Abstract

How does political accountability shape the careers of politicians? We examine a model of accountability under repeated moral hazard, and study the career dynamics of incumbent politicians under the voter-optimal equilibrium. When discounting is low, stationary equilibria are optimal and provide perfect incentives. For higher rates of discounting, however, the equilibrium path features richer dynamics under which a politician’s re-election prospects improve with good performance and deteriorate with bad performance. First-term politicians are among the most electorally vulnerable, while a politician who succeeds in office for sufficiently many terms becomes entrenched, exerting no subsequent effort and never being replaced. In our setting, entrenchment is a necessary consequence of providing politicians with optimal incentives. We therefore cannot conclude from the fact that a politician becomes entrenched that accountability was not at work.

JEL Classification Codes: C73, D72, M51.

Key words: principal-agent model; dynamic games; moral hazard; entrenchment; adverse selection; accountability

preliminary draft; subject to many changes
1 Introduction

Eric Cantor represented Virginia’s 7th Congressional district from 2001 and rose quickly to become House majority leader in his sixth term in office. He had acquired national attention for his leadership in the national Republican party, and the work he had done in Congress. He was considered to be a likely future speaker of the House, and a politician, like many before him, who would remain a fixture of Washington society until retirement or death. But in 2014, the voters of his district denied him this legacy by replacing him with his primary challenger. The shock of Cantor’s primary defeat hit hard in Washington and the national media, with Gail Collins writing in the *New York Times* that “the House majority leader was tossed out of office Tuesday in an apocalyptic, stunning, incredible earthquake of an election in Virginia that has left the nation absolutely floored in shock.”

Cantor’s career in Washington contrasts with the careers of politicians who did eventually become entrenched in office—Beltway “lifers” like Strom Thurmond, Robert Byrd, Orrin Hatch, Jerry Lewis, Joe Biden, and Charlie Rangel. These politicians were able to make it through several terms, and some even survived through some notable political scandals. Many held their seats for life, firmly secure in re-election, especially in the later part of their careers. Some have even passed their seats on to their family members, as with the Dingells of Michigan—John Dingell, Sr., who served in Congress for 22 years and was succeeded by his son, John Dingell, Jr., in 1955, who in turn was succeeded by his wife, Debbie Dingell, in 2015, now totaling 87 years of Dingells in Congress.

The level of political entrenchment that the careers of these politicians reflect is rare, however. Most politicians retire or are booted out of office sometime in the middle or early part of their careers. Data collected by Hibbing (1991) show that while incumbents do very well in general, first-term members of Congress are the most electorally vulnerable. Re-election rates among those seeking a new term are generally increasing in tenure but are relatively stable after the first term.

In this paper, we study the link between political accountability and the career dynamics of incumbent politicians by re-examining the classical Ferejohn (1986) setting of repeated moral hazard that features a (representative) voter and a set of politicians, one of whom is in office each period. In each period of the model, the politician in office privately observes the productivity level, which takes a binary value, high or low. She then chooses her effort level

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1 https://www.nytimes.com/2014/06/12/opinion/gail-collins-putting-a-cap-on-cantor.html

2 Hibbing finds that for the period 1946-1984, the re-election rate of freshman House members seeking re-election is 84%. The rate for sophomores jumps to 93% and is then relatively flat, never dropping below this rate, and climbs to 99% for those in their 14th term.
from a bounded range. Effort, along with productivity, determines performance. Under the low productivity shock, failure is certain; otherwise, performance is increasing in effort. Crucially, the voter cannot tell whether a failure was due to low productivity or to low effort. The voter then decides whether to retain the politician or to replace her with a new one, and the period ends.

We look at the voter-optimal equilibrium of this model. If the discount factor is high, then a stationary equilibrium is optimal for the voter and perfectly resolves the moral hazard problem, achieving the voter’s first-best payoff. For lower levels of the discount factor, however, the voter-optimal equilibrium is necessarily non-stationary. In each term of office, the politician faces an equilibrium-prescribed level of effort. She either succeeds—i.e., delivers an outcome consistent with high productivity and this effort level—or she fails. The consequences of success and failure depend on the continuation value with which she started her present term. Her continuation value improves with every success, and declines with every failure, with some caveats as follows.

On the path of play, the politician’s continuation value ranges in an interval from lowest to highest. Every politician starts her career in office at the lowest possible value in this range. A politician with the highest possible value can never fail, is always re-elected, and therefore exerts zero effort; such a politician has job “tenure.” Success results in certain re-election for all other values as well, and the politician starts her next term with a higher value. Failure, on the other hand, results either in removal from office, or retention at lower value. For a politician that started her current term at the lowest possible value, failure results in certain removal. For one who started her current term with a value that is higher than this but below a certain threshold, failure results in the voter removing her from office with probability that is decreasing in her value. If she is retained she will start her next term at the lowest possible value, as if restarting her career from scratch. For an untenured politician who started the current term at a value that is above the aforementioned threshold, failure results in the voter re-electing her, with a value that is lower than her current value (heralding worse future job security).

There is also another important threshold in addition to the one mentioned above. If the politician starts a term with a value that is above this threshold, then success in that term results in tenure. With enough consecutive successes her value can grow sufficiently that it crosses this key threshold. The voter-optimal equilibrium path of play is unique, and the necessary number of consecutive successes to achieve tenure is finitely bounded, so in any voter-optimal equilibrium, some politician will eventually be tenured.
We also explore three extensions of the model. In the first, we limit the degree to which politicians can become entrenched by assuming that the voter will renegotiate away from the current equilibrium path if his value falls below a pre-specified (positive) value. This assumption can be viewed as a relaxation of the strong commitment assumption that is implicit in our baseline political contract (given our focus on voter-optimal equilibrium). The higher is the payoff threshold at which the voter seeks to renegotiate away from the path, the less commitment power the voter is assumed to have. Nevertheless, we find that the voter-optimal equilibrium subject to this renegotiation threshold has very similar dynamics to that of the baseline model with the exception that no politician is ever tenured on the path of play. However, we stress that the voter’s inability to tenure a politician is harmful to his interest ex ante: Voters who cannot commit to tenure a politician face worse outcomes than those who can.

In the second extension, we allow the politician to voluntarily retire if she encounters an opportunity outside of politics that she deems sufficiently attractive. Political career dynamics in this extension are again similar to that of the baseline model, but the possibility of a sufficiently attractive outside offer can preclude the possibility of politician entrenchment in the voter-optimal equilibrium. The idea is that when a politician voluntarily leaves office, the voter benefits from starting afresh with a new politician to whom he owes nothing—for, our analysis shows that the new politician starts her career with the lowest possible continuation value on the path of play.

Finally, we consider an extension in which we add adverse selection to the baseline setting by assuming that politicians can be either “good” or “bad” types, that effort and ability are complements, and that bad types always fail. Political career dynamics in this extension are again similar to the one in the baseline model, except that the voter may now find it optimal to start a politician’s career at a continuation value that is higher than the lowest possible. The reason for this is that in the voter-optimal equilibrium, the voter cycles through politicians, removing them from office until he discovers a good type. Replacement thus comes with the cost of having to wait longer until a good type is discovered, which happens when the politician generates a success. By offering better job prospects to a newly elected politician conditional on her being a good type, the voter reduces such replacement costs.

Overall, these extensions show that even as we enrich the model with varied realistic features, political career dynamics are qualitatively similar to those in the baseline setting, with interpretable exceptions regarding the possibility of entrenchment.
Related literature—Our work connects to prior work on political careers. Past work has studied political careers mainly through models in which politicians serve only a restricted number of terms, or the relationship between voters and politicians is stationary. Ashworth (2005), for example, studies a three-period career concerns model in which politicians allocate effort across different activities, focusing on how career stage determines this allocation. Mattozzi and Merlo (2008) study a two-period model in which politicians decide whether to enter/remain in politics or work in the private sector.

Models in which politicians are unrestricted in the number of terms they can serve typically focus on stationary equilibria, following the early work by Ferejohn (1986) and Banks and Sundaram (1990, 1993). However, some important exceptions exist. Schwabe (2009), for example, allows politicians to serve an unrestricted number of terms, but restricts attention to a class of equilibria that are sub-optimal for the voter. Anesi and Buisseret (2020) also allow for an unrestricted number of terms, and study a model with adverse selection and moral hazard, but restrict attention to the case of minimal discounting. Kartik and Van Weelden (2019) study an accountability model with adverse selection, studying settings with and without term limits and focusing on a class of Markovian equilibria in the latter. Our paper relates to this prior work, but focuses on studying the relationship between optimal accountability and political careers over the long run in the case where stationary equilibria are not optimal.

Our paper also relates to prior work on repeated moral hazard, and is most closely related to work studying principal-optimal equilibria in settings where the principal is unable to finely adjust the agent’s compensation. This includes the prior work on delegation by Lipnowski and Ramos (2020), Li et al. (2017) and Guo and Hörner (2018), as well as other work in political economy including work on indirect control and war by Padró i Miquel and Yared (2012) and Yared (2010), and a recent paper by Foarta and Sugaya (2019), who look at principal-optimal equilibria in an intervention game. In these contributions, as in our work, the standard recursive toolbox developed by Spear and Srivastava (1987) and Abreu et al. (1990) facilitates the analysis of how the future terms of a relationship can substitute for monetary incentives. These techniques have been applied much more broadly with varied applications, for example in related work by Fong and Li (2017), Clementi and Hopenhayn (2006), DeMarzo and Fishman (2007), Thomas and Worrall (1990) and Atkeson and Lucas (1992), which all share with our work the feature that utility is imperfectly transferable, due to limited liability or risk aversion.

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3See Duggan and Martinelli (2017) and Ashworth (2012) for thorough surveys of this literature.
In the case of our model, the principal has only the simple choice of whether to fire or retain the agent. This choice is the main incentivizing device in many settings other than ours, where the remuneration of workers is fixed and does not vary with performance through options, bonuses, commissions, and the like.\textsuperscript{4}

2 Model

There is a representative voter and an infinite pool of politicians. Time is discrete with an infinite horizon and indexed by \(t = 0, 1, 2 \ldots\) Each period starts with a politician in office. One of the politicians begins in office at date 0. A productivity shock \(\theta_t\) is drawn independently from \(\{0, 1\}\) and privately seen by the politician in office. The high productivity shock, \(\theta_t = 1\), has probability \(\mu \in (0, 1)\) and the low productivity shock, \(\theta_t = 0\), has probability \(1 - \mu\). After seeing the productivity level, the politician chooses how much effort to supply, \(a_t \in [0, 1]\).

The voter next observes only her own payoff, the product \(y_t = \theta_t a_t\), and decides whether to retain the politician or remove her from office. Slightly abusing terminology, we will refer to \(y_t\) as the outcome of period \(t\), and often drop the time subscript when the period is clear. We denote by \(\rho_t\) the probability with which the politician is re-elected. If the voter removes the politician, she is replaced by someone from the pool, and never re-enters office in the future.\textsuperscript{5} In each period \(t\), the politician that is in office gets a payoff \(1 - ca_t\) where \(c > 0\) is the marginal cost of effort. All politicians that are not in office get 0. All individuals discount the future using a common discount factor, \(\delta \in (0, 1)\), and we will express continuation values in terms of average flow payoffs. We look at perfect public equilibria of this model that are optimal from the perspective of the voter.\textsuperscript{6}

Because any deviation by the voter is observable to all politicians, and there is an “always-shirk, always-replace” equilibrium that gives the voter a zero payoff, voter optimal

\textsuperscript{4}In fact, the salaries of low- (and sometimes even high-) level staff even in some private corporations— not just nonprofit organizations and the public sector—are set according to a pre-specified pay-schedule and cannot be (easily) adjusted. Bosses control incentives only in the extent of job security that they provide. And while many such workers, particularly public sector bureaucrats, often have almost full job security and do not face a realistic threat of being fired, they still face the threat of being transferred to undesirable postings. Though these workers may be returned to desirable positions in the future, amending our model to allow this will be without loss for the principal’s utility; see footnote 5. So, although the prominent application of our model is to positions of elected office, the model also applies to other settings in which our substantive assumptions apply.

