Redistribution, Risk Premia, and the Macroeconomy*

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Abstract

We study the effects of redistribution on risk premia and investment in a heterogeneous agent New Keynesian environment. Heterogeneity in agents' marginal propensity to take risk (MPR) summarizes differences in risk aversion, constraints, rules of thumb, and background risk relevant for portfolio choice on the margin. Shocks which redistribute to agents with high MPRs reduce risk premia and, absent a monetary policy tightening, raise investment. We quantitatively evaluate the role of this mechanism for the transmission of conventional monetary policy in the U.S. economy. An unexpected reduction in the nominal interest rate redistributes to agents with high MPRs and lowers the risk premium. This rationalizes the relative roles of dividend growth, risk-free rates, and excess returns in generating an increase in the stock market and contributes substantially to the transmission of monetary policy through investment.

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1 Introduction

The returns on real investment carry large and countercyclical risk premia. Risk premia also systematically respond to macroeconomic policy — with respect to monetary policy, for example, this is evident in the equity premium (Bernanke and Kuttner [2005]), the term premium (Hanson and Stein [2015]), and the external finance premium (Gertler and Karadi [2015]). The basic New Keynesian framework as described in Woodford [2003] and Gali [2008] has little to say regarding these aspects of the business cycle and policy transmission. As noted by Kaplan and Violante [2018], this is equally true for an emerging body of heterogeneous agent New Keynesian (HANK) models whose focus on heterogeneous marginal propensities to consume (MPCs) has substantially enriched the implications for aggregate consumption but less so for asset prices and therefore investment.

We demonstrate that redistribution across agents who differ instead in risk-bearing capacity can generate rich and realistic implications for risk premia and aggregate investment. We first make precise the relevant notion of risk-bearing capacity to evaluate the effects of redistribution: the marginal propensity to take risk (MPR), an agent's marginal propensity to save in capital out of an additional unit of income relative to her marginal propensity to save overall. Heterogeneity in MPRs encodes heterogeneity in risk aversion, rules of thumb, constraints, and background risk relevant for portfolio choice on the margin. Shocks which redistribute to agents with relatively higher MPRs reduce the market price of risk. Given nominal frictions and monetary policy which does not tighten in response, the required return on capital falls and investment rises. We quantitatively explore this mechanism for conventional monetary policy shocks in a calibration to the U.S. economy. High-MPR agents borrow in the bond market from low-MPR agents in order to hold leveraged positions in capital. By devaluing nominal debt through unexpected inflation, reducing the real interest rate, and lowering labor's share of income, an unexpected loosening of monetary policy redistributes to high-MPR investors and lowers the risk premium. This can rationalize the roles of dividend growth, risk-free rates, and excess returns in generating an increase in the stock market, and contributes substantially to the transmission of monetary policy through investment.

We begin by characterizing these effects in a two-period model with simple analytical results. Agents consume, supply labor, and trade nominal bonds and claims on capital. Production is subject to aggregate TFP shocks, rendering the latter claims risky, while agents may also be subject to idiosyncratic income risk. Agents have Epstein and Zin [1991] preferences with arbitrary heterogeneity in discount factors, intertemporal elasticities of substitution, and risk aversion. Agents may also follow rules-of-thumb in their portfolio allocation or face constraints on leverage, nesting models of limited participation. Disentangling risk aversion from intertemporal substitution and accounting for these other determinants of portfolio choice are important given our focus on risk premia rather than just risk-free rates.

In this environment, we first define an individual agent's MPR and contrast it with the MPC. Given a marginal unit of income, an agent can consume it, save in bonds, or save in claims on capital. The MPR summarizes her marginal saving in capital relative to saving overall. To shed light on its structural determinants we characterize its limit as the volatility of aggregate shocks falls to zero, applying the methodology of Devereux and Sutherland [2011] from open-economy macroeconomics to HANK and our particular statistic of interest. If agents only differ in preferences, the limiting MPR of an agent is fully described by her risk tolerance relative to a weighted-average of risk tolerance among all agents in the economy. This contrasts starkly with her limiting MPC, which is invariant to risk tolerance and depends instead on her degree of patience and intertemporal elasticity of substitution. Focusing on MPRs rather than risk aversion is useful because the former are more naturally measurable, and because they remain the relevant notion of heterogeneity in the presence of portfolio constraints, rules-of-thumb, or idiosyncratic risk.

Shocks which redistribute to agents with high MPRs reduce risk premia and, absent a monetary policy tightening, raise investment. The effect on risk premia is a general implication of market clearing in asset markets: to the extent agents on aggregate wish to increase their portfolio share of capital, its expected return must fall relative to that on bonds. The associated effect on investment depends on the degree to which the return on bonds rises versus the return on capital falls. Absent nominal rigidity, this depends on the appropriately weighted average intertemporal elasticity of substitution across agents. With nominal rigidity, it depends on the monetary policy response. In the useful benchmark where monetary policy maintains a constant real interest rate, the required return on capital must fall and hence redistribution to high MPR agents necessarily raises investment.

These results are relevant for a wide class of shocks which redistribute across agents, including TFP, fiscal policy, and monetary policy. The redistributive effects of these shocks can be organized into three groups. First, if the shock changes nominal prices, it will revalue balance sheets through heterogeneous exposures to the nominal bond (as in Doepke and Schneider [2006]). Second, if the shock changes relative prices such as real interest rates or real wages, it will redistribute across agents having heterogeneous exposures to these prices through Slutsky income effects. Third, if the shock changes the composition of profit versus labor income, it will redistribute across agents with heterogeneous claims on such income. While these sources of redistribution are standard in the HANK literature — elucidated in the case of monetary policy shocks, for instance, in Auclert [2018] — it is their covariance with MPRs rather than MPCs which matters for risk premia rather than risk-free rates.

Accounting for the risk premium effects of monetary policy is important given empirical evidence suggesting that this may be a key transmission channel to the real economy. We refresh this point from Bernanke and Kuttner [2005] using the proxy structural vector autoregression (SVAR) approach in Gertler and Karadi [2015]. Using monthly data from July 1979 through June 2012, we run a six-variable VAR including the excess return on the S&P 500 and real return on a T-bill. Using Fed Funds surprises on FOMC days as an instrument, we find that an unexpected loosening of monetary policy resulting in a 25bp reduction in the 1-year Treasury bond leads to an unexpected increase in excess returns of 2pp. Using a Campbell and Shiller [1988] decomposition, 1.1pp (55%) of this increase is driven by lower excess returns whereas only 0.2pp (10%) is driven by lower risk-free rates.[1]

Extending the model to the infinite horizon, we quantitatively demonstrate that a HANK model calibrated to the U.S. economy is capable of rationalizing these facts. Informed by patterns in the Survey of Consumer Finances (SCF) and the need to make the computational burden tractable, we focus on a three-agent model. One type, workers, face a rule-of-thumb in portfolio allocation. The other two types, capitalists, face a rich portfolio choice problem in asset markets but have zero labor productivity. We identify capitalists with the 34% of households in the SCF who have actively traded using a broker in the past year or have a private business.[2] We calibrate the model to be consistent with standard "macro" moments on the business cycle and asset returns as well as "micro" moments summarizing heterogeneity in MPRs inferred from the SCF. We infer these moments using households' observed portfolio holdings and our analytical formulas mapping portfolios to MPRs.

Following an unexpected cut in the nominal interest rate, wealth redistributes to agents with high MPRs. This occurs through each of the three redistributive channels previously described, in all cases reflecting the fact that the high-MPR agents endogenously hold a levered position in capital. First, since the lower nominal interest rate generates unexpected inflation, the high-MPR agent faces a lower real debt burden in the nominal bond. Second, since the lower nominal interest rate implies a lower real interest rate, the high-MPR faces a lower cost of capital when borrowing from the low-MPR agent. Third, since the lower nominal interest rate raises equilibrium production, and furthermore raises markups when nominal rigidity is in wages rather than prices, the wealth share of high-MPR agents rises due to their disproportionate claim on capital.

This mechanism can explain the risk premium effects of monetary policy found in the data and contributes substantially to the transmission of monetary policy through investment.

[1] The remaining 0.7pp (35%) is driven by higher expected dividends.

[2] Since such households receive a lower share of income from labor income, we model them as capitalists and the other set of households as workers.

Using the same Campbell and Shiller [1988] decomposition on model impulse responses as we used on the data, 61% of the excess return on equity in our baseline parameterization arises from a lower future excess returns, compared to 55% in the data and 0% in an alternative parameterization with symmetric capitalists. Comparing the investment responses in our baseline parameterization to those in the case with symmetric capitalists, the heterogeneity in MPRs amplifies the stimulus to investment by more than 10% on impact and substantially more in the medium-term.

Our paper contributes to the rapidly growing literature on heterogeneous agent New Keynesian models by focusing on risk premia and investment rather than risk-free rates and consumption. In this sense, it is especially complementary with the work of Auclert [2018], Kaplan et al. [2018],Wong [2019], and others on monetary policy and consumption, and McKay and Reis [2016], Kekre [2018], Auclert et al. [2018], Hagedorn et al. [2019] and others on fiscal policy and consumption. Our perspective uncovers the role of the MPR in determining the aggregate effects of redistribution, beyond the literature's focus to date on MPCs. Like us, Ottonello and Winberry [2018] study investment and Luetticke [2018] emphasizes the importance of household portfolios. We build on these papers by studying the joint determination of these quantities with risk premia given aggregate risk.

In this sense, we bring to the HANK literature many established insights from heterogeneous agent and intermediary-based asset pricing. We emphasize the relationship between the wealth distribution and asset prices as in Dumas [1989], Longstaff and Wang [2012], and Garleanu and Panageas [2015]. We further emphasize the importance of portfolio constraints and asset prices as in Basak and Cuoco [1998], Gomes and Michaelides [2008], Guvenen [2009], He and Krishnamurthy [2013], and Lenel [2018]. Relative to all these papers, our contribution is to embed these insights in a production economy with nominal rigidities and discipline the calibration with our novel measurement of MPRs. In recent work, Drechsler et al. [2018] also study the risk premium and output responses to monetary policy shocks in a two-agent model. We demonstrate that these results are more general than their specific model of banking, and apply in a more conventional New Keynesian setting.

Indeed, our paper most directly builds on prior work focused on risk premia in New Keynesian economies. It complements Bernanke et al. [1999], Gertler and Karadi [2011] and Gertler and Kiyotaki [2011] by clarifying how and why risk premia may move in markets which may not be intermediated by specialists. It focuses on movements in the market price of risk rather than the quantity of risk in Ilut and Schneider [2014], Fernandez-Villaverde et al. [2015], Basu and Bundick [2017], and other analyses of uncertainty shocks. It makes use of the insight in Caballero and Farhi [2018], Caballero and Simsek [2018], and DiTella [2018] that an increase in the risk premium will induce a recession if the safe interest rate does not sufficiently fall in response. And like the first two as well as Brunnermeier and Sannikov [2012], it emphasizes the role of heterogeneity in asset valuations in driving movements in risk premia. Relative to these papers, our contribution is to summarize the relevant dimension of heterogeneity in terms of the MPR and to explore the importance of this mechanism in a quantitative DSGE in the tradition of Rudebusch and Swanson [2012], Gourio and Ngo [2016], and other analyses of asset prices and quantities in the U.S. economy.

2 Redistribution, risk premia, and investment

We first illustrate the relationship between redistribution, risk premia, and investment in a two-period environment allowing us to obtain simple analytical results.

2.1 Two-period environment and equilibrium

We consider a continuum of measure one agents with Epstein and Zin [1991] preferences

$$v_0^i = \left((1-\beta^i)\left(c_0^i \Phi^i(\ell_0^i)\right)^{1-\frac{1}{\psi^i}} + \beta^i \left(\mathbb{E}_0 (c_1^i)^{1-\gamma^i}\right)^{\frac{1-\frac{1}{\psi^i}}{1-\gamma^i}} \right)^{\frac{1}{1-\frac{1}{\psi^i}}}.$$

Agents may differ in their discount factors $\{\beta^i\}$, intertemporal elasticities of substitution $\{\psi^i\}$, and relative risk aversion $\{\gamma^i\}$. Their flow utility in period 0 is over consumption and labor supply; we need not assume a particular parametric form for $\Phi^i(\cdot)$ at this stage. For simplicity we assume agents' exogenously supply one unit of labor in period 1; this can be relaxed easily. Agents' expectations are taken over z_1, aggregate productivity.

Agents face resource constraints

$$P_0 c_0^i + \frac{B_0^i}{1+i_0} + Q_0 k_0^i \leq W_0 \eta^i \ell_0^i + B_{-1}^i + (\Pi_0 + (1-\delta)Q_0)k_{-1}^i + P_0 \Delta_0^i t_0,$$
$$P_1(z_1)c_1^i(z_1) \leq W_1(z_1)\eta^i + B_0^i + \Pi_1(z_1)k_0^i$$

where $\int_0^1 \Delta_0^i di = 0$. Beyond consuming and supplying labor (given productivity η^i), they purchase a portfolio of bonds B_0^i and capital k_0^i. We index the overall level of transfers with t_0 and the sign and magnitude of transfers for a given agent with Δ_0^i; later we will study shocks to t_0 to understand the effects of redistribution on macroeconomic outcomes.

The representative firm earns profits

$$\Pi_0 k_{-1} = P_0 z_0 \ell_0^{1-\alpha} k_{-1}^{\alpha} - W_0 \ell_0 + Q_0 x_0 - P_0 \left(\frac{\bar{x}_0}{\delta k_{-1}}\right)^{\chi^x} x_0,$$
$$\Pi_1(z_1) k_0 = P_1(z_1) z_1 \ell_1^{1-\alpha} k_0^{\alpha} - W_1(z_1)\ell_1(z_1)$$

after hiring labor and renting capital from workers, and investing x_0 in producing new capital subject to a standard investment technology with adjustment costs χ^x and $\bar{x}_0 = x_0$ taken as given by the representative firm.

Macroeconomic policy is summarized by t_0 as well as monetary policy $\{i_0, P_1(z_1)\}$, where we assume $P_1(z_1) = \bar{P}_1$ to eliminate the inflation risk in the nominal bond for simplicity. Market clearing in goods each period is

$$\int_0^1 c_0^i di + \left(\frac{x_0}{\delta k_{-1}}\right)^{\chi^x} x_0 = z_0 \ell_0^{1-\alpha} k_{-1}^{\alpha},$$
$$\int_0^1 c_1^i(z_1) di = z_1 \ell_1^{1-\alpha} k_0^{\alpha},$$

in labor each period is

$$\int_0^1 \eta^i \ell_t^i di = \ell_t,$$

in the capital rental market is

$$\int_0^1 k_{t-1}^i di = k_{t-1},$$

in the capital claims market is

$$\int_0^1 (1-\delta) k_{-1}^i di + x_0 = \int_0^1 k_0^i di,$$

and in bonds is

$$\int_0^1 B_{t-1}^i di = 0.$$

The definition of equilibrium is then standard. Together these ingredients provide the simplest environment to study our questions of interest. Later in the paper we add constraints on portfolios and idiosyncratic income risk to demonstrate the robustness of our insights.

Our solution approach throughout this section will be to study the economy around the deterministic steady-state (with respect to aggregate risk). We assume

$$\log z_1 \sim N\left(\log \bar{z}_1 - \frac{1}{2}\sigma^2, \sigma^2\right)$$

and approximate the economy around $\{\sigma = 0, t_0 = \bar{t}_0, z_0 = \bar{z}_0, i_0 = \bar{i}_0\}$. Throughout, we denote $\bar{\cdot}$ to be a value at this approximation point, and $\hat{\cdot}$ the log/level deviation (except for σ, which is a perturbation parameter but will not be denoted as $\hat{\sigma}$).

We begin by focusing on redistribution through fiscal policy ($\hat{t}_0$) alone before considering the effects of TFP shocks ($\hat{z}_0$) or monetary policy shocks ($\hat{i}_0$).

2.2 Redistribution and risk premia

We first characterize the effect of redistribution on risk premia. To do so, we combine agents' first-order conditions with respect to bonds and capital to give

$$\mathbb{E}_0 c_1^i(z_1)^{-\gamma^i}\left(r_1^k(z_1) - r_0\right),$$

where we denote $r_1^k(z_1)$ and r_0 the net real returns on the capital claim and bond, respectively:

$$1 + r_1^k(z_1) = \frac{\Pi_1(z_1)}{Q_0}\frac{P_0}{P_1},$$
$$1 + r_0 = (1 + i_0)\frac{P_0}{P_1}.$$

Approximating the above condition up to 3rd order around the deterministic steady-state across all agents and exploiting market clearing in the asset markets, we obtain

$$\mathbb{E}_0 \hat{r}_1^k(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \underbrace{\gamma\sigma^2}_{\substack{\text{steady-}\\\text{state}\\\text{risk}\\\text{premium}}} + \underbrace{\zeta_{t_0}\hat{t}_0\sigma^2}_{\substack{\text{effect}\\\text{of shock}\\\text{on risk}\\\text{premium}}} + o(||\cdot||^4).$$

In the usual way, a second-order approximation is needed to characterize the risk premium holding all other state variables (in this case t_0) at their deterministic steady-state levels, while a third-order approximation is needed to capture the state-dependence of the risk premium. While ζ_{t_0} is thus our coefficient of interest, summarizing the effect of redistribution on risk premia, we can build intuition by first examining the steady-state risk premium γ:

$$\gamma = \left(\int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^i di}\frac{1}{\gamma^i}di\right)^{-1}.$$

As is standard in the heterogenous agent asset pricing literature, the market price of risk is given by the appropriately weighted harmonic mean of risk aversion in the economy. This suggests, then, that redistribution will change risk premia by changing the wealth and thus consumption shares of agents in the economy, thereby affecting the market price of risk. This is formalized in ζ_{t_0}:

$$\zeta_{t_0} = \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \left[\int_0^1 \bar{\xi}_{t_0}^i \left(1 - \frac{\gamma}{\gamma^i}\right) di \right], \tag{1}$$

where

$$\bar{\xi}_{t_0}^i \equiv \overline{\frac{dc_1^i}{dt_0}}.$$

Intuitively, redistributing wealth and thus raising the consumption shares of relatively risk-tolerant investors (for whom $1 - \frac{\gamma}{\gamma^i} < 0$) will lower the risk premium.

