Abstract

This paper investigates how political incentives affect the allocation of public funds using data from Brazil’s Federal Legislature, a setting in which federal politicians representing multi-member districts can issue budgetary amendments for public works in their respective districts. We examine these allocations in the context of a model that features three key aspects of this decision. First, politicians choose the allocation of funds to maximize the share of votes they receive in future elections and therefore their chances of staying in power. Second, while politicians are affected by political incentives, they may also have altruistic motives in deciding where to allocate these public works. Third, politicians take into account the fact that their allocation decisions are likely to influence the choices of the other incumbent politicians, and vice versa. Based on our model estimates, we find that relative to a social planner, political incentives distort around 23 percent of the allocation of public funds. Simulations illustrate the tradeoff between altruistic motives and electoral returns and the extent to which alternative electoral rules can reduce political distortions. Finally, we use the model to evaluate policies aimed at changing the electoral system. We find that a score voting system, which scholars believe is a better form of voting, increases the distortions generated by political incentives.
1 Introduction

In Brazil, as in several other countries, each federal legislator has the power to amend the budget in order to appropriate funds for the provision of public goods in their district. In 2009, these amendments alone totaled over BRL 4.5 billion (US$2.5 billion). This type of discretionary spending when allocated and spent efficiently can be an important driver of economic development and a key determinant of quality of life. The allocation of these public funds is, however, rarely the outcome of a social planning problem, but instead are distorted by political incentives and institutions: an idea that have been well documented theoretically in an extensive literature on redistributive politics.\textsuperscript{1}. But what we ultimately care about is the extent of this distortion and how best to design policy to reduce it. Unfortunately, empirical evidence of this nature has been virtually nonexistent.

In this paper, we build and estimate a structural model of the decisions of federal legislators in Brazil to allocate public funds across municipalities within their state. Brazil’s federal legislature provides an ideal setting to analyze the effects of political incentives on public funds provision.\textsuperscript{2} The Constitution permits each federal legislator a fixed number of budgetary amendments to fund public good projects to municipalities.\textsuperscript{3} This provision allows us to investigate what factors influence these allocation decisions without worrying about the endogeneity of who has access to these funds, which is an important concern in other contexts, such as in the U.S. Congress. Moreover, these budgetary appropriations, which are commonly earmarked for large-scale development projects, have important welfare considerations.

Set within this context, our model highlights two key aspects of this decision. First, we

\textsuperscript{1}Myerson (1993) and Lizzeri and Persico (2001) are important studies on the effects of electoral incentives on how politicians target voters. See Persson and Tabellini (2000) for a review of the literature. The empirical literature has relied mostly on cross-country evidence (e.g. Milesi-Ferretti, Perotti, and Rostagno (2002) and Persson and Tabellini (2005)). One notable is exception is Stromberg (2008), who investigates the impact of the U.S. electoral college system on how U.S. presidential candidates allocate their campaign resources across states.

\textsuperscript{2}The study of Brazil’s budgetary amendments has a long tradition in comparative politics. Since Ames (1995) classic study, there have been a number of empirical studies investigating both the allocation of these budget appropriations, as well as their electoral returns (e.g. Samuels (2003); Pereira and Renno (2003)). The most recent example is Firpo, Ponczek, and Sanfelice (2012). The authors show that politicians tend to reward municipalities that supported them in the previous election, and that among the legislators who seek re-election, voters reciprocate by voting for the candidates who have brought more resources to their localities. Our study complements and extends this body of work. Many of these results, which we replicate in our reduced-form analysis, help to motivate our model and its underlying assumptions.

\textsuperscript{3}Since 1993, federal legislators are typically allowed up to 20 amendments, but this number has varied in some years.
allow for the possibility that politicians may have altruistic motives (in additional to electoral incentives), which may induce them to target these funds to needier or more productive regions. We also allow for heterogeneity among politicians in their degree of altruism. Unobserved heterogeneity has been an important limitation of most reduced-form analysis, and as we later document, allowing for this form of unobserved heterogeneity is an important feature of the model. Second, because these federal legislators are members of a multi-member district, their actions are likely to influence the actions of the other incumbents. This is (to our knowledge) the first empirical study in political economy to explicitly incorporate these strategic interactions, and examine their effects on an incumbent’s allocation decisions. Given this model, we are able to examine the extent to which political incentives distort the allocation of public funds from one in which only municipal welfare is considered, and to investigate how different electoral rules can affect the degree of distortion.

We estimate this model in two steps, adapting some of the techniques used in the industrial organization literature (e.g Bajari et al. (2010)). We assume that politicians are playing a simultaneous move game with imperfect information about the other politicians’ degree of altruism. As a result, politicians must form beliefs about the actions of the other incumbents, and they do so based on an incumbent’s previous election results. This assumed behavior is consistent with a large literature in economics and political science showing that incumbent politicians tend to reward their electoral strongholds, and this is in fact what we also find in our data. Having estimated these beliefs, we then estimate a politician’s decision to allocate his budget appropriations across regions within a state, using simulated maximum likelihood.

Our estimated model yields several interesting findings. Given our parameters, we estimate that 23.4 percent of these public funds are distorted relative to a social planner’s allocation that places equal weights across the regions. This distortion is driven but the incentives of non-altruistic politicians to target regions with more votes, often times at the cost of targeting poor or more productive places. We estimate that non-altruistic politicians, who care mostly about electoral incentives, represent approximately 68 percent of the pool of candidates. We also

\footnote{There is class of "core-supporter" models, which argue that politicians seeking to maximize the political return to public investments, target federal projects and programs to loyal constituents ((Cox and McCubbins 1986); (Dixit and Londregan 1996)). The empirical evidence is also largely consistent with theory. For instance using U.S. intergovernmental transfers from 1957 to 1997, Snyder and Ansolabehere (2006) show that the State’s governing party allocates funds to areas that provided them with the strongest electoral support. Case (2001) also finds support for the core supporter model in the manner in which block grants of an Albanian social assistance program were distributed across communities. Schady (2000) also finds evidence that Fujimorians supporters were disproportionately favored in the allocation of the Peruvian Social Fund (FONCODES).}
find that the beliefs about others actions play an important role in the decisions of politicians. Politicians are less likely to allocate goods to places where they believe others will also allocate their goods, which may explain the bailiwick phenomena that we see in Brazil (Ames 1995). Interestingly, there is heterogeneity in this response as well. The effects of social interactions are driven primarily by the non-altruistic types. Altruistic politician are much less sensitive to the actions of others.

Given the presence of these political pathologies, a natural question to ask is how can we design policy to reduce this distortion. An obvious starting point, which is often suggested by policymakers and academics alike, would be electoral reform. In this vein, we consider the effects of adopting a score voting system. Score voting (also a variant of the the Borda count), which gives each candidate a certain number of points corresponding to the position in which he or she is ranked by each voter, is widely considered a preferred voting system for its degree of expressiveness. Nevertheless, we find that this class of voting rules, if anything increases the degree of distortion, and the intuition for this result is straightforward. Because not placing first still yields some benefits under these alternate voting rules, this provides even more incentives for politicians to target places with larger electorates, and especially if the distribution of votes across regions is highly skewed. Thus, short of drastic changes to the electoral institutions (e.g. the creation single-member districts within the state), simple changes to the electoral rules seem unlikely to lead to large reductions in the distortion.

Overall our study highlights the importance of political institutions for the allocation of public expenditures, and in particular the type of distortion that can arise when the incentives between the politician and a social planner are not aligned. But how much we can change institutions to align incentives is still an open question, and in this case our results suggest that perhaps more emphasis should be place on attracting better types of politicians (Dal Bó, Finan, and Rossi (2013); Ferraz and Finan (2009)). Finally, our approach, although molded to fit the Brazilian context, is nevertheless quite general and can be easily adapted to provide insights into important questions in other settings.

The paper proceeds as follows. The next section describes Brazil’s Federal legislature and provides some institutional background. Section 3 describes the data we use to estimate the model and presents the reduced-form evidence that motivates the study. The model is then presented in Section 4, followed by a discussion of our estimation approach in Section 5. Section 7 presents both our estimation results, as well as our policy simulations. Section 8 concludes the paper.
2 Background

Several features of Brazil’s political institutions facilitate our empirical analysis. This section provides basic background on the Brazil’s federal legislature, and highlights some of its institutional features that we exploit in our analysis.

2.1 Brazil’s Federal Legislature

Brazil’s federal legislature (also referred to as Chamber of Deputies) consists of 513 seats allocated across 27 states according to population size. Each state represents a multimember voting district, where candidates can receive votes from any of its municipalities. As opposed to a single-member district, incumbents not only face competition from new potentials challengers, but also from the other incumbents, which is likely to influence their allocation decisions.

Nationwide elections for the legislature are held on a four-year cycle and incumbents can be elected an unlimited number of times. Despite the lack of any term limits, legislative careerism is surprisingly absent in Brazil. For instance, the percentage of deputies who sought reelection during the 1994 and 1998 elections was 75 percent. Moreover, several elected deputies either take a leave of absence or resign in order to assume another political position in the municipal, state, or national government. When this occurs, a candidate who had been elected as an alternate (suplente), then assumes the office. Surprisingly, the number of suplentes needed to serve office is not insignificant. Of the politicians that served the 49th parliamentary session, at least 20 percent of them were deputies elected as alternates.