\textsuperscript{5}The latter is without loss for voter utility. Specifically, for any equilibrium in which the a politician is re-elected after some delay, there is weakly better equilibrium in which this does not happen.

\textsuperscript{6}Fudenberg and Tirole (1991) (pp. 187-188) provide a definition of perfect public equilibria in their Definitions 5.2 and 5.3, which applies to our model.
equilibria are behaviorally equivalent to play under the voter-optimal strategy profile among those that respect politician incentives.\footnote{This follows directly when the voter uses a pure strategy. Voter incentives might still matter in principle, as the voter must be indifferent when called upon to mix. However, as our analysis establishes that such indifference will automatically be satisfied in the commitment solution, this added constraint imposes no further limits on attainable voter outcomes in this model.} We will then let an incentive-compatible (IC) policy be a strategy profile in the game described above, such that no politician has an incentive to deviate from her prescribed conditional effort choice at any history. In each period $t > 0$, we will use the term \textit{incumbent} to refer to the politician in office in the previous period. For period $t = 0$, the incumbent is the initial politician.

First, note that it is strictly dominated for a politician to work when productivity is low, as that leads to an inferior outcome in the stage game, and does not affect the public outcome of the dynamic game. Thus when productivity is low, $\theta_t = 0$, the politician in office chooses to not to exert any effort, $a_t = 0$. From now on, we will abuse notation, and interpret a politician’s choice at public history $h$ to be her choice at history $h$ conditional on a high-productivity shock in that period.

Also, note that if an equilibrium “prescribes” the politician to exert effort $a$ after a certain history $h$, then it is without loss of generality to assume that continuation play is identical for all outcomes $y \geq a$, and identical again for all outcomes $y < a$. Given the prescribed effort level $a$, we will refer to an outcome $y \geq a$ as a success and outcome $y < a$ as a failure. This implies that for an equilibrium-prescribed effort level, there are only two stage game strategies that the politician in office in a period $t$ will ever use: $a_t = 0$ and $a_t = a[h_t]$, where $a[h_t]$ is the prescribed effort level on the equilibrium path following history $h_t$.\footnote{An alternative assumption (also without loss of generality) is that if any outcome $y \not\in \{0, a\}$ is realized then the politician is replaced. The observable implications of this assumption are identical to the one we make since again the politician would thus not choose any effort levels other than 0 or $a$.} We refer to the former as the \textit{shirking} strategy, and the latter as the \textit{working} strategy. Naturally, different equilibria and different histories may lead to different prescriptions of effort along the path of play.

Finally, note that the voter’s highest feasible payoff is $\mu$. His first-best payoff is equal to $\mu \min\{1, 1/c\}$, and it obtains if and only if every politician in office exerts the highest effort subject to participation ($a = \min\{1, 1/c\}$) in every high-productivity period.

2.1 Stationary equilibria

Since it is without loss to assume that the voter’s behavior at any history is a step function in current outcome $y$ with possible discontinuity only at the prescribed effort level at that
history, a stationary equilibrium is parametrized by a stationary effort prescription \(a\), and a stationary pair of firing probabilities \(\psi_0\) and \(\psi_a\) such that the politician in office is removed with probability \(\psi_0\) following \(y < a\), and with probability \(\psi_a\) following \(y \geq a\). The politician finds it optimal to exert effort \(a_t = a\theta_t\) in all periods \(t\), and the voter’s equilibrium payoff, \(\mu a\), is increasing in the prescribed effort level.

In addition, observe that it is without loss to take \(\psi_0 = 1\) or \(\psi_a = 0\). If both \(\psi_0\) and \(\psi_a\) were interior, then raising \(\psi_0\) while lowering \(\psi_a\) so that the on-path firing rate remains constant would preserve the politician’s incentive to choose \(a\) rather than deviate to a period of shirking. As well, even if \(\psi_0 = 1\), then lowering \(\psi_a\) would strengthen the politician’s incentives, by raising the value of working without changing the value of shirking. Therefore, we may take \(\psi_a = 0\) without loss of optimality. The politician is thus retained for outcome \(a\) and fired with probability \(\psi_0\) for outcome zero. Hereafter, we will drop the subscript and let \(\psi = \psi_0\).

Facing a stationary firing rule, the politician’s best response is stationary. She either works for a value satisfying \(u_W = (1 - \delta)(1 - c\mu a) + \delta[1 - \psi(1 - \mu)]u_W\) or shirks for a value satisfying \(u_S = (1 - \delta) + \delta(1 - \psi)u_S\). Putting these together, we have

\[
\frac{u_W}{u_S} = \frac{(1 - c\mu a)(1 - \delta) + \delta\psi}{(1 - \delta) + \delta\psi(1 - \mu)}.
\]

Work is incentive compatible for the politician if and only if \(u_W/u_S \geq 1\), a condition that becomes easier to satisfy as \(\psi\) is raised. It is therefore optimal to set \(\psi = 1\). In this case, \(u_W/u_S \geq 1\) if and only if \(ca \leq \delta\). Thus the optimal \(a\) is the smaller of \(1\) and \(\delta/c\). The key findings of this analysis are as follows.

**Proposition 1.** In a voter-optimal stationary equilibrium, for a politician in office in period \(t\), we have:

1. The politician chooses effort \(a_t = a\theta_t\), where \(a = \min\{1, \delta/c\}\).

2. The politician is fired if she fails \((y_t < a)\) and retained if she succeeds \((y_t \geq a)\).

The expected equilibrium payoff to the voter is \(\mu a = \mu \min\{1, \delta/c\}\), and therefore there is a stationary equilibrium achieving the voter’s first best payoff if and only if \(\delta \geq c\).

So far, we have replicated the main features of the Ferejohn (1986) analysis, which also focuses on stationary equilibria and shows that they can be characterized by a cutoff retention rule. However, despite the environment being stationary, voter-optimal equilibria need not be stationary for values of the discount factor lower than \(c\).
2.2 Voter-optimal equilibria

In the previous section, we showed that if $\delta \geq c$, the stationary equilibrium achieves the voter’s first-best payoff, so that for this case a stationary equilibrium is a voter-optimal equilibrium. In this section, we complete the characterization of voter-optimal equilibrium by studying the case of $\delta < c$.

We take a recursive approach à la Spear and Srivastava (1987). It is immediate that there is an IC policy generating continuation value $u$ for the politician in office if and only if $u \in [1 - \delta, 1]$. The politician can guarantee herself a payoff of at least $1 - \delta$ by shirking indefinitely, and the maximum payoff of 1 can be obtained if she is always retained and exerts no effort in any period. Any value in the interval can be generated by randomizing between these two incentive compatible extremes—immediate replacement after the first term, and unconditional tenure.

Given the parameters $\delta, c$, for any $u, a$ and outcome $y$, it will be convenient for our main results to define the quantity

$$v_u(a, y) := \begin{cases} \frac{1}{\delta}[u - (1 - \delta)(1 - ca)] & \text{if } y \geq a \\ \frac{1}{\delta}[u - (1 - \delta)] & \text{if } y < a. \end{cases}$$

**Proposition 2.** Suppose that $\delta < c$. In a voter-optimal equilibrium, every first-term politician starts in office with continuation payoff $u = 1 - \delta$, and for any politician who starts any period $t$ with continuation payoff $u \in [1 - \delta, 1]$, we have:

1. The politician chooses $a_t = \theta_t a_u$, where $a_u = \min \left\{ 1, \frac{1-u}{(1-\delta)c} \right\}$.

2. Given the outcome $y$ in period $t$, if $v_u(a_u, y) \geq 1 - \delta$, then the politician is retained with probability 1 at continuation value $v_u(a_u, y)$ starting from the next period. If $v_u(a_u, y) < 1 - \delta$, she is retained with probability $v_u(a_u, y)/(1 - \delta)$ at continuation value $1 - \delta$ from the next period.

We now make a series of observations about this voter-optimal equilibrium. On the equilibrium path, the voter prescribes an effort level $a_u$ to the politician that the politician chooses when productivity is high. When productivity is low, the politician optimally chooses not to exert any effort. Note that if $u = 1$, the equilibrium-prescribed effort level is $a_u = 0$, so that the politician cannot fail. We say that a politician with continuation value $u = 1$ has tenure: She remains in office forever, independent of all current and subsequent outcomes, and therefore never exerts any effort.
Next, note that a politician that is successful is re-elected for sure, with a continuation value higher than her previous period’s continuation value. Failure, on the other hand, results either in removal from office, or retention at a continuation value that is lower than the previous period’s continuation value. In particular, if \( u = 1 - \delta \) (which is the lowest possible continuation value for a politician currently in office, and the one at which every politician starts their career), then failure results in certain removal. If \( u \in (1 - \delta, 1 - \delta^2) \) then failure results in the politician being probabilistically retained, and if she is in fact retained, then she starts the next term at value \( 1 - \delta \), as if restarting her career from scratch. If \( u \geq 1 - \delta^2 \) and she fails, then she is retained for sure at a value that is lower than the one with which she started the term, but higher than the one at which she started her career.\(^9\)

Importantly, as a politician’s continuation value climbs with an increasingly long run of successes, it is possible for her to actually achieve tenure on the path of play. In particular, whenever the politician’s continuation value is above \( 1 - (1 - \delta)c \), being successful in the current term results in tenure. This is because if \( u > 1 - (1 - \delta)c \) then the prescribed effort is \( a_u < 1 \), and hence \( v_u = 1 \) if the politician succeeds. (A related observation is that all untenured politicians that are more than one successful term away from tenure are prescribed the maximum feasible effort level of \( a = 1 \); and if a politician is only one successful term away from tenure, then they are prescribed an effort level that is either strictly less than 1 because \( u \) exceeds \( 1 - (1 - \delta)c \), or equal to 1 because \( u \) exactly equals \( 1 - (1 - \delta)c \).) The corollary below shows that with a long enough run of successes, the politician’s continuation value can indeed cross this threshold.

**Corollary 1.** Suppose that \( \delta < c \). Then the sequence of cutoffs \( \left\{ \frac{\delta}{1 - \delta^k} \right\}_{k=1}^{\infty} \) is such that, if \( \frac{\delta}{1 - \delta^k} < c \leq \frac{\delta}{1 - \delta^{k+1}} \), then in the voter-optimal equilibrium, any politician who has \( k \) successes in a row will be tenured after her next success. If \( c > \frac{\delta}{1 - \delta^k} \), any politician that produces a single success in office will be immediately tenured.

**Proof.** A straightforward recursive calculation shows that if the politician starts with continuation value \( 1 - \delta \) then following \( k \) successes her continuation value will be

\[
 u_k = 1 - \frac{1}{\delta^{k-1}} + \left( \frac{1}{\delta^k} - 1 \right) c
\]

\(^9\)For a politician that starts with value \( u \) and is successful, \( v_u - u \geq \frac{1 - \delta}{\delta} (u - 1 + c) \geq \frac{1 - \delta}{\delta} (c - \delta) > 0 \) since \( u \geq 1 - \delta \) and \( c > \delta \). Thus for every successful politician \( v_u > u \).