There is a deeper point contained in the expression for ζ_{t_0}, however. To elucidate it, it is useful to re-write agents' micro-level optimization problem as

$$\max \left((1-\beta^i)\left(c_0^i \Phi^i(\ell_0^i)\right)^{1-\frac{1}{\psi^i}} + \beta^i \left(\left(\mathbb{E}_0 (c_1^i)^{1-\gamma^i} \right) \right)^{\frac{1-\frac{1}{\psi^i}}{1-\gamma^i}} \right)^{\frac{1}{1-\frac{1}{\psi^i}}} \quad s.t.$$

$$c_0^i + \frac{1}{1+r_0} b_0^i + q_0 k_0^i = w_0 \eta^i \ell_0^i + y_0^i(P_0, \pi_0, q_0, t_0),$$

$$c_1^i = w_1 \eta^i + b_0^i + \pi_1 k_0^i,$$

where we have denominated in lower-case the real analogs to the nominal variables introduced earlier, and we have collected agents' non-labor income — which they take as exogenous — in

$$y_0^i(P_0, \pi_0, q_0, t_0) = \frac{1}{P_0} B_{-1}^i + (\pi_0 + (1-\delta) q_0) k_{-1}^i + \Delta_0^i t_0.$$

Defining

$$a_0^i \equiv \frac{1}{1+r_0} b_0^i + q_0 k_0^i,$$

as the real savings of agent i, agents' optimal policies imply the marginal propensities to consume, work, save in bonds, save in capital, and save overall

$$\left\{ \frac{\partial c_0^i}{\partial y_0^i}, \frac{\partial l_0^i}{\partial y_0^i}, \frac{\partial b_0^i}{\partial y_0^i}, \frac{\partial k_0^i}{\partial y_0^i}, \frac{\partial a_0^i}{\partial y_0^i} \right\},$$

which jointly solve the system

$$\frac{\partial}{\partial y_0^i}\left(\mathbb{E}_0\left[\beta^i (ce_0^i)^{\gamma^i-\frac{1}{\psi^i}}\left(c_1^i(z_1)\right)^{-\gamma^i}(1+r_1^k(z_1))\right]-(1-\beta^i)\left(\Phi^i(\ell_0^i)\right)^{1-\frac{1}{\psi^i}}\left(c_0^i\right)^{-\frac{1}{\psi^i}}\right)=0, \quad (2)$$

$$\frac{\partial}{\partial y_0^i}\left(\mathbb{E}_0\left[\beta^i (ce_0^i)^{\gamma^i-\frac{1}{\psi^i}}\left(c_1^i(z_1)\right)^{-\gamma^i}(1+r_0)\right)\right]-(1-\beta^i)\left(\Phi^i(\ell_0^i)\right)^{1-\frac{1}{\psi^i}}\left(c_0^i\right)^{-\frac{1}{\psi^i}}\right)=0, \quad (3)$$

$$\frac{\partial}{\partial y_0^i}\left(-c_0^i\frac{\Phi^{i'}(\ell_0^i)}{\Phi^i(\ell_0^i)}\right)=0, \quad (4)$$

$$\frac{1}{1+r_0}\frac{\partial b_0^i}{\partial y_0^i}+q_0\frac{\partial k_0^i}{\partial y_0^i}=\frac{\partial a_0}{\partial y_0^i}, \quad (5)$$

$$\frac{\partial c_0^i}{\partial y_0^i}+\frac{\partial a_0^i}{\partial y_0^i}=w_0^i\eta^i\frac{\partial \ell_0^i}{\partial y_0^i}+1, \quad (6)$$

$$\frac{\partial c_1^i}{\partial y_0^i}=\frac{\partial b_0^i}{\partial y_0^i}+\pi_1\frac{\partial k_0^i}{\partial y_0^i}, \quad (7)$$

where the certainty equivalent in the first two conditions is $ce_0^i \equiv \left(\mathbb{E}_0(c_1^i(z_1))^{1-\gamma^i}\right)^{\frac{1}{1-\gamma^i}}$.

We now seek to better understand the structural determinants of these marginal decisions by characterizing their limit as aggregate risk falls to zero. Here we apply to HANK and our particular statistics of interest techniques that were developed in Devereux and Sutherland [2011] in the context of country-level portfolios in open-economy macroeconomics. These authors demonstrate that to characterize equilibrium portfolios as aggregate risk falls to zero, the researcher must employ a second-order approximation to optimal portfolio choice around the deterministic steady-state. Analogously, in the present environment, to characterize the marginal portfolio responses to income, the researcher must employ a second-order approximation to (2), (3), and (6) around the deterministic steady-state. Doing so, we obtain the following characterization of marginal portfolios:

$$\frac{1}{1+\bar{r}_0}\overline{\frac{\partial b_0^i}{\partial y_0^i}}=\left(1-\frac{\gamma}{\gamma^i}\right)\overline{\frac{\partial a_0^i}{\partial y_0^i}},$$

$$\bar{q}_0\overline{\frac{\partial k_0^i}{\partial y_0^i}}=\frac{\gamma}{\gamma^i}\overline{\frac{\partial a_0^i}{\partial y_0^i}}.$$

We can thus characterize an agent's limiting marginal propensity to take risk, defined as follows:

Definition 1. *An agent's* marginal propensity to take risk *(MPR) is given by*

$$mpr_0^i \equiv \frac{q_0\frac{\partial k_0^i}{\partial y_0^i}}{\frac{\partial a_0^i}{\partial y_0^i}}-1.$$

It follows that in this environment:

Lemma 1. *An agent's limiting MPR is given by*

$$\overline{mpr}_0^i = \frac{\gamma}{\gamma^i} - 1.$$

Intuitively, an agent's marginal propensity to save in risky capital relative to save overall is determined by her risk aversion relative to that of the average agent in the economy. In the representative agent benchmark, it follows that $\overline{mpr}_0^i = 0$, or in other words $\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \overline{\frac{\partial a_0^i}{\partial y_0^i}}$: equilibrium prices must be such that the agent will allocate the marginal dollar fully to capital, since capital is in positive supply but bonds are not.

This expression for the limiting MPR clarifies two useful points. First, the MPR captures a dimension of heterogeneity in principle orthogonal to the marginal propensities to consume and save overall: while the latter are determined by agents' attitudes towards consumption across *dates* (discount factors and intertemporal elasticities of substitution), MPRs are governed by attitudes towards consumption across *states* (relative risk aversion). Second, the MPR may be quite distinct from observed portfolios because it captures portfolio allocation on the margin. Indeed, an agent's limiting portfolio share in capital in this environment will be

$$\frac{\bar{q}_0 \bar{k}_1}{\bar{a}_0^i} = \left(\frac{\bar{c}_1^i}{(1+\bar{r}_0)\bar{a}_0^i} \right) \left(\frac{\gamma}{\gamma^i} - \frac{\bar{w}_1 \eta^i}{\bar{c}_1^i} \right).$$

which depends not only on risk aversion but her hedging motives given her non-traded labor income. While this hedging motive matters for equilibrium portfolios, the expression for the limiting MPR clarifies that it is irrelevant on the margin.

Revisiting the expression for ζ_{t_0} in (1) using the limiting MPR, we can more succinctly describe the effect of redistribution on the risk premium as follows.

Proposition 1. *Given*

$$\mathbb{E}_0 \hat{r}_1^k(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \zeta_{t_0}\hat{t}_0\sigma^2 + o(|| \cdot ||^4),$$

the steady-state risk premium is given by

$$\gamma = \left(\int_0^1 \frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^i di} \frac{1}{\gamma^i} di \right)^{-1}$$

and the effect of redistribution on the risk premium is given by

$$\zeta_{t_0} = -\frac{\gamma}{\int_0^1 \bar{c}_1^i di} \left[\int_0^1 \bar{\xi}_{t_0}^i \overline{mpr}_0^i di \right],$$

where $\bar{\xi}_{t_0}^i \equiv \overline{\frac{dc_1^i}{dt_0}}$.

Intuitively, Proposition 1 says that redistributing wealth (and thus raising the consumption) of agents with high MPRs will lower the equilibrium risk premium.

Focusing on MPRs rather than coefficients of relative risk aversion $\{\gamma^i\}$ is useful not only because the former are more readily measurable, but because MPRs remain sufficient to evaluate the effects of redistribution in richer environments. For instance, suppose some agents are not at an interior optimum in their portfolio choice because of the additional constraint

$$q_0 k_0^i = \omega_0^i a_0^i,$$

which can reflect a leverage constraint on their portfolio or rule-of-thumb in their portfolio allocation. And suppose agents are subject to idiosyncratic risk beyond the aggregate risk already described: they receive an endowment ϵ_1^i where $\mathbb{E}_0(\epsilon_1^i|z_1) = 0$. In these cases, the MPR of a given agent is now generalized:

Lemma 2. *Consider the richer environment with portfolio constraints, rules-of-thumb, and idiosyncratic risk. Continuing to study the economy around the steady-state with zero aggregate risk (but now arbitrary idiosyncratic risk), if an agent is unconstrained*

$$\overline{mpr}_0^i = \gamma \left(\frac{\gamma^i + 1}{\gamma^i} \frac{\mathbb{E}_0(\bar{c}_1^i(\epsilon_1^i))^{-(\gamma^i+2)} \mathbb{E}_0(\bar{c}_1^i(\epsilon_1^i))^{-\gamma^i}}{\left(\mathbb{E}_0(\bar{c}_1^i(\epsilon_1^i))^{-(\gamma^i+1)}\right)^2} - 1 \right) - 1,$$

and if an agent is against a binding leverage constraint or follows a rule-of-thumb

$$\overline{mpr}_0^i = \omega_0^i - 1.$$

As is evident, in these richer environments the MPR depends on more than relative risk aversion. Yet it remains the case that the MPR is the relevant dimension of micro heterogeneity to characterize the effects of redistribution on risk premia, as summarized in the following generalization of Proposition 1.

Proposition 2. *In the richer environment with portfolio constraints, rules-of-thumb, and* *idiosyncratic risk, the steady-state risk premium is given by*

$$\gamma = \left(\int_{i \in UC} \frac{1}{\int_{i \in UC} \left[\bar{w}_1 \eta^i + \bar{\pi}_1 \bar{k}_0^i \right] di} \frac{\mathbb{E}_0 \bar{c}_1^i (\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0 (\bar{c}_1^i (\epsilon_1^i)^{-\gamma^i - 1}} \frac{1}{\gamma^i} di \right)^{-1}$$

and the effect of redistribution on the risk premium is given by

$$\zeta_{t_0} = -\frac{\gamma}{\int_{i \in UC} \left[\bar{w}_1 \eta^i + \bar{\pi}_1 \bar{k}_0^i \right] di} \left[\int_0^1 \bar{\xi}_{t_0}^i \overline{mpr}_0^i di \right],$$

where $\bar{\xi}_{t_0}^i \equiv \overline{\frac{dc_1^i}{dt_0}}$ *for* $i \in UC$, $\bar{\xi}_{t_0}^i \equiv \overline{\frac{d(1+\bar{r}_0)a_0^i}{dt_0}}$ *for* $i \notin UC$, *and* $UC = \{i | unconstrained\}$.

This proposition clarifies two important insights. First, despite the increased complexity in this environment, it remains the case that the effects of redistribution on the risk premium are governed by covariance between the change in wealth and MPRs. Second, while the expression for the steady-state risk premium integrates only over agents who are at an interior optimum in their portfolio choice (the set UC), reflecting that these are the only agents pricing both assets, the effects of redistribution continue to depend on the MPRs across the entire distribution of agents. For instance, if wealth transfers to agents who do not participate in the market for capital claims and thus have a relatively low $\overline{mpr}^i$, in equilibrium the remaining agents must be induced to hold a greater fraction of their portfolio in capital and thus the risk premium must rise. This is consistent with insights in prior quantitative models of asset pricing such as Guvenen [2009] and Chien et al. [2012].

2.3 Redistribution and investment

We now ask how redistribution affects equilibrium investment. It is isomorphic and expositionally simpler to focus on new capital k_0 rather than investment x_0, though of course it is easy to go between the two. In terms of the model's state variables, new capital is given by

$$\hat{k}_0 = \underbrace{\delta_{t_0}^{k_0} \hat{t}_0 + \tfrac{1}{2} \delta_{t_0^2}^{k_0} \hat{t}_0^2 + \tfrac{1}{6} \delta_{t_0^3}^{k_0} \hat{t}_0^3}_{\substack{\text{effects absent} \\ \text{aggregate risk}}} + \delta_{\sigma^2}^{k_0} \sigma^2 + \underbrace{\delta_{t_0 \sigma^2}^{k_0} \hat{t}_0 \sigma^2}_{\substack{\text{effect with} \\ \text{aggregate risk}}} + o(|| \cdot ||^4)$$

around the deterministic steady-state, for some coefficients $\delta_{\cdot}^{k_0}$.

The first set of terms reflect the effects of redistribution on investment in environments without aggregate risk, and they are well understood. For instance, absent nominal rigidity and with exogenous labor supply,

$$\delta_{t_0}^{k_0} \propto \int_0^1 \overline{\frac{\partial a_0^i}{\partial y_0^i}} \Delta_0^i di.$$

That is, redistributing to agents with a high marginal propensity to save will lead to an increase in capital and hence investment, with offsetting effects on consumption. With nominal rigidity, the response of investment also depends on how monetary policy responds.

We instead focus on the incremental effects of redistribution in environments with aggregate risk, summarized in the term $\delta_{t_0\sigma^2}^{k_0}$. Equivalently, we seek to understand the investment responses which accompany a change in the risk premium due to redistribution, summarized by the term ζ_{t_0}. Since

$$1 + r_1^k(z_1) = \frac{\alpha z_1 k_0^{\alpha-1}}{q_0},$$

the marginal product of capital falls in k_0, and the price of capital rises in k_0 (reflecting optimal investment among firms), we can conclude

$$\mathbb{E}_0 \hat{r}_1^k(z_1) \propto -\hat{k}_0.$$

Hence, even if redistribution to high MPR agents lowers the risk premium, to evaluate the effects on investment we must determine whether the required return on capital falls (and thus investment rises) or the safe real interest rate simply rises (and thus investment may remain unchanged or even fall).

Absent nominal rigidity, this depends crucially on the average intertemporal elasticity of substitution in the economy. In particular, we can prove the following result:

Proposition 3. *Suppose there is no nominal rigidity and all agents have the same intertemporal elasticity of substitution $\psi^i = \psi$. Then at least in a neighborhood around $\psi = 1$,*

$$\delta_{t_0\sigma^2}^{k_0} \propto \begin{cases} \zeta_{t_0} \;\; \text{if} \;\; \psi < 1, \\ 0 \;\; \text{if} \;\; \psi = 1, \\ -\zeta_{t_0} \;\; \text{if} \;\; \psi > 1. \end{cases}$$

Intuitively, in the benchmark case with $\psi = 1$ (log intertemporal preferences), when agents on aggregate wish to reallocate wealth into the risky asset, the return on the safe asset rises by exactly enough to equilibrate asset markets and leave the overall level of saving in the risky asset (and thus capital accumulation) unchanged. When $\psi > 1$, the return on the safe asset need not rise by as much and hence investment rises; when $\psi < 1$, the return on the safe asset rises by even more and hence investment in fact falls. This parallels existing results in the literature on uncertainty shocks, and reflects the presence of offsetting income and substitution effects in response to the shock. As described in Gourio [2012], since typical macroeconomic models are calibrated with intertemporal preferences close to log, this can explain what Cochrane [2017] calls the "macro-finance separation" in joint analyses of asset pricing and business cycles such as Tallarini Jr. [2000].

With nominal rigidity, the safe real interest rate effectively becomes a policy instrument. If monetary policy simply seeks to replicate the allocation absent nominal rigidity, the above results remain relevant. But if policy does not respond in this way — say, because the redistributive shock in question is in fact one to monetary policy, as will be the case later in this paper — the relevance of the intertemporal elasticity of substitution is diminished. In the useful benchmark case where monetary policy maintains a constant real interest rate $\hat{r}_0 = 0$, we see that $\mathbb{E}_0 \hat{r}_1^k(z_1) - \hat{r}_0 = \mathbb{E}_0 \hat{r}_1^k(z_1)$. It immediately follows that investment rises if redistribution is toward to high MPR agents:

Proposition 4. *Suppose there is nominal rigidity and policy maintains a constant real interest rate $\hat{r}_0 = 0$. Then*

$$\delta^{k_0}_{t_0 \sigma^2} \propto -\zeta_{t_0}$$

regardless of the intertemporal elasticities of substitution $\{\psi^i\}$.

These insights build on Caballero and Farhi [2018] and Caballero and Simsek [2018], who demonstrate that an increase in the risk premium will induce a recession if the safe interest rate does not sufficiently fall in response (as if the nominal interest rate is at the zero lower bound). We demonstrate that their insights more generally distinguish the real effects of shocks to risk premia — in our environment due to redistribution across heterogeneous MPRs — in New Keynesian environments versus those in the real business cycle tradition.

2.4 Redistributive effects of shocks

So far, we have demonstrated that a budget-balanced change in transfers will lower the risk premium if it redistributes wealth to high-MPR agents, and will raise investment through this channel in the presence of nominal rigidity and accommodative monetary policy. In fact these insights extend to a wide class of shocks which naturally redistribute across agents in HANK.

Here we focus on a shock to productivity (z_0) or monetary policy (i_0). Formally, we now treat these as state variables. Employing the same techniques as were used for the transfer shock t_0, we can generalize the response of the risk premium.

