Multidimensional Auctions of Option Values

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[PRELIMINARY AND INCOMPLETE DRAFT]

Abstract

This paper conducts a structural analysis of auctions of contracts when private information is multidimensional. These involve a trade-off between adverse selection and moral hazard. Specifically, we study oil lease auctions where bids consist of both a cash payment and a royalty rate on extraction revenue. Upon modeling the lease as a commodity option - as the winner chooses whether to exercise it ex-post - we derive optimal bids under an unspecified scoring rule. We nonparametrically identify bidders’ types from their bids and develop a nonparametric estimation procedure to recover their joint distribution. Analyzing cash-royalty auctions in Louisiana, our rich model allows us to compare their performance to cash auctions with fixed royalty in terms of moral hazard, allocative distortions, and information rents. In addition, we assess the effects of changing the lease duration and exploiting fluctuations in oil prices.

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1 Introduction

When a principal wishes to contract with an agent, there may be many potential agents from which to choose, who each have different types that the principal cannot observe. The principal must decide how to choose among these agents and set contract terms. In designing such a mechanism, he faces a trade-off between adverse selection and moral hazard. A mechanism that links the principal's payoff more closely to outcomes - e.g. via a sharing rule - reduces agents' information rents, but it also reduces agents' incentives to exert effort. In this context, one mechanism proposed by theory is an auction of contracts in which the auction endogenously determines both an upfront payment (cash) and a sharing rule (royalty). Specifically, McAfee and McMillan (1987) and Laffont and Tirole (1987) explain that auctions of this form can achieve an optimal trade-off between information rents yielded to agents and moral hazard, when agents have one-dimensional types.

Indeed, formal and informal auctions which determine both a cash payment and royalty are commonly observed; examples include government-firm, author-publisher, patent holder-licensee, entrepreneur-acquirer, landlord-sharecropper, and sports association-broadcaster contracting, among others. However, whether these auctions perform well in practice is an empirical question. First, the optimal auction proposed in theory involves the principal designing a menu of cash-royalty contracts; real-world auctions may not be implemented in this way. Second, agents may have multidimensional types, while existing optimality results concern one-dimensional types. Cash-royalty auctions have the potential to perform better than simpler auctions - e.g. which fix the royalty and have bidding on the cash component only - but they may in fact do worse depending on how they are implemented.

This paper conducts a structural analysis of auctions of contracts with multidimensional private information. Specifically, we study government oil lease auctions where bids consist of both a cash payment and a royalty rate on extraction revenue. Each bidder chooses his two-dimensional bid based on his two-dimensional type. Meanwhile, the auctioned leases grant the right, but not the obligation, to develop a tract of land for oil production. As a result, the royalty component of the bid induces moral hazard by reducing the winning firm's incentive to develop the tract. The fact that there is no obligation is of first-order importance empirically; the majority of publicly leased tracts are left undeveloped. Therefore, the contract
takes the form of a real option in our model. This endogenizes a bidder’s value for the contract to explicitly account for the probability that he will not develop the tract and for how the royalty component of his bid affects this probability. Royalties chosen endogenously by bidders also have important implications for adverse selection. While higher royalty levels generally reduce firms’ information rents,\(^1\) we show that endogenous royalties exacerbate adverse selection because the firms most willing to bid up royalties are less desirable types.

The data we use are from Louisiana’s cash-royalty auctions of oil leases on state lands. Though bids are two-dimensional, there is no specified scoring rule, so we cannot rely on properties of specific scoring functions in our analysis. However, much can be learned - by bidders and by the econometrician - from observations of the state’s choice over many auctions. In particular, we document that the state favors both higher cash payments and higher royalties, in contrast to some optimal mechanisms proposed in theory. Another pattern we see in the data is that the within-auction distribution of bids is scattered across two dimensions, in a manner strongly suggestive of multidimensional rather than one-dimensional bidder types.

As mentioned previously, bidders have two-dimensional types \((\theta_1, \theta_2)\) in our model. The first component, \(\theta_1\), represents productivity, and the second component, \(\theta_2\), represents cost. From the perspective of a bidder who has bid royalty rate \(a\), the contract is essentially an option to obtain \((1 - a)\theta_1\) expected units of a commodity at a cost of \(\theta_2\). Whether the bidder will exercise this option depends on the future price of the commodity. We borrow from the option pricing literature to define a closed-form expression representing the value of such an option, the Black (1976) model for commodity options. Building these option values into an auction model, we derive optimal cash-royalty bids as a function of bidders’ types under an unspecified scoring rule. As part of this analysis, we show that bidding a higher royalty percentage is less costly for undesirable “weak” types - those with low productivity and/or high cost - than for “strong” types. This has important implications for adverse selection; the royalty component of bidding effectively provides weak types a cheaper currency with which to bid.

Given this model, we nonparametrically identify bidders’ types from their bids. Specifically, the model yields a system of two first-order conditions that characterizes the bidder’s choice of bid as a function of his type. As the bid is observed, this is a

\(^1\)See DeMarzo et al. (2005).
system of two equations in which the bidder’s type components are the two unknowns. The mathematical intuition for identification is that this system of equations has a unique solution. Economically, the separate identification of $\theta_1$ and $\theta_2$ is related to the fact that bidders’ types cannot be reduced to scalar representations in our model. Royalties are levied on revenue not profit, so the distinctive roles played by productivity versus cost in determining the bid cannot be reduced to or represented by the bidder’s overall value for the contract. Using observed bids, we recover types for any bidder who submits a bid, regardless of whether he won or whether the lease was developed.

Based on the identification argument, we develop a nonparametric estimation procedure to recover the joint distribution of type components. As the probability of winning plays an important role in bidders’ choice of bid, the most important step of estimation is estimating the probability of winning as a function of the cash-royalty bid. To do this, we estimate separately the two structural objects composing this probability: first, we estimate via kernels the bivariate bid distribution; second, we estimate via sieves the probability that the state favors bid A over bid B, as a function of the bid components of A and B. Then, the probability of winning is estimated as their composite: the probability that a given bid is favored over competing bids drawn from the observed bid distribution. After this step, we follow the constructive identification argument to estimate bidders’ two-dimensional types.

Using our estimated structural model, we compare Louisiana’s cash-royalty auction to the counterfactual fixed-royalty auction, in which the principal fixes a common royalty rate, and bidders bid only on the cash payment amount. The model is sufficiently rich to allow comparison in the particulars of moral hazard, allocative distortions, and information rents, the combined effects of which lead to an overall comparison of social surplus and government revenue. A particularly interesting finding is that Louisiana’s auction yields relatively high information rents to firms. Compared to the fixed-royalty auction, the cash-royalty auction in Louisiana disproportionately weakens cash competition relative to the royalty level it achieves. As a consequence, total government revenue - the sum of cash payments and royalties - is actually lower than in fixed-royalty auctions with a comparable royalty level, even though the fixed-royalty auction is a more restricted mechanism. We also assess policy instruments beyond auction design, such as changing the duration of the lease and exploiting fluctuations in oil prices. Higher oil prices have a positive impact on
government revenue that is much more than proportional to the price increase, while
the effect of increasing the lease duration is small.

**Related Literature**  In studying multidimensional auctions of contracts, this paper
primarily relates to two streams of literature: the multi-attribute auctions literature
and the auctions of contracts literature. In addition, since the contract in our model
takes the form of an option, this paper is related to work on real options, and since
our application involves oil leases, our paper is related to work on empirical oil lease
design. To our knowledge, this is the first paper to build multidimensional private
information and bidding into auctions of contracts, which are subject to adverse
selection and moral hazard.