\(^10\)To see all this, note that as we mentioned above, if \( u = 1 \) then a politician cannot fail since \( a_u = 0 \). On the other hand, for \( u = 1 - \delta \) failure results in certain removal since it results in \( v_u = 0 \). For all other \( u \in (1 - \delta, 1) \), the politician’s next period continuation value is either \( 1 - \delta \) if \( v_u = \frac{1}{\delta} (u - 1 - \delta) < 1 - \delta \) which holds iff \( u < 1 - \delta^2 \); or it is \( v_u = \frac{1}{\delta} [u - (1 - \delta)] < u \), which holds since \( u < 1 \).
Thus \( u_k \geq 1 - (1 - \delta)c \) if and only if \( c \geq \delta/(1 - \delta^{k+1}) \). Therefore, if \( \frac{\delta}{1-\delta^{k+1}} < c \leq \frac{\delta}{1-\delta} \) then any politician with \( k \) past successes will be tenured after her next success. If \( c > \delta/(1 - \delta) \) then every politician will be tenured after her first success. ■

Because the sequence of cutoffs in the corollary converges to \( \delta \), for any value of \( c > \delta \) some politician will eventually achieve enough consecutive successes to become tenured. This result implies that the optimal way for the voter to incentivize effort in the early periods is to provide the politician with the promise of tenure. The promise is credible given that the voter can be punished for breaking the promise, for instance, if the relationship stipulates that the continuation equilibrium has every subsequent politician exert zero effort and be replaced with certainty.

The corollary puts an upper bound on the number of consecutive successes needed for the politician to achieve tenure. But, in fact, a politician may achieve tenure with fewer than this number of consecutive successes if she has a sequence of successes following a failure that results in her being re-elected with a continuation value larger than \( 1 - \delta \). As mentioned above, this case arises if the politician’s continuation value has climbed above \( 1 - \delta^2 \). That said, for this to happen, it must be that the politician has not yet been tenured prior to this point. In particular, it is possible that the tenure threshold \( 1 - (1 - \delta)c \) described above is smaller than \( 1 - \delta^2 \), in which case the politician could be tenured before she ever gets to experience a term in which she is re-elected for sure despite failing.

Figure 1 makes our observations concrete by depicting some simulated paths of the politician’s continuation value for a numerical example. The figure shows that some politicians have longer careers than others. Some achieve tenure quicker, others take longer, and still others may be removed from office before they make tenure. In the early stages of a politician’s career, failure is more likely to result in the politician being booted, or the continuation value being “reset” to \( 1 - \delta \). Failure later on is less likely to result in removal or reset, since the continuation value is more likely to be higher. A politician is more likely to serve another term the more successful she has been in the past, but even politicians who start their careers with a good run may have short careers if they are unlucky enough to experience a sequence of bad shocks.

Finally, our observations about the path of play are not merely features of a special voter-optimal equilibrium characterized in Proposition 2. In fact, these observations are strengthened by the fact that all voter-optimal equilibria are equivalent in terms of the paths of play that they generate. We prove this in the appendix by observing that the
voter’s optimal continuation payoff, as a function of the politician’s value $u$, is either affine or strictly concave on $[1 - \delta, 1]$. This then implies that the equilibrium actions solved for in Proposition 2 above are in fact uniquely optimal.

**Proposition 3.** If $\delta < c$, then every voter-optimal equilibrium generates the same distribution over the possible paths of play.

### 3 Extensions

#### 3.1 Limiting entrenchment

After a politician becomes entrenched in the baseline model, the voter’s continuation value from the relationship is zero. A common institutional approach to limiting entrenchment is to impose term limits on political office. A fixed term limit, however, is not the optimal way to protect the voter from politician entrenchment. This is obvious in the case of a one-term limit, as the politicians would always shirk since she is offered no dynamic incentives.

It is also straightforward to see in the case of a two-term limit. If $\delta < c$, then a first-term politician exerts the exact same effort level as under the best stationary equilibrium for the voter. She is retained if she attains at least that outcome, and is replaced otherwise. Therefore, the best equilibrium with a two-term limit gives her a payoff $1/(1 + \delta)$ times
that of the best stationary equilibrium, and provides a worst-case voter utility of $\delta/(1 + \delta)$ times that of the best stationary equilibrium. In a sense, the two equilibria feature the same degree of entrenchment, as all politicians in office in either of these equilibria always have a continuation value of $1 - \delta$. So imposing a two-term limit also does not permit dynamics to outperform a stationary equilibrium, either for the best-history voter value or the worst-history voter value.

Proposition 4 below will imply that the sub-optimality of imposing a fixed term limit extends, in fact, beyond these cases to any finite number of terms.

In light of these observations, we take a different approach to limiting politician entrenchment. We start from the observation that, although political entrenchment is part of the voter's optimal contract with the politician, allowing for entrenchment may be undesirable because the voter's continuation value can drop to zero on the path of play. Suppose we set a threshold $\pi$ on how low we allow the voter's utility to fall from any history. What is the form of the political contract that is optimal for the voter subject to this constraint? The next proposition provides the answer.

**Proposition 4.** Suppose an equilibrium exists such that the voter's continuation payoff after every history is at least $\pi > 0$. Then there is some $\hat{u} \in [1 - \delta, 1)$ such that in any equilibrium that is optimal for the voter subject to the constraint that the voter's continuation value after every history is at least $\pi$, (i) the continuation value at any history of any politician that is in office lies in $[1 - \delta, \hat{u}]$, (ii) every politician that enters office starts with continuation value $1 - \delta$, and (iii) if a politician starts a period $t$ with continuation value $u$, we have:

1. The politician chooses $\theta_t a_{u,\hat{u}}$, where $a_{u,\hat{u}} = \min\{1, \frac{(1-\delta)(1-u)+\delta(\hat{u}-u)}{(1-\delta)c}\}$.

2. Given the outcome $y$ in period $t$, if $v_u(a_{u,\hat{u}}, y) \geq 1 - \delta$, then the politician is retained with probability 1 at continuation value $v_u(a_{u,\hat{u}}, y)$ starting from the next period. If $v_u(a_{u,\hat{u}}, y) < 1 - \delta$, she is retained with probability $v_u(a_{u,\hat{u}}, y)/(1 - \delta)$ at continuation value $1 - \delta$ from the next period.

The proposition states that the optimal way to protect the voter from suffering low payoffs is to directly preclude entrenchment, that is, place a permanent upper bound on the degree of job security a politician can enjoy, whatever is her history in office. For any level

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11Let $\pi^*$ be the voter-optimal equilibrium payoff for the voter if politicians face a two-term limit. Since every second-term politician shirks, the voter's continuation payoff from retaining a politician is $\delta\pi^* < \pi^*$. Therefore, the voter optimally fires the politician for failure and makes IC bind, just as in the stationary equilibrium—and so (maximizing first-term effort subject to these two constraints) a first-term politician chooses the same effort level as in the best stationary equilibrium.
of voter security \( \bar{\pi} \), the theorem says, there is an “entrenchment limit” \( \hat{u} \) that could assure the voter a value of \( \bar{\pi} \) from any history.

The dynamics of a politician’s career implied by the proposition are very similar to those of the baseline model with the exception that no politician is ever tenured. If a politician ever achieves the highest possible continuation value of \( \hat{u} \), then she chooses low effort in that period, and reliably has her continuation value fall to the strictly lower value of \( [\hat{u} - (1 - \delta)]/\delta \).

Finally, it is worth noting that politician entrenchment arises as a result of the voter’s ability to commit to tenure the politician as a reward for producing successes early on in her career.\(^{12}\) Our approach to limiting entrenchment is essentially a relaxation of this commitment assumption. In particular, this approach is reminiscent of the form of renegotiation-proofness introduced by Pearce (1987) under which players are able to renegotiate away from the current equilibrium path only if their value falls below a pre-specified threshold. One important difference, though, is that Pearce’s players set this threshold to be as high as possible, whereas the above proposition applies to arbitrary feasible \( \bar{\pi} \).\(^{13}\) In fact, one can think of the extent to which the commitment assumption is relaxed as being measured by the magnitude of \( \bar{\pi} \): The higher is \( \bar{\pi} \), the more the commitment assumption is being relaxed, with the maximum possible relaxation taking place at Pearce’s bound.

### 3.2 Voluntary retirement

Many politicians voluntarily retire from office and find lucrative positions outside of politics, such as in the private sector, as lobbyists, advisers, industry executives, and board members of major corporations and organizations (see, e.g., Hall and Van Houweling, 1995). Suppose we enrich the baseline model by assuming that in each period, a politician receives with some probability the opportunity to leave their career in politics and work elsewhere. How does this possibility affect accountability and the politician’s career dynamics?

Specifically, we consider a modified model in which a politician randomly receives outside offers, giving her utility \( w > 0 \), and can thus decide to voluntarily retire from politics. The rate of arrival of these offers depends on whether the politician is in office or not. An in-office politician receives an offer with probability \( p \in (0,1) \), while an out-of-office politician receives an offer with probability \( p \in [0,p) \). Without loss of generality, we normalize \( p \) to equal zero—or, equivalently, interpret politician payoffs as being payoffs net of

\(^{12}\) We stress again, however, that Proposition 2 still describes a perfect equilibrium path of play.

\(^{13}\) Another distinction is that Pearce (1987) chiefly focuses on strongly symmetric equilibria of a symmetric game, rather than a principal-agent problem.
her continuation value after being replaced. In addition, we will restrict attention to the interesting case of $w \in [1 - \delta, 1]$. If $w < 1 - \delta$, every offer is turned down and the ability to work outside of politics has no effect on political career dynamics. If $w > 1$, every offer is accepted, and while the arrival of these offers does affect a politician’s continuation value, the politician’s career dynamics are the same as in a version of the baseline model with a lower discount factor, $\delta$, and higher cost of effort, $c$. Finally, in this extension, it will be convenient to give the players access to a public randomization device.\(^{14}\)

Each period proceeds as follows. First, the voter decides whether to re-elect the incumbent or to replace her. Then, the realization of the public random variable is revealed. The politician in office then either privately receives an outside offer of $w$ with probability $p$, or no offer with probability $1 - p$. If the politician has received an outside offer, she decides whether to accept or reject it. If she accepts the offer, a new politician arrives, and herself potentially receives an outside offer to accept or reject. After some politician in office either does not receive or does not accept an outside offer, she receives her office benefit of 1, observes the state $\theta_t$, and makes an effort choice $a_t$ at a private cost of $ca_t$. Finally, the outcome $y_t = \theta_t a_t$ is publicly observed.\(^{15}\)

**Proposition 5.** Suppose that $\delta < c$ and $1 - \delta \leq w \leq 1$. In a voter-optimal equilibrium, every first-term politician starts in office with continuation payoff $(1 - p)(1 - \delta) + pw$, and there are $v^* \in \{w, 1\}$ and $u^* \in [w, 1]$ such that the politician’s continuation payoff at the start of every period lies in the interval $[(1 - p)(1 - \delta) + pw, v^*]$, and for any politician who starts any period $t$ with continuation payoff $\tilde{u}$, we have:

1. If $\tilde{u} \leq w$, then she accepts an outside offer if it arrives, and her continuation value after the outside-offer stage (if she does not accept one) is $u = \frac{\tilde{u} - pw}{1 - p}$. If $\tilde{u} > u^*$, then she does not accept the outside offer even if one arrives, and her continuation value after the outside-offer stage is $u = w$. If $\tilde{u} \in (w, u^*)$, then with probability $\frac{u^* - \tilde{u}}{u^* - w}$, she accepts an outside offer if it arrives, and her continuation value after the outside-offer stage (if she does not accept one) is $u = w$; with probability $\frac{\tilde{u} - w}{u^* - w}$, she does not accept

\(^{14}\)As is typically the case, this assumption is useful technically, but we do not view it as substantively relevant. To interpret it in our setting, the voter and politician could, for example, react to commonly observed global events that are unrelated to the uncertain productivity changes that drive the moral hazard problem. Wolter (2002), for example, finds that voters in gubernatorial elections condition their re-election decision on economic fluctuations that are unrelated to gubernatorial actions. These fluctuations may serve as the sunspots.

