-\theta_1]t} + [1 - \theta_1]^2e^{-[1-\theta_1]t}\xi(0). \]

Figure A-1 plots the calibrated evolution of system costs.

We now solve for $J(V(T), C(T))$ in the setting with forward-looking households. At time $T$, a household that has yet to adopt the technology solves:
\[ J(v_i, C(T)) = \max_{\Psi_i} e^{-\delta(\Psi_i - T)} [v_i - C(\Psi_i)], \]
for $\Psi_i > T$. Substituting from equation (A-2) for $C(\Psi_i)$ (and re-expressing equation (A-2) for an initial condition on $C(T)$ rather than $C(0)$), this becomes:
\[ J(v_i, C(T)) = \max_{\Psi_i} e^{-\delta(\Psi_i - T)} \left[ v_i - \frac{\theta_0}{1 - \theta_1} \left[ 1 - e^{-(1-\theta_1)(\Psi_i - T)} \right] - e^{-\theta_1(\Psi_i - T)}C(T) \right] \quad (A-3) \]
for $v_i > 0$ and, recognizing that installation costs are always positive, becomes $J(v_i, C(T)) = 0$ for $v_i \leq 0$. Assume that $v_i > 0$. The first-order necessary condition for a maximum is
\[ \delta \left[ v_i - \frac{\theta_0}{1 - \theta_1} \left[ 1 - e^{-(1-\theta_1)(v_i - T)} \right] - e^{-(1-\theta_1)(v_i - T)}C(T) \right] \]
\[ = - (1 - \theta_1) \left[ \frac{\theta_0}{1 - \theta_1} e^{-(1-\theta_1)(v_i - T)} - e^{-(1-\theta_1)(v_i - T)}C(T) \right], \]

A-7
which yields:

\[ \Psi_i^* = T - \frac{1}{(1 - \theta_1)} \left\{ \ln \left( -\delta v_i + \delta \frac{\theta_0}{1 - \theta_1} \right) - \ln \left( \delta \frac{\theta_0}{1 - \theta_1} - \delta C(T) + \theta_0 - (1 - \theta_1) C(T) \right) \right\} \]

Substituting into equation (A-3), we have

\[ J(v_i, C(T)) = e^{-\delta(\Psi_i^*-T)} \left[ v_i - \frac{\theta_0}{1 - \theta_1} \left[ 1 - e^{-(1-\theta_1)(\Psi_i^*-T)} \right] - e^{-(1-\theta_1)(\Psi_i^*-T)} C(T) \right] \]

for \( v_i > 0 \). This expression and its analogue for \( v_i \leq 0 \) provide the terminal subsidy \( s(T) \), as described in the main text.

To solve the setting with myopic households, note that equation (1) implicitly defines \( y(t) \) as a function of \( \lambda(t) \) and \( V(t) \). We then have two differential equations (\( \dot{V}(t) = y(t) \) and the costate equation) in two variables. We know \( V(0) \) and \( V(T) \). For any guess for \( \lambda(T) \), we solve the system backwards from time \( T \) to time 0 and compare the obtained \( V(0) \) to the desired \( V(0) \).

To solve the setting with forward-looking households, note that equation (5) implicitly defines \( y(t) \) as a function of \( \lambda(t) \), \( s(t) \), \( \mu(t) \), and \( V(t) \). We then have four differential equations (the transition and costate equations) in the same four variables. We know \( V(0) \), \( \mu(0) \), \( V(T) \), and \( s(T) \). For any guess for \( \lambda(T) \) and \( \mu(T) \), we solve the system backwards from time \( T \) to time 0 and compare the obtained \( V(0) \) and \( \mu(0) \) to the desired \( V(0) \) and \( \mu(0) \).

In both cases we use the Matlab Knitro solver to search for the terminal conditions that yield trajectories that satisfy the initial conditions. We run the solver with a multistart procedure. We use Matlab’s ode45 solver to solve the system of differential equations for any guess of terminal conditions.
C Analyzing a Fixed Budget

We now consider a setting in which the regulator has a fixed budget but is free to choose cumulative adoption. Let $Z(t)$ denote cumulative spending, with $Z_T > 0$ the fixed budget. We assume that the budget is small enough that the regulator wants to exhaust it. Begin with myopic consumers.