**TO BE COMPLETED:**

- Multi-attribute (mostly price-quality) auctions
  - exogenous quality: Nakabayashi (2013), Yoganarasimhan (2016), Krasnokut-
    skaya, Song, and Tang (2018), Laffont, Perrigne, Simioni, and Vuong (2018),
    Andreyanov (2018)
  - quasi-linear scoring rule: Che (1993), Bushnell and Oren (1994), Bushnell and
    Oren (1995), Asker and Cantillon (2008), Asker and Cantillon (2010), Lewis and
    Bajari (2011)
  - price/quality ratios: Takahashi (2018)
  - interdependent scoring rule: Sant’Anna (2018)
  - identification conditions: Hanazono, Hirose, Nakabayashi, and Tsuruoka (2016)

- Auctions of contingent payment contracts (theory)
    (1988), DeMarzo et al. (2005), and others

- Real options

- Empirical oil lease design
  - Bhattacharya, Ordin, and Roberts (2018), Herrnstadt, Kellogg, and Lewis (2019)
2 Data

2.1 Introduction

The Louisiana Department of Natural Resources (DNR) sells oil leases on lands owned by the state of Louisiana and its agencies. We study data from these sales that occurred for onshore\(^2\) leases of at least 1 acre between 1974 and 2003. A lease grants the lessee the right, but not the obligation, to develop the leased tract of land for oil production. The fact that there is no obligation to develop is a key feature of these leases; the Department of the Interior reports that as of the end of 2011, “approximately 56 percent of total acres of public land under lease in the Lower 48 States [...] are not undergoing either production nor exploration activities.”\(^3\) For our analysis, we restrict our attention to auctions in which each bidder name is associated with one bid, and it is clear which bid won.

2.2 Bids

To bid for a tract of land that Louisiana has advertised for leasing, a firm must fill out a bid form specifying the terms it offers for the lease. While a firm may specify that it wants to lease an arbitrary portion of the advertised tract, we will only analyze auctions where all bids are bidding for the entire tract. This ensures that all bidders in an auction are bidding on the same piece of land and lets us focus our analysis on contract terms. The bid form asks the bidder to specify a cash payment, a royalty rate, the duration of the lease (up to a maximum of 3 years), and any additional considerations the bidder wishes to offer. The first two components will emerge as the deciding factors; we discuss each component in turn.

The cash payment is an amount that the winning bidder pays to the state even if he ultimately does nothing with the leased tract; it is non-contingent. It is the sum of two parts: an immediate payment and a “rental” which is paid annually during the life of the lease thereafter. The state stipulates that the rental must equal at least half of the immediate payment; in 93% of bids in our sample, the rental equals half of the immediate payment. We define the cash component of the bid as the immediate payment plus the discounted present value of rentals. Next, the royalty is

\(^2\)With tract kind codes 2 and 4 in the Louisiana DNR categorization.

\(^3\)“Oil and Gas Lease Utilization, Onshore and Offshore: Updated Report to the President,” U.S. Department of the Interior, May 2012.
a percentage of production revenue that the lessee pays to the state, contingent on producing. Production revenue equals production volume times the price obtained for oil. Note that the royalty is levied on revenue, not on profit. This is the norm for mineral leasing across the U.S., likely due to the administrative burden that verifying costs would entail. The minimum acceptable royalty in our sample is 12.5%.

The duration of the lease determines the number of years for which the lessee holds the right to develop the leased tract. If there is no production during that time, the lease expires; if there is production, the lease remains in effect for as long as production continues. For the onshore leases in our sample, Louisiana limits the duration to a maximum of 3 years. Though bidders are free to specify a shorter duration, 99% of leases are for 3 years in practice. Finally, bidders may offer additional consideration, e.g. an obligation to drill a well within a certain period of time or pay a penalty, but none of the bids in our sample offer any.

Therefore, the bids in the auctions we study are effectively two-dimensional; the contracts bid differ in cash payment and in royalty rate. We refer to this two-dimensional auction as the cash-royalty auction. This auction format is an alternative to the one-dimensional auction used by the federal Bureau of Land Management and some other states such as New Mexico, in which the government fixes a common royalty rate, and bidders bid only on the cash payment amount. We refer to the latter auction format as the fixed-royalty auction. Unlike the fixed-royalty auction, the cash-royalty auction allows each bidder to bid his own choice of royalty rate flexibly and endogenously.

Figure 1 displays a scatterplot of the cash-royalty bids observed in our data. Dollar amounts are deflated by the GDP implicit price deflator and stated in 2009 dollars. The median and mean cash bid are $663 and $1,503 per acre, respectively; the cash distribution is skewed and is closer to a log-normal than a normal distribution. The median and mean royalty bid are both 23%. While there are outliers, royalties are concentrated between 15% and 30%. Meanwhile, the cash and royalty components of a bid are positively correlated; the raw correlation coefficient between the log of the cash bid per acre and the royalty bid is 0.47.
2.3 Which bid is chosen?

Unlike in a one-dimensional auction, what constitutes a winning bid is not immediately obvious in a multi-attribute auction. As such, some multi-attribute auctions specify a “scoring rule,” which is a function used to convert a multi-attribute bid into a scalar score for purposes of determining a winner. Other multi-attribute auctions, including this Louisiana auction, do not specify a scoring rule. Here, technical staff make recommendations to the State Mineral and Energy Board regarding which bid is most advantageous to the state, and the Board makes a final decision as to the winner. Our inquiry into the finer details of the process yielded a response that the “geological and engineering staff look at each bid and make the determination [...] by looking at all factors involved.” A few bidders who replied to our inquiries confirmed the absence of a specified scoring rule; when asked if they knew how the cash component of their bid would be weighted versus the royalty component, bidders indicated they were informed by past auctions, but nonetheless had uncertainty regarding exactly how the decision-makers would choose a winner. The bidders’ perspective of the state’s choice is better represented by choice probabilities than by a choice rule.

We turn to the auction data to learn more about these choice probabilities. We first look at the illuminating case of two-bidder auctions. Table 1 provides a summary of the patterns observed in bidding and winning. Defining a dominant bid as one
Table 1: Bidding and winning in two-bidder auctions

<table>
<thead>
<tr>
<th>Condition</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>There is a dominant bid</td>
<td>62%</td>
</tr>
<tr>
<td>Dominant bid won</td>
<td>99%</td>
</tr>
<tr>
<td>Dominant bid lost</td>
<td>1%</td>
</tr>
<tr>
<td>There is no dominant bid</td>
<td>38%</td>
</tr>
<tr>
<td>Higher cash won</td>
<td>59%</td>
</tr>
<tr>
<td>Higher royalty won</td>
<td>41%</td>
</tr>
</tbody>
</table>

which has both a higher cash payment and a higher royalty than the competing bid, we see that there is a dominant bid 62% of the time. Moreover, when there is a dominant bid, the state selects it as the winner 99% of the time. When there is no dominant bid, the bid with higher cash wins 59% of the time, and the bid with higher royalty wins 41% of the time, indicating that the state’s choice is not lexicographic.

It is interesting that there is often a dominant bid and that the state strongly favors it. After all, a high royalty is not an unambiguous good, as it depresses the firm’s incentive to develop the lease. The cash-royalty auction here is clearly implemented in a manner that is different from the auctions proposed by McAfee and McMillan (1987) and Laffont and Tirole (1987). Those auctions involve a menu of contracts where cash and royalty are negatively correlated, and the principal would award the contract to the lowest-royalty/highest cash bid. As McAfee and McMillan (1987) and Laffont and Tirole (1987) are concerned with one-dimensional bidder types only, a departure from their optimal mechanism need not indicate suboptimality here. Nonetheless, this gives us some reason to think Louisiana’s mechanism may not be optimal.

Figure 2 visualizes the state’s choice probabilities using data from all auctions with two or more bids. Regardless of the number of bids, the state’s choice of winner among them contains information about pairwise choices that the state made. For instance, if bid A is chosen over bids B and C, we learn about two pairwise choices: that A is chosen over B and that A is chosen over C. Figure 2 plots each of these pairwise choices, with each pair generating two points to plot: the winner (blue circle) and the loser (red triangle). The $x$ and $y$ coordinates of the point represent the royalty and cash components of the bid in relation to the competing bid, so points in the right quadrants have higher royalty than the competing bid, points in the upper quadrants have higher cash, and points in the upper-right quadrant have both higher royalty and higher cash. The transition from red triangles in the lower-left to blue circles in
the upper-right visualizes the increased probability of winning as a bid moves in that direction. Moreover, starting at any point in the plot and moving up or to the right, the transition to more winning strongly suggests that the probability of being chosen is increasing in both cash and royalty.