The D’hondt open-list proportional representation method determines how many seats in a state each political party earns, while voters’ preference select the individual candidates within each party. Although the electorate can vote for the political party, this option is rarely exercised as elections tend to be highly individualized. This electoral system, which fosters both inter and intra-party competition, has been a source of Brazil’s weak party system (Mainwaring and Scully 1995). It is not unusual for several elected officials to change parties during their electoral terms. In the 49th parliamentary session 55 percent of the deputies switched parties during their term. With such a low degree of party loyalty both from the standpoint of the politician as well as the electorate, our empirical analysis does not focus on party politics but instead on the individual behavior of federal deputies.

\footnote{We use the terms “Deputy” and “Legislator” interchangeably.}
Several aspects of Brazil’s political institutions promote pork-barrel politics. Brazil’s legislature is comparatively weak and seldom legislates on issues of national concern (Ames 1995). While refraining from serious policy making, federal deputies engage in pork-barrel politics. As a Federal Deputy from Ceará stated in the Brazilian newspaper, Folha de São Paulo, on February 21, 1988: “A political career in Brazil is closely connected to success in bringing home material benefits … Especially in the poorest regions, communities judge their deputies on what they bring home”; or Federal Deputy Joaquim Haickel expressed: “The primary function of a deputy is getting resources; legislating comes second.” (Mainwaring 2002). To facilitate these objectives, federal deputies have had the right to submit pork-barrel amendments to the budget since 1988. Before 1996, members of Congress were not limited in the number budgetary proposal and between 1992 and 1995 averaged close to 137 per year per member. In its current form, Brazil’s constitution allows each member of Congress discretion over 20 budgetary amendments per year, totaling to a fixed amount typically of BRL$1.5 million, although this amount can in principal vary by year.

2.2 Data sources

To estimate our model, we assemble an extensive database of political and municipal characteristics for all of Brazil from the period 1996 to 2013. The data used for this study combines secondary data from three sources.

To investigate budgetary allocations, we collected budgetary amendment data from the Federal Chamber of Deputies. Information on the author’s name, the amount and type of public investment, and the recipient municipality is provided for each budgetary amendment issued from 1996 to 2013.

Using the authors’ names, these data are merged with the characteristics and electoral results of each politician. Election data are available from the Tribunal Superior Eleitoral (TSE) in electronic form. These data contain vote total for each candidate by municipality, along with various individual characteristics: including gender, education, occupation, and party affiliation. We use this information to construct our primary measure of political support – municipal vote share – as well as various other measures of electoral performance and competition, such the candidate’s rank and vote total.

Our final data source is from the Brazilian Institute of Geography and Statistics (Instituto Brasileiro de Geografia e Estatística (IBGE)). The 2000 population census provides several
socioeconomic and demographic characteristics such as poverty rates, income inequality, population density, share of the population that is urban, and share of the population that is literate.

### 3 Summary Statistics and Reduced-form Evidence

In this section, we present some summary statistics and reduced-form evidence that will help develop and estimate our structural model. For convenience, we present results based on the 50th legislature, which issued budgetary amendments from 1996-1999 and faced the possibility of re-election in 1998. Although for the reduced-form analysis, we restrict the sample to a single term the patterns we present here are similar when considering alternative or additional terms (see Firpo, Ponczek, and Sanfelice (2012)).

#### 3.1 Summary Statistics

Our estimation sample consists of the 648 politicians that issued a federal budgetary amendment during the 1996-1999 legislative term. The 135 non-elected deputies were originally voted as alternates, but later replaced elected deputies who were unable to fulfill his responsibilities. Elected deputies relinquish their post for a variety of reasons, but typically it is to assume a political position elsewhere. Inclusion of these deputies does not have any significant effect on our estimation results.

Table 1 presents some basic information on the budgetary amendments issued by federal deputies in our estimation sample. On average, each year a deputy proposes 15 budgetary amendments that are approved, with an approximate value of $1.3 million reais. Both the number and the amount decline slightly over the electoral cycle, although differences across years are not large.

The geographic variation in the distribution of these public works is considerable. More than 10 percent of municipalities did not receive a single public work during the 1996-1999 term, with the median municipality only receiving a BRL$280,000 in budgetary amendments. This stands in stark contrast with the BRL$10,000,000 that the top one percent of municipalities receive. The degree of this geographic variation can also be seen in Figure 1.

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6As mentioned previously, federal deputies were allowed up to 20 outlays totaling up to 1.5 million per year. The limits are in general reached however in our estimation we only consider outlays targeted to municipality and exclude the ones that are designed to benefit either the state or the country as a whole.
Table 2 displays summary statistics by state corresponding to the 1998 election. On average, 68 percent of deputies from the 1996-1999 term, who had issued a budgetary amendment, ran for re-election in 1998. Of those, 65 percent were re-elected. These averages however, mask considerable variation across states. In some states reelection rates are above 80 percent, while in others they are even below 50 percent. Elected federal deputies average anywhere between 5 to 21 times the number of votes that non-elected deputies receive. Federal deputies are predominately male (90 percent), and only 60 percent of them have a college degree.

3.2 Descriptive Evidence

There are several features of the data that we attempt to account for with our model. Our most basic assumption is that voters care about these public works and reward the politicians who supply them. This assumption seems reasonable if we simply plot the number of votes an incumbent receives by the amount of public goods they provided to the municipality. As Figure 2 depicts, there is a strong association between the amount of public goods provided and the number of votes. The relationship also appears to be quite linear, which is consistent with politicians targeting larger projects to more populated municipalities. Consistent with this explanation, we do see some concavity when we instead plot on the y-axis the share of votes received from the municipality (see Figure 3). Targeting even larger projects may do little to increase one’s vote share within a municipality, but can increases one’s vote totals if targeted to a more populated municipality. In Figure 4, we do see that larger projects are in fact targeted to more populated places.

In Table 3, we explore the robustness of these correlations to unobserved deputy and municipal characteristics. Because each state is a multi-member district, we can estimate these correlations controlling for both deputy and municipality fixed-effects. As it turns out, controlling for both fixed characteristics of the deputy (e.g. valence, education levels) as well as fixed characteristics of the municipality (e.g. poverty levels, party affiliation) has little impact on the relationship between electoral performance in the municipality and public spending (see columns 1-4).

Another way to visualize the data, which will motivate how we model the voting rule, is to examine how a deputy’s rank within the municipality – in terms of the amount of public goods provided – relates to the vote share he or she receives. In Figure 5, we plot the coefficients

\footnote{We excluded the district of Brasília.}
from a regression that regresses the share of votes within municipality on a set of indicators for the deputy’s rank in the municipality. The regression also controls for municipal fixed-effects. Deputies ranked first receive vote shares that are 27 percent point higher than those that rank above 22. Moreover, the effects of rankings decrease sharply. Second placed deputies receive only 7 percentage point higher vote share, whereas the vote share of deputies ranked third and above are statistically indistinguishable. These results suggest that when casting their ballots, voters do prefer the candidate who provided them with the most public goods.

Thus far, the evidence seems to suggest that voters do care and reward politicians who supply them with more public goods, and that politicians in turn, target larger or more projects to more populated municipalities. But this does not suggest, however, that electoral returns are the only motives when deciding where to target these public works. Politicians may also wish to target their public works based more on the needs of the municipality, especially if they are less concerned with electoral incentives. An example of this is when we compare the distribution patterns of public goods, distinguishing between deputies who ran for re-election and those that do not. In Panel A of Figure 6, we plot the distribution of public goods by poverty levels for those who ran for re-election and those that did not. In Panel B, we plot a similar figure but instead of poverty levels use the municipality’s human development index, which is a commonly used composite index of a country’s (or in our case a municipality’s) life expectancy, educational attainment, and income level. As both graphs indicate, incumbents who do not run for re-election are on average much more likely to target poorer and less developed municipalities, relative to those with electoral concerns.

4 Model

In this section, we develop a model of politicians’ allocation decisions with three objectives in mind. First, the model should be sufficiently rich to enable us to perform interesting policy evaluations. Second, it should have the ability of generating the empirical patterns documented in the previous section: (i) deputies allocate resources to municipalities with high political returns as well as to municipalities with limited political returns; (ii) some deputies choose not to run for reelection and, if they choose not to run, they allocate resources differently; and (iii) residents are more likely to vote for deputies that transferred a larger amount of resources to their municipality during the previous term. Third, we want to allow for strategic interaction among politicians to take into account the common intuition that politicians consider the choices
of their rivals when deciding to which municipalities to allocate the available funds.

To achieve the two goals, we consider an economy in which, in each term $t$, $J$ deputies make two choices. First, they decide how to allocate a fixed amount of resources $\bar{Q}$ among $M$ municipalities. Second, they choose whether to run for reelection at the end of the term. The amount of resources that deputy $j$ allocates to municipality $m$ will be denoted with $q^{j,m}$. Similarly, we will denote with $q^j = \{q^{j,1}, \ldots, q^{j,M}\}$ the collection of allocations chosen by deputy $j$, with $q = \{q^1, \ldots, q^J\}$ the allocations of all deputies, and with $q^{-j} = \{q^1, \ldots, q^{j-1}, q^{j+1}, \ldots, q^J\}$ the allocations of all deputies except $j$. Finally, we will use the notation $Q^m = \sum_{j=1}^J q^{j,m}$ to describe the total amount of funds received by municipality $m$. Each municipality is populated by $N_m$ individuals who have to choose for which politician to vote at the end of the current term.