Proposition 5. *With shocks to productivity z_0 or monetary policy i_0 along with transfers t_0, the risk premium is given by*

$$\mathbb{E}_0\hat{r}_1^k(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \sum_{\theta_0\in\{t_0,z_0,i_0\}} \zeta_{\theta_0}\hat{\theta}_0\sigma^2 + o(||\cdot||^4),$$

where the steady-state risk premium is unchanged from Proposition 2 and the effect of each shock on the risk premium is given by

$$\zeta_{\theta_0} = -\frac{\gamma}{\int_{i\in UC}\left[\bar{w}_1\eta^i + \bar{\pi}_1\bar{k}_0^i\right]di}\left[\int_0^1 \bar{\xi}_{\theta_0}^i\overline{mpr}_0^i di\right],$$

where $\bar{\xi}_{\theta_0}^i \equiv \overline{\frac{dc_1^i}{d\theta_0}}$ for $i \in UC$, $\bar{\xi}_{\theta_0}^i \equiv \overline{\frac{d(1+\bar{r}_0)a_0^i}{d\theta_0}}$ for $i \notin UC$, and $UC = \{i|unconstrained\}$ as before.

The redistributive effects of TFP or monetary shocks are fully contained in the heterogeneous $\{\bar{\xi}_{z_0}^i\}$ and $\{\bar{\xi}_{i_0}^i\}$ along the cross-section of agents. These, in turn, depend on agents' heterogeneous exposures, the degree of nominal rigidity, and the endogenous response of policy to each shock. For instance, if wages are rigid at $W_0 = \bar{W}$ and the labor market clears according to a rationing rule

$$\ell_0^i(\ell_0) := \left\{\int_0^1 \eta^i\ell_0^i di = \ell_0\right\},$$

we can decompose the $\bar{\xi}_{\theta_0}^i$ as in the following result.

Proposition 6. *Given rigid wages and the above labor rationing rule which renders ℓ_0^i exogenous for each agent, and focusing for simplicity on an agent at an interior optimum in portfolio choice ($i \in UC$), then*

$$\begin{aligned}\bar{\xi}_{\theta_0}^i \equiv \overline{\frac{dc_1^i}{d\theta_0}} = (1+\bar{r}_0)\overline{\frac{\partial a_0^i}{\partial y_0^i}}\Bigg[& -\underbrace{\tfrac{1}{P_0}B_{-1}^i\tfrac{1}{P_0}\overline{\tfrac{dP_0}{d\theta_0}}}_{\substack{\textit{unexpected}\\ \textit{inflation}}} + \underbrace{\left(\tfrac{1}{1+\bar{r}_0}\bar{b}_0^i + \bar{q}_0(\bar{k}_0^i - (1-\delta)k_{-1}^i)\right)\tfrac{1}{1+\bar{r}_0}\overline{\tfrac{d(1+r_0)}{d\theta_0}}}_{\textit{terms of trade}} \\ & + \underbrace{k_{-1}^i\left(\overline{\tfrac{d\pi_0}{d\theta_0}} + (1-\delta)\tfrac{1}{1+\bar{r}_0}\overline{\tfrac{d\pi_1}{d\theta_0}}\right) + \eta^i\left(\overline{\tfrac{dw_0\ell_0^i}{d\theta_0}} + \tfrac{1}{1+\bar{r}_0}\overline{\tfrac{dw_1}{d\theta_0}}\right)}_{\textit{claims on aggregate income}} \\ & + \underbrace{\psi^i c_0^i\tfrac{1}{1+\bar{r}_0}\overline{\tfrac{d(1+r_0)}{d\theta_0}} + (\psi^i - 1)\eta^i\bar{w}_0\left(1 - \bar{\tau}^{\ell_0^i}\right)\overline{\tfrac{d\ell_0^i}{d\theta_0}}}_{\textit{substitution effects}}\Bigg]\end{aligned}$$

for $\theta_0 \in \{z_0, i_0\}$ *and* $\bar{\tau}^{\ell_0^i} \equiv 1 - \frac{-\bar{c}_0^i \Phi^{i'}(\bar{\ell}_0^i)/\Phi^i(\bar{\ell}_0^i)}{\eta^i (1-\alpha) z_1 (\bar{\ell}_0)^{-\alpha} k_{-1}^{\alpha}}$.

Anticipating our quantitative evaluation of monetary policy transmission in the next section, it is useful to interpret each term in the case of a shock to i_0 in particular. The first three terms reflect redistribution across agents arising from their heterogeneous exposures to the monetary shock; the final terms reflect differences in consumption responses to this redistribution arising from substitution effects, and are not our primary focus here.

First, if a cut in the nominal interest rate generates unexpected inflation ($\overline{\frac{dP_0}{di_0}} < 0$), it will revalue existing nominal balance sheets and redistribute toward net debtors in the nominal bond. Second, if a cut in the nominal interest rate lowers the equilibrium real interest rate ($\overline{\frac{d(1+r_0)}{di_0}} > 0$), it will redistribute wealth across new borrowers and lenders through Slutsky income effects. In particular, it will lower the cost of capital for agents engaging in new borrowing using the nominal bond, and will lower the rate of return on capital for agents increasing their net position in capital.[3] Third, if a cut in the nominal interest rate changes the composition of profit versus labor income, it will redistribute across agents having heterogenous exposures to these sources of income. Provided wages are sticky, standard arguments imply that $\overline{\frac{d\pi_0}{di_0}} < 0$ and $\overline{\frac{dw_0\ell_0^i}{di_0}} > 0$. Hence, in the short run wealth will redistribute to owners of capital.

These sources of redistribution have been previously exposited in the HANK literature — for instance, in the case of monetary policy shocks, by Auclert [2018]. Our analysis demonstrates that it is their covariance with MPRs rather than MPCs which matters for risk-premia rather than risk-free rates. And as indicated by our results on investment, these effects on risk premia can in turn drive fluctuations in the level of economic activity.

3 Monetary policy transmission in data and model

We now evaluate the relevance of these insights for the transmission of conventional monetary policy in a calibration to the U.S. economy in a richer infinite horizon setting. Before enriching the model, we review empirical evidence on monetary transmission which poses a strong challenge to existing macroeconomic frameworks.

[3] Following Auclert [2018], it is an agent's unhedged exposure to real interest rates ($\frac{1}{1+\bar{r}_0}\bar{b}_0^i + \bar{q}_0(\bar{k}_0^i - (1-\delta)k_{-1}^i)$ in the present setting) rather than the agent's overall level of saving ($\bar{a}_0^i$) which is relevant to evaluate her change in wealth given a change in real interest rates.

3.1 Empirical effects of monetary policy shocks in U.S. data

The effects of an unexpected shock to monetary policy have been the subject of a large literature in empirical macroeconomics. In response to an unexpected loosening, it appears that the price level rises and production expands, consistent with our workhorse macroeconomic models. But, as noted in Bernanke and Kuttner [2005] and a number of subsequent papers using asset pricing data, it appears that changes in risk premia are at the center of policy transmission rather than risk-free rates, posing a challenge to workhorse models where risk premia have a limited role.[4]

We refresh the findings in Bernanke and Kuttner [2005] using the proxy structural vector autoregression (SVAR) approach in Gertler and Karadi [2015]. Using monthly data from July 1979 through June 2012, we first run a six-variable, six-lag VAR using the 1-year Treasury yield, CPI, industrial production, S&P 500 return relative to the 1-month T-bill, 1-month T-bill relative to the change in CPI, and smoothed dividend-price ratio on the S&P 500.[5,6] Over January 1991 through June 2012 we then instrument the residuals in the 1-year Treasury yield (the *monetary policy indicator*) with an external instrument: Fed Futures surprises on FOMC days aggregated to the month level from Gertler and Karadi [2015]. The identification assumptions are that the exogenous variation in the monetary policy indicator in the VAR are due to the structural monetary shock and that the instrument is correlated with this structural shock but not the five others in the VAR. Under these assumptions, a first-stage regression of the monetary policy residual on the surprise, followed by a second-stage regression of all other residuals on the predicted residual, can be used to identify the effects of a monetary policy shock on all variables in the VAR.

We first demonstrate the validity of the first stage in Table 1. We consider 5 possible measures of policy surprises constructed in Gertler and Karadi [2015]: using the current Fed Funds futures contract; the 3-month ahead Fed Funds futures contract; the 2-quarters ahead 3-month Eurodollar contract; the 3-quarters ahead 3-month Eurodollar contract; or the 4-quarters ahead 3-month Eurodollar contract. In all cases, the 1-year Treasury yield rises given a positive surprise, as would be expected. The effects are statistically significant at conventional levels, and for the first two instruments the F statistics above 10 exceed the threshold of a strong instrument recommended by Stock et al. [2002]. Since shocks in our

[4]This conclusion remains debated, however. In recent work, for instance, Nakamura and Steinsson [2018] provide an alternative view arguing that the response of asset prices to monetary shocks is not primarily driven by changes in risk premia but instead changes in beliefs about fundamentals.

[5]The series for the 1-year Treasury yield, CPI, and industrial production are taken from the dataset provided by Gertler and Karadi [2015]. The remaining series are from CRSP.

[6]The smoothed dividend-price ratio is computed as the 3-month moving average of dividend paid in a month divided by the price of the stock at the end of the month, value-weighted over stocks in the S&P 500.

	1-year Treasury yield					
Current Fed Funds	0.85					0.25
	(0.22)					(0.43)
Expected Fed Funds, 3 mos ahead		1.15				1.26
		(0.28)				(0.46)
Expected 3-mo ED rate, 2 qtrs ahead			0.84			1.72
			(0.31)			(1.38)
Expected 3-mo ED rate, 3 qtrs ahead				0.68		-4.90
				(0.30)		(1.82)
Expected 3-mo ED rate, 4 qtrs ahead					0.70	3.04
					(0.31)	(1.05)
Observations	258	258	258	258	258	258
Adj R^2	0.05	0.06	0.03	0.02	0.02	0.07
F-statistic	14.46	16.27	7.59	5.24	5.00	6.39

Table 1: effects of monetary policy instruments on first-stage residuals of VAR

Notes: robust standard errors given in parenthesis.

Current excess return	2.03
	[1.56,2.79]
Dividends	0.71
	[0.37, 1.74]
Real rate	-0.21
	[-0.52, 0.12]
Future excess return	-1.11
	[-2.47, -0.17]

Table 2: effects of 1 SD monetary shock on current excess returns and components

Notes: 95% confidence interval in brackets is computed using the wild bootstrap (to account for uncertainty in the coefficients of the VAR) with 10,000 iterations.

model will be to the current nominal interest rate, to maximize comparability we focus on the current Fed Funds futures contract as our instrument in what follows.

We then plot the impulse responses to a negative monetary policy shock using this instrument in Figure 1. Note that since the structural monetary policy shock is not observed, its magnitude should be interpreted through the lens of its 25bp decrease in the 1-year yield. Consistent with the wider literature, industrial production and the price level rise, and the real interest rate falls. Excess returns rise by 2pp in the first month but fall to be small and negative in the months which follow. This is consistent with a decline in the equity premium, also suggested by the fall in the dividend/price ratio.

Following Bernanke and Kuttner [2005], we can more formally decompose this 2pp excess return on the stock market into the contribution from higher expected dividends, lower

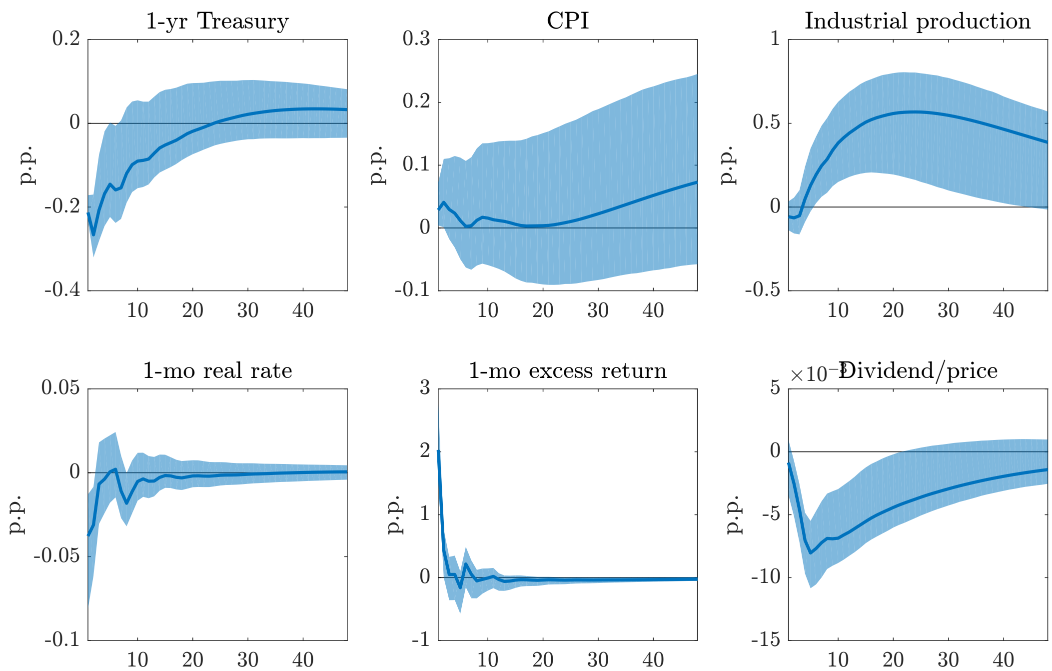

Figure 1: effects of 1 SD monetary shock given current Fed Funds instrument

Notes: 95% confidence interval at each horizon is computed using the wild bootstrap (to account for uncertainty in the coefficients of the VAR) with 10,000 iterations.

real risk-free discount rates, and lower risk premia using a Campbell and Shiller [1988] decomposition:

$$(\text{excess return})_t - \mathbb{E}_{t-1}[(\text{excess return})_t] = (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=0} \rho^j \Delta(\text{dividends})_{t+j}$$
$$- (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=0} \rho^j (\text{real rate})_{t+j} - (\mathbb{E}_t - \mathbb{E}_{t-1}) \sum_{j=1} \rho^j (\text{excess return})_{t+j}$$

Using the proxy SVAR to compute the revised expectations in real rates and excess returns, we obtain the decomposition in Table 2. 1.1pp (55%) of the increase in the stock market is due to news about lower future excess returns, 0.7p (35%) is due to higher dividends, and only 0.2pp (10%) is due to lower risk-free rates. Hence, the proxy SVAR approach validates the original message from Bernanke and Kuttner [2005]: monetary policy shocks primarily change excess returns through their effects on risk premia, not risk-free rates.

3.2 Household portfolios and MPRs from the SCF

To motivate the structure of our infinite horizon model as well as the "micro" targets in our calibration, we now characterize key dimensions of household heterogeneity in portfolio choice using the 2016 SCF. We proceed in three steps.

First, we organize the value of household net worth (A^i) into claims on the economy's capital stock (Qk^i, in positive net supply) and bonds ($\frac{1}{1+i}B^i$, in zero net supply accounting for the government and rest of the world). The key step in doing this is to account for the implicit leverage households have on capital claims through the leverage of firms and of financial intermediaries. In particular, if household i owns \$1 in equity in a firm which has net leverage

$$\frac{\text{assets net of bonds}}{\text{equity}} = lev^f,$$

then we assign the household

$$\{Qk^i = lev^f, \frac{1}{1+i}B^i = 1 - lev^f\}.$$

If household i owns \$1 in equity in an intermediary which has net leverage

$$\frac{\text{assets net of bonds}}{\text{equity}} = lev^I$$

and the intermediary's assets net of bonds are equity claims on the above firm, then we assign the household

$$\{Qk^i = lev^I lev^f, \frac{1}{1+i}B^i = 1 - lev^I lev^f\}.$$

We use the Financial Accounts of the United States as well as analyses of hedge funds and private equity to inform these leverage assumptions. We outline the specific assumptions and present the resulting aggregate decomposition of household net worth in appendix A.

Second, we seek to identify households which may follow a rule-of-thumb in portfolio choice ($\notin UC$), following the language in the two-period model) versus actively rebalance in response to shocks ($\in UC$). We define unconstrained households as those who report that they have bought or sold stocks or other securities through a brokerage over the past year, or those who have an active or nonactively managed business. Table 3 summarizes key statistics of these two groups. Unconstrained households make up only 34% of U.S. households but own 72% of the net worth and 78% excluding the primary residence and vehicles. Aggregating across constrained households, capital claims are 1.19 times their net worth and 0.95 times their net worth excluding the primary residence and vehicles; this

	$i \notin UC$	$i \in UC$
Frac of total households	66%	34%
Frac of total net worth	28%	72%
Frac of total net worth, excl res + vehicles	22%	78%
$\sum_i qk^i / \sum_i a^i$	1.19	1.28
$\sum_i qk^i / \sum_i a^i$, excl res + vehicles	0.95	1.26
Mean wage and salary income to total income	61%	51%
N	20,613	10,627

Table 3: constrained versus unconstrained households

Notes: unconstrained households defined as those who report that they have bought or sold stocks or other securities through a brokerage over the past year, or those who have an active or nonactively managed business. Observations are weighted by SCF sample weights.

suggests that while these households may be passive in financial markets, they are far from constrained in holding capital on aggregate. Finally, labor income is a relatively higher share of income for the average constrained household than unconstrained household. Appendix A provides further detail on the differences between constrained and unconstrained households.

Third, we use our analytical formulas mapping portfolios to MPRs to characterize the distribution of MPRs across U.S. households. Absent idiosyncratic risk, our analytical results in section 2.2 imply that

$$mpr^i = \begin{cases} \frac{Qk^i}{A^i} - 1 & i \notin UC, \\ \left(\frac{Qk^i}{A^i} - 1\right)\left(1 - \frac{\text{wage and salary income}}{\text{total income}}\right) & i \in UC \end{cases}.$$

After computing this for each household using the definition of net worth excluding the primary residence and vehicles, Figure 2 plots the fraction of households by MPR bin conditional on households being constrained or unconstrained, and Table 4 summarizes key moments of the distributions. As is evident, the MPRs among unconstrained households tend to be higher than constrained households, but there is wide dispersion among both groups. Appendix A projects our measure of the MPR on observables such as age, net worth, and income to provide further insight on this distribution.

3.3 Infinite horizon environment and equilibrium revisited

We now extend the model to the infinite horizon to assess whether such heterogeneity in MPRs is capable of rationalizing the Campbell and Shiller [1988] decomposition of the stock market response to a monetary policy shock and — to the extent it does — how important this is for policy transmission through investment.