2.4 Summary

The data suggest some important features for which a model of these auctions should account. First, the auctioned contract is a real option; the winning firm has a choice of whether or not to develop it. Moreover, one of the bid components (the royalty) affects the firm’s incentives to exercise the option, so the probability of option exercise and the value of the contract (or auction item) to the firm are endogenously affected by the royalty he bids. Second, the bids are two-dimensional, and the distribution of bids in Figure 2 is strongly suggestive of bidders having multidimensional rather than one-dimensional types. Third, there is no scoring rule, so we cannot rely on properties of specific scoring functions. However, much can be learned - by bidders and by the econometrician - from observations of the state’s choice over many auctions. In particular, we see that the probability of winning the auction increases in both the cash and royalty components of the bid, and we should not rely on the current mechanism being an optimal one.
3 Model

In this section, we build a model of multidimensional auctions of contracts that incorporates the important features observed in the data. For clarity of exposition, we abstract away from heterogeneity across auction items here, and defer their treatment to the section on estimation.

3.1 Setup

**Auction** The principal seeks to sell a contract to one of \( n \) firms competing to win it. The contract grants the contracted firm the right, but not the obligation, to engage in revenue-generating production of a commodity. Each competing firm makes a two-dimensional bid, \((a, b)\), which leads to a probability of winning given by \( P(a, b) \). This probability \( P(\cdot, \cdot) \) is increasing in both arguments. The bid component \( a \) represents a royalty rate on the firm’s revenue to be paid to the principal, and \( b \) represents a cash payment. The firm must pay \( b \) upon winning the contract regardless of its post-auction actions, but it pays the royalty rate \( a \) only if it generates revenue; i.e. the royalty is a contingent payment.

**Bidders** The bidding firms have two-dimensional types \( \tilde{\theta} = (\theta_1, \theta_2) \). The type component \( \theta_1 \) represents the bidder’s expected production volume. Since \( \theta_1 \) is an expected value, it allows for uncertainty while incorporating the productivity of the firm, e.g. well and field design, recovery rate, and other intensive margin differences among firms. The type component \( \theta_2 \) represents the firm’s economic cost of exercising the option to produce, including opportunity costs. Upon winning the contract, a firm decides whether or not to exercise the option. If it does exercise the option, it expects to produce \( \theta_1 \) units of the good at a cost of \( \theta_2 \).

**Value of the contract** From a bidder’s perspective, the contract is essentially a real option; it is an option to obtain \((1 - a)\theta_1\) units of the good at a cost of \( \theta_2 \), where \((1 - a)\) represents the portion of revenue kept by the firm after paying royalties. Critically, the bidder’s value for the contract must internalize the fact that its probability of exercise is less than one and depends on the future price of the good, which is uncertain at the time of bidding. Suppose the price of the good is \( s \) when the firm decides whether to exercise the option. Then the firm will only exercise if
\(s(1-a)\theta_1 > \theta_2\); that is, if its share of revenue after paying royalties to the principal exceeds the cost of option exercise.

We borrow from the option pricing literature to define a closed-form expression representing the value of such an option. In particular, since the good produced is a commodity in our application, we value the contract according to the Black (1976) model for valuing commodity options. We present the model first and discuss the underlying assumptions afterwards. Let \(t\) be the duration in years until the option expires, \(F\) the \(t\)-year futures price of the good, \(\sigma\) the volatility of \(F\), \(r\) the continuously compounded one year risk-free interest rate, and \(\Phi(\cdot)\) the standard normal cdf. Then a bidder’s value for the option at the time of its auction is given by

\[
V(a, \bar{\theta}) = e^{-rt} \left[ F(1-a)\theta_1 \Phi(x) - \theta_2 \Phi(x - \sigma \sqrt{t}) \right],
\]

where

\[
x \equiv \frac{\ln(F(1-a)\theta_1/\theta_2) + \sigma^2 t/2 \sigma \sqrt{t}}.
\]

The expression \(\Phi(x - \sigma \sqrt{t})\) represents the probability of option exercise, or the probability that the price of the commodity will be high enough to make the firm’s share of revenue exceed its cost. This helps us understand the expression for \(V(a, \bar{\theta})\). Intuitively, if option exercise occurred exogenously with probability \(\Phi(x - \sigma \sqrt{t})\), the value of the option would be given by \(e^{-rt}(F(1-a)\theta_1 - \theta_2)\Phi(x - \sigma \sqrt{t})\): the expected profit from exercise times the probability of exercise, with some present-value discounting for time. However, option exercise is in fact endogenous and occurs when the price of the commodity is higher than some threshold. So the expected price conditional on exercise is actually higher than the unconditional futures price \(F\), and therefore \(F(1-a)\theta_1\) is multiplied by \(\Phi(x)\) rather than \(\Phi(x - \sigma \sqrt{t})\) to account for this.

Adapting well-known results from the option-pricing literature, Table 2 summarizes how the value of the option and the probability of exercising it are affected by the constituent parameters. Holding all else constant, higher productivity type \(\theta_1\) has a positive effect on both option value and exercise probability, while a higher cost type \(\theta_2\) has a negative effect, in line with intuition. Also as expected, a higher royalty rate \(a\) has a negative effect on both option value and exercise probability.
Table 2: Effect on option value and probability of exercise

<table>
<thead>
<tr>
<th>Effect of</th>
<th>on $V(a, \theta)$</th>
<th>on $\Pr(\text{exercise})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\theta_1$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$a$</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$F$</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>+</td>
<td>ambiguous</td>
</tr>
<tr>
<td>$t$</td>
<td>+</td>
<td>ambiguous</td>
</tr>
</tbody>
</table>

**Discussion**  
A number of model features and assumptions deserve further discussion. First, the model and subsequent analysis can be consistent with both private and interdependent value paradigms regarding production volume, depending on the interpretation of $\theta_1$. We know from the existing empirical literature that estimation of auction models proceeds through first-order conditions in the style of Guerre, Perrigne, and Vuong (2000) for both private and interdependent values, but the interpretation of what is obtained differs by paradigm. Under private values, the value obtained is interpreted as the bidder’s primitive valuation; under interdependent values, it is interpreted as an adjusted valuation, adjusted in anticipation of the winner’s curse. For one-dimensional auctions, theory tells us the exact form of this winner’s curse adjustment. For multidimensional auctions, the theory literature so far does not inform us as to the exact form.

Meanwhile, when it comes to firms’ post-auction actions, the model focuses on the extensive margin decision of firms, i.e. of whether they exercise the option, which may be viewed as a discrete form of effort. This is motivated by the fact that well over a majority of leases in our data are not developed, so the extensive margin decision of firms is the one of first-order interest. Intensive margin differences between firms are included in $\theta_1$.

Finally, the Black (1976) option pricing model is built on the approximation that oil futures prices follow a geometric Brownian motion, and that volatility is constant and known for the duration of the option. Also, the option price represents that of a European option, which may only be exercised at expiration, rather than an American option, which may be exercised at any time until expiration and for which there is no closed form expression of value. The difference in value between a European and American option is likely to be small in our application; Kellogg (2014) references Hilliard and Reis (1998) in noting that the American premium is no more than 2%
of the European option price for volatilities typical of oil futures. Also, Herrnstadt, Kellogg, and Lewis (2019) find that a large share of wells are drilled just prior to lease expiration, even though they have the option to drill at any time before. What the Black (1976) model buys us is a closed-form expression for option value. The closed form has benefits for tractability when solving a two-dimensional bidding problem, which we turn to in the next section.

3.2 Cash-royalty bidding

Given his type \((\theta_1, \theta_2)\), a bidder chooses the two components of his bid \((a, b)\) to maximize his expected utility from the auction. This maximization problem can be written as

\[
\max_{a, b} [V(a, \bar{\theta}) - b] P(a, b).
\]

To gain an intuitive understanding of the maximization problem, it helps to break it into two steps: in step 1, we consider which combination of cash and royalty the bidder would choose in order to achieve an arbitrary probability of winning, \(p\), since there are in general many elements in the set \(\{(a, b) | P(a, b) = p\}\). Then given step 1, we consider the bidder’s choice of \(p\). Conceptually separating the bidder’s choice of \(p\) from how it chooses to achieve that \(p\) will make explicit the effect of the bidder’s type on its preference for bidding more royalty versus more cash.