\(^{15}\)Various assumptions in this setup are irrelevant. For example, results are identical if the politician’s offer is observed by the voter; if the sunspot realizes at the end of the period; if a newly elected politician cannot condition on a sunspot; or if a newly elected politician cannot receive an outside offer.
the outside offer even if it arrives, and her continuation value after the outside-offer stage is \( u = u^* \).

2. Given the politician’s continuation value \( u \) after the outside-offer stage, she chooses
\[ a_t = \theta_t a_{u,v^*}, \]
where \( a_{u,v^*} := \min \{ 1, \frac{(1-\delta) + \delta v^* - u}{(1-\delta)c} \} \).

3. Given the outcome \( y \) in period \( t \), if \( v_u(a_{u,v^*}, y) \geq (1-p)(1-\delta) + pw \), then the politician is retained with probability 1 at continuation value \( v_u(a_{u,v^*}, y) \) starting from the next period. If \( v_u(a_{u,v^*}, y) < (1-p)(1-\delta) + pw \), she is retained with probability \( v_u(a_{u,v^*}, y)/(1-p)(1-\delta) + pw \) at continuation value \( (1-p)(1-\delta) + pw \) from the next period.

The proposition characterizes a voter-optimal equilibrium up to two details: First, is the highest admitted continuation value for a politician \( w \) or 1? Second, if it is the latter, then how big is the interval \((w, u^*)\) on which the politician mixes between taking and leaving the outside offer?

When the highest possible continuation value for the politician is 1, effort decisions are the same as in the main model conditional on the politician’s continuation value. The politician and voter use the public randomization device only to coordinate on continuation play in the interval above \( w \) on which the politician mixes between taking and leaving the outside offer. In this case, note that the voter has a discrete benefit of having the politician leave with an outside offer and starting afresh with a new politician to whom she owes nothing.

When the politician’s highest possible continuation value is \( w \), effort is reined in, just as in the extension on limiting entrenchment, to keep the politician’s value from exceeding \( w \). This way, the voter benefits from the politician taking the outside offer whenever it arrives, again due to the benefit of getting to start with a new politician who enters office at the lowest possible continuation value on the equilibrium path. Here, because values stay below \( w \), the public randomization device is never used.

As Hall and Van Houweling (1995) have noted, politicians that are particularly electorally insecure are more likely to retire, perhaps a strategic choice on their part. This is also true in our extension since the probability of replacement (i.e. the politician’s electoral insecurity) is decreasing in the politician’s continuation value from staying in politics.

Finally, note that when \( v^* = w \) politicians never get entrenched, and they always accept the outside offer. But when is this the case? The next proposition establishes that is a sufficient condition is that \( w < 1 \) is high enough.
Proposition 6. Suppose that $\delta < c$. If $w < 1$ is sufficiently high, then a voter-optimal equilibrium exists such that every politician has a hazard rate of at least $p$ of leaving office in any given period. In particular, no politician is ever fully entrenched.

Holding the politician to maximal continuation value $w$ rather than 1 entails a cost of reducing short-term effort that can be sustained, but entails a benefit of allowing the voters a fresh start with a new politician rather than retaining an entrenched one. To see why $w \approx 1$ should be sufficient, then, note that the incentive cost is very small in this case, while the benefit is bounded away from zero.

3.3 Adding adverse selection

We now enrich the model by assuming that each politician has a perfectly persistent skill level $\omega \in \{0, 1\}$, which only she knows as of her first term in office. Skill is independent across politicians and takes the high value of 1 with probability $q \in (0, 1)$. The politician’s payoffs are as before, but the outcome in each period $t$ produced by a type $\omega$ office-holder is $y_t = \omega a_t \theta_t$. Thus, low skilled politicians are not able to produce successes. An important consequence of this assumption is that skill and effort are complements, and therefore the usual tension between selecting good types and sanctioning poor performance highlighted by Fearon (1999) will not be at play.\textsuperscript{16}

As in the previous extension, it will be convenient for this extension as well to assume that the voter and politicians have access to a public correlation device.\textsuperscript{17} So we will assume that a uniformly distributed random variable realizes after the politician’s effort decision but before the voter’s retention decision.

A starting observation is that stationary play is again suboptimal if $\delta < c$, as in the baseline model. In particular, there are now two reasons for dynamics under a voter-optimal equilibrium—resolving moral hazard as in our baseline model, and a learning problem that is itself inherently dynamic.

Another observation is that learning in this model features, in the terminology of the strategic experimentation literature, “perfectly revealing good news.” Under this special structure, the politician, after her first term in office, is either known to be the high skill type (due to a past success) or is less likely to be a high skill type than anyone from the pool

\textsuperscript{16}Ashworth et al. (2017) show that whether there is a tradeoff between sanctioning and selection depends on whether skill and effort are complements or substitutes. Duggan and Martinelli (2015) study the features of this tradeoff when skill and effort are substitutes.

\textsuperscript{17}Footnote 21 in the appendix explains how we use the public randomization device in this extension.
of challengers. A feature of the baseline model that carries over to this model is, therefore, that if a politician fails in her first term, then she is replaced.

**Lemma 1.** In a voter-optimal equilibrium, any politician who fails in her first term is removed from office.

The observation implies that the same methods that we used to characterize the voter-optimal equilibrium in the baseline setup can be adopted here as well. Other than a politician’s first term in office, she is known to be good. Therefore, the learning problem does not make an appearance in the recursive analysis.

**Proposition 7.** Suppose $\delta < c$. There exist $u^* \in [1 - \delta, 1]$ such that in the voter-optimal equilibrium:

1. In her first term in office, a politician of type $\omega$ chooses $\omega a_\omega$ if the state is $\theta$, where $a_\omega := \min\{1, \frac{1}{(1-\delta)c}\}$.

2. If a politician fails in their first term in office ($y_0 < a_\omega$) then she is replaced, and if she succeeds ($y_0 \geq a_0$) then she is retained with continuation value $u^*$. Thus, her continuation payoff at the start of her career is $u_0 = (1 - \delta)(1 - \mu c a_\omega) + \mu \delta u^*$ if she is of high skill and $1 - \delta$ if she is of low skill.

3. Whenever a politician who has previously produced a success starts a term in office with continuation value $u \in [1 - \delta, 1]$, then like in the baseline model:

   (a) The politician chooses $\theta a_u$ when the state is $\theta$, where $a_u := \min\{1, \frac{1-u}{(1-\delta)c}\}$.

   (b) Given the outcome $y$ in period $t$, if $v_u(a_u, y) \geq 1 - \delta$, then the politician is retained with probability 1 at continuation value $v_u(a_u)$ from the next period. If $v_u(a_u, y) < 1 - \delta$, then the politician is retained with probability $v_u(a_u, y)/(1 - \delta)$ at continuation value $1 - \delta$.

The proposition implies that as long as the voter does not know the type of the politician in office, he cycles through politicians until he discovers one that is good. Since bad types always fail, this discovery happens the moment a politician succeeds. After that point, the good politician’s career is similar to that of the baseline model without adverse selection, with one important difference: It may be optimal for the voter to start the politician’s career at a continuation value that is higher than $1 - \delta$ or, equivalently, to provide a newly elected politician strict incentives to work. The reason is an effective replacement cost. When the voter boots the incumbent out of office, he gets a flow payoff of zero until he draws a skilled
type. Since this may take some time, removing a politician that is known to be good is costly to the voter. The following proposition provides a sufficient condition for \( u_0 > 1 - \delta \).

**Proposition 8.** If \( \delta < c \) is close enough to \( c \), and \( \mu \) and \( q \) are small enough, then \( u^s > c(1 - \delta)/\delta \) so that \( u_0 > 1 - \delta \), where \( u^s \) and \( u_0 \) are the quantities in Proposition 7.

The proposition concerns which continuation values for the politician best serve the voter. If that value is low, the politician will likely soon be fired. If it is high, she will likely soon be tenured. The voter-optimal continuation value for the politician optimally resolves this tradeoff. In particular, the proposition provides sufficient conditions for this optimal value to be strictly greater than \( c(1 - \delta)/\delta \), that is, higher than the continuation value of a politician who, in the previous period, was to be fired for a failure, was made indifferent between maximum and minimum effort, and was successful.

To obtain such sufficient conditions, the proposition considers the case that \( q \) and \( \mu \) are low. For low values of \( q \), the cost of firing the politician is high, as it is unlikely the voter will discover a high-skill politician soon afterwards. For low values of \( \mu \), the risk of having to soon provide the politician with tenure (with the costs to the voter that this entails) is also low, as the likelihood of a success—even if the politician is of high quality—is low. Therefore, at least in the range where the politician exerts full effort, the voter prefers the politician’s continuation value to be as high as possible, as this provides more periods of productive cooperation.

Finally, if \( \delta \) is close to \( c \), then the range in which the politician exerts full effort will have \( c(1 - \delta)/\delta \) in its interior. When this is the case, it is possible to give a skilled politician strict incentives to work in the first term and still incentivize maximal effort in the next. This makes it sufficient for the “one-success” continuation payoff to be less than the voter would want the politician to have when the cost of firing is high and risk of tenure is low. The upshot is that under the conditions of Proposition 8, the working strategy is strictly incentive compatible for a high-skilled first-term politician.

