A Dynamic Model of Censorship

Yiman Sun*
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Abstract

We model censorship as a dynamic game between an agent and an evaluator. Two types of public news, good and bad news, are informative about the agent’s ability. However, the agent can hide bad news from the evaluator, at some cost, and will do so if and only if this secures her a significant increase in tenure. Thus, the evaluator faces a bandit problem with endogenous news processes. When bad news is conclusive, the agent always censors when the public belief is sufficiently high, but below a threshold, she entirely or partially stops censoring. The possibility of censorship hurts the evaluator and the good agent, and it may also hurt the bad agent. However, when bad news is inconclusive, we show that the good agent censors bad news more aggressively than the bad agent does. This improves the quality of public information and may benefit all players.

Keywords: Censorship, Information Manipulation, Learning, Dynamic Games

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*Toulouse School of Economics, University of Toulouse Capitole. Email: yiman.sun@tse-fr.eu

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1 Introduction

Individuals in positions of power, be they political leaders or the managers of firms, often suppress or censor bad news in order to improve their standing and prevent any threats to their authority. Such censorship is widely regarded to be undesirable. Nonetheless, we can imagine situations where the suppression of bad news may lead to better outcomes. For example, a political leader may be embarking on a radical reform that has the potential to be transformative. Being radical, the reform is also subject to teething troubles, and if the public were to become aware of all the difficulties, it might prematurely lose faith in the leader and replace her.

The decades of the 1960’s - 1980’s witnessed rapid industrialization and exceptionally high growth rates in many Asian developing countries and regions, including Singapore, South Korea, and Taiwan. This has been called an economic “miracle.” Controversially, all these regions were under authoritarian rule at the time. One hypothesis from a review by Sirowy and Inkeles (1990) links economic growth with authoritarianism: “the superior ability of an authoritarian regime to govern that facilitates economic growth is expressed indirectly by the social and political stability it fosters” (p. 130). This “political stability” may be sustained by different means, one of which is censorship. As Rodan (2004) noticed, “[a]lmost by definition, authoritarian regimes involve censorship” (p. 1). On the other hand, we have many examples of leaders who persist with foolhardy projects, hiding all negative evidence. For instance, during China’s Great Leap Forward (1958-1962), the central government’s failure to access up-to-date local information due to local officials’ concealment was partially responsible for the ensuing famine. These considerations suggest that it is important to examine the implications of censorship in a formal model.

This paper studies the interplay between learning and censorship in a dynamic environment. We consider a relationship between an agent who seeks to remain in office, and an evaluator who learns the competence of the agent from public news about the quality of the agent’s project. The evaluator wants to retain the competent agent and dismiss the incompetent one, thereby terminating her project. The project of the agent gives rise to two news processes – good news and bad news. Good news is publicly observed, and confirms that the project is good. However, bad news can also arise, and is more likely when the project is bad. The agent can always suppress bad news when it materializes, but this is costly. Thus, the information that the evaluator receives is endogenously determined by the agent’s censorship policy. We assume that the agent

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1See Guriev and Treisman (2018) for other means, including repression, co-option, propaganda, and censorship.

2See Li and Yang (2005) for details.

3We assume that the authentic news processes are correlated with the agent’s type and do not consider the moral hazard problem in which the agent may exert private effort in advance to promote good news or to delay bad news. In our model, the only decision that the agent makes is whether to suppress bad news whenever it materializes. Thus, our model focuses exclusively on the ex post
knows her own competence level, and consequently, competent agents and incompetent ones may well censor differently.

The political censorship story might be appealing, but in fact our model can be applied more broadly to a general reputation management context where the concealment of information is concerned. The agent can be a manager of a division of a large firm, while the evaluator is the firm’s CEO. Alternatively, the agent might be an entrepreneur, with the evaluator being a venture capitalist who is funding the project. Finally, the agent might be a political leader, with the evaluator standing for the population. In these contexts, bad news can take several forms – mechanical breakdowns, reports of malpractice or customer complaints. Bad news can be suppressed – accounts can be “cooked,” log files can be faked, and unhappy customers could be mollified with refunds or gifts. In the rapid expansion of digital economy, a new technique called Reverse Search Engine Optimization has also been developed to manage reputation and suppress negative results, for example angry customer reviews, on Internet search engines. None of these measures are costless; they take time and money, and psychological costs may be associated with dishonest behavior.  