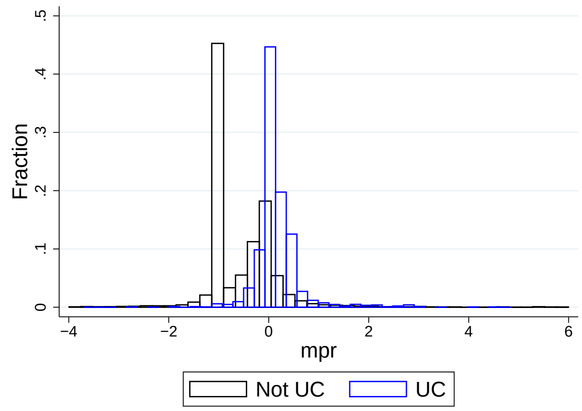

Figure 2: fraction of households by MPR, by unconstrained status

Notes: unconstrained households defined as those who report that they have bought or sold stocks or other securities through a brokerage over the past year, or those who have an active or nonactively managed business.

	$i \notin UC$	$i \in UC$
p1	-4.0	-1.2
p5	-1.2	-0.4
p50	-1.0	0.0
p95	0.5	0.6
p99	5.1	2.9
p99.75	20.3	11.9
N	20,613	10,627

Table 4: percentiles of the MPR distribution, by unconstrained status

Notes: unconstrained households defined as those who report that they have bought or sold stocks or other securities through a brokerage over the past year, or those who have an active or nonactively managed business. Observations are weighted by SCF sample weights.

To render the computational burden manageable we assume a finite number of types of agents. Informed by the patterns in the SCF, we assume three types. The first two types ($\{a, b\}$) map to the unconstrained agents while the third type (c) map to the constrained agents following a rule-of-thumb in portfolio choice. Taking the evidence in Table 3 to its limit, we further assume that the first two types have no labor productivity ($\eta^a = \eta^b = 0$), so we refer to them as capitalists whereas the c types with $\eta^c = 1$ are workers. We will distinguish the two types of capitalists by their targeted MPR, seeking to replicate the wide heterogeneity in MPRs evident in the SCF. To formalize nominal rigidity we assume workers supply differentiated labor varieties, but due to symmetry this will not be a meaningful source of heterogeneity within this group.

The environment is a natural extension of the two-period setting except that to facilitate a steady-state wealth distribution we assume that agents may die in each period with probability ς, passing on their wealth to their newborn offspring with fully altruistic bequest motive. We assume that workers are reborn as workers, while capitalists' children are reborn as type a with probability λ and as type b with probability b. Because capitalists do not receive labor income and because of the homotheticity of the utility function their consumption, saving, and portfolio choice problem will be homogeneous in wealth,[7] allowing us to focus on a representative capitalist of each type.

Defining $p^a = (1 - \varsigma(1 - \lambda))$ and $p^b = (1 - \varsigma\lambda)$, the representative capitalist $i \in \{a, b\}$ thus maximizes Epstein and Zin [1991] preferences

$$v_t^i = \left((1 - \beta^i) \left(c_0^i \right)^{1 - \frac{1}{\psi^i}} + \beta^i \left(\mathbb{E}_0 \left[p^i (v_{t+1}^i)^{1-\gamma^i} + (1 - p^i)(v_{t+1}^j)^{1-\gamma^i} \right] \right)^{\frac{1 - \frac{1}{\psi^i}}{1-\gamma^i}} \right)^{\frac{1}{1 - \frac{1}{\psi^i}}}$$

subject to the resource constraint

$$P_t c_t^i + \frac{B_t^i}{1 + i_t} + Q_t k_t^i \leq B_{t-1}^i + (\Pi_t + (1 - \delta)Q_t)k_{t-1}^i + P_t t_t^i.$$

The representative worker family also maximizes Epstein and Zin [1991] preferences

$$v_t^c = \left((1 - \beta^c) \left(c_0^c \Phi^c \left(\int_0^1 \ell_0^c(j) \right) \right)^{1 - \frac{1}{\psi^c}} + \beta^c \left(\mathbb{E}_0 (v_{t+1}^c)^{1-\gamma^c} \right)^{\frac{1 - \frac{1}{\psi^c}}{1-\gamma^c}} \right)^{\frac{1}{1 - \frac{1}{\psi^c}}}$$

subject to the resource constraint

$$P_t c_t^c + \frac{B_t^c}{1 + i_t} + Q_t k_t^c \leq$$
$$\int_{j=0}^1 (1 - \tau_w) W_t(j) \ell_t^c(j) + B_{t-1}^c + (\Pi_t + (1 - \delta)Q_t)k_{t-1}^c - (1 - \tau_w) W_t \ell_t \frac{\chi^w}{2} \left(\frac{W_t(j)}{W_{t-1}} - 1 \right)^2 + P_t t_t^c,$$

labor demand

$$\ell_t^c(j) = \left(\frac{W_t(j)}{W_t} \right) \ell_t$$

reflecting Rotemberg [1982] adjustment costs in wages, and the rule-of-thumb constraint

$$Q_t k_t^c = \omega^c \left(\frac{B_t^c}{1 + i_t} + Q_t k_t^c \right).$$

[7] Indeed, it is for this reason that we assume $\eta^a = \eta^b = 0$ rather than simply $\eta^a, \eta^b < \eta^c$.

For concreteness we now assume a disutility of labor as in Shimer [2010]

$$\Phi^c(\ell_t^c) = \left(1 + \left(\frac{1}{\psi^c} - 1\right)\frac{\bar{\xi}\xi}{1+\xi}(\ell_t^c)^{\frac{1+\xi}{\xi}}\right)^{\frac{\frac{1}{\psi^c}}{1-\frac{1}{\psi^c}}},$$

where ξ controls the Frisch elasticity of labor supply. A labor packer hires the different varieties of workers to produce the aggregate

$$\ell_t = \left[\int_0^1 \ell_t^c(j)^{\frac{\epsilon^w}{\epsilon^w-1}}dj\right]^{\frac{\epsilon^w-1}{\epsilon^w}}.$$

The representative firm hires labor, rents capital, and invests to maximize spot profits

$$\Pi_t k_t = P_t z_t \ell_t^{1-\alpha} k_{-1}^{\alpha} - W_t \ell_t + Q_t x_t - P_t \left(\frac{\bar{x}_t}{\delta k_{t-1}}\right)^{\chi^x} x_t$$

where $\bar{x}_t = x_t$ is taken as given by the representative firm. Market clearing in goods is

$$c_t^a + c_t^b + c_t^c + \left(\frac{x_t}{\delta k_{t-1}}\right)^{\chi^x} x_t = z_t \ell_t^{1-\alpha} k_{t-1}^{\alpha},$$

in the capital rental market is

$$k_{t-1}^a + k_{t-1}^b + k_{t-1}^c = k_{t-1},$$

in the capital claims market is

$$k_t^a + k_t^b + k_t^c = (1-\delta)\left(k_{t-1}^a + k_{t-1}^b + k_{t-1}^c\right) + x_t,$$

and in the bond market is

$$B_t^a + B_t^b + B_t^c = 0.$$

Finally, among capitalists the government rebates the wealth of capitalists who die, so

$$P_t t_t^a = \lambda \sum_{i\in\{a,b\}} \left[B_{t-1}^i + (\Pi_t + (1-\delta)Q_t)k_{t-1}^i\right],$$
$$P_t t_t^b = (1-\lambda) \sum_{i\in\{a,b\}} \left[B_{t-1}^i + (\Pi_t + (1-\delta)Q_t)k_{t-1}^i\right],$$

and rebates the Rotemberg [1982] adjustment costs in wages as well as ad-valorem taxes on wages back to workers, so

$$P_t t_t^c = W_t \ell_t \frac{\chi^w}{2} \left(\frac{W_t}{W_{t-1}} - 1 \right) + \tau^w W_t \ell_t.$$

We assume that monetary policy is now specified by a standard Taylor [1993] rule

$$1 + i_t = \left(\frac{P_t}{P_{t-1}} \right)^{\phi^\pi} m_t,$$

and that the two driving forces in the economy $\{\log z_t, \log m_t\}$ follow independent AR(1) processes with persistence $\{\rho_z, \rho_m\}$ and standard deviation $\{\sigma_z, \sigma_m\}$. To offer the model another margin with which to improve its asset pricing predictions, we allow for a small probability of a very negative TFP realization $\underline{\sigma}_z$ which can occur each period with probability $\underline{p}$.

The definition of an equilibrium naturally generalizes that in the two-period environment. We can obtain a recursive representation in terms of the states

$$\{a_{t-1}^a, a_{t-1}^b, a_{t-1}^c, W_{t-1}, z_{t-1}, m_{t-1}\},$$

where $a_{t-1}^i \equiv \frac{1}{P_t} \left[B_{t-1}^i + (\Pi_t + (1-\delta) Q_t) k_{t-1}^i \right]$ is again real net worth of agent i. We solve the model globally, slowly updating the mapping between states and aggregate prices to ensure that market clearing is satisfied at each gridpoint in the state space.

3.4 Parameterization and the stochastic steady-state

We now parameterize the model to be consistent with standard business cycle and asset pricing moments for the U.S. economy, as well as key distributional moments using the SCF summarized above. We implement a quarterly calibration.

We first externally set certain model parameters to standard values, summarized in Table 5. Among the model's preference parameters, we set the IES to $\psi^i = 0.75$ across all agents and the Frisch elasticity of labor supply to $\xi = 0.75$, consistent with the micro evidence in Chetty et al. [2011]. We set the fraction of capitalists reborn as a types $\lambda = 0.005$, so that we will later map a types to the 0.5% of unconstrained agents with the highest MPRs. Among the model's technological parameters, we set $\alpha = 0.33$ so that labor's share of income is 67% and $\delta = 0.025$, standard values in the literature. We choose an elasticity of substitution across worker varieties $\epsilon = 10$ and Rotemberg wage adjustment costs of $\chi^w = 400$, which together imply a Calvo [1983]-equivalent frequency of wage adjustment around 7 quarters, slightly higher than that reported in Grigsby et al. [2019]. We assume that the monetary authority responds to inflation with $\phi^\pi = 1.5$. We set the persistence of TFP shocks to

	Description	Value	Notes
$\psi^{a,b,c}$	IES	0.75	
ξ	Frisch elast	0.75	Chetty et al (11)
λ	fraction reborn a	0.005	
α	1 - labor share	0.33	
δ	depreciation rate	0.025	
ϵ	elast of subs across workers	10	
φ	Rotemberg wage adj costs	400	$\approx \mathbb{P}(\text{adjust}) = 7$ qtrs
ϕ^π	Taylor coeff on inflation	1.5	
ρ_z	persistence TFP shock	0.95	
$\underline{\sigma}_z$	downside TFP shock	-0.07	
$\underline{p}$	downside probability	0.008	
σ_m	stdev MP shock	0.0006	
ρ_m	persistence MP shock	0	
τ_w	undoes wage markup	-0.11	

Table 5: externally set parameters

$\rho_z = 0.95$ and the downside TFP shock $\underline{\sigma}_z = -0.07$ occurring with probability $\underline{p} = 0.008$. We set the standard deviation of monetary shocks $\sigma_m = 0.000625$, consistent with a 25bp change in the annualized nominal rate, and the persistence $\rho_m = 0$. Finally, we assume that the wage markup is perfectly offset by $\tau^w = -\frac{\epsilon}{\epsilon-1}$.

We calibrate the remaining parameters to target key macro and micro moments, summarized in Table 6. Among macro moments, we target $\ell = 0.25$ (a normalization), the volatility of the log difference in consumption (0.6%) and investment (2.4%) from the Bureau of Economic Analysis' National Income and Product Accounts (NIPA) over Q1 1947 through Q1 2018, and annualized real interest rate (1.4%) and annualized excess return of the S&P 500 (7.1%) over July 1979 through June 2012 used in section 3.1. Among micro moments, we target the difference in the median MPR among the top 0.5% of unconstrained households (11.9) and remaining set of unconstrained households (0) in Table 4, yielding an MPR difference of roughly 11.9. The aggregate wealth held by the top 0.5% of unconstrained households by MPR is 1.6% of the total wealth of unconstrained households. Finally, we target the 22% of aggregate wealth held by constrained agents as well as the 95% of wealth held in capital among constrained agents reported in Table 3. Table 6 characterizes the values of 9 parameters chosen to match these 9 targets.

Table 7 compares the model's business cycle and asset pricing moments with the data. The first panel of 7 describes targeted moments in the calibration while the second panel describes untargeted moments. While the model closely matches the volatility of the log difference in output and labor as well as the volatility of the annualized real interest rate,

	Description	Value	Moment	Target	Model
$\bar{\xi}$	ℓ disutility	21	ℓ	0.25	0.25
σ_z	std dev TFP	0.005	$\sigma(\Delta \log c)$	0.6%	0.7%
χ	elast q^k to $\frac{x}{k_{-1}}$	8	$\sigma(\Delta \log x)$	2.4%	2.4%
$\beta^{a,b,c}$	discount factor	0.99	$\mathbb{E}r_{+1}$	1.4%	1.4%
γ^b	RRA b	800	$\mathbb{E}\left[r^e_{+1} - r_{+1}\right]$	7.1%	7.6%
γ^a	RRA a	5	$mpr^a - mpr^b$	12	8
γ^c	RRA c	250	$\sum_{i \notin UC} a^i / \sum_i a^i$	22%	14%
ω^c	portfolio c	0.95	$\sum_{i \notin UC} qk^i / \sum_{i \notin UC} a^i$	0.95	0.95
ς	death prob	0.07	$a^a / \sum_{i \in UC} a^i$	1.6%	1.4%

Table 6: targets and calibrated parameters

Notes: targeted business cycle moments are from Q1/47-Q1/18 NIPA and targeted asset pricing moments are from 7/79-6/12 data underlying VAR. Comparable estimates obtained in the model assuming a debt/equity ratio of 2 on a stock market claim.

Moment (ann.)	Data	Model
$\sigma(\Delta \log c)$	0.6%	0.7%
$\sigma(\Delta \log x)$	2.4%	2.4%
$\mathbb{E}r_{+1}$	1.4%	1.4%
$\mathbb{E}\left[r^e_{+1} - r_{+1}\right]$	7.1%	7.6%
$\sigma(\Delta \log y)$	0.9%	1.2%
$\sigma(\Delta \log \ell)$	0.9%	1.1%
$\sigma\left(\mathbb{E}r_{+1}\right)$	0.8%	0.7%
$\sigma\left(\mathbb{E}\left[r^e_{+1} - r_{+1}\right]\right)$	5.4%	1.2%

Table 7: business cycle and asset pricing moments

Notes: business cycle moments are from Q1/47-Q1/18 NIPA and asset pricing moments are from 7/79-6/12 data underlying VAR. Comparable estimates obtained in the model assuming a debt/equity ratio of 2 on a stock market claim.

it still underestimates the time-variation in the equity premium. Table 8 compares the distributional moments in the model with those in a calibration with symmetric capitalists ($a = b$), where $\gamma^a = \gamma^b = 420$ is chosen to match the same equity premium as the baseline. Relative to the symmetric case, the high-MPR a agents hold a leveraged position in capital, borrowing in the bond market from b agents. This will play a key role in driving the risk premium and investment effects from a monetary policy shock which follow.

3.5 Impulse responses in model vs. data

We now simulate the effects of a negative innovation to the nominal interest rate.

Across parameterizations, Figure 3 demonstrates that a loosening of monetary policy

Moment	Model	$a = b$
$a^a / \sum_i a^i$	1.2%	0.5%
$a^b / \sum_i a^i$	84.9%	93.3%
$qk^a / \sum_i a^i$	10.5%	0.5%
$qk^b / \sum_i a^i$	76.3%	93.6%
$\partial a^a / \partial y^a$	0.99	0.99
$\partial a^b / \partial y^b$	0.99	0.99
$\partial a^c / \partial y^c$	0.99	0.99
$\partial k^a / \partial y^a$	8.58	0.99
$\partial k^b / \partial y^b$	0.89	0.99
$\partial k^c / \partial y^c$	0.94	0.94
mpr^a	7.67	0.00
mpr^b	-0.10	0.00
mpr^c	-0.05	-0.05

Table 8: exposures and MPRs in the stochastic steady-state

Notes: consistent with Definition 1, we compute $mpr^i \equiv \frac{q_0 \frac{\partial k^i}{\partial y^i}}{\frac{\partial a^i}{\partial y^i}} - 1 \approx \frac{\frac{\partial k^i}{\partial y^i}}{\frac{\partial a^i}{\partial y^i}} - 1$, where the approximation follows from $q_0 \approx 1$ in the stochastic steady-state.

generates an unexpectedly positive return on capital claims. But unlike in the model with symmetric capitalists, the baseline features consistently negative excess returns in the quarters which follow. Indeed, Figure 4 demonstrates that there is a meaningful fall in expected returns. Applying a Campbell and Shiller [1988] decomposition on the model like that applied to the data earlier, Table 2 demonstrates that the baseline model matches the respective roles of news about future dividend growth, risk-free rates, and excess returns in explaining the stock market reaction to the policy shock.

Consistent with the analytical results, the reduction in risk premia is accompanied by an amplification of the investment effects from monetary policy shocks. Comparing the investment response in the baseline to the parameterization with symmetric capitalists, Figure 3 demonstrates that the stimulus to investment is 10% greater on impact, and several times that in the medium-term. Taken together, these results suggest that MPR heterogeneity is necessary to rationalize the evidence accumulated since Bernanke and Kuttner [2005], and that the transmission through risk premia matters not only for prices but also for investment.