$$\max_{y(t), V(T)} \int_0^T e^{-rt} \left[ B \left( 1 - F(V(t)) \right) - G \left( -[C(t) - V(t)] f(V(t)) y(t) \right) \right] dt$$

s.t. $\dot{V}(t) = y(t)$

$\dot{Z}(t) = -[C(t) - V(t)] f(V(t)) y(t)$

$V(0) = F^{-1}(1 - Q_0)$

$Z(0) = 0, \; Z(T) = Z_T.$

The Hamiltonian is:

$$H(t, y(t), V(t), Z(t), \lambda_V(t), \lambda_Z(t)) = e^{-rt} \left[ B \left( 1 - F(V(t)) \right) - G \left( -[C(t) - V(t)] f(V(t)) y(t) \right) \right] + e^{-rt} \lambda_V(t) y(t) - e^{-rt} \lambda_Z(t) [C(t) - V(t)] f(V(t)) y(t).$$

$\lambda_V(t)$ gives the (current) shadow value of $V(t)$, and $\lambda_Z(t)$ gives the (current) shadow value of $Z(t)$. The necessary conditions for a maximum are:

$$\lambda_V(t) = [C(t) - V(t)] f(V(t)) \left( \lambda_Z(t) - G' \left( -[C(t) - V(t)] f(V(t)) y(t) \right) \right), \quad (A-4)$$

$$-\dot{\lambda}_V(t) + r \lambda_V(t) = -B' \left( 1 - F(V(t)) \right) f(V(t))$$

$$+ \left[ \lambda_Z(t) - G' \left( -[C(t) - V(t)] f(V(t)) y(t) \right) \right] \left[ f(V(t)) y(t) - [C(t) - V(t)] f'(V(t)) y(t) \right]$$

$$-\dot{\lambda}_Z(t) + r \lambda_Z(t) = 0,$$

$\lambda_V(T) = 0,$

along with the transition equations, the initial conditions, and the terminal condition on $Z(\cdot)$. Extending the problem to allow the regulator to obtain benefits after time $T$ would only change the final, transversality condition, which does not affect our analysis of $\dot{s}(t)$. 
Differentiate equation (A-4) with respect to time and suppress the argument of $G(\cdot)$:

$$
\dot{\lambda}_V(t) = \left[ \dot{C}(t) - y(t) \right] f(V(t)) \left[ \lambda_Z(t) - G' \right] - \left[ C(t) - V(t) \right] f'(V(t)) y(t) \left[ \lambda_Z(t) - G' \right] \\
- \left[ C(t) - V(t) \right] f'(V(t)) \dot{G}'' \\
\left[ - \left[ \dot{C}(t) - y(t) \right] f(V(t)) y(t) - \left[ C(t) - V(t) \right] f'(V(t)) y(t)^2 - \left[ C(t) - V(t) \right] f(V(t)) \dot{y}(t) \right] \\
+ \left[ C(t) - V(t) \right] f(V(t)) \dot{\lambda}_Z(t).
$$

Substitute for $\dot{\lambda}_V(t)$ and $\dot{\lambda}_Z(t)$ from the costate equations and rearrange to obtain:

$$
\dot{s}(t) = \frac{-r \lambda_V(t) f(V(t)) + r \lambda_Z(t) s(t) - B' + [G' - \lambda_Z(t)] y(t) + [s(t)]^2 G'' \left[ f(V(t)) \dot{y}(t) + f'(V(t)) [y(t)]^2 \right]}{G' - \lambda_Z(t) - s(t) f(V(t)) \dot{G}'' y(t)}
$$

where we use $\dot{s}(t) = \dot{C}(t) - y(t)$ from the definition of $V(t)$ and suppress the argument of $B(\cdot)$. We see the same effects as in the main text’s setting with a fixed adoption target $Q_T$, plus an additional effect. $\lambda_Z(t) < 0$ is the shadow cost of additional spending, so that $\lambda_Z(t) s(t)$ is the shadow cost of time $t$ spending. This shadow cost is driven by the scarcity of funds. We see the price discrimination channel become amplified by this additional cost of funds. We also see a new channel that works to make the efficient subsidy decline over time. The regulator has a fixed budget, and all else equal, an impatient regulator chooses to consume more of this budget earlier.

Combine this new channel with the first (Hotelling) channel and substitute for $\lambda_V(t)$ from equation (A-4) to obtain:

$$
\dot{s}(t) = \frac{r \cdot s(t) G' - B' + [G' - \lambda_Z(t)] y(t) + [s(t)]^2 G'' \left[ f(V(t)) \dot{y}(t) + f'(V(t)) [y(t)]^2 \right]}{G' - \lambda_Z(t) - s(t) f(V(t)) \dot{G}'' y(t)}.
$$

We have replaced the Hotelling channel and the new channel with $r \cdot s(t) G'$. This remaining term is the exact same term as the Hotelling channel in the main text, once we substitute for $\lambda(t)$ in equation (2) from equation (1). Thus, the main text’s analysis of $\dot{s}(t)$ also applies to a setting with a fixed budget instead of a fixed adoption target. The analysis for the case with forward-looking households is similar.