Before we can characterize the optimal decisions of deputies, we have to introduce the preferences of deputies, the preferences of the voters, the electoral , and how each deputy interact with his or her political rivals. This will be the subject of the next three subsections.

4.1 Preferences

The allocation of deputy $j$ to municipality $m$ has a positive and increasing effect on the welfare of its residents. We will assume that the welfare of an individual in a municipality depends on the total amount of resources allocated to the municipality by the $J$ deputies, $Q^m$, a set of municipality characteristics $X_m$, and a variable, $K_m$, which captures every other factor that affects the residents’ welfare. This implies that the welfare of each resident of municipality $m$ can be written in the following form:

$$w_m (Q^m, X_m, K_m).$$

The total welfare in municipality $m$ can then be easily calculated by multiplying the individual welfare by the number of individuals living in the municipality:

$$W_m (Q^m, X_m, K_m) = N_m w_m (Q^m, X_m, K_m).$$

Deputies choose the allocations to the $M$ municipalities and whether to run for reelection so as to maximize their utility. The politicians have a different utility function depending on their decision to run for reelection. If they choose to run, their utility is comprised of four
parts. First, they derive utility from the opportunity of being in power. Second, the deputies derive utility from increasing the welfare of the individuals living a municipality through their allocations. Finally, there is a utility cost of running for reelection and a preference shock.

Specifically, let \( p_j \) be the probability of winning the next elections, \( v^p_j \) the utility from being in power, \( v^np_j \) the egoistic part of the utility if the deputy is not in power, \( C_R \) the cost of running for reelection, \( \tilde{\epsilon}_j \) the preference shock, and \( \alpha_j \) the weight that deputy \( j \) assigns to the altruistic motive. Then, for a particular allocation of resources by all deputies \( q = \{ q^1, \ldots, q^J \} \), politician \( j \)'s utility can be written in the following way:

\[
\bar{U}_R^j(q) = p^j(q)v^p_j + (1 - p^j(q))v^np_j + \alpha_j \sum_m W_m \left( \sum_j q_{j,m}, X_m, K_m \right) - \tilde{C}_R + \tilde{\epsilon}_j
\]

The first part of the utility, \( p^j(q)v^p_j + (1 - p^j(q))v^np_j \), captures the egoistic motives of politician \( j \). Provided that \( v^p_j \geq v^np_j \), it enables us to generate the documented fact that politicians tend to allocate more resources to municipalities with higher electoral returns. Observe that generally the probability of winning the election, \( p^j \), depends on the allocations chosen by all deputies, \( q \). This probability will be derived in the next subsection. The second part of the utility, \( \alpha_j \sum_m W_m \) describes the altruistic motives of politicians and it allows us to produce the observed fact that politicians transfers part of their funds to municipalities with limited political returns.

To allow for sufficient heterogeneity, we will assume that the shock \( \tilde{\epsilon}_j \) is composed of two parts. The first part depends on the allocation chosen by deputy \( j \) and will be denoted with \( \tilde{\epsilon}_j(q^j) \). The second part does not vary with the chosen allocation, but it is specific to the decision of running for reelection. It will be denoted with \( \tilde{\nu}_R \).

Using simply manipulations, deputy \( j \)'s utility can be written in the following alternative form:

\[
\bar{U}_R^j(q) = p^j(q)(v^p_j - v^np_j) + \alpha_j \sum_m W_m - \bar{C}_R + \tilde{\epsilon}_j(q^j) + \tilde{\nu}_R,
\]

where now \( \bar{C}_R \) also includes the value of not being in power. The utility from being in power, \( v^p_j \), and the utility from not in power, \( v^np_j \), are assumed not to vary with the allocation chosen by deputy \( j \). We can therefore divide the politician’s utility by \( \alpha^j + v^p_j - v^np_j \) and obtain

\[
U^j_R(q_1, \ldots, q_J) = (1 - \beta_j)p^j(q) + \beta_j \sum_m W_m - C_R + \tilde{\epsilon}_j(q^j) + \nu_R.
\]
This alternative formulation of the politician’s utility makes clear the trade off that the deputy faces when choosing how to allocate his or her budget across municipalities. The politician can allocate resources in ways that increase the probability he or she will remain in power. Alternatively, resources can be allocated to increase the welfare of the state residents. This formulation also clarifies that the significance of the two components in the deputy’s decision depends on the parameter \( \beta_j = \frac{\alpha_j}{\alpha_j + v_p - v_{np}} \) which can be interpreted as the degree of altruism of deputy \( j \). We will refer to \( \beta_j \) as \( j \)’s type.

If deputy \( j \) decides not too run, her or his utility will not depend on the value of being in power and on the cost of running. It will therefore take the following form:

\[
\bar{U}_{NR}^j(q) = v_{np}^j + \alpha^j \sum_m W_m \left( \sum_j q_{jm}, X_m, K_m \right) + \bar{\varepsilon}^j (q^j) + \bar{\nu}_{NR}.
\]

If we divide the utility by the same value used for a deputy who runs for reelection, \( \alpha^j + v_p^j - v_{np}^j \), we have a utility function that depends on the type \( \beta_j \):

\[
\bar{U}_{NR}^j(q) = v_{np}^j + \beta^j \sum_m W_m \left( \sum_j q_{jm}, X_m, K_m \right) + \bar{\varepsilon}^j (q^j) + \nu_{NR}.
\]

Given this utility function, the decisions of a deputy who chooses not to run for reelection will only be affected by welfare considerations. In this case, political incentives will play no role in the decision process. This enables us to generate the documented fact that deputies who choose not to run for reelection are more likely to allocate resources to poorer municipalities with a small number of votes.

### 4.2 Residents’ Voting Decisions and Deputies’ Strategic Interactions

The previous discussion has emphasized that, if deputy \( j \) chooses to run for reelection, her utility and therefore her decisions depend on the probability of winning the next election. In this subsection, we will describe how residents choose for which deputy to vote and hence how the probability of winning the election is determined.

The probability that politician \( j \) wins the election is an increasing function of the total number of residents who decides to vote for \( j \) in the district. Let \( q_{jm} \) be a random variable
which denotes deputy $j$’s allocation to municipality $m$ in the next term in case he wins the election. Then, resident $i$ of municipality $m$ votes based on two variables: ideology $\xi_{i,j,m}$ and the amount of resources they expects to receive during the next term by deputy $j$, $E(q_{j,m}^{'} | Z)$, where $Z$ is the set of variables used to form the expectation. Specifically, resident $i$ votes for deputy $j$ if

$$j = \text{argmax}_{j \in J} \left\{ E\left(q_{1,m}^{'} | Z\right) + \xi_{i,1,m}, \ldots, E\left(q_{J,m}^{'} | Z\right) + \xi_{i,J,m} \right\}.$$  

We now describe how these expectations are formed. Two kinds of politicians compete in an election: incumbents and politicians who run for the first time, challengers. Consider first incumbent $j$. Voters predict the amount politician $j$ will transfer in the next term using politician $j$’s current allocation $q_{j,m}$ and the deputy’s observable characteristics $X_j$. As a consequence,

$$E\left(q_{j,m}^{'} | q_{j,m}, X_j\right) = f(q_{j,m}, X_j)$$

In each election, there are also $J_c$ challengers who generally have no record on previous allocations to municipality $m$. To determine how voters predict the allocation that will be chosen by challengers in the future term we make the following assumption. Voters have no information on the challengers’ type and therefore assign equal probability to the event that a challenger chooses each one of the feasible allocations.

The probability that individual $i$ in municipality $m$ votes for politician $j$, $Pr_m(j)$, can therefore be computed as follows:

$$Pr_m(j) = P\left( f(q_{m,j}, X_j) + \xi_{i,j,m} > f(q_{m,k}, X_k) + \xi_{i,k,m}, \forall k \neq j \right).$$

We can now compute the number of votes a politician receives from municipality $m$, $nv_m(j)$. It is simply the number of voters times $Pr(j)$:

$$nv_m(j) = N_mPr_m(j)$$

It is then straightforward to calculate the total number of votes in the district for each candidate.

Given the total number of votes, we determine the probability that a candidate wins the

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8The data support this assumption. Conditional on legislator fixed-effects, a region that received an amendment in the previous election is 45.8 (robust standard error = 0.011) percentage points more likely to receive another amendment in the next election.
election in a way that approximates the D’hondt open-list proportional representation election system used in Brazil. Specifically, for each realization of shocks, we consider the set of allocations chosen by the incumbents and the set of allocations voters believes will be selected in the future term by the challengers. For these allocations, we compute the number of votes received by each candidate in the district. We then rank the candidates starting with the one who was awarded the largest number of votes. The probability of winning the election is then set equal to 1 for all candidates with a ranking smaller or equal to the number of seats assigned in the election and to 0 for everyone else. Although the lack of party politics is one potential limitation in the way we have specified the probability of winning, the data do seem to support this omission, as approximately only 1 out of 10 legislators in a given election is elected through his party. This is again consistent with decades of research on Brazil arguing for its relatively weak party system.

The previous discussion emphasizes that in our model the utility and therefore the choices of deputy $j$ depend on the decisions of all other deputies. To deal with these strategic interactions, we make two assumptions. First, deputies make simultaneous decisions. Second, deputies do not know the type of other deputies, $\beta$. They only know the distribution function, $\pi(\beta)$, from which the types are independently drawn.