At the core of these results is the endogenous redistribution of wealth to high MPR agents after a loosening of monetary policy, evident in the first panel of Figure 3. We can make sense of this redistribution through the lens of Proposition 6 in our analytical results. Since the high MPR agents endogenously hold a levered position in the economy's capital stock, following a loosening of monetary policy (i) the unexpected increase in the price level erodes

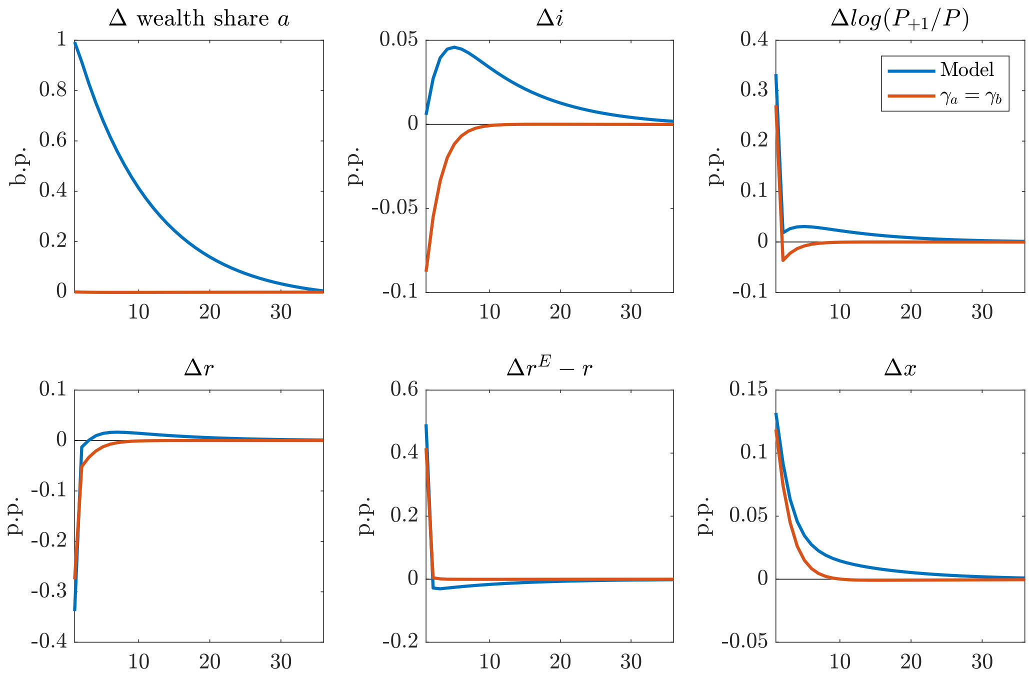

Figure 3: negative monetary policy shock

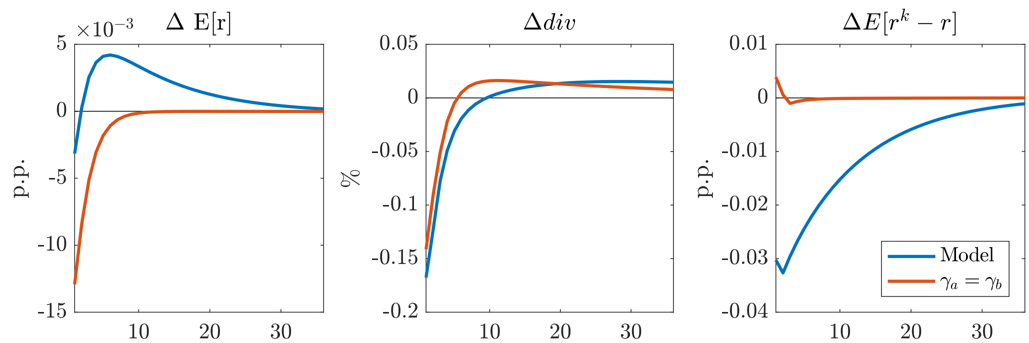

Figure 4: negative monetary policy shock

the real value of their debt in period 0, (ii) their real cost of capital to borrow has fallen, and (iii) their share of aggregate income rises, given that wages are sticky instead of prices. Figure 5 computes the difference in impulse responses between the parameterizations with MPR heterogeneity and symmetric capitalists. Using a one-time transfer of wealth in the latter parameterization, the difference in impulse responses is almost fully accounted for by the redistribution of wealth. Of these sources of redistribution, the deflation of their nominal debt accounts for most of the wealth transferred to the high MPR agents.

4 Conclusion

In this paper we study the effects of redistribution on risk premia and investment in a New Keynesian environment with heterogeneous propensities to bear risk. To evaluate the effects of redistributive shocks in this setting, the relevant notion of heterogeneity is in the marginal portfolio responses to an additional dollar of income, summarized in the MPR. Redistribution to agents with high MPRs lowers the risk premium and, if monetary policy is accommodative, raises investment. In a calibration to the U.S. economy, heterogeneity in MPRs explains the risk premium effects of monetary policy which have eluded existing models and contributes substantially to its transmission to the real economy.

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% Excess return	Data	Model	$a = b$
Δ Dividends	35%	27%	67%
$-$Real rates	10%	12%	33%
$-$Excess returns	55%	61%	0%

Table 9: Campbell and Shiller [1988] decomposition of excess returns after monetary shock

Notes: estimates from data correspond to Table 2. Comparable estimates obtained in the model assuming a debt/equity ratio of 2 on a stock market claim.

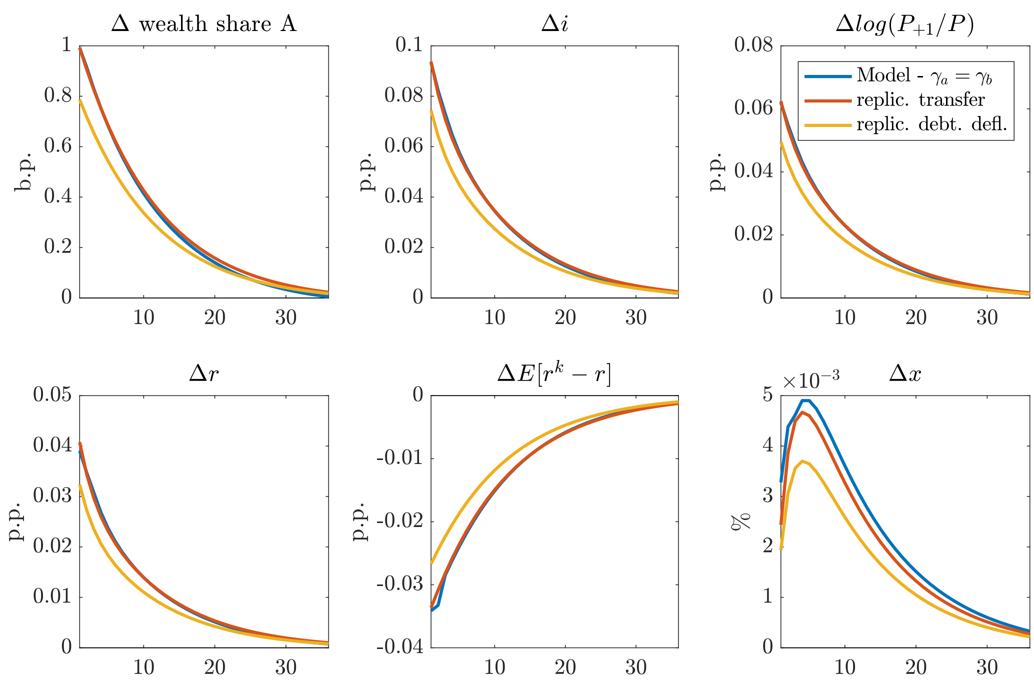

Figure 5: redistributive effects of negative monetary policy shock

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Proofs

Lemma 1

Proof. Substituting (7) into equations (2) and (3) yields

$$\begin{aligned}
0 = &\frac{1}{\psi^i}\frac{1}{c_0^i}\frac{\partial c_0^i}{\partial y_0^i} + \left(\frac{1}{\psi^i} - 1\right)\frac{\Phi^{i\prime}(l_0^i)}{\Phi^i(l_0^i)}\frac{\partial l_0^i}{\partial y_0^i} \\
&+ \left[\left(\gamma^i - \frac{1}{\psi^i}\right)(ce_0^i)^{\gamma^i - 1}\mathbb{E}_0(c_1^i(z_1))^{-\gamma^i} - (1+r_0)\mathbb{E}_0 m_{0,1}^i(z_1)\frac{\gamma^i}{c_1^i(z_1)}\right]\frac{\partial b_0^i}{\partial y_0^i} \\
&+ \left[\left(\gamma^i - \frac{1}{\psi^i}\right)(ce_0^i)^{\gamma^i - 1}\mathbb{E}_0(c_1^i(z_1))^{-\gamma^i}\pi_1 - (1+r_0)\mathbb{E}_0 m_{0,1}^i(z_1)\frac{\gamma^i}{c_1^i(z_1)}\pi_1\right]\frac{\partial k_0^i}{\partial y_0^i}
\end{aligned}$$

and

$$\begin{aligned}
0 = &\frac{1}{\psi^i}\frac{1}{c_0^i}\frac{\partial c_0^i}{\partial y_0^i} + \left(\frac{1}{\psi^i} - 1\right)\frac{\Phi^{i\prime}(l_0^i)}{\Phi^i(l_0^i)}\frac{\partial l_0^i}{\partial y_0^i} \\
&+ \left[\left(\gamma^i - \frac{1}{\psi^i}\right)(ce_0^i)^{\gamma^i - 1}\mathbb{E}_0(c_1^i(z_1))^{-\gamma^i} - \mathbb{E}_0(1+r_1^k(z_1)) m_{0,1}^i(z_1)\frac{\gamma^i}{c_1^i(z_1)}\right]\frac{\partial b_0^i}{\partial y_0^i} \\
&+ \left[\left(\gamma^i - \frac{1}{\psi^i}\right)(ce_0^i)^{\gamma^i - 1}\mathbb{E}_0(c_1^i(z_1))^{-\gamma^i}\pi_1 - \mathbb{E}_0(1+r_1^k(z_1)) m_{0,1}^i(z_1)\frac{\gamma^i}{c_1^i(z_1)}\pi_1\right]\frac{\partial k_0^i}{\partial y_0^i},
\end{aligned}$$

where the stochastic discount factor between periods 0 and 1 is

$$m_{0,1}^i(z_1) \equiv \frac{\beta^i}{1-\beta^i}(c_0^i)^{\frac{1}{\psi^i}}(\Phi^i(l_0^i))^{\frac{1}{\psi^i}-1}(ce_0^i)^{\gamma^i - \frac{1}{\psi^i}}(c_1^i(z_1))^{-\gamma^i}.$$

As is evident, these equations are not linearly independent absent aggregate risk, when $1 + r_0 = 1 + r_1^k$. Subtracting them, we have

$$0 = \mathbb{E}_0 m_{0,1}^i(z_1)\frac{\gamma^i}{c_1^i(z_1)}(r_1^k(z_1) - r_0)\left(\frac{\partial b_0^i}{\partial y_0^i} + \pi_1(z_1)\frac{\partial k_0^i}{\partial y_0^i}\right). \qquad (8)$$

A second-order approximation of the right-hand side around the deterministic steady-state then implies

$$0 = \left(\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right) \left(\mathbb{E}_0 \left(\hat{r}_1^k(z_1) - \hat{r}_0\right) + \frac{1}{2}\sigma^2\right) - \left(\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}}\right) \frac{\gamma^i + 1}{\bar{c}_1^i} \left(\bar{w}_1 \eta^i + \bar{\pi}_1 \bar{k}_0^i\right) \sigma^2 + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}} \sigma^2 + o(||\cdot||^3). \quad (9)$$

Now note that returning to the agent's FOCs with respect to k_0^i and b_0^i which imply the optimal portfolio choice condition

$$\mathbb{E}_0 (c_1^i(z_1))^{-\gamma^i} (r_1^k(z_1) - r_0) = 0,$$

a second-order approximation of the left-hand side around the deterministic steady-state implies

$$\mathbb{E}_0 \left(\hat{r}_1^k(z_1) - \hat{r}_0\right) + \frac{1}{2}\sigma^2 = \frac{\gamma^i}{\bar{c}_1^i} \left(\bar{w}_1 \eta^i + \bar{\pi}_1 \bar{k}_0^i\right) \sigma^2 + o(||\cdot||^3).$$

Anticipating the characterization of the steady-state equity premium

$$\mathbb{E}_0 \left(\hat{r}_1^k(z_1) - \hat{r}_0\right) + \frac{1}{2}\sigma^2 = \gamma \sigma^2 + o(||\cdot||^3)$$

in the next result, it follows from (9) that

$$\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \frac{\gamma}{\gamma^i} \overline{\frac{\partial a_0^i}{\partial y_0^i}},$$

using $\bar{q}_0 = \frac{\bar{\pi}_1}{1+\bar{r}_0}$ and $\frac{1}{1+\bar{r}_0} \overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \overline{\frac{\partial a_0^i}{\partial y_0^i}}$. The expression for $\overline{mpr}_0^i \equiv \frac{\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}}}{\overline{\frac{\partial a_0^i}{\partial y_0^i}}}$ then follows. □

Proposition 1

Proof. Optimal portfolio choice is

$$\mathbb{E}_0 (c_1^i(z_1))^{-\gamma^i} (r_1^k(z_1) - r_0) = 0.$$

We successively consider higher-order approximations and repeatedly make use of the method of undetermined coefficients and market clearing.

Up to first-order around the deterministic steady-state, optimal portfolio choice yields

$$\mathbb{E}_0 \hat{r}_1^k(z_1) - \hat{r}_0 = o(||\cdot||^2).$$

It follows that given a first-order expansion in terms of the states

$$\hat{r}_1^k(z_1) = \hat{z}_1 + \delta_{t_0}^{r_1^k} \hat{t}_0 + o(||\cdot||^2),$$
$$\hat{r}_0 = \delta_{t_0}^{r_0} \hat{t}_0 + o(||\cdot||^2),$$

with coefficients $\delta_{\cdot}^{\cdot}$, we can conclude

$$\delta_{t_0}^{r_1^k} = \delta_{t_0}^{r_0}$$

by the method of undetermined coefficients.

Up to second-order around the deterministic steady-state, optimal portfolio choice yields

$$\mathbb{E}_0 \hat{r}_1(z_1) - \hat{r}_0 + \frac{1}{2}\mathbb{E}_0 \hat{r}_1(z_1)^2 - \frac{1}{2}\hat{r}_0^2 = \gamma^i \mathbb{E}_0 \hat{c}_1^i(z_1)\left(\hat{r}_1(z_1) - \hat{r}_0\right) + o(||\cdot||^3).$$

Using the above first-order approximations of $\hat{r}_1^k(z_1)$ and $\hat{r}_0$ in terms of the underlying states, and the first-order approximation of $\hat{c}_1^i(z_1)$

$$\hat{c}_1^i(z_1) = \delta_{z_1}^{c_1^i} \hat{z}_1 + \delta_{t_0}^{c_1^i} \hat{t}_0 + o(||\cdot||^2),$$

it follows that optimal portfolio choice implies

$$\mathbb{E}_0 \hat{r}_1(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \gamma^i \delta_{z_1}^{c_1^i} \sigma^2 + o(||\cdot||^3). \tag{10}$$

By the period 1 resource constraint and equilibrium wages and profits

$$w_1(z_1) = (1-\alpha) z_1 k_0^\alpha,$$
$$\pi_1(z_1) = \alpha z_1 k_0^{\alpha-1},$$

the method of undetermined coefficients implies

$$\delta_{z_1}^{c_1^i} = \frac{\bar{w}_1 \eta^i + \bar{\pi}_1 \bar{k}_0^i}{\bar{c}_1^i}.$$

Hence, multiplying both sides of (10) by $\frac{\bar{c}_1^i}{\gamma^i}$, integrating over all agents i, and making use of the market clearing conditions which imply that

$$\int_0^1 \bar{c}_1^i di = \int_0^1 \left(\bar{w}_1 \eta^i + \bar{\pi}_1 \bar{k}_0^i\right) di,$$

we obtain

$$\mathbb{E}_0 \hat{r}_1(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \left(\frac{\bar{c}_1^i}{\int_0^1 \bar{c}_1^i di} \frac{1}{\gamma^i}\right)^{-1} \sigma^2 + o(||\cdot||^3), \tag{11}$$

defining γ as in the claim. Furthermore, by (10) we can conclude

$$\delta_{z_1}^{c_1^i} = \frac{\gamma}{\gamma^i}. \tag{12}$$

Finally we consider the third-order approximation of optimal portfolio choice around the deterministic steady-state:

$$\begin{aligned}
&\mathbb{E}_0 \hat{r}_1(z_1) - \hat{r}_0 + \frac{1}{2}\mathbb{E}_0 \hat{r}_1(z_1)^2 - \frac{1}{2}\hat{r}_0^2 \\
&\quad = \gamma^i \mathbb{E}_0 \hat{c}_1^i(z_1) \left(\hat{r}_1(z_1) - \hat{r}_0\right) - \frac{1}{2}\left(\gamma^i\right)^2 \mathbb{E}_0 \left(\hat{c}_1^i(z_1)\right)^2 \left(\hat{r}_1(z_1) - \hat{r}_0\right) \\
&\qquad + \frac{1}{2}\gamma^i \mathbb{E}_0 \hat{c}_1^i(z_1) \left(\hat{r}_1(z_1)^2 - \hat{r}_0^2\right) - \frac{1}{6}\left(\mathbb{E}_0 \hat{r}_1(z_1)^3 - \hat{r}_0^3\right) + o(||\cdot||^4).
\end{aligned} \tag{13}$$

A second-order expansion of $\hat{r}_1^k(z_1)$ and $\hat{r}_0$ in terms of the underlying states yields

$$\begin{aligned}
\hat{r}_1^k(z_1) &= \hat{z}_1 + \delta_{t_0}^{r_0} \hat{t}_0 + \frac{1}{2}\delta_{t_0^2}^{r_0} \hat{t}_0^2 + \left(-\frac{1}{2} + \gamma + \frac{1}{2}\delta_{\sigma^2}^{r_0}\right)\sigma^2, \\
\hat{r}_0 &= \delta_{t_0}^{r_0} \hat{t}_0 + \frac{1}{2}\delta_{t_0^2}^{r_0} \hat{t}_0^2 + \frac{1}{2}\delta_{\sigma^2}^{r_0} \sigma^2
\end{aligned}$$

where we have already made use of the fact that, by the method of undetermined coefficients, (17) implies

$$\begin{aligned}
\frac{1}{2}\delta_{t_0^2}^{r_1^k} &= \frac{1}{2}\delta_{t_0^2}^{r_0}, \\
\frac{1}{2}\delta_{\sigma^2}^{r_1^k} - \frac{1}{2}\delta_{\sigma^2}^{r_0} + \frac{1}{2} &= \gamma.
\end{aligned}$$