Similarly, politicians can arrest reporters, bribe witnesses, or shut down Internet forums, but this is also costly. Basic economic intuition suggests that concealing information necessarily hurts the evaluator. Moreover, the possibility of censorship makes the evaluator more suspicious about the agent’s performance. He does not know whether the reason that no bad news arrives is because the agent is competent, or because the agent is censoring. Thus the possibility of censorship also hurts a competent agent who has no way to prove that she was not hiding anything. It can only benefit an incompetent agent, since censorship helps her survive bad news. If the above intuition is correct, then the policy implication would be to reduce censorship by making it as hard and costly as possible.

However, we show that the above intuition is only partially correct, and it crucially depends on the details of the information structure. Specifically, it depends on whether negative evidence is **conclusive**, i.e., it can only arise when the agent is incompetent. Indeed, when bad news can also arise when the agent is competent, we show that censorship may potentially increase the welfare of all parties, including the evaluator.

We now turn to the details of our model and its basic insights. All news is modeled as exponential/Poisson news. Good news can only arise for a competent agent, and is therefore conclusive. We consider two qualitatively different information structures. We first assume that bad news is also conclusive, i.e., it can only arise for an incompetent agent. We then consider the more general case of inconclusive bad news. The evolution of the evaluator’s posterior belief about the agent’s competence in the absence of information manipulation. This is in line with the literature on political censorship and on disclosure games in Section 2. By contrast, a literature on reputation is concerned with the ex ante moral hazard problem, where exerting effort can improve performance/quality and promote good news, e.g., Holmström (1999), Mailath and Samuelson (2001), Board and Meyer-ter Vehn (2013).

See Rosenbaum, Billinger, and Stieglitz (2014) for a review on honesty experiments.
Table 1: Main results of the paper

<table>
<thead>
<tr>
<th>Faster good news</th>
<th>Slower good news</th>
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<tr>
<td>Conclusive bad news</td>
<td>Censorship may hurt all parties</td>
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<tr>
<td>Independent dismissal threshold</td>
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<tr>
<td>Inconclusive bad news</td>
<td>Censorship may benefit all parties</td>
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<tr>
<td>Independent dismissal threshold</td>
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of news depends on the agent’s censorship policy, and on the (exogenous) parameters of the news processes. In our analysis, it will be useful to distinguish information structures according to whether the absence of news is “good” or “bad,” i.e., whether the evaluator’s posterior belief drifts up or down.\(^5\) We say that *good news arrives faster* when good news arrives faster than the incompetent agent’s bad news. If good news arrives faster, then, regardless of the censoring decisions of the agent, the evaluator’s belief is updated downwards in the absence of news. We say that *good news arrives slower*, when it arrives slower than the incompetent agent’s bad news. In this case, the direction in which the evaluator’s belief moves in the absence of news depends on the censorship policy of the agent. Table 1 summarizes our main results in different information structures.

The simplest case is when bad news is conclusive, i.e., it only arises for the incompetent agent, and when good news arrives faster. Thus the public belief about the agent being competent will drift down in the absence of news regardless of the censorship policy, but censorship will accelerate the downward process. Our first observation is that the belief threshold at which the evaluator fires the agent is *independent* of the (incompetent) agent’s censorship strategy, due to the fact that the option value of continuation depends only on the possibility that good news arises. This observation allows us to use the logic of backward induction to pin down behaviors in any equilibrium. Since censorship is costly, the agent will incur the cost if and only if this secures her a sufficient increase in tenure. Thus, there is a unique belief threshold, at which the agent switches from *Full-Censorship* to *No-Censorship*. Obviously, the evaluator and the competent agent are worse off with censorship, which exactly confirms our initial intuition. However, the incompetent agent may also be worse off with censorship, since she has a shorter tenure, conditional on no bad news occurring, than when censorship is not possible. This happens whenever the Full-Censorship period is short. The benefit from censorship for the incompetent agent is that she survives bad news in