Step 1: Bidder’s choice of \((a, b)\) for arbitrary \(p\)

Now consider said step 1. Fixing \(p\), the bidder chooses its preferred bid from the set \(\{(a, b) | P(a, b) = p\}\). Since \(P(a, b)\) is increasing in both arguments, a choice of \(a\) immediately determines the \(b\) required to achieve \(P(a, b) = p\). Hence, we can define \(b(a, p)\) as a function of \(a\) and \(p\) and focus on the bidder’s choice of \(a\). The bidder’s choice of royalty \(a\) given \(p\) is then

\[
a(p, \bar{\theta}) \equiv \max_a V(a, \bar{\theta}) - b(a, p),
\]

Since the probability of winning is fixed at \(p\), the bidder chooses royalty \(a\) to maximize his payoff conditional on winning, which is the value of the contract at that royalty minus the cash payment required to achieve \(p\) with that royalty. This yields the
first-order condition

\[ V_1(a, \tilde{\theta}) = b_1(a, p). \]  

(3)

Intuitively, the first-order condition balances the marginal cost of a higher royalty against the marginal benefit. On the one hand, bidding a higher royalty allows a bidder to bid less cash (right-hand side of (3)), but it also reduces the value of the contract to him (left-hand side of (3)).

The proof of the following proposition shows that bidding a higher royalty percentage is less costly for undesirable “weak” types - those with low productivity \((\theta_1)\) and/or high cost \((\theta_2)\) - than for “strong” types. Mathematically, the effect of a higher royalty on contract value, \(V_1(a, \tilde{\theta})\), becomes more negative with \(\theta_1\) and less negative with \(\theta_2\). This has important implications for adverse selection; the royalty component of bidding effectively provides weak types a cheaper currency with which to bid. Naturally, this leads strong types to favor a low-royalty high-cash bid and weak types to favor a high-royalty low-cash bid from among the set \(\{(a,b) | P(a,b) = p\}\).

**Proposition 1.** *The marginal effect of royalty \(a\) on contract value, \(V_1(a, \tilde{\theta})\), is decreasing in \(\theta_1\) and increasing in \(\theta_2\). As a result, the bidder’s choice of royalty as a function of \(p\) and \(\tilde{\theta}\), \(a(p, \tilde{\theta})\), is decreasing in \(\theta_1\) and increasing in \(\theta_2\).*

Note that the royalty \(a\) is a function of both \(\theta_1\) and \(\theta_2\). The same goes for the cash payment \(b\). This is not a model in which each bid component is purely a function of one type component.

**Step 2: Bidder’s choice of \(p\) given \(a(p, \tilde{\theta})\)**

Now consider step 2, the bidder’s choice of probability of winning, \(p\). Knowing from step 1 that the bidder will choose to bid royalty \(a(p, \tilde{\theta})\) and cash \(b(a(p, \tilde{\theta}), p)\) given \(p\), we define the payoff conditional on winning as a function of \(p\) and \(\tilde{\theta}\) as follows:

\[ \pi(p, \tilde{\theta}) \equiv V(a(p, \tilde{\theta}), \tilde{\theta}) - b(a(p, \tilde{\theta}), p). \]

Then the bidder chooses \(p\) to maximize his expected utility from the auction, \(\pi(p, \tilde{\theta})p\). Formally, the maximization problem and the corresponding first-order condition are

\[ p(\tilde{\theta}) \equiv \max_p \pi(p, \tilde{\theta})p, \]
\[
\frac{1}{p} = -\frac{\pi_1(p, \tilde{\theta})}{\pi(p, \tilde{\theta})}.
\]

As is standard in auctions, the bidder would like to increase his probability of winning, but achieving a higher \( p \) requires a more competitive bid, which decreases his payoff conditional on winning. The bidder’s choice of \( p \) optimally balances these two forces.

As we look ahead to bringing our bidding model to data, it is helpful to rewrite the first-order conditions from steps 1 and 2 in terms of observables. If the observed bid corresponds to the bidder’s optimal choice, we can replace \( a(p, \tilde{\theta}) \) with observed \( a, b(a(p, \tilde{\theta}), p) \) with observed \( b, \) and \( p \) with \( P(a, b) \). Further, we replace \( \pi(p, \tilde{\theta}) \) with its definition, use \( b_1(a, p) = -P_1(a, b)/P_2(a, b) \) by properties of implicit derivatives, and use \( \pi_1(p, \tilde{\theta}) = -b_2(a(p, \tilde{\theta}), p) = -1/P_2(a, b) \) by the envelope theorem. Then the bidder’s choice of two-dimensional bid \( (a, b) \) as a function of his two-dimensional type \( (\theta_1, \theta_2) \) is characterized by the following system of two first-order conditions:

\[
V_1(a, \tilde{\theta}) = -\frac{P_1(a, b)}{P_2(a, b)}, \quad (4)
\]

\[
V(a, \tilde{\theta}) = b + \frac{P(a, b)}{P_2(a, b)}. \quad (5)
\]

In particular, equation (5) is familiar in form; it resembles the first-order condition for first-price auctions as written in Guerre, Perrigne, and Vuong (2000).

In auctions with a known scoring rule, a multidimensional auction is outcome-equivalent to an auction in which bidders bid on the scalar score only and the highest score wins. The existing literature has exploited this idea to characterize equilibrium bidding for a restricted class of scoring auctions, including auctions with quasi-linear scoring rules. To our knowledge, all work that characterizes equilibrium in multidimensional auctions has relied on this scalar reduction. In the absence of a known scoring rule, we do not characterize equilibrium bidding. Rather, we characterize the bidder’s optimal bid in response to the probability of winning \( P(a, b) \), which is observed by bidders and the econometrician through auction outcomes. This indirect approach is in the spirit of Guerre, Perrigne, and Vuong (2000). It is also related to Larsen and Zhang (2018), who exploit observed outcomes of agents’ actions to identify their types when the precise rules of the game are unknown.
4 Identification

We show that the type \((\theta_1, \theta_2)\) associated with each observed bid \((a, b)\) is identified nonparametrically from the observables: the joint distribution of bids \(G(a, b)\) and whether each bid wins or loses.

Identification requires that there be a unique \((\theta_1, \theta_2)\) that rationalizes bid \((a, b)\) given the observed patterns of winning and losing. Now consider equations (4) and (5) that characterize the mapping between types and optimal bids. In these equations, \(V(\cdot, \cdot)\) is a known function, and \(P(\cdot, \cdot)\), the probability of winning as a function of the bid, is identified immediately from observations of bids and whether they win. Then \(\theta_1\) and \(\theta_2\) are the only remaining unknowns. Equations (4) and (5) thus constitute a system of two nonlinear equations in two unknowns, and we can show identification by proving that the system has a unique solution. The proof of the following proposition does exactly this.

**Proposition 2.** The two-dimensional bidder type \((\theta_1, \theta_2)\) associated with bid \((a, b)\) is identified nonparametrically.

Intuitively, the separate identification of \(\theta_1\) and \(\theta_2\) implies that bidders’ types cannot be reduced to scalar representations in this model. To provide a contrasting example, suppose royalties were levied on profit instead of revenue and development was mandatory. In this case, \(V(a, \bar{\theta}) = (1 - a)(F\theta_1 - \theta_2)\) and \(V_1(a, \bar{\theta}) = -(F\theta_1 - \theta_2)\). Then in (4) and (5), \(\bar{\theta}\) could be replaced by the scalar value \((F\theta_1 - \theta_2)\) without loss, and all \((\theta_1, \theta_2)\) pairs in the set \{\((\theta_1, \theta_2)\)|\((F\theta_1 - \theta_2) = v)\} would be observationally equivalent; i.e. separate identification of \(\theta_1\) and \(\theta_2\) would fail. In reality, royalties are levied on revenue not profit, so the distinctive roles played by \(\theta_1\) versus \(\theta_2\) in determining the bid cannot be reduced to or represented by the bidder’s overall value for the contract. Continuing with the mandatory development example for sake of intuition, royalty being levied on revenue implies \(V(a, \bar{\theta}) = (1 - a)(F\theta_1 - \theta_2)\) and \(V_1(a, \bar{\theta}) = -F\theta_1\). Then, the separate identification of \(\theta_1\) and \(\theta_2\) is apparent given the system of (4) and (5).