**Appendix**

**A Proofs**

**A.1 Proof of Proposition 2**

For any continuation value \( u \in [1 - \delta, 1] \) for the incumbent politician, let \( \pi(u) \) denote the voter’s optimal continuation value among all feasible contracts that give the politician a
value of \( u \). For any average continuation value \( v \in [0, 1] \) for the politician to have starting in the following period, let \( \tilde{\pi}(v) \) denote the voter’s optimal continuation value among all feasible contracts that give the agent an average value of \( v \). These continuation values for the voter are defined by the following Bellman equation:

\[
\pi(u) = \Phi \tilde{\pi}(u) := \sup_{a \in [0, 1], \, v_s, v_f \in [0, 1]} \left( 1 - \delta \right) \mu a + \delta \left[ \mu \tilde{\pi}(v_s) + (1 - \mu) \tilde{\pi}(v_f) \right]
\]

subject to \( u = (1 - \delta)(1 - \mu c a) + \delta [\mu v_s + (1 - \mu) v_f] \) \hspace{1cm} (PK)

and \( (1 - \delta)1 + \delta v_f \leq (1 - \delta)(1 - c a) + \delta v_s \); \hspace{1cm} (IC)

\[
\tilde{\pi}(v) = \tilde{\Phi} \pi(v) := \sup_{\rho \in [0, 1], \, u, u_0 \in [1 - \delta, 1]} \rho \pi(u) + (1 - \rho) \pi(u_0)
\]

subject to \( \rho u = v \); \hspace{1cm} (PK)

where \( \Phi, \tilde{\Phi} \) are operators on the functions \( \tilde{\pi} \) and \( \pi \) respectively. The first of the two constraints defining \( \pi \) above is the voter’s promise-keeping (PK) constraint, saying that the politician’s current continuation value is indeed \( u \) if she follows the prescribed action and the continuation values following success and failure are \( v_s \) and \( v_f \), respectively. The second is the incentive-compatibility (IC) constraint, saying that the politician would rather follow her prescribed strategy than engage in the one-time deviation of shirking today. For the latter, the politician trades off the myopic benefit of shirking following a good shock against the expected gain in future value from playing her on-path action rather than failing. There is no incentive compatibility constraint in the definition of \( \tilde{\pi} \) because the firing choice is fully under the voter’s control, and he can commit. But there is the promise-keeping (PK) constraint that says that the politician’s average continuation value is a combination of the zero value she gets from being fired and the positive value she gets from being retained.

Standard arguments can be used to show that an optimal policy exists; that the value it generates to the voter, as a function of the politician’s continuation value, is the unique solution to the above Bellman equation; and that \( \pi \) and \( \tilde{\pi} \) are both continuous and concave.\(^\dagger\)

With these observations, we proceed to prove the proposition.

\(^\dagger\)These arguments proceed as follows. The voter’s optimal payoff function \( \pi : [1 - \delta, 1] \rightarrow \mathbb{R} \) must satisfy the Bellman equation \( \pi = \Phi \tilde{\pi} \). As for the operators \( \Phi \) and \( \tilde{\Phi} \) between the spaces of real bounded functions on \([0, 1]\) and \([1 - \delta, 1]\) (the latter being viewed as metric spaces with respect to uniform convergence), it is straightforward to see that \( \Phi \) is a contraction of modulus \( \delta \), that \( \tilde{\Phi} \) is a weak contraction, and that both take concave continuous functions to concave continuous functions. The limit of concave continuous functions is itself concave and continuous, so the voter’s optimal value functions, \( \pi \) and \( \tilde{\pi} \), yield a unique solution to the Bellman equation, and both are concave and continuous.
We start by observing that there is some $v_s \in [1 - \delta, 1]$ such that $\pi$ is affine on $[0, v_s]$ and coincides with $\pi$ on $[v_s, 1]$. Indeed, the Bellman equation tells us that, for a given $v \in [1 - \delta, 1]$, we have $\tilde{\pi}(v) = \pi(v)$ if and only if $u \pi(v) \geq v \pi(u) + (u - v)\tilde{\pi}(0)$ for every $u \in (v, 1]$. But this condition rearranges to the requirement that

$$\pi(v) \geq \tilde{\pi}(0) + v\frac{\pi(u) - \pi(v)}{u - v}, \quad \forall u \in (v, 1].$$

Since $\pi$ is concave, the inequality is more stringent the closer $u$ is to $v$. Therefore, $\tilde{\pi}(v) = \pi(v)$ if and only if either $v = 1$ or $\pi(v) \geq \tilde{\pi}(0) + v \pi'(v^+)$. Again because $\pi$ is concave, it now follows that the set of $v$ at which $\tilde{\pi}(v) = \pi(v)$ is of the form $[v_s, 1]$ for some $v_s \in [1 - \delta, 1]$. If then follows that, for any $v \in [0, v_s]$, the program defining $\tilde{\pi}(v) = \tilde{W}\pi(v)$ has $u = v_s$ as an optimum, so that $\tilde{\pi}$ is affine on $[0, v_s]$.

Now, concavity of $\tilde{\pi}$ tells us that $v_s$ and $v_f$ are chosen to make (IC) hold with equality, except perhaps in the case that $v_s = v_f$. If neither of these conditions held, we could modify the contract to one of this form without loss, bringing $v_s$ and $v_f$ closer together, holding $\alpha$ fixed, and maintaining (PK), for a weakly higher value. But even in the case that $v_s = v_f$, (IC) implies $\alpha = 0$. Therefore, we can restrict attention to the case that (IC) holds with equality. Then combining the (IC) equation with (PK) immediately gives a solution for $v_s$ and $v_f$. Specifically,

$$v_s = \underline{\pi}_a := \frac{1}{\delta}[u - (1 - \delta)(1 - ca)] \quad \text{and} \quad v_f = u := \frac{1}{\delta}[u - (1 - \delta)]$$

Finally, since every $u \geq 1 - \delta$ and $a \in [0, 1]$, we have $0 \leq u \leq \underline{\pi}_a$. It follows that $u, \underline{\pi}_a \in [0, 1]$ if and only if $\underline{\pi}_a \leq 1$. Summarizing these observations yields

$$\pi(u) = \Phi \tilde{\pi}(u) = \max_{a \in [0, 1]} (1 - \delta)\mu a + \delta [\mu \tilde{\pi}(\underline{\pi}_a) + (1 - \mu)\tilde{\pi}(u)] \quad \text{subject to } \underline{\pi}_a \leq 1.$$  

Since $\underline{\pi}_a$ is an increasing and affine function of $a$, and $\tilde{\pi}$ is concave, the objective in the maximization problem above is concave in $a$. Moreover, its derivative with respect to $a$ is simply $(1 - \delta)\mu[1 + c\tilde{\pi}'(\underline{\pi}_a)].$\footnote{In general $\tilde{\pi}$ may fail to be differentiable. But since it is concave, it has one-sided derivatives defined everywhere, and these are sufficient to characterize the optimal $a$ via a first-order condition. Whenever we refer to a derivative in this appendix without specifying a direction, we mean that the relevant claim applies to each one-sided derivative.} Now, let $v^* = 1$ if $\tilde{\pi}'(1) \geq -1/c$, and otherwise, let $v^*$ be the highest $v \in [0, 1]$ such that $\tilde{\pi}'|_{[0,v]} \geq -1/c$. By definition, notice that this means $\tilde{\pi}$ cannot be affine in a neighborhood of $v^*$, which in turn implies that $v^* \geq v_s$.  

21
The values of $u$ such that $\bar{u}_1 = v^*$ and $\underline{u} = v^*$ are, respectively,

$$u_L := 1 - \delta(1 - v^*) - (1 - \delta)c \quad \text{and} \quad u_R := 1 - \delta(1 - v^*).$$

In addition, the value of $a$ such that $\bar{a} = v^*$ is $a = [(1 - \delta) + \delta v^* - u] / [(1 - \delta)c]$. Therefore, the politician’s optimal action takes the form

$$a_u = \begin{cases} 
1 & \text{if } u < u_L, \\
\frac{(1-\delta)+\delta v^*-u}{(1-\delta)c} & \text{if } u \in [u_L, u_R], \\
0 & \text{if } u > u_R.
\end{cases}$$

Substituting the optimal choice of current action into the Bellman equation and differentiating gives us

$$\pi'(u) = \begin{cases} 
\mu \bar{\pi}'(\bar{u}_1) + (1 - \mu)\bar{\pi}'(\bar{u}) & \text{if } u < u_L, \\
\mu \left(-\frac{1}{c}\right) + (1 - \mu)\bar{\pi}'(\bar{u}) & \text{if } u \in (u_L, u_R), \\
\bar{\pi}'(\bar{u}) & \text{if } u > u_R.
\end{cases}$$

If $v^*$, the formula for $a_u$ will then follow directly. Assume now, for a contradiction, that $v^* < 1$. Then $v^* < u_R$, and every $u \in [v^*, u_R)$ has

$$\bar{\pi}'(u) = \pi'(u) = \mu \frac{-1}{c} + (1 - \mu)\bar{\pi}'(\bar{u}) \geq \frac{-1}{c},$$

where the first equality follows from $u > u_R \geq v^* \geq v_*$, and the inequality follows from $\underline{u} < v^*$. But then $\bar{\pi}'|_{[v^*, u_R)} \geq \frac{-1}{c}$, contradicting the definition of $v^*$. Thus, $v^* = 1$.

Finally, we verify that $v_* = 1 - \delta$, so that the voter’s optimal retention rule will be as desired. To see this, it is enough to observe that both operators $\Phi, \bar{\Phi}$ clearly take nonincreasing concave functions to nonincreasing functions (in fact, $\bar{\Phi}$ takes every concave function to a nonincreasing function), and that a limit of nonincreasing functions is itself nonincreasing. The contraction property then tells us that $\pi$ is nonincreasing. It follows that, in the optimization defining $\bar{\pi}(v) = \Phi \pi(v)$, the voter optimally takes $u = \max\{1-\delta, v\}$, i.e. takes $u \in [1 - \delta, 1]$ as small as possible subject to $(P\bar{K})$.

\hspace{1cm}$\Box$

A.2 Proof of Proposition 3

Let us establish the result for two different cases. First, suppose $\delta \leq c/(1 + c)$, so that some voter-optimal equilibrium entails $a_u =$ being played at every $u \in [1 - \delta, 1]$. By direct
computation, \( \pi \) is affine and \( \tilde{\pi} \) is constant on \([0, 1 - \delta]\) and affine on \([1 - \delta, 1]\), since the Bellman operators \( \Phi, \tilde{\Phi} \) preserve this property. But \( \pi \) is affine, nonnegative, not globally 0, and takes value 0 value at 1, implying it is strictly decreasing. Therefore, the voter retention rule that does as little firing as possible subject to promise keeping is uniquely optimal. All that remains is to show it is uniquely optimal to have a politician choose \( a_{1 - \delta} \) when in office at continuation value \( 1 - \delta \). To that end, note that any strict incentive to work would give the politician a value strictly above \( 1 - \delta \), as would a strictly positive continuation value from shirking. Therefore, it must be that her incentive constraint is binding and she is fired for failure. Rearranging the promise keeping and incentive constraints, and monotonically transforming the objective, the voter-optimal effort level for a newly elected politician therefore solves

\[
\max_{a,v_s \in [0,1]} (1 - \delta)\mu a + \delta [\mu \tilde{\pi}(v_s)]
\]

subject to \( 1 - \delta = (1 - \delta)(1 - ca) + \delta v_s \)

But the objective is affine on its one-dimensional domain, strictly positive at the maximum feasible effort level \( a_{1 - \delta} \), and takes value 0 at the minimum feasible effort level of 0. It is therefore uniquely maximized at \( a = a_{1 - \delta} \), as desired.

We now turn to the \( c/(1 + c) < \delta < c \) case. Note that uniqueness follows if \( \pi \) is strictly concave on its entire domain. To see this, begin with the observation that the optimal politician action \( a_u \) that we solved for is unique for every \( u \in [1 - \delta, 1] \) since it derives from maximizing a strictly concave objective over a convex domain. Optimality requires that the constraint (IC) hold with equality, because \( \tilde{\pi} \) is weakly concave on its domain, and \( \tilde{\pi} \) is not affine between \( u \) and \( \tilde{\pi}_{a_u} \) for any \( u \in [1 - \delta, 1) \). Finally, \( \pi \) is strictly decreasing because it is nonincreasing and strictly concave, so that firing as infrequently as \( (PK) \) allows is uniquely optimal for the voter.