\(^5\)A similar distinction can be found in the strategic experimentation literature. For example, the posterior belief drifts towards to the stopping region in Keller, Rady, and Cripps (2005), while the posterior belief drifts away from the stopping region in Keller and Rady (2015).
the Full-Censorship period. A short Full-Censorship period means that the benefit is small, thus it will be overcome by the negative consequence from censorship, i.e., the acceleration of the downward drifting of public beliefs.

We next keep the assumption that good news arrives faster, but consider the case where bad news is inconclusive, i.e., it can arise for both types of agent. Our main insight arises from the fact that the competent agent has a greater incentive to censor bad news than the incompetent one does. This is due to the fact that the competent agent knows that good news may arise and secure her permanency in tenure, which is not possible for the incompetent agent. Furthermore, the competent agent is also less likely to get further bad news to censor. Therefore, the competent agent has a higher continuation value in the job, and the belief threshold at which she stops censoring, $p^G$, is lower than the threshold at which the incompetent agent stops censoring, $p^{B \dagger}$. Consequently, the two types of agent separate in censorship policies in the interval $(p^G, p^{B \dagger})$, and bad news now becomes endogenously conclusive – any bad news may only come from the incompetent agent since the competent one always censors it. Thus, censorship improves the quality of public information and increases the evaluator’s payoff.\(^6\)

Finally, we examine the case where good news arrives slower. When bad news is conclusive, we cannot have a pure strategy equilibrium where the incompetent agent stops censoring at some threshold – if this were the case, the evaluator’s belief would drift upwards, which would then make censoring bad news attractive. We show there is a critical belief threshold such that when this threshold is reached, both parties randomize. The incompetent agent randomizes between censoring and not, while the evaluator randomizes between firing the agent and retaining her.\(^7\) Above this belief threshold, the incompetent agent censors for sure. It is noteworthy that the evaluator dismisses the agent at a higher threshold belief than he does in the absence of censorship, since the existence of censorship reduces future bad news, and since the evaluator’s option value of continuation depends on the arrival of bad news. This only happens when good news arrives slower, because the dismissal threshold depends critically on the arrival of bad news only if the belief updating process in the absence of news is upward, but not downward. We call the increase in the dismissal threshold belief the discouragement effect since censorship discourages the agent’s incentive for learning. When bad news is inconclusive, we show that the very similar equilibrium still exists if the censoring cost is low. The evaluator and the incompetent agent use the same equilibrium strategies as in the conclusive bad news case, but the competent agent censors with probability one even when the belief is at the dismissal threshold. In this equilibrium, almost all bad news from both types of agent will be censored, except for some bad news from the incompetent agent when the belief is at the dismissal threshold. The worsened information quality hurts the evaluator. However, if the censoring cost is intermedi-

\(^6\)Indeed, for some parameter values it can also improve the payoffs of both types of agent, and therefore benefit all players.

\(^7\)More precisely, in our continuous time model, the evaluator randomizes over stopping times.
ate, there exists an equilibrium in which only the competent agent finds it optimal to censor bad news, but the incompetent agent never censors it since the cost is too high for her. This separation again improves the quality of public information and benefits the evaluator. We also find an encouragement effect; that is, the evaluator’s dismissal threshold belief decreases since his incentive for learning is encouraged.

Our model has policy implications on censorship. We may interpret the censoring cost as reflecting institutional structures against it. In some circumstances, i.e., when bad news only arises for the incompetent agent, we find that censorship could be unambiguously bad, and thus the censoring cost should be made as high as possible. However, when bad news also arises for the competent agent, the evaluator may prefer neither a too strong institution that prevents censorship, nor a too weak institution that allows too much censorship. An institution that tolerates some mild censorship may be more preferred.