5 Estimation

We develop a multi-step nonparametric estimation procedure, which entails estimating the probability of winning \(P(a, b)\) and subsequently solving for \((\theta_1, \theta_2)\) using (4)
and (5). A constructive and detailed description of how to solve for \((\theta_1, \theta_2)\) is provided in the proof of Proposition 2. Therefore, this section focuses on the estimation of \(P(a, b)\).

In principle and given enough data, \(P(a, b)\) can be estimated directly via a nonparametric regression of win/loss dummies on bid components \((a, b)\). This approach treats \(P(\cdot, \cdot)\) as a “blackbox” and does not reveal what is generating \(P(\cdot, \cdot)\). In our estimation, we choose to be more explicit about what generates \(P(\cdot, \cdot)\) by estimating it as the composite of two components: the bid distribution and the state’s pairwise choice probability, described earlier in section 2.3. Before we get to the details, we first address heterogeneity across auction items.

5.1 Auction-level heterogeneity

All empirical auction studies of non-homogeneous goods must address auction-level heterogeneity before estimating the bid distribution a bidder competes against. No two auction items are exactly the same, so the heterogeneity needs to be controlled for in some way. Let \(z\) represent the vector of descriptive auction-level covariates that differ among auction items. As nonparametric conditioning on \(z\) would suffer from the curse of dimensionality, we propose the following.

For each auction item, predict the median of \(a\) conditional on \(z\) and the median of \(b\) conditional on \(z\) from “leave-one-out” quantile regression of bid components \(a\) and \(b\) on \(z\), respectively. Leave-one-out here refers to not using own-auction bids in the regression, rather using \(a, b, z\) data from all the other auctions. Then, for purposes of estimating the bid distribution only, convert each \(a\) to its deviation from the (auction-specific) conditional median royalty, and convert each \(b\) to its log deviation from the conditional median cash bid. In other words, the bid’s competitive position is characterized by how many more percentage points it bids in royalty and how much more (in logs) it bids in cash relative to the conditional median. This procedure normalizes bids with respect to the estimated median bid conditional on \(z\) and allows pooling of bids across heterogeneous auctions when estimating the bid distribution \(G(a, b)\).
5.2 Estimation of the probability of winning \(P(a, b)\)

To estimate \(P(a, b)\), we first estimate the bivariate bid distribution \(G(a, b)\) via kernels. Second, we estimate the state’s pairwise choice probability \(C(a, b, a', b')\) via sieve maximum likelihood, where \(C(a, b, a', b')\) is defined as the probability that the state chooses bid \((a, b)\) over a competing bid \((a', b')\). Finally, we estimate \(P(a, b)\) as the probability that bid \((a, b)\) is chosen over \(n\) competing bids drawn from \(G(a, b)\).

First, let \((\tilde{a}, \tilde{b})\) denote the bids normalized according to section 5.1. We estimate the bivariate distribution of \((\tilde{a}, \tilde{b})\), denoted \(\tilde{G}(\cdot, \cdot)\), nonparametrically via kernel estimation. Then, if \(\bar{a}(z)\) and \(\bar{b}(z)\) are the medians conditional on \(z\) as described in section 5.1, we can define \(G(\cdot, \cdot)\) as a simple transformation of \(\tilde{G}(\cdot, \cdot)\): \(G(a, b) \equiv \tilde{G}(a - \bar{a}(z), \ln b - \ln \bar{b}(z))\). Thus we estimate \(G(a, b)\). As is usual with auctions, the bid distribution should be estimated fixing the number of bidders \(n\), as it is likely to change with \(n\).

Second, we estimate the state’s pairwise choice probability \(C(a, b, a', b')\) nonparametrically via sieve maximum likelihood. Recall from section 2.3 that “pairwise” does not refer to the number of bidders \(n\) equaling two; each \(n\)-bidder auction yields \(n - 1\) observed pairwise choices made by the state between the winning bid and each of the losing bids, and all of these observations can be used for estimation. For each observed pairwise choice between \((a, b)\) and \((a', b')\), the bids composing the pair are the inputs to \(C(\cdot, \cdot, \cdot, \cdot)\), and an indicator for \((a, b)’s\) win, \(w\), is the binary outcome. We reduce the dimensionality of this function by assuming the state compares two bids based on their component-wise differences; that is, \(C(a, b, a', b') \equiv C(a - a', \ln b - \ln b', b')\).

The log-likelihood, over all observed pairwise choices, of observed outcome \(w\) given bids \((a, b)\) and \((a', b')\) is

\[
\sum_{k=1}^{K} w_k \ln C(a_k - a'_k, \ln b_k - \ln b'_k, b') + (1 - w_k) \ln [1 - C(a_k - a'_k, \ln b_k - \ln b'_k, b')],
\]

where \(K\) is the number of observed pairwise choices. Sieve maximum likelihood approximates \(C(\cdot, \cdot, \cdot)\) with sieves - we use multivariate Bernstein polynomials - and estimates the polynomial parameters that maximize the likelihood. For regularity, we restrict \(C(\cdot, \cdot, \cdot)\) to increase in its first two arguments, reflecting the state’s preference for higher bid components. Since the domain of Bernstein polynomials is \([0, 1]\), we
convert each argument of $C(\cdot, \cdot, \cdot)$ to its quantile in the observed distribution of that argument for purposes of computing the polynomials.

Finally, having estimated $G(\cdot, \cdot)$ and $C(\cdot, \cdot, \cdot)$, we estimate $P(\cdot, \cdot)$ as their composite. Using the case of two-bidder auctions as an illustrative example, the probability of winning with bid $(a, b)$ is the expectation of pairwise choice probability over the distribution of competing bids,

$$P(a, b) = \int C(a - a', \ln b - \ln b', b') dG(a', b').$$

6 Application to Louisiana cash-royalty auctions

6.1 Details

We bring our model to the Louisiana cash-royalty auctions and apply our estimation method. Our sample, described in section 2, constitutes 1,279 auctions with at least 2 bidders. We use the pairwise choices observed from all of these auctions to estimate the state’s pairwise choice probability $C(\cdot, \cdot)$. Meanwhile, the bid distribution $G(\cdot, \cdot)$ should be estimated for a fixed number of bidders. 987 out of 1,279 auctions in our sample have two bidders, so we estimate $G(\cdot, \cdot)$ and $P(\cdot, \cdot)$ using these 987 auctions. After estimating $P(\cdot, \cdot)$, solving the system of first-order conditions for types $(\theta_1, \theta_2)$ requires historical data on 3-year oil futures and implied volatilities, which begin in 1989. Therefore, $(\theta_1, \theta_2)$ are estimated for bids from 1989-2003, a period encompassing 260 auctions with $n = 2$ in our sample.

To obtain the one year risk-free interest rate $r$, we take nominal 1-year treasury rates provided by FRED and convert them to real rates via application of the Fisher equation, using percentage changes in the GDP implicit price deflator as the measure of inflation. For $F$, the 3-year futures price of oil, we use data on crude oil futures (CL36) provided by Quandl. For $\sigma$, the volatility of $F$, we follow Kellogg (2014) in deriving implied volatility from prices of crude oil options by inverting Black (1976). For that purpose, we purchased historical prices of crude oil options from the CME Group; further details on deriving implied volatility are provided in the appendix.

To control for auction-level heterogeneity as described in section 5.1, our auction covariates $z$ include tract acreage, an indicator for royalty recipient type,$^4$ auction

$^4$Specifically, tract kind code 4 indicates that the royalty recipient is a state agency other than
year and calendar month to account for heterogeneity across time, and two variables to account for geological and geographic heterogeneity, constructed as follows. First, we use historical production data from Drillinginfo to compute a production index for each township\(^5\) in Louisiana; it is the log of barrel-of-oil-equivalents (BOE) produced per well in each township. Second, we perform a local linear regression of observed bid components on the geographic coordinates of the associated lease’s township; this surface fit is done once for each auction, excluding own-auction bids, to produce a fitted royalty index and cash payment index for each auction based on geographic location. This is a control meant to capture any remaining geological and geographic heterogeneity not captured by the production index.