Given these assumptions, when deputy $j$ chooses the optimal allocation, he does not know and cannot calculate the optimal allocations of the other deputies. Deputy $j$ can only compute the probability that his political rivals will choose one of the possible allocations given the opponents’ characteristics. We will denote with $\sigma(q^h \mid X^h)$ the probability that deputy $j$ assigns to deputy $h$ choosing allocation $q^h$ given characteristics $X^h$. The probability that $j$’s rivals choose the sequence of allocations $q^{-j} = \{q^1, \ldots, q^{j-1}, q^{j+1}, \ldots, q^J\}$ can therefore be written in the following form

$$\sigma_{-j}(q^{-j} \mid X_m) = \Pi_{h \neq j} \sigma(q^h \mid X^h).$$

### 4.3 Deputies’ Optimal Decisions

Now that preferences, the voting decision, and the strategic interactions have been outlined, we can write down the maximization problem of deputy $j$. We will do this in two step. First, conditional on the running decision, we describe how the deputy chooses the optimal allocation of resources. Second, we then determine whether it is optimal for the deputy to run for reelection.
Consider first the case in which it is optimal for politician $j$ to run for reelection. Conditional on running, he chooses the allocation that maximizes the expected value of his utility, where the expectation is taken over the allocations of his political rivals. Specifically, he selects the $q^j = \{q^{j,1}, \ldots, q^{j,M}\}$ that solves the following problem:

$$V_R^j (X_m, X, \beta^j) = \max_{q^j} \int_{q^{-j}} \left[ (1 - \beta^j) p^j (q) + \beta^j \sum_{m=1}^{M} W_m \right] \sigma_{-j}(q^{-j}|X)dq^{-j} - C_R + \varepsilon^j (q^j) + \nu_R$$

$$s.t. \sum_{m} q^{j,m} \leq \bar{Q},$$

where $V_R^j (X_m, X, \beta^j)$ is the value of running for election.

Consider now the case in which it is optimal for deputy $j$ not to run for reelection. Similar to the previous case, the optimal allocation maximizes the deputy’s expected utility. The only difference is that, conditional on not running, his utility does not depend on the political incentives. This implies that the optimal allocation solves the following problem:

$$V_{NR}^i (X_m, X, \beta^j) = \max_{q^j} \int_{q^{-j}} \left[ \bar{v}_{np}^j + \beta^j \sum_{m=1}^{M} W_m \right] \sigma_{-j}(q^{-j}|X)dq^{-j} + \varepsilon^j (q^j) + \nu_{NR}$$

$$s.t. \sum_{m} q^{j,m} \leq \bar{Q},$$

where $V_{NR}^j (X_m, X, \beta^j)$ denotes the value of not running for election.

It is now straightforward to determine whether deputy $j$ will run for reelection. He will take part in the election if the value of running is greater than the value of not running, i.e. if

$$V_R^j (X_m, X) \geq V_{NR}^j (X_m, X).$$

In the rest of the paper we will denote with $d_R^j$ the running decision of deputy $j$.

We conclude the theoretical section by outlining the timing of the game played by the politicians and by providing a definition of its equilibrium. It is straightforward to describe the timing. First, nature reveals $\beta^j$ to the politicians, which as discussed above is private information. Then, the politicians simultaneously decide how much to invest in each municipality subject to their resource constraint and whether to run for re-election. Finally, voters cast their ballots at the polls.
We can now define the Bayesian Nash equilibrium that characterizes our model.

**Definition 1** Allocations $q_1^*, \ldots, q_J^*$ and the deputies’ decisions to run for reelection $d_{R1}^*, \ldots, d_{RJ}^*$ are a Bayesian Nash equilibrium if, for each deputy $j$, conditional on $q_j^*$ and $d_{Rj}^*$, the decisions $q_j^*$ and $d_{Rj}^*$ maximize deputy $j$’s utility.

The following Proposition establishes that in our setting a Bayesian Nash equilibrium in pure strategies exists.

**Proposition 1** The model developed in this paper has a Bayesian Nash equilibrium in pure strategies.

**Proof.** In the Appendix. ■

The rest of the paper is devoted to the estimation of the model and to the evaluation of policies aimed at mitigating the effect of the political incentives.

## 5 Econometric Implementation

In this section, we discuss the additional assumptions made to estimate the model and the estimation method.

To estimate the proposed model, we need to assume a specific functional form for the probability that a resident votes for deputy $j$, $Pr_m(j)$, for the welfare function, $w_m$, and for the distributions from which the shocks are drawn. To characterize the voting probability, we make the following four assumptions. First, we assume that the expected allocation function $f(q_{m,j}, X_j)$ has a polynomial form. Second, we make the simplifying assumption that $f(q_{m,j}, X_j)$ is independent of deputy $j$ characteristics. Third, we allow the constant in the polynomial form of $f(q_{m,j}, X_j)$ to vary between incumbents and challengers, but we restrict this constant to be identical within the group of incumbents and within the group of challengers. Finally, the ideology variable $\xi_{i,k,m}$ is assumed to be distributed according to an extreme value distribution. Under these assumption, the probability that a resident living in municipality $m$ votes for deputy $j$ takes a standard logit form. Without loss of generality, we can normalize the choices with respect to the allocation selected by one of the challengers. We denote this allocation by $\bar{q}$. The probability that an individual in municipality $m$ votes for incumbent $j$
can then be written as follows:

\[ Pr_m(j) = \frac{\exp^{\gamma_0+\gamma_1(q^{m,j}-\bar{q})}}{1 + \sum_{h=2}^{J_c} \exp^{\gamma_1(q^{m,h}-\bar{q})} + \sum_{h=1}^{J} \exp^{\gamma_0+\gamma_1(q^{m,h}-\bar{q})}} \]

where \( \gamma_0 \) is the difference between the constant that characterizes the incumbents and the constant that characterizes the challengers.

The welfare function was chosen with the following objectives in mind. First, we would like to have some concavity in the function to capture decreasing returns to public works. Second, the public works allocated by deputies are not the only public goods that affect an individual’s welfare. Transfers from other public offices and public goods generated by private organizations also enter the welfare functions. Our specification must be able to account for these other sources of public goods. Finally, the welfare function must be able to account for the possibility that some public works are more productive and therefore generate higher welfare in some municipalities than in others. To capture these feature, we employ the following welfare function:

\[ w_m = \log \left( \delta_m y_m + \tilde{\rho}_m \sum_{j=1}^{J} q_{j,m} \right) \]

where \( y_m \) is per-capita income of municipality \( m \). The logarithm enables us to capture decreasing returns. The municipality per-capita income \( y_m \) accounts for the existence of other public goods and for the possibility that their value varies by municipality. Finally, the coefficient \( \tilde{\rho}_m \) enables us to introduce productivity differences across municipalities.

It is important to discuss three points related to the identification of the parameters of the welfare function. First, if one divide and multiply by \( \delta_m \) inside the logarithm, the welfare function can be rewritten in the following way:

\[ w_m = \log \left( \frac{\delta_m y_m + \tilde{\rho}_m \sum_{j=1}^{J} q_{j,m}}{\delta_m} \right) - \log (\delta_m) \]

Since the term \( \log (\delta_m) \) affects only the constant of each deputy’s objective function, it has no effect on the deputies’ decisions. As a consequence, \( \delta_m \) and \( \tilde{\rho}_m \) cannot be separately identified, only their ratio \( \rho_m = \frac{\tilde{\rho}_m}{\delta_m} \) can be identified. This result is intuitive. If there are no electoral incentives, differences in allocations across municipalities may be driven by the fact that a municipality has higher needs for the transfer or by the fact that the transfer is more productive.
in that municipality. Differences in allocations can therefore provide information only on the relative value of these two forces. For this reason, in the rest of the paper we will make use of the following welfare function:

\[ w_m = \log \left( y_m + \rho_m \sum_{j=1}^{J} q_{j,m} \right) \]

As a second remark, notice that in principle all productivity parameters \( \rho_m \) for \( i = 1, \ldots, M \) can be identified using variation in per-capita income \( y_m \). In the data, however, there is little geographical variation in \( y_m \). For this reason, we normalize the productivity parameter of the most populated region \( \rho_1 \) to 1 and estimate the remaining productivity of the remaining municipalities relative to region 1. As a final remark, observe that we cannot distinguish between differences in productivity and differences in preferences. A higher \( \rho_m \) could be the result of municipality \( m \) being more productive in the use of the public resources or having stronger preferences for public works. This is a standard result and, as a consequence, the parameter \( \rho_m \) is likely to capture preference as well as technology differences.

A preference shock affects the politicians’ decisions which is drawn from a log-normal distribution with mean equal to 0 and variance \( \sigma^2 \).

In addition to the assumptions implicit in the functional forms of the voting probability and welfare function, we impose other assumptions to make the estimation computationally more tractable. First, we assume that there are two types of deputies: deputies with low \( \beta \), the egoistic types, and deputies with high \( \beta \), the altruistic types. Second, we discretize the provision of public goods into four choices. Specifically, a deputy can choose to give 0 percent, 33.33 percent, 66.66 percent, or 100 percent of the budget to a given municipality subject to the constraint that the allocations must add up to BRL$1.5 million. Third, as it is standard in the estimation of games, we will assume that only one equilibrium is observed in the data (Draganska et al. 2008). Fourth, in the model, the probability of winning is determined under the assumption that challengers will choose each possible allocation with equal probability.