The rest of the paper is organized as follows. Section 2 discusses the related literature. Section 3 introduces the model. Section 4 and Section 5 characterize equilibria and discuss welfare effects for the conclusive bad news case and the inconclusive bad news case, respectively. Section 6 concludes. The Appendix provides all proofs.

2 Related Literature

This paper studies costly censorship in a dynamic environment. It mainly relates to four strands of literature.

First, it relates to a literature on political censorship. Shadmehr and Bernhardt (2015) study the interaction between a ruler and a representative citizen. The ruler can censor a bad media report at a cost in order to mitigate the likelihood of revolution. Their paper, as well as ours, assumes a passive role of the media. Thus the focus is solely on the relation between the ruler and the citizen. They find that at the ex ante stage, before the ruler knows her type, she can benefit by committing to reducing censorship slightly. We establish a similar result. Moreover, we show that even for a ruler who knows her type being bad, she may still benefit by committing to no censorship.

Besley and Prat (2006) study the media capture problem, where a media outlet maximizes his profits either from his audience who values information, or from a government who bribes him to keep silent about their negative news. Their paper focuses on the role of media outlets, and explicitly models the censoring cost as a direct or indirect transfer to a media outlet who is willing to forgo its readership and take a bribe. Gehlbach and Sonin (2014) and Eraslan and Ozerturk (2017) also examine the role of media outlets. The former takes a Bayesian persuasion approach where a government

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8 The institutional structures may include the political system, the media industry structure, and the accessibility of digital technologies.
can commit to an editorial policy for news release, while the latter studies a media outlet’s reputation concerns that arise due to an information gatekeeping policy.

Egorov, Guriev, and Sonin (2009) consider the role of media outlets in monitoring bureaucrats. In their model, a dictator needs a bureaucrat to implement her policy, but faces a moral hazard problem. To incentivize the bureaucrat to work, the dictator relies only on media reports. However, a free media that solves the monitoring problem also exposes the dictator’s incompetence to the public. Thus, if the dictator’s income mainly depends on the bureaucrat’s performance, she would rather risk being overthrown but allow a free media to monitor the bureaucrat. Similarly, Lorentzen (2014) studies a regime change game in which a regime trades off the risk being overthrown and an informative media capable of monitoring the lower-level officials.

Edmond (2013) and Redlicki (2017) also study information manipulation in politics. They consider a global game played by citizens to attack a regime, in which the private signals received by citizens can be manipulated at a cost by the regime either through shifting the mean of a signal in Edmond (2013) or through increasing the noise of a signal in Redlicki (2017). Guriev and Treisman (2018) examines how a dictator employs different means – propaganda, censorship, co-optation, and repression to survive.

Most papers on censorship consider one-shot interaction in a political setting. Smirnov and Starkov (2018) and Hauser (2019) are exceptions. Smirnov and Starkov (2018) study censorship in product reviews. The main difference with our setting is that they assume censorship is costless, and some consumers are naive – they believe that no reviews are censored. By contrast, we examine censorship with a strategic evaluator, and study the interdependence of censorship and learning, where costs play a crucial role. Hauser (2019) studies the interaction between investment and costly censorship of a firm in a dynamic environment. Under the assumption that bad news only arises when the quality of the project is low, he finds that a firm may benefit from sufficiently expensive censorship, which echoes our results in the conclusive bad news case.

Second, another related literature is on disclosure of verifiable information, beginning from Grossman (1981) and Milgrom (1981). A sub-class of the literature following Dye (1985) and Jung and Kwon (1988) studies the scenario where a receiver is uncertain about whether a sender possesses a piece of evidence. Our setting shares the same feature – the lack of news could happen either because no news has arrived or because it has been censored. While the early literature exclusively focuses on static models, later papers extend them into different dynamic settings where multiple signals may arise gradually, e.g., Shin (2003), Grubb (2011), Acharya, DeMarzo, and Kremer (2011), Guttman, Kremer, and Skrzypacz (2014).