### 6.2 Estimation results

Figures 3 and 4 plot level contours of the estimated bid density and pairwise choice probability, respectively, along with the observations used to estimate them. Figure 3 shows that the bid distribution is densest near the conditional median bid and gets sparser farther from the median. Most bids are within 5 percentage points of the conditional median royalty and within a log deviation of 1 from the conditional median cash payment. The contours in Figure 4 show the transition from low probability to high probability of being favored by the state as bid components increase, which directly follows the transition from losses (red triangles) to wins (blue circles) in the underlying data.

Figures 5 and 6 plot the densities of estimated productivity type \(\hat{\theta}_1\) and per-unit cost \(\hat{\theta}_2/\hat{\theta}_1\), respectively. Note that these are raw values across all auctions in the estimation sample; they are not conditioned on any particular value of auction covariates. The ratio \(\hat{\theta}_2/\hat{\theta}_1\) represents the bidder’s cost per unit of production (barrel), which is a more useful measure of cost than \(\hat{\theta}_2\) on its own. Figure 6 includes for comparison a red dotted line representing the average of \((1-a)\times\text{oil price}\) in the estimation sample; \((1-a)\times\text{oil price}\) is the revenue a firm would keep per barrel if it developed the tract. Supposing \((1-a)\times\text{oil price}\) were fixed, firms with per-unit cost to the left of the dotted line would exercise their option, while firms to the right of it would not. As there is substantial mass to the right of the red line, we begin to see why so many leases are not developed.

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\(^5\)A township is a 6-mile square designated by the Public Land Survey System.
Figure 3: Estimated bivariate bid density $\hat{g}(\tilde{a}, \tilde{b})$

Figure 4: Estimated pairwise choice probability $\hat{C}(a - a', \ln b - \ln b', b')$
Figure 5: Kernel density of estimated $\ln \hat{\theta}_1$

Figure 6: Kernel density of estimated $\hat{\theta}_2/\hat{\theta}_1$
7 Counterfactual analysis

7.1 Cash-royalty auctions versus fixed-royalty auctions

Fixed-royalty auctions – the prevailing auction design for public mineral leasing in the U.S. – are a simpler alternative to cash-royalty auctions. In that auction format, the principal fixes a common royalty rate, and bidders bid only on the cash payment amount. Cash-royalty auctions have the potential to yield better outcomes for the principal than fixed-royalty auctions, but how they perform in practice is an empirical question that depends on the details of implementation. Using our estimated structural model, we compare Louisiana’s cash-royalty auction to the fixed-royalty benchmark. The model is sufficiently rich to allow comparison in the particulars of allocative distortions, moral hazard, and information rents, the combined effects of which lead to an overall comparison of social surplus and government revenue. To conduct this counterfactual comparison, we simulate fixed-royalty auctions with fixed royalty rates ranging from 0% to 50%. All evaluations are ex-ante with respect to oil price realizations, so results are not tied to a specific path of oil prices.

In the graphs that follow, the solid blue curve represents the fixed-royalty auction. The movement of the solid curve from left to right shows how the quantity of interest is affected by the level of the fixed royalty, which is indicated on the x-axis. Meanwhile, the dotted horizontal line marks the value of the quantity of interest in Louisiana’s cash-royalty auction. Since the royalty is not fixed in a cash-royalty auction, the x-axis is irrelevant for the dotted horizontal line. We are interested in comparing the level of the dotted line to the solid curve.

**Allocative distortions** In standard auctions with one-dimensional types and bids, ex-ante symmetry of bidders implies allocative efficiency; the bidder with the highest value always wins. Here, generalizing the type space to two dimensions allows us to meaningfully analyze the allocative effects of auction design even when bidders are ex-ante symmetric. To do this, we first take note of which bidder would have won in a hypothetical zero royalty auction, i.e. an auction in which the mineral rights could have been purchased outright. This is the socially optimal allocation. Using this as the benchmark, we then compute under different auction designs the fraction of auctions that retain the socially optimal allocation.

The blue curve in Figure 7 shows that allocative distortions increase with royalty
for fixed-royalty auctions. Intuitively, as royalties are levied on revenue, high royalties are especially punishing for bidders who have both high productivity and high cost, compared to bidders who have low productivity and low cost. This leads to flips in the winner’s identity as the fixed royalty increases. Meanwhile, we see from the orange dotted line that Louisiana’s auction system yields allocative distortions that are worse than any of the fixed-royalty auctions plotted. This is because the endogenous choice of royalty by bidders in Louisiana exacerbates adverse selection. As shown in Proposition 1, royalties are less costly for low productivity and/or high cost types and serve as a cheaper currency with which to bid. As a result, these socially suboptimal types win more often than when royalty is fixed by the government.

**Moral hazard** For the designers of the Louisiana leasing system, one intended benefit of allowing the “market” to determine royalties would have been to avoid the incidence of inflexible fixed royalties on tracts that are not productive enough to support them. Such flexibility could potentially increase the rate of option exercise compared to a rigid fixed-royalty system in which one royalty rate – such as the federal BLM’s 12.5% – is applied universally. It would also lift the government’s burden of having to determine the fixed royalty. However, we see in Figure 8 that the Louisiana auction does not achieve any special benefit for the probability of option exercise; the average royalty in the Louisiana leasing system is 23%, while its probability of option exercise (orange dotted line) is similar to what it would have been had the government
just fixed the royalty at 24%. As expected, the blue curve shows that the probability of option exercise is decreasing in fixed royalty, as the firm’s incentive to exercise the option declines with the portion of revenue transferred to the government.

**Social surplus** Social surplus in this context is defined as all production revenues minus costs. It incorporates the effects of the allocative distortions and moral hazard which were discussed above. The designers of Louisiana’s leasing system may have intended its flexibility to yield benefits for social surplus. However, as was the case in our investigation of moral hazard, there does not seem to be any special benefit; Figure 9 shows that the social surplus in Louisiana (orange dotted line), which has average royalty of 23%, is similar to what it would have been had the government just fixed the royalty at 25%. As expected given the allocative distortion and moral hazard associated with royalties, the blue curve shows that social surplus is declining in royalty.

**Cash payment from winning firm to government** We now turn our attention to the transfers between agent and principal. Figure 10 plots the cash component of the winner’s bid, which is paid by the winning firm to the government. Naturally, the blue curve shows that among fixed-royalty auctions, a higher fixed royalty decreases the value of the lease to the bidder, resulting in lower cash bids. In Louisiana’s cash-royalty auction, though, the cash payment is even lower than the average royalty level
(23%) would lead us to expect. Specifically, Figure 10 shows that the average cash payment in Louisiana (orange dotted line) is about $940 per acre. In a 23% fixed-royalty auction, it would have been about $1,500 per acre. Compared to having one-dimensional competition in cash only, two-dimensional competition as implemented in Louisiana leads to a disproportionate weakening of cash competition relative to the royalty level it achieves.

**Information rents (winning firm’s surplus)** To get a sense of the firms’ surplus under each auction design, we consider their information rents. Information rents are defined as the firm’s ex-ante value for the lease, evaluated at the relevant royalty rate, minus the cash component of its bid. We know from theory that agents’ information rents are decreasing in royalty; the more the principal’s payoff is tied to the actual outcome, the less room there is for the agent to reap information rents. The blue curve in Figure 11 confirms this. Meanwhile, we see that as a consequence of weaker cash competition as discussed above, bidders in Louisiana reap higher information rents than the average royalty level (23%) would lead us to expect. The winning firm’s average surplus is about $2,000 per acre in Louisiana’s cash-royalty auction. In a 23% fixed-royalty auction, it would have been only $1,600 per acre. So agents’ information rents are 25 percent higher in Louisiana’s cash-royalty auction than in a fixed-royalty auction of similar royalty levels.
Figure 10: Cash payment from winning firm to government

![Graph showing cash payment from winning firm to government.]

Figure 11: Information rents

![Graph showing information rents.]

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Total government revenue (sum of cash and royalties) Combining the effects of allocative distortions, moral hazard, and firms’ information rents, we now assess the effect of auction design on overall government revenue, defined as the sum of cash payments and ex-ante expected royalties. In line with the high information rents discussed above, Louisiana’s cash-royalty auction performs poorly in terms of total government revenue, as shown in Figure 12. In fact, a fixed-royalty auction with royalty fixed at any rate between 3% and 47% would yield higher total revenue for the government! This interval includes the federal BLM’s fixed rate of 12.5% as well as the 25% rate which is common on privately owned lands. In particular, implementing a 25% fixed-royalty auction rather than the current design would yield a 14% increase in revenue from $3,367 to $3,842 per acre.