We now argue that \( \pi \) is strictly concave when \( c/(1 + c) < \delta < c \). Following the proof of Lemma 2 in Guo and Hörner (2018), it follows that \( \pi \) is either affine or strictly concave when \( \delta < c \). But observe that, for \( c/(1 + c) < \delta < c \) and \( u \in (0, 1 - \delta] \) sufficiently small,

\[
\pi(u) = (1 - \delta)\mu + \delta \left( (1 - \mu)\tilde{\pi} \left( \frac{u - (1 - \delta)}{\delta} \right) + \mu \tilde{\pi} \left( \frac{u - (1 - \delta)(1 - c)}{\delta} \right) \right)
\]

\[
= (1 - \delta)\mu + \delta \left( (1 - \mu)\pi(1 - \delta) + \mu \pi \left( \frac{u - (1 - \delta)(1 - c)}{\delta} \right) \right)
\]

We now turn to the \( c/(1 + c) < \delta < c \) case. Note that uniqueness follows if \( \pi \) is strictly concave on its entire domain. To see this, begin with the observation that the optimal politician action \( a_u \) that we solved for is unique for every \( u \in [1 - \delta, 1] \) since it derives from maximizing a strictly concave objective over a convex domain. Optimality requires that the constraint (IC) hold with equality, because \( \tilde{\pi} \) is weakly concave on its domain, and \( \tilde{\pi} \) is not affine between \( u \) and \( \tilde{\pi}_{a_u} \) for any \( u \in [1 - \delta, 1) \). Finally, \( \pi \) is strictly decreasing because it is nonincreasing and strictly concave, so that firing as infrequently as \( (PK) \) allows is uniquely optimal for the voter.
and therefore,
\[ \pi'(u) = (1 - \mu)\pi' \left( \frac{u - (1 - \delta)(1 - c)}{\delta} \right) \neq \pi' \left( \frac{u - (1 - \delta)(1 - c)}{\delta} \right). \]
So \( \pi \) cannot be affine, and therefore must be strictly concave. \( \Box \)

A.3 Proof of Proposition 4

Let \( \mathcal{U} \) be the set of all incumbent politician continuation values from any history in an equilibrium at which the voter’s continuation payoff is at least \( \pi \) after every history, and let \( \hat{u} \in [1 - \delta, 1] \) be the supremum of \( \mathcal{U} \).

The alternative equilibrium we now consider is the voter’s optimal equilibrium among all those that always yield an incumbent politician value in \( [1 - \delta, \hat{u}] \). Essentially the exact same proof as that of Proposition 2 shows that the form described in the statement of this proposition is an optimal such contract. In particular, notice, the modified limit on \( a_{u, \hat{u}} \) is chosen to ensure \( \bar{\pi}_{a_{u, \hat{u}}} \leq \hat{u} \). Let \( \pi_{\hat{u}} : [1 - \delta, \hat{u}] \to \mathbb{R} \) denote its induced value function, so \( \pi_{\hat{u}}(u) \) is the voter’s optimal value over all equilibria which (i) give the incumbent a continuation value of \( u \), and (ii) never give any incumbent politician a continuation value greater than \( \hat{u} \). Again as in the proof of Proposition 2, the function \( \pi_{\hat{u}} \) is continuous and nonincreasing.

That the original equilibrium respected the threshold \( \pi \) implies, given optimality of the modified equilibrium, that \( \pi_{\hat{u}}(u) \geq \pi \) for every \( u \in \mathcal{U} \). But then, that \( \pi_{\hat{u}} \) is continuous implies \( \pi_{\hat{u}}(\hat{u}) \geq \pi \), and that it is nonincreasing implies \( \pi_{\hat{u}} \geq \pi \), as required. \( \Box \)

A.4 Proof of Proposition 5

The proof of Proposition 5 follows similar lines to the proof of Proposition 2. We begin by defining optimal value functions for the voter as a function of the politician’s continuation value. Because the stage game has more components, it is now convenient to define three value functions rather than two.

First, for any continuation value \( u \in [1 - \delta, 1] \) for a politician who has either just turned down or not received an outside offer, let \( \pi(u) \) denote the voter’s optimal continuation value among all feasible contracts that give the politician a value of \( u \). For any continuation value \( \hat{u} \in [pw + (1 - p)(1 - \delta), 1] \) for the politician to have after seeing the realized public randomization outcome, but before learning whether an outside offer realized or not, let \( \hat{\pi}(\hat{u}) \) denote the voter’s optimal continuation value among all feasible contracts that give the politician a continuation value of \( \hat{u} \). Note that the optimal value of the voter when
a politician has continuation value \( \tilde{u} \) before the public randomization device realizes is therefore \( \text{cav} \tilde{\pi}(\tilde{u}) \), where \( \text{cav} \tilde{\pi} \) is the concave envelope of \( \tilde{\pi} \). Finally, after the current period’s outcome \( y \) has realized, for any average continuation value \( v \in [0, 1] \) for a politician to have starting in the following period, let \( \tilde{\pi}(v) \) denote the voter’s optimal continuation value among all feasible contracts that give the agent an average value of \( v \).

Now consider the following system of Bellman equations:

\[
\pi(u) = \Phi \tilde{\pi}(u) := \sup_{a \in [0,1], v_s, v_f \in [0,1]} (1 - \delta)\mu a + \delta [\mu \tilde{\pi}(v_s) + (1 - \mu) \tilde{\pi}(v_f)]
\]

subject to

\[
u = (1 - \delta)(1 - \mu ca) + \delta [\mu v_s + (1 - \mu )v_f] \quad \text{(PK)}
\]

and

\[
(1 - \delta)ca \leq \delta(v_s - v_f); \quad \text{(IC)}
\]

\[
\tilde{\pi}(\tilde{u}) = \tilde{\Phi}(\tilde{\pi}, \tilde{\pi})(v) := \sup_{\lambda \in [0,1], u \in [1-\delta,1], \tilde{u}_0 \in [pw+(1-p)(1-\delta),1]} (1 - p\lambda)\pi(u) + p\lambda \text{cav} \tilde{\pi}(\tilde{u}_0)
\]

subject to

\[
\tilde{u} = (1 - p\lambda)u + p\lambda w, \quad \text{(PK)}
\]

\[
\lambda = 1 \text{ if } u < w \text{ and } \lambda = 0 \text{ if } u > w; \quad \text{(IC)}
\]

\[
\tilde{\pi}(v) = \tilde{\Phi}(v) := \sup_{\rho \in [0,1], \tilde{\pi}(\tilde{u}) \in [pw+(1-p)(1-\delta),1]} \rho \text{cav} \tilde{\pi}(\tilde{u}) + (1 - \rho)\text{cav} \tilde{\pi}(\tilde{u}_0)
\]

subject to

\[
v = p\tilde{u}. \quad \text{(P\text{K})}
\]

Let us explain this system. The operator \( \Phi \), describing optimal incentive provision for politician effort, is exactly as in the proof of Proposition 2. The operator \( \tilde{\Phi} \), describing the optimal retention rule for a given politician continuation value is essentially identical to that from Proposition 2, with two small differences. First, the possibility of an outside offer raises the minimum possible continuation value of a retained politician—it is now \( \tilde{u}_L := (1 - p)(1 - \delta) + pw \) rather than only \( 1 - \delta \). Second, because play can condition on a sunspot after a given politician is elected or re-elected, continuation values are delivered via \( \text{cav} \tilde{\pi} \) rather than \( \tilde{\pi} \). Now, we turn to the operator \( \tilde{\Phi} \). The associated promise-keeping condition (P\text{K}) expresses the politician’s continuation value \( \tilde{u} \) as a weighted average of \( w \) and \( u \), where \( w \) is her continuation value if she accepts an outside offer of \( w \), and \( u \) is her continuation value in the complementary case. Letting \( \lambda \) denote her (chosen) probability of leaving for the private sector conditional on receiving such an offer, the total probability of receiving and accepting an offer is \( p\lambda \). As a politician who contemplates leaving office

---

\[\text{cav} f \] is the pointwise smallest concave function \([a, b] \to \mathbb{R}\) that is pointwise above \( f \).
is comparing value $w$ to $u$, it follows that she will willingly accept [resp. reject] an outside offer if and only if $w \geq [\leq] u$, which justifies the incentive-compatibility condition (IC).

We proceed in a similar fashion to the proof of Proposition 2. First, we note standard recursive arguments show that an optimal policy exists and that the value it generates to the voter, as a function of the politician’s continuation value, is the unique bounded solution to the above system of Bellman equations. Furthermore, as all three operators above take weakly decreasing functions to weakly decreasing functions and take upper semicontinuous to upper semicontinuous functions, the four functions $\pi$, $\hat{\pi}$, cav $\hat{\pi}$, and $\tilde{\pi}$ are all weakly decreasing and upper semicontinuous. That they are weakly decreasing implies that setting $\tilde{u}_0 = \tilde{u}_L$ is optimal in the optimizations determining $\hat{\pi}(\tilde{u})$ and $\tilde{\pi}(v)$. Also, since the concave envelope of any upper semicontinuous function on a compact interval is concave and continuous, it follows that $\tilde{\Phi}$ takes any upper semicontinuous function to a concave and continuous one, so that that $\tilde{\pi}$ and $\pi$ are concave and continuous.

Next, because cav $\hat{\pi}$ is concave and weakly decreasing, just as in the proof of Proposition 2, it is optimal in the program determining $\tilde{\pi}(v)$ to set the re-election probability $\rho$ as high as the promise-keeping constraint will allow. Then, without loss of optimality, the incentive-compatibility condition (IC) in the equation defining $\pi$ holds with equality, for because moving $(v_f, v_s)$ closer together while satisfying the promise-keeping condition will weakly improve the voter’s objective by inducing a mean-preserving contraction over continuation values evaluated via the concave function $\tilde{\pi}$. Combining binding incentive compatibility with promise keeping yields exact formulas for the success and failure continuation values as a function of the action taken—$v_f = u$ and $v_s = \bar{u}_a$, where both quantities are given in the proof of Proposition 2—with the only constraint on the action being that $\bar{u}_a \leq 1$.

We make three additional simplifying observations. First, cav $\hat{\pi}$ necessarily agrees with $\hat{\pi}$ at $\tilde{u}_L := \tilde{u}_L = pw + (1 - p)(1 - \delta)$, which is an extreme point of the two functions’ domain. Second, that setting $\tilde{u}_0 = u = w$ is feasible in the program determining $\hat{\pi}(w)$ implies cav $\hat{\pi}(\tilde{u}_L) \geq \pi(w)$, so that setting $\lambda = 1$ is optimal when $\tilde{u} = w$ (i.e. for the one value of $\tilde{u}$ at which (IC) does not determine $\lambda$). Intuitively, because the voter can, at worst, continue the same play with a newly elected politician, it always weakly benefits him when the politician takes her outside offer. Third, applying what we have learned to $\hat{\pi}(\tilde{u}_L)$ yields the equation $\hat{\pi}(\tilde{u}_L) = (1 - p)\pi(\frac{\tilde{u}_L - pw}{1 - p}) + p\hat{\pi}(\tilde{u}_L)$, which can be rearranged to $\hat{\pi}(\tilde{u}_L) = \pi(1 - \delta)$.  