A second-order expansion of $\hat{c}_1^i(z_1)$ in terms of the underlying states yields

$$\hat{c}_1^i(z_1) = \delta_{t_0}^{c_1^i} \hat{t}_0 + \delta_{z_1}^{c_1^i} \hat{z}_1 + \frac{1}{2}\delta_{t_0^2}^{c_1^i} \hat{t}_0^2 + \delta_{t_0 z_1}^{c_1^i} \hat{t}_0 \hat{z}_1 + \frac{1}{2}\delta_{z_1^2}^{c_1^i} \hat{z}_1^2 + \frac{1}{2}\delta_{\sigma^2}^{c_1^i} \sigma^2 + o(||\cdot||^3).$$

Substituting these into (13) and collecting terms, we obtain

$$\mathbb{E}_0 \hat{r}_1^k(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = $$
$$\gamma^i \delta_{z_1}^{c_1^i}\sigma^2 + \left[\gamma^i \left(\delta_{t_0}^{c_1^i}\gamma + \delta_{t_0 z_1}^{c_1^i}\right) - \left(\gamma^i\right)^2 \delta_{t_0}^{c_1^i}\delta_{z_1}^{c_1^i} + \gamma^i \delta_{z_1}^{c_1^i}\delta_{t_0}^{r_0} - \gamma\delta_{t_0}^{r_0}\right]\hat{t}_0\sigma^2 + o(||\cdot||^4).$$

Making use of (12) substantially simplifies this to

$$\mathbb{E}_0 \hat{r}_1^k(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \gamma^i \delta_{z_1}^{c_1^i}\sigma^2 + \gamma^i \delta_{t_0 z_1}^{c_1^i}\hat{t}_0\sigma^2 + o(||\cdot||^4).$$

Again multiplying both sides by $\frac{\bar{c}_1^i}{\gamma^i}$, integrating over all agents i, and making use of the market clearing conditions, we obtain

$$\mathbb{E}_0 \hat{r}_1^k(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 + \frac{\gamma}{\int_0^1 \bar{c}_1^i di}\left(\int_0^1 \bar{c}_1^i \delta_{t_0 z_1}^{c_1^i} di\right)\hat{t}_0\sigma^2 + o(||\cdot||^4).$$

Then, by the period 1 resource constraint and equilibrium wages and profits, the method of undetermined coefficients implies

$$\bar{c}_1^i \delta_{t_0 z_1}^{c_1^i} + \bar{c}_1^i \delta_{t_0}^{c_1^i}\delta_{z_1}^{c_1^i} = \alpha\bar{w}_1\eta^i\delta_{t_0}^{k_0} + \bar{\pi}_1\delta_{t_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{t_0}^{k_0}.$$

It follows that

$$\int_0^1 \bar{c}_1^i \delta_{t_0 z_1}^{c_1^i} di = -\int_0^1 \bar{c}_1^i \delta_{t_0}^{c_1^i}\delta_{z_1}^{c_1^i} di + \bar{\pi}_1 \int_0^1 \delta_{t_0}^{k_0^i} di,$$

using, by market clearing and the assumed technology[8],

$$\int_0^1 \alpha\bar{w}_1\eta^i\delta_{t_0}^{k_0} di = \alpha\bar{w}_1\delta_{t_0}^{k_0} = \alpha(1-\alpha)\bar{z}_1\bar{k}_0^\alpha\delta_{t_0}^{k_0},$$
$$\int_0^1 (1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{t_0}^{k_0} di = (1-\alpha)\bar{\pi}_1\bar{k}_0\delta_{t_0}^{k_0} = \alpha(1-\alpha)\bar{z}_1\bar{k}_0^\alpha\delta_{t_0}^{k_0}.$$

Since a first-order approximation to capital claims market clearing implies

$$\int_0^1 \delta_{t_0}^{k_0^i} di = \bar{k}_0\delta_{t_0}^{k_0},$$

[8]Note that we linearize rather than log-linearize with respect to k_0^i, b_0^i, t_0 since in principle these may be negative.

it further follows that

$$\int_0^1 \bar{c}_1^i \delta_{t_0 z_1}^{c_1^i} di = -\int_0^1 \bar{c}_1^i \delta_{t_0}^{c_1^i} \delta_{z_1}^{c_1^i} di + \bar{\pi}_1 \bar{k}_0 \delta_{t_0}^{k_0}.$$

Moreover, since a first-order approximation to goods market clearing implies

$$\int_0^1 \bar{c}_1^i \delta_{t_0}^{c_1^i} di = \alpha \bar{z}_1 \bar{k}_0^{\alpha} \delta_{t_0}^{k_0}$$

and $\bar{\pi}_1 = \alpha \bar{z}_1 \bar{k}_0^{\alpha-1}$, it further follows that

$$\begin{aligned}
\int_0^1 \bar{c}_1^i \delta_{t_0 z_1}^{c_1^i} di &= -\int_0^1 \bar{c}_1^i \delta_{t_0}^{c_1^i} \delta_{z_1}^{c_1^i} di + \int_0^1 \bar{c}_1^i \delta_{t_0}^{c_1^i} di, \\
&= \int_0^1 \bar{c}_1^i \delta_{t_0}^{c_1^i} \left(1 - \delta_{z_1}^{c_1^i}\right) di, \\
&= \int_0^1 \bar{c}_1^i \delta_{t_0}^{c_1^i} \left(1 - \frac{\gamma}{\gamma^i}\right)
\end{aligned}$$

where the final line uses (12). Hence, we can conclude

$$\begin{aligned}
\zeta_{t_0} &= \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \left(\int_0^1 \bar{c}_1^i \delta_{t_0 z_1}^{c_1^i} di\right), \\
&= \frac{\gamma}{\int_0^1 \bar{c}_1^i di} \int_0^1 \bar{c}_1^i \delta_{t_0}^{c_1^i} \left(1 - \frac{\gamma}{\gamma^i}\right).
\end{aligned}$$

Denoting $\bar{\xi}_{t_0}^i \equiv \bar{c}_1^i \delta_{t_0}^{c_1^i} = \overline{\frac{dc_1^i}{dt_0}}$ and using the result from Lemma 1 that $\overline{mpr}_0^i \equiv \frac{\gamma}{\gamma^i} - 1$ completes the expression for ζ_{t_0} given in the claim. □

Lemma 2

Proof. First consider the case of an agent facing a binding leverage constraint or rule-of-thumb. If the agent maintains

$$q_0 k_0^i = \omega_0^i a_0^i$$

in response to a marginal change in income, clearly

$$q_0 \frac{\partial k_0^i}{\partial y_0^i} = \omega_0^i \frac{\partial a_0^i}{\partial y_0^i}.$$

Provided this holds in the limit of zero aggregate risk, it follows that

$$\bar{q}_0 \overline{\frac{\partial k_0^i}{\partial y_0^i}} = \omega_0^i \frac{\partial a_0^i}{\partial y_0^i}.$$

Now consider the case of an agent at an interior solution in portfolio choice but also facing idiosyncratic income risk, so that the period 1 resource constraint is

$$c_1^i(\epsilon_1^i, z_1) = \epsilon_1^i + w_1(z_1)\eta^i + b_0^i + \pi_1(z_1)k_0^i.$$

The analog of (8) in this case is

$$0 = \mathbb{E}_0 m_{0,1}^i(\epsilon_1^i, z_1) \frac{\gamma^i}{c_1^i(\epsilon_1^i, z_1)} (r_1^k(z_1) - r_0) \left(\frac{\partial b_0^i}{\partial y_0^i} + \pi_1(z_1) \frac{\partial k_0^i}{\partial y_0^i} \right), \tag{14}$$

where the stochastic discount factor is

$$m_{0,1}^i(\epsilon_1^i, z_1) \equiv \frac{\beta^i}{1-\beta^i} (c_0^i)^{\frac{1}{\psi^i}} (\Phi^i(l_0^i))^{\frac{1}{\psi^i}-1} (ce_0^i)^{\gamma^i - \frac{1}{\psi^i}} (c_1^i(\epsilon_1^i, z_1))^{-\gamma^i}$$

and the certainty equivalent is evaluated over both the idiosyncratic and aggregate states

$$ce_0^i = \left(\mathbb{E}_0 (c_1^i(\epsilon_1^i, z_1))^{1-\gamma^i} \right)^{\frac{1}{1-\gamma^i}}.$$

A second-order approximation of the right-hand side of (14) around the deterministic steady-state (with respect to only the aggregate states) then implies

$$\begin{aligned} 0 = &\left(\mathbb{E}_0 \bar{m}_{0,1}^i(\epsilon_1^i) \frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)} \right) \left(\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}} \right) \left(\mathbb{E}_0 \left(\hat{r}_1^k(z_1) - \hat{r}_0 \right) + \frac{1}{2}\sigma^2 \right) \\ &- \left(\mathbb{E}_0 \bar{m}_{0,1}^i(\epsilon_1^i) \frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)} \frac{\gamma^i + 1}{\bar{c}_1^i(\epsilon_1^i)} \right) \left(\overline{\frac{\partial b_0^i}{\partial y_0^i}} + \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}} \right) \left(\bar{w}_1 \eta^i + \bar{\pi}_1 \bar{k}_0^i \right) \sigma^2 \\ &+ \left(\mathbb{E}_0 \bar{m}_{0,1}^i(\epsilon_1^i) \frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)} \right) \bar{\pi}_1 \overline{\frac{\partial k_0^i}{\partial y_0^i}} \sigma^2 + o(|| \cdot ||^3). \end{aligned} \tag{15}$$

Again returning to the agent's FOCs with respect to k_0^i and b_0^i which imply the optimal portfolio choice condition

$$\mathbb{E}_0 (c_1^i(\epsilon_1^i, z_1))^{-\gamma^i} (r_1^k(z_1) - r_0) = 0,$$

a second-order approximation of the left-hand side around the deterministic steady-state (again with respect to only the aggregate states) implies

$$\mathbb{E}_0\left(\hat{r}_1^k(z_1)-\hat{r}_0\right)+\frac{1}{2}\sigma^2=\frac{1}{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)}\left(\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)}\right)\left(\bar{w}_1\eta^i+\bar{\pi}_1\bar{k}_0^i\right)\sigma^2+o(||\cdot||^3).$$

Anticipating the characterization of the steady-state equity premium

$$\mathbb{E}_0\left(\hat{r}_1^k(z_1)-\hat{r}_0\right)+\frac{1}{2}\sigma^2=\gamma\sigma^2+o(||\cdot||^3)$$

in the next result, it follows that

$$\left(\bar{w}_1\eta^i+\bar{\pi}_1\bar{k}_0^i\right)=\frac{\gamma}{\gamma^i}\frac{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)}{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{1}{\bar{c}_1^i(\epsilon_1^i)}}$$

and hence from (15) that

$$\bar{q}_0\overline{\frac{\partial k_0^i}{\partial y_0^i}}=\left[\frac{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)}\frac{\gamma^i+1}{\bar{c}_1^i(\epsilon_1^i)}}{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)}}\frac{\gamma}{\gamma^i}\frac{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)}{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{1}{\bar{c}_1^i(\epsilon_1^i)}}-\gamma\right]\overline{\frac{\partial a_0^i}{\partial y_0^i}}$$

using $\bar{q}_0=\frac{\bar{\pi}_1}{1+\bar{r}_0}$ and $\frac{1}{1+\bar{r}_0}\overline{\frac{\partial b_0^i}{\partial y_0^i}}+\bar{q}_0\overline{\frac{\partial k_0^i}{\partial y_0^i}}=\overline{\frac{\partial a_0^i}{\partial y_0^i}}$. Given the definition $\overline{mpr}_0^i\equiv\frac{\bar{q}_0\overline{\frac{\partial k_0^i}{\partial y_0^i}}}{\overline{\frac{\partial a_0^i}{\partial y_0^i}}}-1$, it follows that

$$\overline{mpr}_0^i=\gamma\left[\frac{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)}\frac{\gamma^i+1}{\bar{c}_1^i(\epsilon_1^i)}}{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)}}\frac{1}{\gamma^i}\frac{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)}{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{1}{\bar{c}_1^i(\epsilon_1^i)}}-1\right]-1.$$

Finally, using

$$\bar{m}_{0,1}^i(\epsilon_1^i)\equiv\frac{\beta^i}{1-\beta^i}(\bar{c}_0^i)^{\frac{1}{\psi^i}}(\Phi^i(\bar{l}_0^i))^{\frac{1}{\psi^i}-1}(\bar{ce}_0^i)^{\gamma^i-\frac{1}{\psi^i}}(\bar{c}_1^i(\epsilon_1^i))^{-\gamma^i},$$

we have that

$$\frac{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)}\frac{\gamma^i+1}{\bar{c}_1^i(\epsilon_1^i)}}{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{\gamma^i}{\bar{c}_1^i(\epsilon_1^i)}}\frac{1}{\gamma^i}\frac{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)}{\mathbb{E}_0\bar{m}_{0,1}^i(\epsilon_1^i)\frac{1}{\bar{c}_1^i(\epsilon_1^i)}}=\frac{\mathbb{E}_0(\bar{c}_1^i(\epsilon_1^i))^{-\gamma^i-2}\mathbb{E}_0(\bar{c}_1^i(\epsilon_1^i))^{-\gamma^i}}{\left(\mathbb{E}_0(\bar{c}_1^i(\epsilon_1^i))^{-\gamma^i-1}\right)^2}\frac{\gamma^i+1}{\gamma^i},$$

completing the claim. □

Proposition 2

Proof. Optimal portfolio choice of an agent at an interior optimum is

$$\mathbb{E}_0(c_1^i(\epsilon_1^i, z_1))^{-\gamma^i}(r_1^k(z_1) - r_0) = 0.$$

As in the proof of Proposition 1, we successively consider higher-order approximations and repeatedly make use of the method of undetermined coefficients and market clearing.

A first-order approximation implies

$$\delta_{t_0}^{r_1^k} = \delta_{t_0}^{r_0}$$

as before.

A second-order approximation implies

$$\mathbb{E}_0\hat{r}_1^k(z_1) - \hat{r}_0 + \frac{1}{2}\mathbb{E}_0\hat{r}_1^k(z_1)^2 - \frac{1}{2}\hat{r}_0^2 = \gamma^i\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\hat{c}_1^i(\epsilon_1^i, z_1)\left(\hat{r}_1^k(z_1) - \hat{r}_0\right) + o(||\cdot||^3).$$

The analog to (10) is now

$$\mathbb{E}_0\hat{r}_1(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \gamma^i\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\delta_{z_1}^{c_1^i(\epsilon_1^i)}\sigma^2 + o(||\cdot||^3). \tag{16}$$

The method of undetermined coefficients now implies

$$\delta_{z_1}^{c_1^i(\epsilon_1^i)} = \frac{\bar{w}_1\eta^i + \bar{\pi}_1\bar{k}_0^i}{\bar{c}_1^i(\epsilon_1^i)}.$$

Hence, multiplying both sides of (10) by $\frac{1}{\gamma^i}\frac{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}$ and integrating over agents i at an interior optimum $UC \equiv \{i|unconstrained\}$, we obtain

$$\mathbb{E}_0\hat{r}_1(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \left(\frac{1}{\int_{i\in UC}\left[\bar{w}_1\eta^i + \bar{\pi}_1\bar{k}_0^i\right]}\frac{1}{\gamma^i}\frac{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}\right)^{-1}\sigma^2 + o(||\cdot||^3), \tag{17}$$

defining γ as in the claim. Furthermore, by (16) we can conclude

$$\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\delta_{z_1}^{c_1^i(\epsilon_1^i)} = \frac{\gamma}{\gamma^i}. \tag{18}$$

A third-order approximation implies

$$
\begin{aligned}
&\mathbb{E}_0\hat{r}_1^k(z_1)-\hat{r}_0+\frac{1}{2}\mathbb{E}_0(\hat{r}_1^k(z_1))^2-\frac{1}{2}\hat{r}_0^2+\frac{1}{6}\mathbb{E}_0(\hat{r}_1^k(z_1))^3-\frac{1}{6}\hat{r}_0^3\\
&=\gamma^i\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\hat{c}_1^i(\epsilon_1^i,z_1)\left(\hat{r}_1^k(z_1)-\hat{r}_0\right)-\frac{1}{2}\left(\gamma^i\right)^2\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\left(\hat{c}_1^i(\epsilon_1^i,z_1)\right)^2\left(\hat{r}_1^k(z_1)-\hat{r}_0\right)\\
&\qquad+\frac{1}{2}\gamma^i\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\hat{c}_1^i(\epsilon_1^i,z_1)\left(\hat{r}_1^k(z_1)^2-\hat{r}_0^2\right)+o(||\cdot||^4).
\end{aligned}
$$

A second-order expansion of $\hat{r}_1^k(z_1)$ and $\hat{r}_0$ in terms of the underlying states again yields

$$
\begin{aligned}
\hat{r}_1^k(z_1)&=\hat{z}_1+\delta_{t_0}^{r_0}\hat{t}_0+\frac{1}{2}\delta_{t_0^2}^{r_0}\hat{t}_0^2+\left(-\frac{1}{2}+\gamma+\frac{1}{2}\delta_{\sigma^2}^{r_0}\right)\sigma^2,\\
\hat{r}_0&=\delta_{t_0}^{r_0}\hat{t}_0+\frac{1}{2}\delta_{t_0^2}^{r_0}\hat{t}_0^2+\frac{1}{2}\delta_{\sigma^2}^{r_0}\sigma^2.
\end{aligned}
$$