\footnote{They also extend their one-shot censorship model into a dynamic setting by assuming that the dictator faces a stationary environment where censoring public bad news can effectively keep the ruler in power, and obtain a similar result as in their static model. By contrast, we analyze the (non-stationary) dynamics of the public confidence about the agent’s competence.}
Although the literature on disclosure games is large, very little attention has been given to the case where concealing information is costly. Three recent papers explore different implications of such a cost, all of which are very different from those that we focus on. Dye (2017) studies a model of voluntary disclosure, in which a seller who has withheld information may be caught by a fact finder after the sale of an asset. In such an event, she has to make a damages payment to the buyer, the amount of which equals the product of the buyer’s overpayment and a “damages multiplier.” An important implication from such a cost function is that the seller would withhold more information as the withholding punishment goes up. Daughety and Reinganum (2018) study the problem of suppression of exculpatory evidence in prosecutions. In their model, a prosecutor wants to convict a defendant but also incurs a moral cost if she convicts an innocent defendant. The prosecutor also receives a penalty if she is caught for suppressing evidence. They extend their model to incorporate the teamwork of two prosecutors and show that this results in the concentration of authority regarding suppressing evidence. Kartik, Lee, and Suen (2017) provide a result on Bayesian updating, and apply it to a multi-sender disclosure game. They show that competition leads to more disclosure in the presence of a concealment cost.

Third, a recent literature on dynamic information design is also related, e.g., Ely (2017), Renault, Solan, and Vieille (2017), Che and Hörner (2018). Their papers, as well as ours, study information manipulation in a dynamic environment. Their models rely on the principal’s commitment power to design a flexible information disclosure policy to induce an agent to choose a desirable action for the principal, and assume that information manipulation is costless. By contrast, we focus on one particular kind of information manipulation – censorship. In addition, we do not assume the commitment power, and assume that censorship is costly.

Finally, this paper relates to two-armed Poisson bandit models in continuous time, e.g., Presman (1991), Keller, Rady, and Cripps (2005), Keller and Rady (2010, 2015). The evaluator here faces a two-armed bandit problem, in which the information generated by the risky arm is endogenous and controlled by the agent. We will borrow results from this literature to solve for the benchmark case in the absence of censorship.

3 The model

Two risk neutral players, an agent (she) and an evaluator (he), play a game in continuous time \( t \in [0, \infty) \). Their discount rates are \( \rho_0 > 0 \) and \( \rho_1 > 0 \), respectively.

The agent has a project. At time \( t = 0 \), nature chooses the type \( \theta \) of the project from the set \( \Theta = \{G, B\} \). We assume that the agent observes nature’s choice, while the evaluator does not. Let \( p_0 \in (0, 1) \) denote the probability that nature chooses a type \( G \) project. Thus, the project’s type is the agent’s private information or her type. We will use the terms the agent’s type and the project’s type interchangeably.
The agent enjoys a flow payoff \( w > 0 \) that is independent of her type while she stays in her job, and has no payoffs after she is dismissed by the evaluator. Only the type \( G \) agent can succeed in her project.\(^{10}\) A success is publicly observable, and it arrives at each jumping time of a Poisson process \( S = \{S_t\}_{t \geq 0} \) with an arrival rate \( \gamma > 0 \). The first success reveals that the type of the agent is \( G \).

A success yields a lump-sum payoff \( k > 0 \) to the evaluator. The evaluator can choose a time \( t \geq 0 \) to irreversibly dismiss the agent. After the dismissal, the game ends, and the evaluator receives his outside option, the total discounted value of which is normalized to \( m \).\(^{11}\) We assume \( h := \gamma k > m > 0 \). Thus, the evaluator prefers the type \( G \) agent to the outside option, and prefers the outside option to the type \( B \) agent.

The agent’s project also potentially produces adverse news events, such as breakdowns. The arrival rate of such news depends on the type of the project. If the project’s type is \( G \), it generates a piece of news at each jumping time of a Poisson process \( N^G = \{N^G_t\}_{t \geq 0} \) with an arrival rate \( \beta^G \geq 0 \). Conditional on the project’s type being \( G \), the success process \( S \) and the news process \( N^G \) are independent. If the project’s type is \( B \), it generates a piece of news at each jumping time of a Poisson process \( N^B = \{N^B_t\}_{t \geq 0} \) with an arrival rate \( \beta^B \), where \( \beta^B > \beta^G \). We call a piece of such news bad news since it happens more often to the type \( B \) agent, although it has no payoff consequence. We also call a success good news.