26
Collecting the above observations yields a simplified system of Bellman equations:

\[ \pi(u) = \sup_{a \in [0, 1]} (1 - \delta)u + \delta [\mu \hat{\pi}(\bar{u}_a) + (1 - \mu)\hat{\pi}(\bar{u})] \]

subject to \( \bar{u}_a \leq 1; \)

\[ \hat{\pi}(\bar{u}) = \begin{cases} (1 - p)\pi\left(\frac{\bar{u} - pw}{1 - p}\right) + p\pi(1 - \delta) & \text{if } \bar{u} \leq w \\ \pi(\bar{u}) & \text{if } \bar{u} > w; \end{cases} \]

\[ \hat{\pi}(v) = \text{cav } \hat{\pi}\left(\max\{v, \bar{u}_L\}\right). \]

We now detail the circumstances under which public randomization is used. To that end, note that \( \pi \) being concave and continuous implies the restrictions \( \hat{\pi}|_{[\bar{u}_L, w]} \) and \( \hat{\pi}|_{(w, 1]} \) are both concave and continuous as well.

Let us now establish some \( u^* \in [w, 1] \) exists for which

\[ \text{cav } \hat{\pi}(\bar{u}) = \begin{cases} \hat{\pi}(\bar{u}) & \text{if } \bar{u} \notin (w, u^*) \\ \lambda \hat{\pi}(w) + (1 - \lambda)\hat{\pi}(u^*) & \text{if } \bar{u} = \lambda w + (1 - \lambda)u^* \text{ for some } \lambda \in [0, 1]. \end{cases} \]

It is immediate that the given functional form is a lower bound for \( \text{cav } \hat{\pi} \), so our goal is to find some \( u^* \in [w, 1] \) such that it forms an upper bound. To show this, one can find a slope \( m \in \mathbb{R} \) such that \( \pi(x) \leq \hat{\pi}(w) + m(\bar{u} - w) \) for every \( \bar{u} \in [w, 1] \), with equality at some \( u^* \in [w, 1] \), which is sure to exist because \( \pi \) is continuous. Using this \( u^* \), let us observe that the given function lies weakly above \( \text{cav } \hat{\pi} \), and hence coincides with it. Indeed, because \( \hat{\pi}(w) \geq \hat{\pi}(w^+) \) and \( \hat{\pi}'(w^-) \geq \lim_{a \to w^+} \hat{\pi}'(x) \) if \( w < 1 \), it is straightforward to show that this function is concave and above \( \hat{\pi} \), hence above \( \text{cav } \hat{\pi} \). The characterization of \( \text{cav } \pi \) follows.

Finally, we turn to the optimal level of effort, that is, the optimal choice of \( a \) in the optimization problem determining \( \pi(u) \). Because the associated objective function

\[ a \mapsto (1 - \delta)mu + \delta [\mu \hat{\pi}(\bar{u}_a) + (1 - \mu)\hat{\pi}(\bar{u})] \]

is concave and continuous, one can solve for the optimal action (which exists by continuity) via a first-order approach. Specifically, letting \( u_L, v^*, u_R \) be exactly as defined in the proof of Proposition 2 and reasoning exactly as in that proof, an optimal choice of action at
continuation value $u$ for the politician is

\[
a_u = \begin{cases} 
1 & \text{if } u < u_L, \\
\frac{(1-\delta)+\delta u^*-u}{(1-\delta)c} & \text{if } u \in [u_L, u_R], \\
0 & \text{if } u > u_R.
\end{cases}
\]

Let us now observe that $v^* \in \{w, 1\}$. By definition, $v^*$ cannot have an open interval around it on which $\hat{\pi}$ is affine, implying $v^* \notin (w, u^*)$. Next, from the hypothesis that $v^* \in [u^*, 1)$, we can derive a contradiction by the argument (verbatim) in the proof of Proposition 2 that $v^* = 1$, because the three functions $\pi$, $\tilde{\pi}$, and $\hat{\pi}$ all agree on $[u^*, 1]$. Finally, we can adapt the same argument as follows to show that $v^* \geq w$. Indeed, assume for a contradiction that $v^* < w$. First, because $\tilde{\pi}$ is affine on $[0, \tilde{u}_L]$, the definition of $v^*$ guarantees that $v^* \geq w$. Therefore, any $u \in [v^*, w)$ close enough to $v^*$ has $u < v^*$, so that

\[
\tilde{\pi}'(u) = \pi'(u) = \pi\left(\frac{u-pw}{1-p}\right) \geq \pi'(u) = \mu \frac{-1}{e} + (1-\mu)\tilde{\pi}'(u) \geq \frac{-1}{e},
\]

where the second equality follows from the chain rule and the inequalities follows from concavity of $\pi$ and $\tilde{\pi}$. However, that $\tilde{\pi}' \geq \frac{-1}{e}$ in a neighborhood to the right of $v^*$ contradicts the definition of $v^*$.

In summary, there exist $v^* \in \{w, 1\}$ and $u^* \in [w, 1]$ such that

\[
\pi(u) = (1-\delta)\mu a_{u,v^*} + \delta \left[ \mu \tilde{\pi}(\tilde{u}_{a_{u,v^*}}) + (1-\mu)\hat{\pi}(u) \right],
\]

\[
\tilde{\pi}(\tilde{u}) = \begin{cases} 
(1-p)\pi(\frac{\tilde{u}-pw}{1-p}) + p\pi(1-\delta) & \text{if } \tilde{u} \leq w \\
\pi(\tilde{u}) & \text{if } \tilde{u} > w;
\end{cases}
\]

\[
\hat{\pi}(v) = \begin{cases} 
\hat{\pi}(\tilde{u}_L) & \text{if } v < \tilde{u}_L \\
\frac{u^*-v}{u^*-w} \hat{\pi}(w) + \frac{v-w}{u^*-w} \hat{\pi}(u^*) & \text{if } w < v < u^* \\
\hat{\pi}(v) & \text{otherwise.}
\end{cases}
\]

where

\[
a_{u,v^*} = \max \left\{ 0, \min \left\{ 1, \frac{(1-\delta)+\delta u^*-u}{(1-\delta)c} \right\} \right\}
\]

The result then follows from observing that the equilibrium described in the proposition generates these optimal values.
A.5 Proof of Proposition 6

Throughout this proof, we take for granted the structure of optimal equilibrium given by Proposition 5, and we follow the notation of its proof.

To prove the result, it suffices to show that it is optimal to the voter to have \( v^* = w \), rather than having \( v^* = \bar{w} \), when \( w \) is close enough to 1. In this case, no politician’s continuation value will ever exceed \( w \) in the voter-optimal equilibrium described by Proposition 5. To show this, it suffices to establish that, when \( w < 1 \) is close enough to 1, \( \pi(\cdot) \) evaluated at \( w \) is higher when using the effort level implied by \( v^* = w \) than when using the effort level implied by \( v^* = 1 \). That is, we wish to show

\[
(1 - \delta) \mu a_w + \delta \left[ \mu \tilde{\pi}(\bar{w} a_w) + (1 - \mu) \tilde{\pi}(\bar{w}) \right] > (1 - \delta) \mu a_w + \delta \left[ \mu \tilde{\pi}(\bar{w} a_w) + (1 - \mu) \tilde{\pi}(w) \right],
\]

where \( a_w \) denotes \( a_{w,1} \) as defined in Proposition 5, \( \bar{w} a_w \) denotes \( \bar{u}|_{u=w_{a_w}} \), and \( w \) denotes \( w_{a_{w,1}} \), also defined in the proof of Proposition 5.

To show the desired inequality, focus on the case of \( w \in [1 - \delta, 1) \) has \( w \geq 1 - c(1 - \delta) \), so that \( a_w = 1 \). Using the expressions for the value functions at the end of the proof of Proposition 5, we find that the left hand side and right hand side of the centered inequality above differ by

\[
LHS - RHS = \left\{ (1 - \delta) \mu a_w + \delta \left[ \mu \tilde{\pi}(\bar{w} a_w) + (1 - \mu) \tilde{\pi}(w) \right] \right\} - \left\{ (1 - \delta) \mu a_w + \delta \left[ \mu \tilde{\pi}(\bar{w} a_w) + (1 - \mu) \tilde{\pi}(w) \right] \right\} \\
= \mu \delta \left[ \tilde{\pi}(\bar{w} a_w) - \tilde{\pi}(\bar{w} a_w) \right] - \mu (1 - \delta) (a_w - a_{w,w}) \\
= \mu \delta \left[ \tilde{\pi}(w) - \tilde{\pi}(1) \right] - \mu (1 - \delta) \left[ \frac{(1-\delta) + \delta}{(1-\delta)c} - \frac{(1-\delta) + \delta}{(1-\delta)c} \right] \\
= \mu \delta \left[ \tilde{\pi}(w) - 0 \right] - \mu \left[ \frac{\delta}{c} - \frac{\delta w}{c} \right] \\
= \mu \delta \left[ (1 - p) \pi(w) + p \pi(1 - \delta) \right] - \frac{\mu \delta}{c} (1 - w) \\
\geq \mu \delta p(1 - \delta) \mu a_{1 - \delta} - \frac{\mu \delta}{c} (1 - w),
\]

which converges to \( \mu^2 \delta p(1 - \delta) a_{1 - \delta} > 0 \) as \( w \to 1 \). Therefore, the difference is strictly positive when \( w < 1 \) is close enough to 1, as required.

A.6 Proof of Lemma 1

Consider optimal play for the incumbent at start of her term, if she is a high skill type. As in the baseline model, we can focus on pure strategy equilibria. If the politician were to choose zero effort, then the voter’s optimal value would be zero, which is not the case. Therefore,
any newly elected politician chooses strictly positive effort if she is a high skill type. Let $v^s$ and $v^f$ be the politician’s continuation value from the next period following success and failure, respectively. An improvement for the voter, which would relax the politician’s incentive constraint, would be to instead remove the politician and put in a new one at starting value $v^f$ following failure. This relaxes the high skill type politician’s incentive to work, and improves the voter’s utility by increasing the odds that the politician in office tomorrow is the high type. The result follows by continuing the argument recursively on the histories of the game.

A.7 Proof of Proposition 7

For any continuation value $u \in [1 - \delta, 1]$ for an incumbent high-type politician, let $\pi_1(u)$ denote the voter’s optimal continuation value (computed under the belief that the current politician is certain to be a high type) among all continuation equilibria that give the politician a value of $u$. For any average continuation value $v \in [0, 1]$ for the politician to have starting in the following period, let $\tilde{\pi}_1(v)$ denote the voter’s optimal continuation value among all feasible contracts that give the agent an average value of $v$. Finally, let $\pi_q$ denote the voter’s optimal continuation value when a new politician (who has never before acted, and is therefore a high type with probability $q$) is in office.

Given Lemma 1, the voter will accrue zero flow payoffs and draw a new politician each period until the first time a high type is drawn. Therefore, these continuation values for the voter are defined by the following Bellman equation:

$$
\pi_q = \Psi (\tilde{\pi}_1, \pi_q) := \sup_{a^*, v^s \in [0, 1]} \left( 1 - \delta \right) q \mu a^* + \delta \left[ (1 - q \mu) \pi_q + q \mu \tilde{\pi}_1(v^s) \right]
$$

subject to $1 - \delta \leq (1 - \delta) (1 - ca^*) + \delta v^s$; (IC$_0$)

$$
\pi_1(u) = \Phi \tilde{\pi}_1(u);
$$

---

21It is standard that these optimal values would be characterized by the given Bellman equation if the voter could commit. As we have observed, any observable deviation by the voter can be made unprofitable because there is an equilibrium—for instance, one in which all future politicians shirk and the voter replaces politicians with a rate independent of performance—that provides min-max payoffs to the voter. The only potentially profitable voter deviations, then, take place when the voter is expected to mix. Hence, with a sunspot available (which can be conditioned upon rather than having the voter privately mix), the given Bellman equation characterizes voter-optimal equilibrium payoffs.
\[
\tilde{\pi}_1(v) = \Phi_q(\pi_1, \pi_q)(v) := \sup_{\rho \in [0,1], \ u \in [1-\delta,1]} \rho \pi_1(u) + (1-\rho)\pi_q \\
\text{subject to } \rho u = v \tag{PK}
\]

where \(\Phi\) is defined as in the proof of Proposition 2 and \(\tilde{\Phi}_q\) and \(\Psi\) are operators on \((\pi_1, \pi_q)\) and \((\tilde{\pi}_1, \pi_q)\), respectively.