A second-order expansion of $\hat{c}_1^i(\epsilon_1^i,z_1)$ in terms of the underlying states now yields

$$
\hat{c}_1^i(\epsilon_1^i,z_1)=\delta_{t_0}^{c_1^i(\epsilon_1^i)}\hat{t}_0+\delta_{z_1}^{c_1^i(\epsilon_1^i)}\hat{z}_1+\frac{1}{2}\delta_{t_0^2}^{c_1^i(\epsilon_1^i)}\hat{t}_0^2+\delta_{t_0z_1}^{c_1^i(\epsilon_1^i)}\hat{t}_0\hat{z}_1+\frac{1}{2}\delta_{z_1^2}^{c_1^i(\epsilon_1^i)}\hat{z}_1^2+\frac{1}{2}\delta_{\sigma^2}^{c_1^i(\epsilon_1^i)}\sigma^2+o(||\cdot||^3).
$$

Substituting these into (13) and collecting terms, we obtain

$$
\begin{aligned}
&\mathbb{E}_0\hat{r}_1^k(z_1)-\hat{r}_0+\frac{1}{2}\sigma^2=\left(\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\gamma^i\delta_{z_1}^{c_1^i(\epsilon_1^i)}\right)\sigma^2\\
&+\left(\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\left[\gamma^i\left(\delta_{t_0}^{c_1^i(\epsilon_1^i)}\gamma+\delta_{t_0z_1}^{c_1^i(\epsilon_1^i)}\right)-\left(\gamma^i\right)^2\delta_{t_0}^{c_1^i(\epsilon_1^i)}\delta_{z_1}^{c_1^i(\epsilon_1^i)}+\gamma^i\delta_{z_1}^{c_1^i(\epsilon_1^i)}\delta_{t_0}^{r_0}\right]-\gamma\delta_{t_0}^{r_0}\right)\hat{t}_0\sigma^2+o(||\cdot||^4).
\end{aligned}
$$

Now consider the coefficient on $\hat{t}_0\sigma^2$. Making use of (18) simplifies this to

$$
\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\left[\gamma^i\left(\delta_{t_0}^{c_1^i(\epsilon_1^i)}\gamma+\delta_{t_0z_1}^{c_1^i(\epsilon_1^i)}\right)-\left(\gamma^i\right)^2\delta_{t_0}^{c_1^i(\epsilon_1^i)}\delta_{z_1}^{c_1^i(\epsilon_1^i)}\right],
$$

which can be equivalently written

$$
\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\left[\gamma^i\left(\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\gamma+\bar{c}_1^i(\epsilon_1^i)\delta_{t_0z_1}^{c_1^i(\epsilon_1^i)}\right)-\left(\gamma^i\right)^2\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\delta_{z_1}^{c_1^i(\epsilon_1^i)}\right].
$$

By agents' period 1 budget constraint and the method of undetermined coefficients, we have

$$
\bar{c}_1^i(\epsilon_1^i)\delta_{t_0z_1}^{c_1^i(\epsilon_1^i)}+\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\delta_{z_1}^{c_1^i(\epsilon_1^i)}=\alpha\bar{w}_1\eta^i\delta_{t_0}^{k_0}+\bar{\pi}_1\delta_{t_0}^{k_0^i}-(1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{t_0}^{k_0}
$$

so that we can write the coefficient on $\hat{t}_0\sigma^2$ as

$$\mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}} \left[\gamma^i \bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\left(\gamma - (\gamma^i+1)\delta_{z_1}^{c_1^i(\epsilon_1^i)}\right) + \gamma^i\left(\alpha\bar{w}_1\eta^i\delta_{t_0}^{k_0} + \bar{\pi}_1\delta_{t_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{t_0}^{k_0}\right)\right].$$

Adding and subtracting $\mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\gamma^i\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}$, we equivalently write this as

$$\mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}} \Big[\gamma^i \bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\left(\gamma - (\gamma^i+1)\delta_{z_1}^{c_1^i(\epsilon_1^i)} + 1\right) \\ + \gamma^i\left(-\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)} + \alpha\bar{w}_1\eta^i\delta_{t_0}^{k_0} + \bar{\pi}_1\delta_{t_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{t_0}^{k_0}\right)\Big].$$

Since agents' period 1 budget constraint and the method of undetermined coefficients implies

$$\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)} = \alpha\bar{w}_1\eta^i\delta_{t_0}^{k_0} + \delta_{t_0}^{b_0^i} + \bar{\pi}_1\delta_{t_0}^{k_0^i} - (1-\alpha)\bar{\pi}_1\bar{k}_0^i\delta_{t_0}^{k_0},$$

we can further write the coefficient on $\hat{t}_0\sigma^2$ as

$$\mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}} \Big[\gamma^i \bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\left(\gamma - (\gamma^i+1)\delta_{z_1}^{c_1^i(\epsilon_1^i)} + 1\right) - \gamma^i\delta_{t_0}^{b_0^i}\Big].$$

Hence, optimal portfolio choice up to third order is given by

$$\mathbb{E}_0\hat{r}_1^k(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \left(\mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}\gamma^i\delta_{z_1}^{c_1^i(\epsilon_1^i)}\right)\sigma^2 \\ + \mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}} \Big[\gamma^i \bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\left(\gamma - (\gamma^i+1)\delta_{z_1}^{c_1^i(\epsilon_1^i)} + 1\right) - \gamma^i\delta_{t_0}^{b_0^i}\Big]\hat{t}_0\sigma^2 + o(||\cdot||^4).$$

Now multiply both sides by $\frac{1}{\gamma^i}\frac{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}$ and integrate over the measure of unconstrained agents, yielding

$$\mathbb{E}_0\hat{r}_1^k(z_1) - \hat{r}_0 + \frac{1}{2}\sigma^2 = \gamma\sigma^2 \\ + \frac{\gamma}{\int_{i\in UC}\left[\bar{w}_1\eta^i + \bar{\pi}_1\bar{k}_0^i\right]di}\int_{i\in UC}\left[\mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}\Big[\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\left(\gamma - (\gamma^i+1)\delta_{z_1}^{c_1^i(\epsilon_1^i)} + 1\right) \\ - \delta_{t_0}^{b_0^i}\Big]\right]\hat{t}_0\sigma^2 + o(||\cdot||^4).$$

Since $\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}$ is independent of ϵ_1^i, we have

$$\mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\left(\gamma-(\gamma^i+1)\delta_{z_1}^{c_1^i(\epsilon_1^i)}+1\right)= \bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\left[(\gamma+1)-(\gamma^i+1)\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}\delta_{z_1}^{c_1^i(\epsilon_1^i)}\right].$$

Then note that

$$\begin{aligned}\mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}\delta_{z_1}^{c_1^i(\epsilon_1^i)} &= \mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-2}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}\bar{c}_1^i\delta_{z_1}^{c_1^i(\epsilon_1^i)},\\ &= \frac{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-2}\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i}}{\left(\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}\right)^2}\frac{\gamma}{\gamma^i}\end{aligned}$$

where the second equality uses (18) and the fact that $\bar{c}_1^i\delta_{z_1}^{c_1^i(\epsilon_1^i)}$ is independent of ϵ_1^i, so that

$$\mathbb{E}_0 \frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\left(\gamma-(\gamma^i+1)\delta_{z_1}^{c_1^i(\epsilon_1^i)}+1\right) = -\bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)}\overline{mpr}_0^i$$

using the result for unconstrained agents in Lemma 2. Finally, note that

$$\begin{aligned}-\int_{i\in UC}\mathbb{E}_0\frac{\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}{\mathbb{E}_0\bar{c}_1^i(\epsilon_1^i)^{-\gamma^i-1}}\delta_{t_0}^{b_0^i} &= -\int_{i\in UC}\delta_{t_0}^{b_0^i},\\ &= \int_{i\notin UC}\delta_{t_0}^{b_0^i},\\ &= -\int_{i\notin UC}\left((1+\bar{r}_0)\bar{a}_0^i\delta_{t_0}^{r_0}+(1+\bar{r}_0)\delta_{t_0}^{a_0^i}\right)\left(\omega_0^i-1\right),\\ &= -\int_{i\notin UC}\left((1+\bar{r}_0)\bar{a}_0^i\delta_{t_0}^{r_0}+(1+\bar{r}_0)\delta_{t_0}^{a_0^i}\right)\overline{mpr}_0^i,\end{aligned}$$

where the second equality uses bond market clearing, the third uses the nature of the binding constraint for constrained agents

$$\frac{1}{1+r_0}b_0 = (1-\omega_0^i)a_0^i,$$

and the fourth uses the definition of the $\overline{mpr}_0^i$ for constrained agents in Lemma 2. Defining

$$\bar{\xi}_{t_0}^i \equiv \bar{c}_1^i(\epsilon_1^i)\delta_{t_0}^{c_1^i(\epsilon_1^i)} = \overline{\frac{dc_1^i(\epsilon_1^i)}{dt_0}}, \ \ i \in UC,$$
$$\bar{\xi}_{t_0}^i \equiv (1+\bar{r}_0)\bar{a}_0^i\delta_{t_0}^{r_0} + (1+\bar{r}_0)\delta_{t_0}^{a_0^i} = \overline{\frac{d(1+r_0)a_0^i}{dt_0}}, \ \ i \notin UC$$

completes the stated result for ζ_{t_0} in the claim. □

Proposition 3

[TO BE ADDED]

Proposition 4

Proof. As derived in the main text,

$$\mathbb{E}_0\hat{r}_1^k \propto -\hat{k}_0.$$

Given nominal rigidity and $\hat{r}_0 = 0$ implied by monetary policy, it follows that

$$\mathbb{E}_0\hat{r}_1^k - \hat{r}_0 \propto -\hat{k}_0.$$

By the method of undetermined coefficients, it follows that

$$\delta_{t_0\sigma^2}^{k_0} \propto -\zeta_{t_0}$$

given the results in Proposition 2. □

Proposition 5

Proof. The proof proceeds analogously to Proposition 2, using a third-order approximation to portfolio choice. Note that in the third-order approximation, all interaction terms between $\hat{t}_0$, $\hat{z}_0$, and $\hat{i}_0$ will cancel out in the resulting expression for the equity premium. The coefficients on $\hat{z}_0\sigma^2$ and $\hat{i}_0\sigma^2$ can then be characterized in an analogous way as the coefficient on $\hat{t}_0\sigma^2$, repeatedly exploiting the method of undetermined coefficients and market clearing. □

Proposition 6

Proof. Assuming that $\frac{dc_1^i}{d\theta_0}$ is continuous in σ, it is equivalent to characterize $\frac{dc_1^i}{d\theta_0}$ and then evaluate its limit at the deterministic steady-state ($\sigma = 0$) or simply compute $\frac{d\bar{c}_1^i}{d\bar{\theta}_0}$ at this limit. It is expositionally simpler to do the latter, so we do that here.

Re-consider agents' micro-level optimization problem given $\sigma = 0$:

$$\max \left((1-\beta^i) \left(\bar{c}_0^i \Phi^i(\bar{\ell}_0^i)\right)^{1-\frac{1}{\psi^i}} + \beta^i \left(\left(\mathbb{E}_0(\bar{c}_1^i(\epsilon_1^i))^{1-\gamma^i} \right) \right)^{\frac{1-\frac{1}{\psi^i}}{1-\gamma^i}} \right)^{\frac{1}{1-\frac{1}{\psi^i}}} \quad s.t.$$

$$\bar{c}_0^i + \bar{a}_0^i = \bar{w}_0 \eta^i \bar{\ell}_0^i + \bar{y}_0^i(\bar{P}_0, \bar{\pi}_0, \bar{q}_0, \bar{t}_0),$$

$$\bar{c}_1^i(\epsilon_1^i) = \epsilon_1^i + \bar{w}_1 \eta^i + (1+\bar{r}_0)\bar{a}_0^i,$$

defining policy functions

$$\bar{c}_1^i(\epsilon_1^i; \bar{y}_0^i(\bar{P}_0, \bar{\pi}_0, \bar{q}_0, \bar{t}_0), \bar{\ell}_0^i, \bar{w}_0, 1+\bar{r}_0, \bar{w}_1),$$

where the exogenous components of income in period 0 are

$$\bar{y}_0^i(\bar{P}_0, \bar{\pi}_0, \bar{q}_0, \bar{t}_0) = \frac{1}{\bar{P}_0} B_{-1}^i + (\bar{\pi}_0 + (1-\delta)\bar{q}_0) k_{-1}^i + \Delta_0^i \bar{t}_0$$

and we recall that $\bar{\ell}_0^i$ is taken as given by the agent under the rationing rule. It follows that

$$\frac{d\bar{c}_1^i(\epsilon_1^i)}{d\bar{\theta}_0} = \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{y}_0^i} \left[-\frac{1}{\bar{P}_0} B_{-1}^i \frac{1}{\bar{P}_0} \frac{d\bar{P}_0}{d\bar{\theta}_0} + \left(\frac{d\bar{\pi}_0}{d\bar{\theta}_0} + (1-\delta)\frac{d\bar{q}_0}{d\bar{\theta}_0} \right) k_{-1}^i \right] + \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{\ell}_0^i} \frac{d\bar{\ell}_0^i}{d\bar{\theta}_0} + \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{w}_0} \frac{d\bar{w}_0}{d\bar{\theta}_0} + \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial (1+\bar{r}_0)} \frac{d\bar{r}_0}{d\bar{\theta}_0} + \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{w}_1} \frac{d\bar{w}_1}{d\bar{\theta}_0} \quad (19)$$

for $\theta_0 \in \{z_0, i_0\}$, where each of the partial derivatives is evaluated with respect to the policy function above. We now characterize each of these partial derivatives in turn.

First note that it is clearly the case that

$$\frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{w}_0} = \eta^i \bar{\ell}_0^i \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{y}_0^i},$$

$$\frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{w}_1} = \frac{1}{1+\bar{r}_0} \eta^i \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{y}_0^i}.$$

Then define the expenditure minimization problem dual to the utility maximization problem above

$$\min \bar{c}_0^{i,h} - \bar{w}_0 \eta^i \bar{\ell}_0^i + \bar{a}_0^{i,h} \ \ s.t.$$

$$\left((1-\beta^i) \left(\bar{c}_0^{i,h} \Phi^i(\bar{\ell}_0^{i,h}) \right)^{1-\frac{1}{\psi^i}} + \beta^i \left(\left(\mathbb{E}_0 (\bar{c}_1^{i,h}(\epsilon_1^i))^{1-\gamma^i} \right) \right)^{\frac{1-\frac{1}{\psi^i}}{1-\gamma^i}} \right)^{\frac{1}{1-\frac{1}{\psi^i}}} \geq \bar{u}^i,$$

$$\bar{c}_1^{i,h}(\epsilon_1^i) = \epsilon_1^i + \bar{w}_1 \eta^i + (1+\bar{r}_0)\bar{a}_0^{i,h},$$

where we use h superscripts to denote compensated (Hicksian) policies. Letting

$$\bar{e}_0^i(\bar{u}^i, \bar{\ell}_0^i, \bar{w}_0, 1+\bar{r}_0, \bar{w}_1)$$

denote the level of period 0 expenditure solving this problem, duality implies

$$\bar{c}_1^i(\epsilon_1^i; \bar{e}_0^i(\bar{u}^i, \bar{w}_0, 1+\bar{r}_0, \bar{w}_1), \bar{\ell}_0^i, \bar{w}_0, 1+\bar{r}_0, \bar{w}_1) = \bar{c}_1^{i,h}(\epsilon_1^i; \bar{u}^i, \bar{\ell}_0^i, \bar{w}_0, 1+\bar{r}_0, \bar{w}_1).$$

This leads to Slutsky identities

$$\begin{aligned}
\frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{\ell}_0^i} &= \frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i; \cdot)}{\partial \bar{\ell}_0^i} - \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{y}_0^i} \frac{\partial \bar{e}_0^i}{\partial \bar{\ell}_0^i}, \\
\frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial (1+\bar{r}_0)} &= \frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i; \cdot)}{\partial (1+\bar{r}_0)} - \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{y}_0^i} \frac{\partial \bar{e}_0^i}{\partial (1+\bar{r}_0)}.
\end{aligned}$$

By the Envelope Theorem,

$$\begin{aligned}
\frac{\partial \bar{e}_0^i}{\partial \bar{\ell}_0^i} &= -\eta^i \bar{w}^i - \bar{c}_0^{i,h} \frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}, \\
\frac{\partial \bar{e}_0^i}{\partial (1+\bar{r}_0)} &= -\frac{1}{1+\bar{r}_0} \bar{a}_0^{i,h},
\end{aligned}$$

so that we may further write the above identities as

$$\begin{aligned}
\frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{\ell}_0^i} &= \frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i; \cdot)}{\partial \bar{\ell}_0^i} + \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{y}_0^i} \left(\eta^i \bar{w}^i + \bar{c}_0^{i,h} \frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)} \right), \\
\frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial (1+\bar{r}_0)} &= \frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i; \cdot)}{\partial (1+\bar{r}_0)} + \frac{\partial \bar{c}_1^i(\epsilon_1^i; \cdot)}{\partial \bar{y}_0^i} \frac{1}{1+\bar{r}_0} \bar{a}_0^{i,h}.
\end{aligned}$$

Substituting the above results into (19), using $\bar{c}_0^{i,h} = \bar{c}_0^i$ and $\bar{a}^{i,h} = \bar{a}^i$ implied by duality, and collecting terms, we obtain

$$\frac{d\bar{c}_1^i(\epsilon_1^i)}{d\bar{\theta}_0} = \frac{\partial \bar{c}_1^i(\epsilon_1^i;\cdot)}{\partial \bar{y}_0^i}\Bigg[-\frac{1}{\bar{P}_0}B_{-1}^i\frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{\theta}_0} + \left(\frac{d\bar{\pi}_0}{d\bar{\theta}_0} + (1-\delta)\frac{d\bar{q}_0}{d\bar{\theta}_0}\right)k_{-1}^i + \eta^i\bar{\ell}_0^i\frac{d\bar{w}_0}{d\bar{\theta}_0} + \frac{1}{1+\bar{r}_0}\eta^i\frac{d\bar{w}_1}{d\bar{\theta}_0}$$
$$+\eta^i\bar{w}_0\frac{d\bar{l}_0^i}{d\bar{\theta}_0} + \frac{1}{1+\bar{r}_0}\bar{a}_0^i\frac{d\bar{r}_0}{d\bar{\theta}_0} + \bar{c}_0^i\frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}\frac{d\bar{\ell}_0^i}{d\bar{\theta}_0}\Bigg] + \frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i;\cdot)}{\partial \bar{l}_0^i}\frac{d\bar{l}_0^i}{d\bar{\theta}_0} + \frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i;\cdot)}{\partial(1+\bar{r}_0)}\frac{d\bar{r}_0}{d\bar{\theta}_0} \quad (20)$$