Finally, the model will be estimated for the state of Roraima. This state is comprised of 15 municipalities and is representative of a group of states that are poorer and less populated for which understanding how resources are allocated and reducing the effect of inefficiencies should have large and positive effects on welfare. For instance, the poverty rate is high with 56% of households living with less than a Real a day. Correspondingly, average per-capita income is low at R$133 per month. The 15 municipalities have on average 18,200 residents.
and a municipal budget of R$5.5 million. Residents and therefore votes are geographically skewed with 71,691 individuals living in the capital city and 2,826 people living on average in the other municipalities. Elections in the state of Roraima are highly competitive. In 1998, for instance, 42 candidates competed in the average municipality with an average vote share of 8%, a minimum vote share of 0%, and a maximum vote share of 29%. The average difference in votes between the top two candidates in a municipality was 6%. The characteristics of the politicians running for election in Roraima are similar to the national average. For example, in 1998 in Roraima 100% of politicians were males and 75% had a college degree, whereas at the national average 95% of politicians were males and 78% had a college degree.

In the estimation, we aggregate the 15 municipalities into 4 macro-regions. We do this for two reasons. The first one is that public goods provided to one municipality are likely to benefit the surrounding municipalities as well. Typical examples are hospitals and schools. The aggregation of the municipalities into macro-regions allows us to mitigate the effects of these spillovers. The second reason is that computationally the estimation becomes manageable. With four regions and four possible choices for each region, each of 8 deputies can select among 35 feasible allocations. To construct the deputy’s expected utility for one particular allocation, we have to consider all possible combinations that can be selected by the deputy’s rivals. This implies that to solve the deputy’s problem we have to consider $35^8 \approx 2.2 \times 10^{12}$ combinations. Even with the use of Message Passing Interface (MPI), which allows us to use simultaneously multiple processors in the estimation, and a supercomputer, the problem we are attempting to solve is computationally demanding. To reduce its complexity, we make use of the following simple sample approximation method. For each deputy, we draw a subset of the the $35^7$ feasible allocations that can be chosen by the deputy’s rivals. For each set of parameter, we then solve the deputies’ problem using the subset of allocations until convergence. After convergence, we re-solve the problem at the estimated set of parameters and at sets of parameters that are close to the solution using the entire set of feasible allocations. If the likelihood function does not change significantly, we stop and use the estimated parameters in our counterfactuals. If the changes in the likelihood function are significant, we increase the size of the subset of feasible allocations and re-estimate the parameters. We then repeat the previous step.

Given these assumptions, there are 11 parameters that need to be estimated. They can be divided into five sets. The first set includes the three parameters related to the type of politician: $\beta_L$, the degree of altruism of the egoistic type, $\beta_H$, the degree of altruism of the altruistic type, and $\pi$, the fraction of altruistic types in the candidate pool. The second set is composed of the
two parameters that characterize the voting decision: $\gamma_0$, the constant in the logit formulation, and $\gamma_1$, the coefficient capturing the effect of the transfers on the voting probability. The third set comprises the 3 parameters of the welfare function: $\rho_m$, the productivity coefficient, where $m = 2,\ldots,4$. The fourth set includes the two parameters that affect the decision to run for reelection: $\nu_L$ and $\nu_H$, the cost of running for egoistic and altruistic types. The last set is composed of parameter that characterizes the distribution from which the preference shocks are drawn: $\sigma^2$.

The parameters are estimated using the two-step method initially developed by Hotz and Miller (1993) and then adapted to the estimation of games by Bajari, Benkard, and Levin (2007). In the first step, we estimate the deputies’ beliefs $\sigma \left( q^h \mid X^h \right)$ using data on allocations for each deputy over a period of 16 years. The beliefs are estimated under two assumptions. First, we impose the restriction that the beliefs take a logistic functional form. Second, as an exclusion restriction needed for the identification of the model, we assume that vote shares in the previous election affect the deputies’ beliefs but not the remaining components of the model. The intuition behind this restriction is that the outcome of previous elections help deputies infer the type of their political rivals before they make decisions on allocations and whether to run for reelection. Vote shares in previous election, however, provide no additional information. In particular, conditional on the allocations chosen in the current term by deputies, voters receive no additional information from past vote shares about the deputy’s type. That is, if $nv_m^p(j)$ denotes vote shares in the previous election, then

$$
E \left( q^{j,m} \mid q^{j,m}, nv_m^p(j) \right) = E \left( q^{j,m} \mid q^{j,m} \right) = f \left( q^{j,m} \right).
$$

Under these two assumptions, we estimate the following multinomial logit in the first stage. Let $A$ be the number of feasible allocations among which a deputy $j$ can choose. Denote with $a_i$ the integer value between zero and $A$ that corresponds to one of these allocation, i.e. $a_i \in \{0,1,\ldots,A\}$. With four regions and four possible choices for each region, $\{0\%,33.33\%,66.66\%,100\%\}$, each deputy can select among 35 allocations that satisfy the feasibility constraint. In our model, allocation 1 corresponds to the allocation that assigns all funds to region 4 or, equivalently, to allocation $q_i = \{0,0,0,1\}$; allocation 2 refers to the allocation that assigns one third of the budget to region 3 and two thirds to region 4, i.e. $q_i = \{0,0,0.33,0.66\}$; and so forth up to allocation 35 which corresponds to $q_i = \{1,0,0,0\}$.

\footnote{In practice, we only see 20 out of the 35 possible allocations in the data.}
Let \( Z^j_i \) be the share of votes that deputy \( j \) received in the last elections in the municipalities to which positive resources are allocated according to allocation \( i \). Then, the probability that a deputy \( j \) chooses a particular allocation \( i \) is given by

\[
Pr(a_i = i | Z^j_i = z) = \frac{exp(\phi' z)}{1 + \sum_{i=1}^{A} exp(\phi' z)} \quad \text{for } i = 1, \ldots, A
\]
\[
Pr(a_i = 0 | Z^j_i = z) = \frac{1}{1 + \sum_{i=1}^{A} exp(\phi' z)}
\]

In Figure 13 in the appendix, we plot the predicted probabilities for each allocation and by legislators. As the figure depicts, there is considerable variation in the probability that a deputy chooses a particular allocation. For instance, some legislators (e.g., 2373 and 2668) are predicted to allocate their funds to allocation 1 which favors region 4, whereas other legislators prefer the higher-numbered allocations, which allocate more resources to region 1.

In the second step, given the beliefs, we estimate the parameters of interest using simulated maximum likelihood (SML) and data on allocation choices by deputies and decisions on whether to run for reelection. Let \( d^k_w \) be a dummy equal to 1 if deputy \( k \) wins the election and denote with \( g_k (q^k, d^k_R, d^k_W | \beta_H) \) the probability of observing one of the possible discrete allocations, the decision to run, and the outcome of the election conditional on the type. Then, given the available data and the assumption that there are two types of deputies, the likelihood function takes the following form:

\[
L = \prod_{k=1}^{T} \left[ \pi g_k (q^k, d^k_R, d^k_W | \beta_H) + (1 - \pi) g_k (q^k, d^k_R, d^k_W | \beta_L) \right],
\]

where \( T \) is the total number of observations. To describe how the conditional probabilities are computed, observe that \( g \) can be decomposed in the following way:

\[
g_k (q^k, d^k_R, d^k_W | \beta) = P_{k,W} (d^k_W | q^k, d^k_R, \beta) P_{k,q} (q^k | d^k_R, \beta) P_{k,R} (d^k_R | \beta),
\]

where \( P_{k,-} \) is the probability associated with one of the outcomes conditional on the type. The contribution to the likelihood of a deputy who chooses to run for reelection can therefore be
written as follows:

$$
\mathcal{L}_{k|R} = \pi \left[ P_{k,W} \left( d^k_W | q^k, d^k_R, \beta_L \right) P_{k,q} \left( q^k | d^k_R, \beta_L \right) P_{k,R} \left( d^k_R | \beta_L \right) \right] \\
+ (1 - \pi) \left[ P_{k,W} \left( d^k_W | q^k, d^k_R, \beta_H \right) P_{k,q} \left( q^k | d^k_R, \beta_H \right) P_{k,R} \left( d^k_R | \beta_H \right) \right]
$$

To describe the contribution of a deputy who chooses not to run for reelection, notice that in this case the electoral outcome is irrelevant. As consequence, the contribution takes the following form:

$$
\mathcal{L}_{k|NR} = \pi \left[ P_{k,q} \left( q^k | d^k_R = 0, \beta_L \right) P_{k,R} \left( d^k_R = 0 | \beta_L \right) \right] \left[ P_{k,q} \left( q^k | d^k_R = 0, \beta_H \right) P_{k,R} \left( d^k_R = 0 | \beta_H \right) \right] .
$$

The likelihood function can therefore alternatively be written as follows:

$$
\mathcal{L} = \prod_{k=1}^{T} \mathcal{L}_{k|R} d^k_R \mathcal{L}_{k|NR}^{1-d^k_R} .
$$