The entire proof of Proposition 2, excluding its final paragraph, applies essentially verbatim to \(\pi_1\) and \(\tilde{\pi}_1\). In particular: both functions are concave and continuous; some \(v_\ast \in [1-\delta,1]\) exists such that \(\tilde{\pi}_1|_{[0,v_\ast]}\) is affine and \(\tilde{\pi}_1|_{[v_\ast,1]} = \pi_1|_{[v_\ast,1]}\); and the optimal politician effort and continuation values prescribed by \(\Phi\) for any given value of the incumbent high-type politician are exactly those prescribed by Proposition 2. In particular, this establishes part 3 of the proposition.

In what follows, we establish parts 1 and 2, assuming without loss that \(v_\ast \in [1-\delta,1]\) is as large as possible subject to \(\tilde{\pi}_1|_{[0,v_\ast]}\) being affine.

For part 1, let us study the optimization problem defining \(\Psi\), rewriting it as

\[
\Psi(\tilde{\pi}_1, \pi_q) = \delta(1-q\mu)\pi_q + q \sup_{a_\ast, v_\ast \in [0,1]} \{(1-\delta)\mu a_\ast + \delta \mu \tilde{\pi}(v_\ast)\} \\
\text{subject to } v_\ast \geq c a_\ast (1-\delta)/\delta
\]

As a continuous optimization problem with compact domain, this problem has a maximizer \((a_\ast, v_\ast)\). That \(c a_\ast (1-\delta)/\delta \leq v_\ast \leq 1\) implies \(a_\ast \leq a_{1-\delta}\) for any such maximizer. To show that \(a_\ast \geq a_{1-\delta}\) in some solution to this program, we separately consider two cases. On the one hand, if \(v_\ast > c a_\ast (1-\delta)/\delta\) for such a maximizer, then it must be that \(a_\ast = 1\), for otherwise \(a_\ast\) could be slightly raised (maintaining feasibility) for a strictly higher objective. On the other hand, if \(v_\ast = c a_\ast (1-\delta)/\delta\), then \((a_\ast, v_\ast, v_f)\) is optimal among all feasible \((a, v^s, v^f)\) in the program defining \(\Phi \tilde{\pi}_1(1-\delta)\) in which the IC constraint binds (which we showed in the proof of Proposition 2 is without loss of optimality). Therefore, following the proof of Proposition 2, \((a_{1-\delta}, c a_{1-\delta} (1-\delta)/\delta)\) is optimal in the same program, which establishes part 1 of the proposition.

Now for part 2, notice that \(\pi_q \leq \max_{v \in [0,1]} \pi_1(v)\), because the voter is (in a best equilibrium) better off starting with a high-type politician than with a politician of uncertain type.\(^{22}\). We also know that \(\pi_q > 0\) since, for instance, any positive-value equilibrium play from the environment without adverse selection can yield a positive (though lower) profit in

\(^{22}\)Given any feasible contract for a voter facing a politician of unknown type, a voter who knows the newly elected politician to be the high type can randomize play to simulate the firing rule of a voter who is uncertain of the initial politician’s type, and get a weakly higher payoff along every path of play.
the current environment with adverse selection. Therefore, it must be that \( v_* < 1 \). Then, for \( u \in (v_*, 1) \) high enough,

\[
\tilde{\pi}_1(u) = \pi_1(u) = \mu \left( \frac{1-u}{c} \right) + \delta \left[ (1 - \mu) \tilde{\pi}_1 \left( \frac{u-(1-\delta)}{\delta} \right) + \mu \tilde{\pi}_1(1) \right]
\]

Therefore,

\[
\tilde{\pi}_1'(u) = \mu \left( -\frac{1}{c} \right) + (1 - \mu) \frac{u-(1-\delta)}{\delta} \geq \mu \left( -\frac{1}{c} \right) + (1 - \mu) \tilde{\pi}_1'(u)
\]

where the inequality follows since \( \tilde{\pi}_1 \) is concave. Therefore, \( \tilde{\pi}_1'(u) \geq -1/c \).

Thus, we can extend \( \tilde{\pi}_1 \) to a concave function on \([1 - \delta, \infty)\) by letting it take value \( \tilde{\pi}_1(u) = \tilde{\pi}_1(1) + (1 - u)/c \) for \( u > 1 \). But then, notice that if the interval \((1 - \delta, v_*)\) is nonempty, then for all \( u \in (1 - \delta, v_*) \) we have

\[
\tilde{\pi}_1'(u) = \mu \left( -\frac{1}{c} \right) + (1 - \mu) \frac{u-(1-\delta)(1-c)}{\delta} \geq \mu \left( -\frac{1}{c} \right) + (1 - \mu) \tilde{\pi}_1'(u)
\]

where the first line inequality follows from the fact that \( \tilde{\pi}_1|_{[0, v_*]} \) is affine, \( \pi_1 \) is concave, and \( \tilde{\pi}_1(v_*) = \pi_1(v_*) \), and the second line follows since again \( \tilde{\pi}_1|_{[0, v_*]} \) is affine. Therefore,

\[
\tilde{\pi}_1'(u) \leq \tilde{\pi}_1' \left( \frac{u-(1-\delta)(1-c)}{\delta} \right)
\]

which implies that \( u \geq \frac{[u - (1 - \delta)(1 - c)]}{\delta} \), and thus \( u \leq 1 - c < 1 - \delta \), a contradiction. Therefore follows that \( v_* = 1 - \delta \), establishing part 2.

### A.8 Proof of Proposition 8

Our first step is to consider a modified contracting problem which differs from the original one in two ways. First, in the modified problem the voter knows the current politician to be a high type and knows all other politicians to be low types, hence gets zero continuation value from replacement. Second, the voter gets the payoff associated with a politician’s intended effort in a given period even if the state turns out to be the low-productivity state.\(^{23}\) Moreover, we will consider this modified contracting problem for all values of \( \mu \), including the value of \( \mu = 0 \) that our model precludes.

\(^{23}\)Instead of the second modification, one could interpret this contracting problem as one with the voter’s payoff scaled up by a factor of \( 1/\mu \) whenever \( \mu > 0 \). This scaling is strategically irrelevant.
We will recursively define the associated value functions, denoted \( \pi_\mu^0 \) and \( \tilde{\pi}_\mu^0 \) for this modified problem, parametrized by the voter’s probability of having to reward, rather than punish, the politician in office. While we find the interpretations of \( \pi_\mu^0 \) and \( \tilde{\pi}_\mu^0 \) above to be instructive (and arguments essentially identical to those supporting Proposition 7 show that the optimal value functions from this modified contracting problem solve the recursive equations that define \( \pi_\mu^0 \) and \( \tilde{\pi}_\mu^0 \) as well), these interpretations are unnecessary for establishing Proposition 8. The rest of the proof can take \( \pi_\mu^0 \) and \( \tilde{\pi}_\mu^0 \) as purely mathematical objects defined by the given recursive equations below. Given such functions, we proceed to show that \( \tilde{\pi}_1^0 \) is maximized to the right of the continuation value \( c(1 - \delta)/\delta \). When \( q \) and \( \mu \) are small, then, we show \( \tilde{\pi}_1^1 \) is also maximized to the right of the same continuation value because this function is well-approximated by \( \tilde{\pi}_1^0 \).

Suppose \( \delta \in (0, c) \) is close enough to \( c \) to ensure \( 1 - (1 - \delta)c > c(1 - \delta)/\delta \). For any \( \mu \in [0, 1) \), let the bounded functions \( \pi_\mu^0 : [1 - \delta, 1] \to \mathbb{R} \) and \( \tilde{\pi}_\mu^0 : [0, 1] \to \mathbb{R} \) be defined by the recursive equations

\[
\tilde{\pi}_\mu^0(v) = \frac{v}{\max\{v, 1 - \delta\}} \pi_\mu^0(\max\{v, 1 - \delta\})
\]
\[
\pi_\mu^0(u) = (1 - \delta)a_u + \delta \left((1 - \mu)\tilde{\pi}_\mu^0 \left(\frac{u-(1-\delta)}{\delta}\right) + \mu\tilde{\pi}_\mu^0 \left(\frac{u-(1-\delta)(1-ca_u)}{\delta}\right)\right),
\]

where \( a_u \) is as defined in the proposition’s statement. By routine use of the contraction mapping theorem, several results are straightforward to establish. First, there is a unique pair of functions, \((\pi_\mu^0, \tilde{\pi}_\mu^0)\), that solves this system, and \( \pi_\mu^0 \) and \( \tilde{\pi}_\mu^0 \) are both continuous and concave functions. Next, the mapping \( \mu \mapsto \tilde{\pi}_\mu^0 \) is a continuous mapping with respect to the supremum-norm on the space of continuous functions. More substantively, whenever \( \mu > 0 \), we have \( 0 \leq \pi_q \leq q \mu \) and \( \mu \tilde{\pi}_\mu^0 \leq \tilde{\pi}_1 \leq \mu \tilde{\pi}_\mu^0 + \pi_q \). Finally, focusing on \( \mu = 0 \), we have \( \tilde{\pi}_0^0(v) = v \) for \( v \in [0, 1 - (1 - \delta)c] \).

Using the above observations, we can now establish the proposition. Given the form of \( \Psi \), we must show that, when \( q, \mu \in (0, 1) \) are both sufficiently low, we have

\[
\bar{u}_1 \notin \arg \max_{v^* \in [0, 1]} \frac{(1 - \delta)q \mu + \delta [(1 - q \mu)\pi_q + q \mu \tilde{\pi}_1(v^*)]}{v^* \in [0, 1]} \text{ subject to } 1 - \delta \leq (1 - \delta)(1 - c) + \delta v^*
\]

Transforming the objective and simplifying the constraint, the goal is to show that

\[
\bar{u}_1 \notin \arg \max_{v^* \in [0, 1]} \frac{\tilde{\pi}_1(v^*)}{\mu} \text{ subject to } (1 - \delta)c/\delta \leq v^*.
\]
But this follows from the fact that since $v^s = 1 - (1 - \delta)c > (1 - \delta)c/\delta = \bar{u}_1$, we have

\[
\frac{1}{\mu} \pi_1(v^s) - \frac{1}{\mu} \tilde{\pi}_1(\bar{u}_1) \geq [\tilde{\pi}_{\mu}^s(v^s) + \pi_q/\mu] \geq [\tilde{\pi}_{\mu}^s(v^s) - \tilde{\pi}_{\mu}^s(\bar{u}_1)] - q
\]

and the expression on the very right converges to $\tilde{\pi}_{\mu}^s(v^s) - \tilde{\pi}_{\mu}^s(\bar{u}_1) > 0$ as $q, \mu \to 0$. □

References