The agent observes the bad news process. When a piece of bad news arrives, she can choose to incur a lump-sum cost \( c > 0 \) to censor it. The evaluator observes bad news if and only if the agent does not censor it, but he cannot distinguish whether a piece of bad news comes from \( N^G \) or \( N^B \) unless \( \beta^G = 0 \). Let \( X^\theta_t \in \{1, 0\} \) be censoring decision of the type \( \theta \) agent at time \( t \geq 0 \) when a piece of bad news has arrived at time \( t \); \( X^\theta_t = 1 \) denotes censoring. A piece of bad news is publicly revealed to the evaluator if it is not censored by the agent.

**Histories and Strategies** – Some care must be taken while defining the agent’s and the evaluator’s decision nodes in the game. Before the game ends,\(^{12}\) the agent needs to make her censoring decisions only at dates when a piece of bad news arrives. On the other hand, the evaluator can dismiss the agent at any time.

At time \( t \), a private history of the type \( \theta \) agent is denoted by \( h^\theta_t \). It consists of a finite sequence of news realizations before and including date \( t \), and her censoring decision for those bad news realizations before but not including date \( t \). Let \( \tilde{h}^\theta_t \) be a typical private history for her at \( t \) when a piece of bad news has just arrived at \( t \). Her strategy specifies a censoring probability \( x^\theta_t \in [0, 1] \) at time \( t \) for each history \( \tilde{h}^\theta_t \); this strategy \( \bar{x}^\theta = \{x^\theta_t\}_{t \geq 0} \) is progressively measurable with respect to the filtration induced by those

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\(^{10}\)This assumption is made for simplicity as our focus is on the concealment of bad news.

\(^{11}\)It can also be interpreted as the flow payoff from the outside option, since they are equal after normalization (i.e., \( \int_0^\infty \rho_1 e^{-\rho_1 t} m \, dt = m \)).

\(^{12}\)All strategies are defined conditional on the game has not yet ended.
continuously according to the following ordinary differential equation (ODE),

If no news is publicly revealed over the time interval of bad news makes the public belief jump from those public histories. The evaluator’s pure strategy at time $t$ for a public history $h_t$ is an $\mathcal{F}$-stopping time $T^t$, at which time he dismisses the agent.

Pure strategies are not sufficient to study equilibria. We now introduce mixed strategies, which are cumulative distribution functions over stopping times. To be specific, a mixed strategy of the evaluator at time $t$ for a public history $h_t$ is an $\mathcal{F}$-adapted process $r^t = \{r^t_\nu\}_{\nu \geq t}$, such that pathwisely,

(a) $r^t_\nu$ is non-decreasing and right continuous in $\nu \geq t$, and takes values in $[0, 1]$;

(b) for any time $t' \geq t$, and $\nu \geq t'$, $r^t_\nu$ is related to $r^t_{t'}$ as follows:

$$r^t_\nu = r^t_{t'} + (1 - r^t_{t'})r^t_{t'}, \quad \text{if } r^t_{t'} < 1,$$

where $r^t_{t'} = \lim_{s \uparrow t'} r^t_s$ for $s > t$, and $r^t_{t'} = 0$.

(a) requires $r^t$ to be a cumulative distribution function over the stopping times $T^t$. (b) requires $r^t$ to be time consistent, but imposes no restriction on $r^t_\nu$ if $r^t_{t'} = 1$. We also call $\frac{dr^t_\nu}{1-r^t_{t'}}$ the (instantaneous) hazard rate whenever it exists.