Under the assumption that the ideology variable affecting the voters' decisions is distributed according to an extreme value distribution, we can compute directly the probability that an incumbent wins an election. But, given the complexity of the model, the functional form of the remaining conditional probabilities is not known. For a given set of parameters, they are therefore replaced by empirical distributions based on simulated values of the shocks. Since $q^k$ and $d^k_R$ are discrete variables, a natural choice of simulators is represented by

$$
\hat{P}_{k,q} \left( q^k | d^k_R, \beta \right) = \sum_{s=1}^{S} \mathbb{I} \left( q^k | d^k_R, \beta \right), \quad \text{and} \quad \hat{P}_k \left( d^k_R | \beta \right) = \sum_{s=1}^{S} \mathbb{I} \left( d^k_R | \beta \right),
$$

where $\mathbb{I}$ is the indicator function and $S$ represents the number of simulations.\(^{10}\)

\(^{10}\)We have also experimented with an alternative definition for the simulators following McFadden (1989). He suggests that the indicator function be replaced by a logit kernel smoother. This approach requires the use of the value for deputy $k$ of choosing an allocation $q^k$ conditional on the decision to run, $V_k \left( q^k | d^k_R = 1, \beta \right)$ and $V_k \left( q^k | d^k_R = 0, \beta \right)$, and the value of running and not running for the same deputy, $V^R_k \left( \beta \right)$ and $V^NR_k \left( \beta \right)$. This alternative simulators are problematic in our context because they require the use of value functions for two different types of individuals: deputies who run and deputies who do not run. For different parameters, these two value functions can be similar or very different in size. For smoother parameters that are constant over the choice of preference and technology functions, this variation creates changes in the probabilities that have nothing to do with the choice of the model parameters and make the estimation of the model difficult.
6 Identification Discussion

In this section, we discuss the identification of the parameters we plan to estimate. The model is sufficiently complex that it is difficult to mathematically prove identification of our parameters. We can, however, provide an intuition about the variation in the data that enables us to identify the parameters of our model.

We start with the parameters of the welfare function \( \rho_i, i = 2, \ldots, 4 \). In our model, deputies who choose not to run for reelection allocate resources based only on welfare considerations. The allocation patterns observed in the data are therefore informative about the parameters that characterize the welfare function. Specifically, the difference between the average allocation to region \( i \) and the average allocation to region 1 inform us on the value of \( \rho_i \), for each \( i \). The identification of the welfare function is based on our assumption that deputies who do not run for reelection only care about welfare. This is clearly a simplifying assumption since those deputies may choose to run for a different office and may therefore still have some electoral motives when choosing how to allocate funds. Theoretically, it is straightforward to account for this possibility. All we have to do is to allow deputies who do not run for reelection to care about the possibility of being elected to a different office and about welfare, but with different utilities of being in power and of not being in power. This is equivalent to using the same preferences for deputies who run and deputies who do not run, but with different altruism parameter \( \beta_j \). The main limitation of this modeling choice is that we lose clear identification of the welfare parameters. We have analyzed the data to understand whether our simplifying assumption is a good approximation of deputies preferences. In the data we observe that most deputies who decide to run for reelection choose to go back to their former occupation and not run for a new office. There is one exception. One deputy decided to run for governor in the state he represented as a deputy. In this case, our assumption is not realistic since this deputy is likely to choose the optimal allocation considering also the number of votes he will receive in the gubernatorial election.

Given the parameters of the welfare function, we can identify the altruism parameters \( \beta_L \) and \( \beta_H \) and the probability that a deputy has low altruism \( \pi \). To see which variation in the data allows us to identify those parameters, consider first the case in which there is only one type of deputy. In this case, the difference in allocations between deputies who choose to run for reelection and deputies who choose not to run identifies the parameter \( \beta \). If there is no difference, the degree of altruism \( \beta \) will be identified to be 1. If deputies who compete in the
election transfer a larger fraction of resources to municipalities with high political gains, $\beta$ will be identified to be lower than 1, with a coefficient that will be closer to 0 the larger the difference. The generalization to two types allow us to consider situations in which in the data there are two distinct groups of deputies who run for reelection. A first one in which deputies allocates a much large fraction of funds to regions with high political gains than deputies who do not run for reelection. A second group composed of deputies whose allocation differ less from the allocations of politicians who do not compete in the election. If in the data there is only one group, $\beta_L$ and $\beta_H$ will be estimated to be statistically equal. We experimented with more than two types, but the data does not support this extension.

The parameters of the voting function can be identified using data on the probability that a deputy is elected. The incumbency effect, $\gamma_0$, can be identified by the difference between the average probability that an incumbents is elected and the same average probability for a challenger. The parameter that measures the effect of public funds on number of votes, $\gamma_1$, can be identified by studying how changes in resource allocations affect the probability of winning an election of an incumbent.

The last three parameters of the model are the cost of running for the two deputy types, $\nu$, and the variance of the preference shocks, $\sigma$. The fraction of individuals who choose to run for reelection for two experience levels inform us on the value of the cost of running for the two types. The variance of the allocations computed across deputies enable us to identify the variance of the preference shocks.

7 Results

In this section, we present the estimates of our structural model and their implications for the allocation of these public goods.

7.1 Parameter Estimates

The parameters estimated using data from Roraima are presented in Table 4. According to these estimates, the data are explained by two distinct types of candidates. There are the altruistic politicians who put most of the weight on the district’s welfare ($\beta = 0.96$), and the egoistic type, who in contrast to the altruistic candidates, care mostly about votes in their decision to allocate the public funds ($\beta = 0.05$). As it turns out, 32% of the pool of candidates
is altruistic.

The coefficient on the public good in the voting function, $\gamma_1 = 0.13$, implies that an extra dollar allocated to a municipality increases a deputies share of votes by 0.5 percentage points. The constant in the voting function $\gamma_0$ provides a value for the incumbency advantage. Under the normalizing assumption that the constant for the challengers is equal to zero, we find that if an incumbent chooses the same allocation that is expected from the challengers, he has an advantage over them of 4.8 percentage points.

Three parameters characterize the welfare function. These coefficients help to capture why regions with few votes and hence limited political gains receive a significant share of resources. To understand our estimation results, it is important to remember that deputies base their choice of where to allocate their budget on the number of votes they can receive in exchange for the transfer and on the amount of welfare they can generate with the transfer. If political incentives dominate the deputies’ decisions, then we would observe the majority of resources being allocated to region 1, which contains the capital of Roraima and is the richest and most populated of the four geographical areas. Instead, we observe a significant fraction of the allocations given to poor and small regions with limited political gains, suggesting that welfare considerations are important. However, for the welfare considerations to explain the transfers to less populated areas, it must be that resources transferred to less populated regions generate a larger increase in welfare than if the same resources were transferred to region 1. In our model this is the case if $\rho_m$, the coefficient on the the total amount of resources allocated to region $m$, is larger than 1, the normalized value for region 1. If this condition is satisfied, then deputies limited incentives to transfer to region 1 for welfare reasons because the resources will not be as productive as in other regions. Our estimates of the $\rho$'s suggest the welfare returns to an extra dollar of public funds are highest in regions 3 and 4 , which helps explain why region 4 despite its low population count receives close to 30 percent of the funds (as we depict below). The welfare returns of these two regions are about 3.5 times larger than for region 2 and 40 time larger than for region 1 (lowest).

To provide some insight on the size of the cost of running we proceed in two steps. We first compute the average value of running for a particular type of politician. We can then determine what fraction of the average value of running the cost represents. We find that egoistic types have a lower cost of running and that it represents 3% of the benefits of running. The cost of running for altruistic types is computed to be 24% of the value of running. Intuitively, it is plausible that individuals that care more about political success have lower financial and
psychological costs of running for a seat.

The last parameter we estimate is the standard deviation of the preference shock $\sigma_{\epsilon}$. We estimate that the standard deviation is equal to 6.82. To give some insight on the importance of the preference shocks in our model, we first compute the average deputy’s utility and then the average deputy’s utility when the shock is equal to one standard deviation. Finally, we then compute the percentage increase in deputy’s utility when we introduce the shock. We find that the welfare increases by 12 percentage points. This is a relatively low increase which suggests that the main source of uncertainty in our model does not play an important role in explaining the data.

### 7.2 Insights from the model

To investigate the implications of the model, we begin by simulating each deputy’s allocation decisions given our parameter estimates. From these allocation choices, we can compute the share of federal expenditures that each region receives. We plot the average (across simulations) of these shares in Figure 7, along with the expenditure shares as computed from the actual data. On average, deputies allocate most of their funds to region 1 and region 4, with region 1 receiving 37 percent of their funds and region 4 receiving 31 percent of the funds. Our model, although quite parsimonious, matches the data reasonably well. As we see in the figure, we match extremely well the allocations to regions 1 and 2. The match for regions 3 and 4 is not as good, since we overpredict the amount of funds transferred to region 3 and underpredict the resources allocated to region 4.

In Figure 8, we recompute these expenditure shares distinguishing between our two types of politicians. The differences across politicians are quite stark: deputies with low levels of altruism allocate more than 40 percent of their funds to region 1, where the electoral incentives are the strongest. Alternatively, altruistic deputies allocate most of their funds to regions 3 and 4, where the welfare returns to an extra dollar are highest. Altruistic deputies also give a significant share of their funds to region 1 and region 2, which is the poorest region in the state.

Another important feature of our model is the ability to compute the share of resources that are distorted due to political incentives. Suppose that a social planner, who only cared about aggregate welfare and placed equal weight on each district, allocated these funds. How would the actual allocation of these funds, when electoral incentives are present, differ from this
counterfactual? Figure 9 displays these differences by district. We find that the electoral incentives generate a significant amount of inefficiencies with 23.4% of resources being mis-allocated. Given the differences between altruistic and non-altruistic deputies in their allocations, it is not surprising to see that compared to our social planner the actual distribution is distorted towards district 1, at the cost of the poorest and less populated districts.