7.2 Effects of lease duration and timing

The previous section showed that fixed-royalty auctions perform better than Louisiana’s cash-royalty auction in terms of government revenue. Having estimated bidders’ types and equipped with our option model, we are also in a position to address policy questions beyond auction design. One such question has to do with the duration of the lease. To determine the effects of a longer lease on government revenue and probability of option exercise, we counterfactually simulate fixed-royalty auctions of a 6-year lease, doubling the observed duration of 3 years. Recall from Table 2 that the effect
of contract duration on option value is positive, while its effect on option exercise is theoretically ambiguous. The left-side plot in Figure 13 shows that for Louisiana, doubling the lease duration would in fact decrease the ex-ante probability of option exercise at most royalty values. While the longer duration would still increase option values and hence cash bids, the right-side plot in Figure 13 shows that the overall increase in government revenue is minor and much less than proportionate to the duration increase. This policy would not be worth pursuing, especially if leases can be re-auctioned shortly after termination.

Meanwhile, better exploiting fluctuations in oil prices is a policy that may yield substantial benefits. As Louisiana has control over auction offerings, they can withhold leases when oil futures are low and release them when oil futures are higher. Figure 14 counterfactually simulates what government revenue would have been had oil futures been 20% higher than what they were at the time of each observed auction. The resulting increase in government revenue is much more than proportional, exceeding 50%. This is because higher oil prices increase not only the royalty dollars conditional on option exercise but also the probability of exercise itself – as shown in Table 2 – as well as option value and hence cash bids.
Figure 14: Effect of 20% higher futures price on government revenue
Appendix

Proof of Proposition 1  First, we show that \( V_1(a, \tilde{\theta}) \) is decreasing in \( \theta_1 \) and increasing in \( \theta_2 \). Let \( \theta_k \) denote the \( k \)th dimension of \( \tilde{\theta} \). Also define \( S \equiv F(1-a) \theta_1 \). Then
\[
\frac{\partial}{\partial \theta_k} V_1(a; \tilde{\theta}) = \frac{\partial}{\partial \theta_k} \left( \frac{\partial V}{\partial S} \frac{\partial S}{\partial a} \right) = \frac{\partial}{\partial \theta_k} \left( \frac{\partial V}{\partial S} \frac{\partial S}{\partial a} \right) + \frac{\partial}{\partial \theta_k} \left( \frac{\partial S}{\partial a} \frac{\partial V}{\partial S} \right). \]
It is well known from the option pricing literature that \( \frac{\partial V}{\partial S} = e^{-rt} \Phi(x) \). So \( \frac{\partial}{\partial \theta_1} \left( \frac{\partial V}{\partial S} \frac{\partial S}{\partial a} \right) = \frac{1}{\theta_1 \sigma \sqrt{t}} e^{-rt} \phi(x) \) and \( \frac{\partial}{\partial \theta_2} \left( \frac{\partial S}{\partial a} \frac{\partial V}{\partial S} \right) = \frac{e^{-rt} \phi(x)}{\theta_2 \sigma \sqrt{t}} \). Meanwhile, \( \frac{\partial S}{\partial a} = -F \theta_1 \), so \( \frac{\partial}{\partial \theta_1} \left( \frac{\partial S}{\partial a} \right) = -F \) and \( \frac{\partial}{\partial \theta_2} \left( \frac{\partial S}{\partial a} \right) = 0 \). Then, \( \frac{\partial}{\partial \theta_1} V_1(a; \tilde{\theta}) = \frac{e^{-rt} \phi(x)}{\theta_1 \sigma \sqrt{t}} F \theta_1 - F e^{-rt} \Phi(x) < 0 \), and \( \frac{\partial}{\partial \theta_2} V_1(a; \tilde{\theta}) = \frac{e^{-rt} \phi(x)}{\theta_2 \sigma \sqrt{t}} F \theta_1 + 0 > 0 \).

Next, we show that the choice of royalty \( a(p, \tilde{\theta}) \) is decreasing in \( \theta_1 \) and increasing in \( \theta_2 \). We know from (3) that \( V_1(a(p, \tilde{\theta}), \tilde{\theta}) = b_1(a(p, \tilde{\theta}), p) \forall \tilde{\theta} \). Taking a total derivative of both sides with respect to \( \theta_k \) gives \( \frac{\partial}{\partial \theta_k} V_1(a; \tilde{\theta}) + \frac{\partial^2 V(a; \tilde{\theta})}{\partial a^2} \frac{\partial a(p, \tilde{\theta})}{\partial \theta_k} \bigg|_{a=a(p, \tilde{\theta})} = \left. \frac{\partial^2 b(a(p, \tilde{\theta}))}{\partial a^2} \frac{\partial a(p, \tilde{\theta})}{\partial \theta_k} \right|_{a=a(p, \tilde{\theta})} \right. \).

Rearranging this gives \( \left. \left\{ \frac{\partial^2 b(a(p, \tilde{\theta}))}{\partial a^2} - \frac{\partial^2 V(a; \tilde{\theta})}{\partial a^2} \right\} \frac{\partial a(p, \tilde{\theta})}{\partial \theta_k} \right|_{a=a(p, \tilde{\theta})} \right. = 0. \) Therefore, \( \frac{\partial}{\partial \theta_1} V_1(a; \tilde{\theta}) < 0 \) from the previous paragraph implies \( \frac{\partial a(p, \tilde{\theta})}{\partial \theta_1} < 0 \), and similarly \( \frac{\partial}{\partial \theta_2} V_1(a; \tilde{\theta}) > 0 \) implies \( \frac{\partial a(p, \tilde{\theta})}{\partial \theta_2} > 0 \).

Proof of Proposition 2  A constructive proof of identification proceeds by proving that there is a unique solution \( (\theta_1, \theta_2) \) to the system (4) and (5). Since \( x \) as defined in (2) is a monotonic function of the ratio \( \theta_1/\theta_2 \), we use a change of variables, solving (4) and (5) for \( \theta_1 \) and \( x \) (instead of \( \theta_1 \) and \( \theta_2 \)) for algebraic convenience. First, as is typically done in solving systems of equations, we use (4) to write \( \theta_1 \) in terms of \( x \). Second, we plug this expression for \( \theta_1 \) into (5), yielding an equation with one unknown variable, \( x \). Third, we show that there is a unique solution \( x \) to this equation. Finally, we give closed-form expressions for \( \theta_1 \) and \( \theta_2 \) as functions of the solution \( x \).

First: Define \( S \equiv F(1-a) \theta_1 \). Then the derivative \( V_1(a, \tilde{\theta}) = \frac{\partial V}{\partial S} \frac{\partial S}{\partial a} = -e^{-rt} F \theta_1 \Phi(x) \), using a well-known result from option pricing that \( \frac{\partial V}{\partial S} = e^{-rt} \Phi(x) \). Substituting this in to the left-hand side of (4), we have \( -e^{-rt} F \theta_1 \Phi(x) = -P_1(a, b)/P_2(a, b) \). This allows us to write \( \theta_1 \) in terms of \( x \): \( \theta_1 = P_1(a, b)/P_2(a, b) e^{-rt} F \Phi(x) \). The expression \( P_1(a, b)/P_2(a, b) e^{-rt} F \) is a positive, known constant; we abbreviate this as \( C_1 \), so \( \theta_1 = C_1 / \Phi(x) \).