**Beliefs** — Given a strategy $x^\theta$ of the type $\theta$ agent, we define the public conjectured strategy $\tilde{x}^\theta = \{\tilde{x}_t^\theta\}_{t \geq 0}$ of the type $\theta$ agent by $\tilde{x}_t^\theta := \mathbb{E}[x_t^\theta|\mathcal{F}_t]$. The public belief at time $t$ about the agent being type $\tilde{G}$ is denoted by $p_t := \mathbb{P}\{\tilde{\theta} = \tilde{G}|\mathcal{F}_t\}$, where the probability measure is induced by the public conjectured strategies. When a success arrives, the public belief jumps to 1. If a piece of bad news is publicly revealed at time $t$, the public belief jumps from $p_t$ to

$$J(p_t, \tilde{x}_t^\tilde{G}, \tilde{x}_t^B) := \frac{p_t - \beta^G (1 - \tilde{x}_t^G)}{p_t - \beta^G (1 - \tilde{x}_t^G) + (1 - p_t - \beta^B (1 - \tilde{x}_t^B))},$$

when the denominator is non-zero. In the absence of censorship ($\tilde{x}_t^G = \tilde{x}_t^B = 0$), a piece of bad news makes the public belief jump from $p_t$ to $J(p_t, 0, 0) < p_t$.

If no news is publicly revealed over the time interval $(t, t + dt)$, the public belief evolves continuously according to the following ordinary differential equation (ODE),

$$\dot{p}_t = g(p_t, \tilde{x}_t^\tilde{G}, \tilde{x}_t^B) := -p_t (1 - p_t) \left[\gamma + (1 - \tilde{x}_t^G) \beta^G - (1 - \tilde{x}_t^B) \beta^B\right]. \quad \text{(ODE)}$$

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$^{13}$The superscript $t$ means the strategy is defined for a history at time $t$. This also applies to the strategy $r^t$ which will be defined later.

$^{14}$See Laraki, Solan, and Vieille (2005) and Riedel and Steg (2017) for details.
To ensure the above (ODE) with an initial condition \( p_0 \in (0, 1) \) admits a well-defined solution, we restrict \( \hat{x}^\theta \) to be admissible as follows. For \( \theta \in \Theta \), we say \( \hat{x}^\theta \) is admissible if for any public history \( h_t \) and \( t' > 0 \), \( \{\hat{x}^\theta_t\}_{t' \leq t} \) is piecewise continuous in \( t \) with a finite number of cutoffs, and it is right continuous with left limits at the cutoffs. Given any admissible \( \hat{x}^G \) and \( \hat{x}^B \), the (ODE) with an initial condition \( p_0 \in (0, 1) \) admits a unique solution \( p = \{p_t\}_{t \geq 0} \) which is piecewise continuously differentiable.\(^{15}\)

**Payoffs** – Given \( r^t \) and \( x^\theta_t = \{x^\theta_{\nu}\}_{\nu \geq t} \), the type \( \theta \) agent’s expected payoff at time \( t \) in a private history \( \bar{h}^\theta_t \) is

\[
v^r_{\theta}(x^\theta_{\bar{h}^\theta_t}) = \mathbb{E} \left[ -\rho_0 \ c \ X^\theta_t + \int_t^{T^t} \rho_0 \ e^{-\rho_0(\nu-t)}(w \ d\nu - c \ X^\theta_\nu \ dN^\nu_\nu) \bigg| \bar{h}^\theta_t \right],
\]

where the expectation is taking over \( T^t \) and \( \{N^\nu_\nu, X^\theta_\nu\}_{\nu \geq t} \).

Given \( r^t \) and \( \bar{x}^\theta_t = \{\bar{x}^\theta_{\nu}\}_{\nu \geq t} \), the evaluator’s expected payoff at time \( t \) in a public history \( h_t \) is

\[
u^r_{\theta}(\bar{x}^\theta_t(h_t)) = \mathbb{E} \left[ \int_t^{T^t} \rho_1 \ e^{-\rho_1(\nu-t)} \mathbbm{1}_{G} \ dS_\nu + e^{-\rho_1(T^t-t)} \ m \bigg| h_t \right],
\]

where \( \bar{x} := \{\bar{x}^G, \bar{x}^B\} \), and the expectation is taking over \( T^t \), \( \{S_\nu\}_{\nu \geq t} \), and \( \{N^\nu_\nu, \bar{x}^\theta_\nu\}_{\nu \geq t, \theta \in \Theta} \).