Before examining how different policies can affect the allocation of budget outlays, in Figure 10 we investigate how deputies’ beliefs about the actions of the other deputies affect their allocations. In this figure, we plot, along with their 95 percent confidence intervals, the predicted values of two linear regressions that estimate the correlation between the amount that a deputy allocated to a particular municipality and how much he believed other deputies would allocate. One regression line is estimated only for altruistic deputies, and the other is estimated only for non-altruistic deputies. Both regression lines control for municipal fixed effects. In both cases, we see a negative relationship between the amount that the deputy allocates and how much others will allocate. However, this negative relationship is much more pronounced for non-altruistic deputies who care more strongly about electoral incentives. In fact, the regression estimates for egoistic types (point estimate = -0.025; robust standard error = 0.003) imply that for every BRL$100,000 that a deputy expects other to allocate, he will reduce his amount by BRL$2,500.

7.3 Policy Evaluations

The results presented in the previous section suggest that the model is a good approximation of politicians’ behavior, since it is able to replicate the main patterns observed in the data. We can therefore use it to perform counterfactual exercises. For each counterfactual, the deputies’ beliefs must be recalculated since the ones estimated using the observed allocations correspond to the equilibrium that characterizes the data. In each counterfactual, we use the estimated parameters and find the deputies’ equilibrium beliefs that match these parameters using fixed point iterations.

Given our parameter estimates and the equilibrium beliefs, we can use our model to investigate how different electoral rules might affect the distribution of public goods. One set of such alternative electoral rules can be constructed using score voting. Score voting allows voters to rank all or a subset of the candidates, where the ranking is associate with a given number of points. Specially, let $J$ be the set of candidates, $J = \{x_1, \ldots, x_J\}$. Then a scoring
rule is defined by a vector of scores \((s_1, \ldots, s_J)\), with \(s_1 \geq \ldots \geq s_J\) and \(s_1 > s_J\), where for each voter’s ranking, \(s_1\) points is assigned to the top ranked alternative, \(s_2\) points is assigned to the second-ranked alternative, and so forth. The \(n\) seats allocated in the election are won by the politicians with the \(n\) highest total number of points.

Given the current system in Brazil, voters can only vote for a single candidate, which corresponds to the following scoring rule \((1,0,\ldots,0)\). A natural alternative to this current system would be to allow voters to rank a subset of the candidates, which scholars often argue is a much more expressive form of voting, and encourages voters to vote sincerely. As such, we consider three different scoring rules, all of which allow voters to rank their top 3 choices. The first scoring rule puts equal weight on the top three candidates, i.e. \((1,1,1,0,\ldots,0)\). The second scoring rule weights the top three candidates in a linear fashion: \((3,2,1,0,\ldots,0)\). The third scoring rules weights the top three candidates in a convex manner: \((5,0.5,0.5,0,\ldots,0)\).

Figure 11 presents distributions of public goods according to each of these voting rules relative to the base case distribution. Interestingly, despite the fact that such scoring rules are widely regarded as preferred methods of collective decision making, these rules if anything move the allocation of public goods further away from the social optimum. As we see from the figure, allowing voters to rank their top 3 choices provides deputies with even more incentive to target their funds to district one. The intuition behind these results is straightforward, and driven in part by the skewness of the electorate size towards district one. Because voters can now choose 3 candidates, the returns to coming into second or third in district one are quite high. In fact, for many potential allocations, the returns to coming in second in district one are much larger than coming in first in the other districts, such as in district 4. As a result, deputies are much more willing to compete over district under these alternative electoral rules.

In Figure 12, we investigate how a doubling in the number of challengers would affect the allocation of public goods. Interestingly, we again find that more competition distorts the distribution of public goods away from the social optimum. At first glance this may seem like a curious result, but the underlying explanation is quite sensible. By increasing the number of challengers and hence competition in each of districts, the deputies are less willing to target multiple districts, and prefer to concentrate all of their allocations to a single district.

\[\text{For each of these voting rules, we have to compute the probability that a candidate is ranked first, second, and third. We use the exploded logit for this computation.}\]
8 Conclusions

A central question in redistributive politics is how do politicians target public funds. In this paper, we present a novel approach to the empirical analysis of how politicians allocate public funds in an environment in which other politicians are behaving strategically. This approach allows us, among other things, to compute the extent to which political incentives distort the allocation of these public funds from a social planner’s problem, and to investigate the effects of electoral rules and political competition in exacerbating this distortion.

Using data from Brazil’s federal legislature, we find that at least 23 percent of these budget amendments are distorted from the social planner’s problem. This distortion is driven by the behavior of non-altruistic politicians, who put little weight on the welfare of the municipalities. According to our estimates, fraction of egoistic politicians represent 68 percent of the candidate pool. We also explore the effects of score voting, a widely endorsed form of voting, on the allocation of public goods. Interestingly, we find that the allocation of public goods, if anything, becomes more distorted since it increases the incentives politicians have to target more populous regions.

Although our model fits the data well, it is quite parsimonious and can be extended and generalized in several directions that represent exciting possibilities of future research. One possible extension would be to make the game dynamic. As Diermeier, Keane, and Merlo (2005) correctly emphasize, politicians are forward-looking agents who career choices are dynamic in nature. Although our model in some respects captures this behavior in the decision to run for re-election, it would be interesting to model these decisions more explicitly, such as the decision to enter higher offices. Another extension would be to add political parties into the model. Although we do not think that this is an important feature of Brazilian politics, one could potentially exploit a nested structure to extend the model in this direction. Other directions of future research will ultimately depend on the collection of new data. For instance, with data on campaign spending, one could easily extend our model to examine whether budgetary amendment complement or substitute campaigning. One could then investigate the impact of campaigning financing laws on not only electoral performance but also public goods allocation.
References


9 Tables and Figures

<table>
<thead>
<tr>
<th>Year</th>
<th>Total</th>
<th>Average per Deputy</th>
<th>Average Amount per Deputy</th>
<th>Standard Deviation</th>
</tr>
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<td>1,370,380</td>
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<td>1999</td>
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<td>20195</td>
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<td>253048.100</td>
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Notes: These data are available: [http://www2.camara.leg.br/](http://www2.camara.leg.br/)
### Table 2: Summary Statistics the 1998 Elections for Federal Deputies

<table>
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<tr>
<th></th>
<th>Number of legislators</th>
<th>Number of Candidates</th>
<th>Number of Parties</th>
<th>Reelection Rates</th>
<th>Share ran for reelection</th>
<th>Average Votes Among Non-elected</th>
<th>Average Votes Among Elected</th>
<th>Share male</th>
<th>Share college degree</th>
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<td>8</td>
<td>51</td>
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<td>0.40</td>
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<td>0.47</td>
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<td>9</td>
<td>52</td>
<td>20</td>
<td>0.54</td>
<td>0.86</td>
<td>5,549</td>
<td>49,648</td>
<td>0.88</td>
<td>0.71</td>
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<td>62,900</td>
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<td>17</td>
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<td>8,180</td>
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<td>0.77</td>
<td>0.76</td>
<td>8,293</td>
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<td>0.81</td>
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<td>0.92</td>
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<td>20</td>
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<td>0.50</td>
<td>6,837</td>
<td>49,380</td>
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Notes: Data are available: [http://www.tse.jus.br/](http://www.tse.jus.br/)
Table 3: Relationship between Electoral Performance and Allocation of Budget Amendments

<table>
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<tr>
<th>Dependent variable</th>
<th>Number of votes per municipality</th>
<th>Vote share per municipality</th>
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<td></td>
<td>(1)</td>
<td>(2)</td>
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<td>679.28</td>
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<td>[49.705]</td>
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<td>Rank within the municipality</td>
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<td>-0.027</td>
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<td></td>
<td>[35.468]</td>
<td>[0.001]</td>
</tr>
<tr>
<td>Municipal intercepts</td>
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<td></td>
</tr>
<tr>
<td>Deputy intercepts</td>
<td>N  Y  Y  N  Y  Y</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>154,139  154,139  154,139  154,139  154,139  154,139</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.47  0.47  0.36  0.2  0.16  0.12</td>
<td></td>
</tr>
</tbody>
</table>

Notes: Column 1 reports the unadjusted correlation between the number of votes a deputy received and the amount of his budgetary amendment approved in the municipality. Column 2 reports the same relationship as in Column 1 but adjusts for both deputy and municipal fixed-effects. Column 3 reports the relationship between the number of votes a deputy received and his ranking in the municipality with respect to the amount of public goods he provided in his budgetary amendment. Columns 4-6 replicate the regressions in columns 1-3 but use the deputy’s vote share in the municipality as the dependent variable. The estimation has been restricted to only those incumbents that ran for re-election. Robust standard errors in brackets.
Table 4: Parameter Estimates