Since interiority in portfolio choice yields

$$\bar{q}_0 = \frac{\bar{\pi}_1}{1+\bar{r}_0}$$

in the deterministic steady-state, we have that

$$\frac{d\bar{q}_0}{d\bar{\theta}_0} = \frac{(1+\bar{r}_0)\frac{d\bar{\pi}_1}{d\bar{\theta}_0} - \bar{\pi}_1\frac{d\bar{r}_0}{d\bar{\theta}_0}}{(1+\bar{r}_0)^2},$$
$$= \frac{1}{1+\bar{r}_0}\frac{d\bar{\pi}_1}{d\bar{\theta}_0} - \frac{\bar{q}_0}{1+\bar{r}_0}\frac{d\bar{r}_0}{d\bar{\theta}_0}.$$

It follows that the expression in brackets in (20) can be written

$$-\frac{1}{\bar{P}_0}B_{-1}^i\frac{1}{\bar{P}_0}\frac{d\bar{P}_0}{d\bar{\theta}_0} + \left(\bar{a}_0^i - (1-\delta)\bar{q}_0k_{-1}^i\right)\frac{1}{1+\bar{r}_0}\frac{d\bar{r}_0}{d\bar{\theta}_0}$$
$$+\left(\frac{d\bar{\pi}_0}{d\bar{\theta}_0} + \frac{1}{1+\bar{r}_0}(1-\delta)\frac{d\bar{\pi}_1}{d\bar{\theta}_0}\right)k_{-1}^i + \eta^i\left(\frac{d\bar{w}_0\bar{\ell}_0^i}{d\bar{\theta}_0} + \frac{1}{1+\bar{r}_0}\frac{d\bar{w}_1}{d\bar{\theta}_0}\right) + \bar{c}_0^i\frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}\frac{d\bar{\ell}_0^i}{d\bar{\theta}_0},$$

where we can further note by the definition of total saving that

$$\bar{a}_0^i - (1-\delta)\bar{q}_0k_{-1}^i = \frac{1}{1+\bar{r}_0}\bar{b}_0^i + \bar{q}_0\left(\bar{k}_0^i - (1-\delta)k_{-1}^i\right).$$

We finally characterize the compensated derivatives $\frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i;\cdot)}{\partial \bar{l}_0^i}$ and $\frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i;\cdot)}{\partial(1+\bar{r}_0)}$. The compensated policies solve the system

$$(1-\beta^i)(\Phi^i(\bar{l}_0^i))^{1-\frac{1}{\psi^i}}(\bar{c}_0^{i,h})^{-\frac{1}{\psi^i}} = \beta^i(\bar{ce}_0^{i,h})^{\gamma^i-\frac{1}{\psi^i}}\mathbb{E}_0(\bar{c}_1^{i,h}(\epsilon_1^i))^{-\gamma^i}(1+\bar{r}_0),$$
$$\left((1-\beta^i)\left(\bar{c}_0^{i,h}\Phi^i(\bar{\ell}_0^{i,h})\right)^{1-\frac{1}{\psi^i}} + \beta^i\left(\left(\mathbb{E}_0(\bar{c}_1^{i,h}(\epsilon_1^i))^{1-\gamma^i}\right)\right)^{\frac{1-\frac{1}{\psi^i}}{1-\gamma^i}}\right)^{\frac{1}{1-\frac{1}{\psi^i}}} = \bar{u}^i,$$
$$\bar{c}_1^{i,h}(\epsilon_1^i) = \epsilon_1^i + \bar{w}_1\eta^i + (1+\bar{r}_0)\bar{a}_0^{i,h},$$
$$\bar{ce}_0^{i,h} = \left(\mathbb{E}_0(\bar{c}_1^{i,h}(\epsilon_1^i))^{1-\gamma^i}\right)^{\frac{1}{1-\gamma^i}}.$$

Straightforward differentiation of this system yields

$$\frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i;\cdot)}{\partial \bar{l}_0^i} = \frac{-\frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}}{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^{i,h}}\frac{1}{1+\bar{r}_0} - \left(\gamma^i - \frac{1}{\psi^i}\right)(\bar{ce}_0^{i,h})^{\gamma^i-1}\mathbb{E}_0\left(\bar{c}_1^{i,h}(\epsilon_1^i)\right)^{-\gamma^i} + \gamma^i \frac{1}{\mathbb{E}_0\left(\bar{c}_1^{i,h}(\epsilon_1^i)\right)^{-\gamma^i}}\mathbb{E}_0\left(\bar{c}_1^{i,h}(\epsilon_1^i)\right)^{-\gamma^i-1}},$$

$$\frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i;\cdot)}{\partial \bar{r}_0} = \frac{\frac{1}{1+\bar{r}_0}}{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^{i,h}}\frac{1}{1+\bar{r}_0} - \left(\gamma^i - \frac{1}{\psi^i}\right)(\bar{ce}_0^{i,h})^{\gamma^i-1}\mathbb{E}_0\left(\bar{c}_1^{i,h}(\epsilon_1^i)\right)^{-\gamma^i} + \gamma^i \frac{1}{\mathbb{E}_0\left(\bar{c}_1^{i,h}(\epsilon_1^i)\right)^{-\gamma^i}}\mathbb{E}_0\left(\bar{c}_1^{i,h}(\epsilon_1^i)\right)^{-\gamma^i-1}}.$$

Differentiating the system defining uncompensated policies

$$\begin{aligned}
(1-\beta^i)(\Phi^i(\bar{l}_0^i))^{1-\frac{1}{\psi^i}}(\bar{c}_0^i)^{-\frac{1}{\psi^i}} &= \beta^i(\bar{ce}_0^i)^{\gamma^i-\frac{1}{\psi^i}}\mathbb{E}_0(\bar{c}_1^i(\epsilon_1^i))^{-\gamma^i}(1+\bar{r}_0),\\
\bar{c}_0^i + \bar{a}_0^i &= \bar{w}_0\eta^i\bar{l}_0^i + \bar{y}_0^i\\
\bar{c}_1^i(\epsilon_1^i) &= \epsilon_1^i + \bar{w}_1\eta^i + (1+\bar{r}_0)\bar{a}_0^i,\\
\bar{ce}_0^i &= \left(\mathbb{E}_0(\bar{c}_1^i(\epsilon_1^i))^{1-\gamma^i}\right)^{\frac{1}{1-\gamma^i}}
\end{aligned}$$

implies that

$$\frac{\partial \bar{c}_1^i(\epsilon_1^i;\cdot)}{\partial \bar{y}_0^i} = \frac{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^i}}{\frac{1}{\psi^i}\frac{1}{\bar{c}_0^i}\frac{1}{1+\bar{r}_0} - \left(\gamma^i - \frac{1}{\psi^i}\right)(\bar{ce}_0^i)^{\gamma^i-1}\mathbb{E}_0\left(\bar{c}_1^i(\epsilon_1^i)\right)^{-\gamma^i} + \gamma^i \frac{1}{\mathbb{E}_0\left(\bar{c}_1^i(\epsilon_1^i)\right)^{-\gamma^i}}\mathbb{E}_0\left(\bar{c}_1^i(\epsilon_1^i)\right)^{-\gamma^i-1}}.$$

Hence, making use of duality ($\bar{c}_0^i = \bar{c}_0^{i,h}$ and so on), we can more succinctly write

$$\begin{aligned}
\frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i;\cdot)}{\partial \bar{l}_0^i} &= \frac{\partial \bar{c}_1^i(\epsilon_1^i;\cdot)}{\partial \bar{y}_0^i}\left(-\psi^i\bar{c}_0^i\frac{\Phi^{i'}(\bar{l}_0^i)}{\Phi^i(\bar{l}_0^i)}\right),\\
\frac{\partial \bar{c}_1^{i,h}(\epsilon_1^i;\cdot)}{\partial \bar{r}_0} &= \frac{\partial \bar{c}_1^i(\epsilon_1^i;\cdot)}{\partial \bar{y}_0^i}\left(\psi^i\bar{c}_0^i\frac{1}{1+\bar{r}_0}\right).
\end{aligned}$$

Combining the prior results and using

$$\frac{\partial \bar{c}_1^i(\epsilon_1^i;\cdot)}{\partial \bar{y}_0^i} = (1+\bar{r}_0)\frac{\partial \bar{a}_0^i}{\partial \bar{y}_0^i}$$

and the definition of the static labor wedge in this environment

$$\bar{\tau}^{\ell_0^i} = 1 - \frac{-\bar{c}_0^i\Phi^{i'}(\bar{l}_0^i)/\Phi^i(\bar{l}_0^i)}{\eta^i\bar{w}_0^i}$$

yields the stated result in the claim. □

Appendix

A Supplemental evidence from the SCF

In this appendix we provide supplemental details on our measurement of household portfolios and MPRs using the 2016 SCF:

- Table 10 summarizes the assumptions we use on leverage and portfolio shares for each of the components of household net worth reported in the SCF.
- Table 11 decomposes the aggregate net worth of U.S. households into claims on capital and bonds using the assumptions in Table 10 and methodology in the main text.
- Figure 6 projects our binary indicator for a household being unconstrained on (binned) observables.
- Figure 7 projects our measure of households' MPR on (binned) observables.

Moment	Value	Source
Firm net leverage (except private business)	1.5	FFA, Nonfinancial corporate business, 2016
Active managed private business net leverage	1.5	FFA, Nonfinancial noncorporate business, 2016
Non-active managed private business net leverage	4	Axelson et al (2013)
Other mutual fund leverage	1.36	Ang et al (2011)
Quasi-liquid retirement account equity share	0.57	FFA, Private and public pension fund holdings, 2016
Combination mutual fund equity share	0.67	FFA, Mutual fund holdings, 2016
Other mutual fund equity share	0.67	assumed same as above
Other managed assets equity share	0.67	assumed same as above

Table 10: assumptions used to decompose household net worth in SCF

	$2016bn		
	$\sum_i \frac{1}{1+i} B^i$	$\sum_i Qk^i$	$\sum_i A^i$
1 Transaction accounts	4,940	0	4,940
2 CDs	620	0	620
3 Stock mutual funds	-3,123	9,062	5,939
4 Tax-free bond mutual funds	1,329	0	1,329
5 Govt bond mutual funds	276	0	276
6 Other bond mutual funds	404	0	404
7 Combination mutual funds	-12	769	757
8 Other mutual funds	-386	1,397	1,011
9 Savings bonds	104	0	104
10 Directly held stocks	-3,019	8,761	5,742
11 Directly held bonds	1,179	0	1,179
12 Cash value life insurance	914	0	914
13 Other managed assets	-53	3,284	3,231
14 Quasi-liquid ret assets	1,934	13,067	15,001
15 Other misc financial assets	0	659	659
16 Vehicles	0	2,717	2,717
17 Primary residence	0	24,176	24,176
18 Res RE excl primary res	0	6,301	6,301
19 Non-res RE	0	3,694	3,694
20 Actively-managed businesses	-8,538	25,552	17,015
21 Non-active-managed businesses	-6,997	9,329	2,332
22 Other misc non-fin assets	0	559	559
23 Mortgage on primary res	-8,310	0	-8,310
24 Mortgage excl primary res	-1,128	0	-1,128
25 Other lines of credit	-127	0	-127
26 Credit card balance	-316	0	-316
27 Installment loans	-1,976	0	-1,976
Vehic installment	-733	0	-733
28 Other debt	-176	0	-176
29 Total	-22,462	109,327	86,865
30 Total, excl primary residence and vehicles	-13,419	82,434	69,015

Table 11: decomposition of household net worth in SCF

Notes: observations are weighted by SCF sample weights.

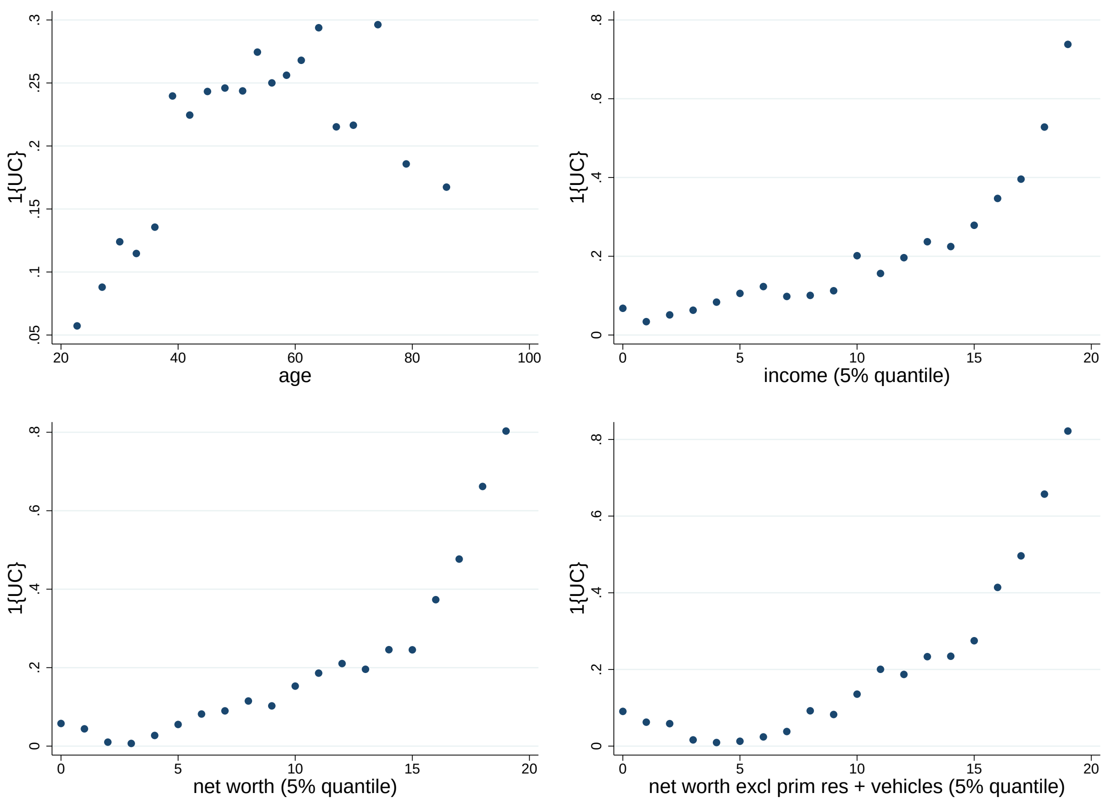

Figure 6: binary indicator for unconstrained households on binned observables

Notes: observations are weighted by SCF sample weights.

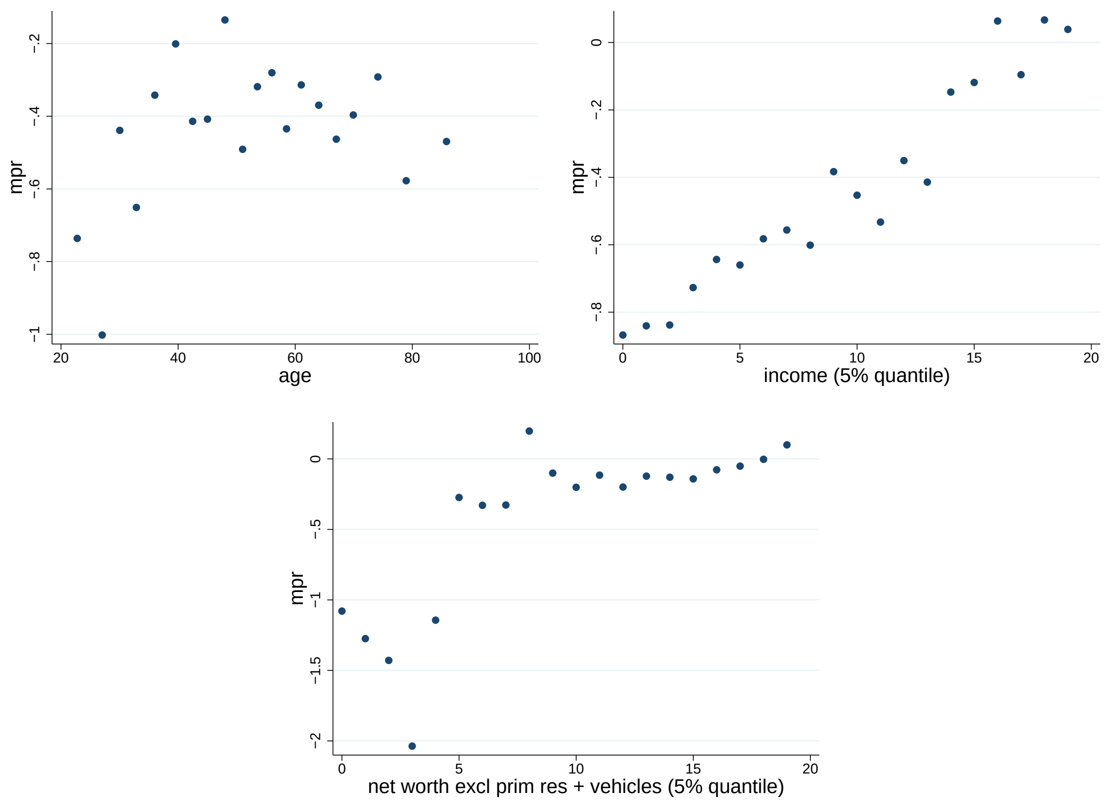

Figure 7: MPR on binned observables

Notes: observations are weighted by SCF sample weights.