Second: Now we plug this expression for \( \theta_1 \) into the left-hand side of (5) after some rearranging:
\[ V(a, \tilde{\theta}) = e^{-rt}(1 - a) F_{\theta_1} \Phi(x) - \theta_2 \Phi(x - \sqrt{t}) \]
\[ = e^{-rt}(1 - a) F_{\theta_1} \Phi(x) - \frac{\theta_2}{(1-a)F_{\theta_1}} \Phi(x - \sqrt{t}) \]
\[ = \frac{e^{-rt}(1-a)FC_1}{\Phi(x)} \Phi(x) - \frac{\theta_2}{(1-a)F_{\theta_1}} \Phi(x - \sqrt{t}) \]
\[ = e^{-rt}(1 - a) FC_1 \left[ 1 - \frac{\theta_2}{(1-a)F_{\theta_1}} \frac{\Phi(x - \sqrt{t})}{\Phi(x)} \right] \]

where the last line comes from plugging in a rearrangement of the definition of \(x\), \(\frac{\theta_2}{(1-a)F_{\theta_1}} = \exp(-\sigma \sqrt{tx} + \sigma^2 t/2)\). \(x\) is the only unknown in the last line. Now (5) becomes

\[ e^{-rt}(1 - a) FC_1 \left( 1 - e^{-\sigma \sqrt{tx}} e^{\sigma^2 t/2} \frac{\Phi(x - \sqrt{t})}{\Phi(x)} \right) = b + \frac{P(a, b)}{P_2(a, b)}. \]

Collecting \(x\) on the left-hand side,

\[ e^{-\sigma \sqrt{tx}} \frac{\Phi(x - \sigma \sqrt{t})}{\Phi(x)} = e^{-\sigma^2 t/2} \left( 1 - \frac{b + P(a, b)}{P_2(a, b)} \right) \frac{1}{e^{-rt}(1 - a) FC_1}. \]

Third: The right-hand side is known, so we abbreviate it as \(C_2\). If the left-hand side is strictly monotonic in \(x\), there is a unique value of \(x\) that satisfies the equation. To check this, we derive the derivative of the left-hand side with respect to \(x\) and simplify:

\[ \frac{\partial}{\partial x} \left( e^{-\sigma \sqrt{tx}} \frac{\Phi(x - \sigma \sqrt{t})}{\Phi(x)} \right) = -\sigma \sqrt{t} e^{-\sigma \sqrt{tx}} \frac{\Phi(x - \sigma \sqrt{t})}{\Phi(x)} + e^{-\sigma \sqrt{tx}} \left( \frac{\Phi(x - \sigma \sqrt{t}) \Phi(x) - \Phi(x - \sigma \sqrt{t}) \Phi(x)}{\Phi(x)^2} \right) \]
\[ = \left( e^{-\sigma \sqrt{tx}} / \Phi(x) \right) \left( -\sigma \sqrt{t} \Phi(x - \sigma \sqrt{t}) + \phi(x) - \sigma \sqrt{t} \phi(x - \sigma \sqrt{t}) \right) \]
\[ = \left( e^{-\sigma \sqrt{tx}} \phi(x - \sigma \sqrt{t}) / \Phi(x) \right) \left( -\sigma \sqrt{t} \phi(x - \sigma \sqrt{t}) + 1 - \frac{\phi(x) \Phi(x - \sigma \sqrt{t})}{\phi(x - \sigma \sqrt{t})} \right) \]
\[ = \left( e^{-\sigma \sqrt{tx}} \phi(x - \sigma \sqrt{t}) / \Phi(x) \right) \left( 1 - \frac{\Phi(x - \sigma \sqrt{t})}{\phi(x - \sigma \sqrt{t})} \left( \sigma \sqrt{t} + \frac{\phi(x)}{\Phi(x)} \right) \right). \]

We show in Remark 2 that as a property of the standard normal distribution, the derivative of \(\frac{\phi(x)}{\Phi(x)} > -1\) for any value of the argument. Therefore, \(\sigma \sqrt{t} + \frac{\phi(x)}{\Phi(x)} > 0\) for any value of \(\sigma \sqrt{t}\). Then it follows that \(\frac{\partial}{\partial x} \left( e^{-\sigma \sqrt{tx}} \frac{\Phi(x - \sigma \sqrt{t})}{\Phi(x)} \right) < 0\), and there is a unique \(x\) that satisfies (6). This \(x\), as the only unknown in an equation, is easy to solve for numerically.

Finally, as functions of the solution \(x, \theta_1 = P_1(a, b)/(P_2(a, b)e^{-rt}F\Phi(x))\) and \(\theta_2 = (1 - a)F\theta_1 \exp(-\sigma \sqrt{tx} + \sigma^2 t/2)\).
Remark 1. Let \( h(x) \equiv \frac{\phi(x)}{\Phi(x)} \). Then \( h(x) + x \geq 0 \) for \( x \in (-\infty, \infty) \).

Proof. Define \( g(x) \equiv (h(x) + x)\Phi(x) = \phi(x) + x\Phi(x) \). Then \( g'(x) = \phi'(x) + \Phi(x) + x\phi(x) \). Using \( \phi'(x) = -x\phi(x) \), \( g'(x) = \Phi(x) > 0 \). Meanwhile, using L'Hôpital’s rule, \( \lim_{x \to -\infty} g(x) = \lim_{x \to -\infty} \frac{x}{\Phi(x)} = \lim_{x \to -\infty} \frac{-\phi(x)^2}{\phi(x)} = \lim_{x \to -\infty} \frac{-2\phi(x)}{-x} = 0 \).

Therefore, \( g(x) \geq 0 \) and \( g(x)/\Phi(x) = h(x) + x \geq 0 \). \( \square \)

Remark 2. Let \( h(x) \equiv \frac{\phi(x)}{\Phi(x)} \). Then \( h'(x) > -1 \) for \( x \in (-\infty, \infty) \).

Proof. First, we derive \( h'(x) + 1 = (\phi'(x)\Phi(x) - \phi(x)^2 + \Phi^2(x))/\Phi^2(x) \)
\[
= \frac{\phi(x)}{\Phi(x)} \left( \frac{\phi'(x)}{\phi(x)} \Phi(x) - \phi(x) + \frac{\Phi(x)^2}{\phi(x)} \right) = \frac{\phi(x)}{\Phi(x)} \left( -x\Phi(x) - \phi(x) + \frac{\Phi(x)^2}{\phi(x)} \right).
\]
Now define \( f(x) \equiv -x\Phi(x) - \phi(x) + \frac{\Phi(x)^2}{\phi(x)} \). Then \( f'(x) = -\Phi(x) - x\phi(x) + x\phi(x) + 2\Phi(x) + x\Phi(x)^2/\phi(x) = \Phi(x) + x\Phi(x)^2/\phi(x) = \frac{\Phi(x)^2}{\phi(x)} (\phi(x)/\Phi(x) + x) = \frac{\Phi(x)^2}{\phi(x)} (h(x) + x) \).

By Remark 1, \( f'(x) \geq 0 \). Meanwhile, using L'Hôpital’s rule and \( \lim_{x \to \infty} x\Phi(x) = 0 \) shown in Remark 1,
\[
\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{\phi(x)^2}{\phi(x)} = \lim_{x \to -\infty} \frac{2\Phi(x)\phi(x)}{-x\phi(x)} = \lim_{x \to -\infty} \frac{2\Phi(x)}{-x} = 0 \.
\]
and \( h'(x) + 1 = \frac{\phi(x)}{\Phi(x)^2} f(x) \geq 0 \). \( \square \)

**Deriving implied volatility from historical prices of crude oil options**

We purchase historical prices of crude oil options from the CME Group. For every call option traded in the data, we invert the Black (1976) option pricing equation to back out the expected volatility implied by its price. We take the median of these implied volatilities in each trade month \( m \) for option maturity \( \tau \) (in months) to be the implied volatility in month \( m \) of \( \tau \)-month futures. As options of 36-month maturity are traded relatively sparsely, we adapt Kellogg (2014)’s method of using the term structure of realized volatility to infer long term volatilities from those of shorter term futures, for which options are traded in higher volumes. The term structure of realized volatility that is relevant for each historical month is estimated as follows. For each month, we use daily realized volatilities of oil futures within the surrounding 1-year window to estimate the fixed effects regression \( \ln rvol_{t,\tau} = \eta_t + \delta_t + \epsilon_{t,\tau} \), where \( rvol_{t,\tau} \) is the realized volatility at date \( t \) of the \( \tau \)-month futures contract. The maturity fixed effect \( \eta_t \) captures the term structure. Finally, the implied volatility of 36-month futures in month \( m \) is inferred from a shorter \( \tau \)-month implied volatility as \( \sigma_{m,36} = \sigma_{m,\tau} \exp(\eta_{m,36} - \eta_{m,\tau}) \).
References


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