<table>
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<tr>
<th>Parameter</th>
<th>Estimate</th>
<th>Std Err</th>
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<td><strong>Altruism</strong></td>
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<tr>
<td>Proportion of Altruistic Deputies</td>
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</tr>
<tr>
<td>Altruistic</td>
<td>$\beta_H$</td>
<td>0.96</td>
</tr>
<tr>
<td>Non-altruistic</td>
<td>$\beta_L$</td>
<td>0.05</td>
</tr>
<tr>
<td><strong>Voting Function</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Effects of public goods on vote shares</td>
<td>$\gamma_1$</td>
<td>0.13</td>
</tr>
<tr>
<td>Incumbency Advantage</td>
<td>$\gamma_0$</td>
<td>0.47</td>
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<tr>
<td><strong>Welfare</strong></td>
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<td></td>
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<tr>
<td>Productivity in region 1</td>
<td>$\rho_1$</td>
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</tr>
<tr>
<td>Productivity in region 2</td>
<td>$\rho_2$</td>
<td>8.87</td>
</tr>
<tr>
<td>Productivity in region 3</td>
<td>$\rho_3$</td>
<td>34.53</td>
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<td>Productivity in region 4</td>
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<tr>
<td><strong>Decision to Run and Shocks</strong></td>
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<td>Cost of running if altruistic</td>
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<tr>
<td>Cost of running if non-altruistic</td>
<td>$\nu_H$</td>
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<tr>
<td>Variance of preference shocks</td>
<td>$\sigma_\epsilon$</td>
<td>6.82</td>
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Figure 1: Distribution of Budgetary Amendments Per Capita
Figure 2: Budgetary Amendments and Electoral Performance
Figure 3: Budgetary Amendments and Vote Share
Figure 4: Distribution of Budgetary Amendments By Population
Figure 5: Vote Share and Rank of Budgetary Amendment Amount
Figure 6: Distribution of Budgetary Amendments by Welfare Levels

Distribution of Public Works

- - - Did not Run for Reelection
- - - Ran for Reelection

Distribution of Poverty Rates:

Did not Run for Reelection
Ran for Reelection

Distribution of Human Development Index:

- - - Did not Run for Reelection
- - - Ran for Reelection
Figure 7: Comparison Between Model’s Prediction and Actual Allocations

Allocation of Public Goods

<table>
<thead>
<tr>
<th>Income Level</th>
<th>Predicted</th>
<th>Actual</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Income</td>
<td>0.4</td>
<td>0.6</td>
</tr>
<tr>
<td>Low Income</td>
<td>0.2</td>
<td>0.3</td>
</tr>
<tr>
<td>U-Middle Income</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>L-Middle Income</td>
<td>0.4</td>
<td>0.5</td>
</tr>
</tbody>
</table>

- **Predicted** represents the model's prediction.
- **Actual** represents the observed allocation.
Figure 8: Distribution of Allocations By Politician Type

**Allocation of Public Goods by Type**

<table>
<thead>
<tr>
<th>Type</th>
<th>Altruistic</th>
<th>Less Altruistic</th>
</tr>
</thead>
<tbody>
<tr>
<td>High Income</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Low Income</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>U-Middle Income</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>L-Middle Income</td>
<td>0.2</td>
<td>0.1</td>
</tr>
</tbody>
</table>

- **Share of public goods**
- **Altruistic**
- **Less Altruistic**
Figure 9: Deviation from the Social Planner

Change in the share of public goods

Deviation from Social Planner

Change in the share of public goods

High Income Low Income U-Middle Income L-Middle Income
Figure 10: Effects of Other Deputies Actions on Allocation Decisions By Politician Type

The figure shows the effects of other deputies' actions on allocation decisions by politician type. The x-axis represents the expected allocation from other deputies, while the y-axis represents the allocation decisions. Two lines are plotted: one for altruistic deputies and another for non-altruistic deputies. The altruistic deputies' line is represented by a solid blue line, and the non-altruistic deputies' line is represented by a dashed green line. The line for altruistic deputies is located above the line for non-altruistic deputies, indicating a greater allocation tendency for altruistic deputies. The shaded area around each line represents the variation or uncertainty in the allocation decisions.
Figure 11: Effects of Electoral Rules on Allocation Decisions

Allocation of Public Goods by Type

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<thead>
<tr>
<th>Income Level</th>
<th>3-equal</th>
<th>3-linear</th>
<th>3-convex</th>
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</thead>
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<td>High Income</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Low Income</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Middle Income</td>
<td></td>
<td></td>
<td></td>
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</table>
Figure 12: Effects of Challenger Competition on Allocation Decisions

Increasing Candidate Competition

<table>
<thead>
<tr>
<th>Change in the share of public goods</th>
<th>High Income</th>
<th>Low Income</th>
<th>Middle Income</th>
<th>Middle Income</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>−0.04</td>
<td>0.02</td>
<td>−0.02</td>
<td>0.04</td>
</tr>
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</table>
A Appendix: Proof of Proposition 1

The solution concept for our game is a Perfect Bayesian Equilibria. Recall the sequence of decisions for our model:

- A deputy $j$ will run for re-election if the value of running $V_R^j$ is greater than the value of not running $V_{NR}^j$, and vice versa.

- A deputy $j$ will choose a set of allocations $q^j$ to maximize the value of running, $V_R^j$ or not running $V_{NR}^j$.

**Definition 2** A strategy profile $\sigma = (\sigma_1, \ldots, \sigma_J)$ is Perfect Bayesian Equilibrium if it induces a Bayesian Nash equilibrium in every subgame.

Given this definition, an equilibrium exists for model if we can show that a Bayesian Nash equilibrium exists for the value of running and the value of not running. Let us first consider the value of running.

Let’s denote the vector $\beta = (\beta_1, \ldots, \beta_J) \in \Xi$ as a profile of types, and the vector $q = (q_1, \ldots, q_J) \in Q$ as a profile of actions.

**Definition 3** A pure strategy Bayesian Nash equilibrium is an element $f = (f_1, \ldots, f_J) \in F$ s.t. for every $j \in J$

$$f_j(\beta_i) \in \arg \max_{q_j \in Q_j} \left\{ \int u_j(q_j, f_{-j}(\beta_{-j}); \beta_j, \beta_{-j}) d\eta(\beta_j|\beta_{-j}) \right\}$$

According to Meirowitz (2003), a game with a finite set of agents, but non-finite type and action spaces has a Bayesian Nash equilibrium in pure strategies if the following five conditions are satisfied:

1. $\Xi_i$ and $Q_i$ are nonempty, convex and compact subsets of Euclidean space.

2. $u_j(q, \beta)$ is continuous
3. For every $\beta$ and measurable function $f_{-j}(\beta_{-j})$
\[
\int u_j(q_j, f_{-j}(\beta_{-j}); \beta_j, \beta_{-j}) d\eta_j(\beta_{-j}|\beta_j))
\] (1)
is strictly quasiconcave in $q_j$.

4. For every $\epsilon_j > 0$, there exists some constant $\delta_j$ s.t. if
\[
g_j(\beta_j) \in \arg \max_{q_j \in Q_j} \left\{ \int u_j(q_j, f_{-j}(\beta_{-j}); \beta_j, \beta_{-j}) d\eta_j(\beta_{-j}|\beta_j)) \right\}
\] (2)
for some $f_{-j}(\beta_{-j}) \in F_{-j}$ then
\[
\sup_{\{(\beta_j', \beta_j'') \in \Xi_j : \|\beta_j' - \beta_j''\| < \delta\}} \| g_j(\beta_j') - g_j(\beta_j'') \| < \epsilon_j.
\] (3)

5. For a.e. measurable set $Q \subset \Xi_{-j}$, $\eta_j(Q|\beta_j)$ is continuous in $\beta_j$.

Conditions 1 and 2 are standard and clearly satisfied. Condition 3 holds by virtue of the fact that the probability of election $p^j(q)$ is assumed to be a non-decreasing function and $W_m(q)$ is assumed to be strictly concave. As discussed in Meirowitz (2003), condition 4 requires that our response functions are continuous and defined on a compact domain. It is easy to see that the response functions are defined on a compact set since the type $\beta_i \in [0, 1]$, which is closed and bounded. If the response functions are also continuous, then the function $g_j(\beta_j') - g_j(\beta_j'')$ is also continuous and defined on a compact domain. Moreover, continuous functions on a compact domain are uniformly continuous, which satisfies the following condition: For every $\epsilon_j > 0$, there exists some constant $\delta_j$ such that for every $\beta_j' \in \Xi_j$ and every $\beta_j'' \in \Xi_j$,
\[
\| \beta_j' - \beta_j'' \| < \delta \Rightarrow \| g_j(\beta_j') - g_j(\beta_j'') \| < \epsilon_j.
\] (4)

All that remains is to show that $g_j(\beta_j)$ is continuous of $\beta_j$. According to the Theorem of the Maximum, the optimal action $q_j$ exists for every $\beta_j \in \Xi_j$ and every $f_{-j} \in F_j$ and the correspondence $\xi_j(\beta_j, f_{-j})$ is upper hemicontinuous. Together with Condition 3, and the fact
that $Q_i$ is convex, the optimal action $q_j$ is unique of each $\beta_j$ and $f_{-j}$. Thus for each $j$ and every $f_{-j}$, $g_j(\beta_j)$ is continuous of $\beta_j$.

With regards the value of not running, note that the only difference between the value of running and non-running is the objective function. Since objective function for not running a satisfies Condition 3, the proof for the existence of a Bayesian Nash equilibrium for not running is immediate. Given that each subgame has a Bayesian Nash equilibrium its from the definition above that an equilibrium exists.

B Appendix: Figures
Figure 13: Predicted Probabilities by Allocation Decisions

Figure Notes: Each figure represents the predicted probabilities of each allocation for a single legislator. The numbers at the top of each figure correspond to the candidate’s electoral number.