Given others’ strategies, let \( V^r_{\theta}(\bar{h}^\theta_t) = \sup_{\{x^\theta_{\nu}\}_{\nu \geq t}} \nu^r_{\theta}(x^\theta_{\bar{h}^\theta_t}) \) and \( U^\bar{x}(h_t) = \sup_{\{r^\nu\}_{\nu \geq t}} \nu^r_{\theta}(\bar{x}^\theta_t(h_t)) \) be the value functions of the type \( \theta \) agent and the evaluator, respectively.

**Equilibria** – We study Perfect Bayesian Equilibria (PBE), which consist of strategies of each type \( \theta \in \Theta \) of agent \( x^\theta = \{x^\theta_{\nu}\}_{\nu \geq t} \) and the evaluator \( r = \{r^\nu\}_{\nu \geq t} \), a public belief \( p = \{p_t\}_{t \geq 0} \), and a public conjectured strategy \( \bar{x}^\theta = \{\bar{x}^\theta_{\nu}\}_{t \geq 0} \) such that

1. For each \( \bar{h}^\theta_t \), \( x^\theta_t \) is optimal for the type \( \theta \) agent, given \( p \) and \( r^t \);

2. For each \( h_t \), \( r^t \) is is optimal for the evaluator, given \( p \) and \( \bar{x}^\theta \) of each \( \theta \in \Theta \);

3. \( \bar{x}^\theta = \mathbb{E}[x^\theta | F] \) and \( p \) is updated according to \( \bar{x}^\theta \) and Bayes rule whenever possible.

It is well known that the optimal allocation rule in a two-armed bandit problem is a cutoff rule with respect to the belief. In our setup, cutoff strategies are of particular interest. We say the evaluator’s strategy is a pure cutoff strategy with a cutoff public belief \( \hat{\rho} \in (0, 1) \) if it is a pure strategy \( T^t = \inf \{\nu \geq t : p_\nu \leq \hat{\rho}\} \) for any public history \( h_t \). This means that the evaluator dismisses the agent whenever the public belief is below or equal to the cutoff \( \hat{\rho} \). The evaluator’s strategy is called a mixed cutoff strategy with

\(^{15}\)Given that \( \bar{x}^\theta \) is piecewise continuous, it is easy to see that \( g(p, \bar{x}^G_\nu, \bar{x}^B_\nu) \) and \( g_\nu(p, \bar{x}^G_\nu, \bar{x}^B_\nu) \) are piecewise continuous in \( t \) for a fixed \( p \), and \( g(p, \bar{x}^G_\nu, \bar{x}^B_\nu) \) is continuously differentiable in \( p \) for a fixed \( t \). Thus, the standard local existence and uniqueness results for ODE ensure a unique solution in the interval where both \( \bar{x}^G_\nu \) and \( \bar{x}^B_\nu \) are continuous. Also, a unique global solution can be obtained through concatenation.
a cutoff \( \hat{p} \in (0, 1) \) if it is a mixed strategy and for any public history \( h_t \) and \( \nu \geq t \),

\[
rt_{\nu} = \begin{cases} 
1, & \exists s \in [t, \nu], \ p_s < \hat{p}, \\
0, & \forall s \in [t, \nu], \ p_s > \hat{p}.
\end{cases}
\]

This means that the evaluator never dismisses the agent when the public belief is above the cutoff \( \hat{p} \), but dismisses the agent immediately when the belief is below \( \hat{p} \).

Similarly, we say the type \( \theta \) agent’s strategy is a pure (resp. mixed) cutoff strategy with a cutoff \( \hat{p} \in (0, 1) \) if it is a pure (resp. mixed) strategy and for any history \( \bar{h}_t^{\theta} \),

\[
xt_{\nu} = \begin{cases} 
1, & \text{if } p_t \in (\hat{p}, 1), \\
0, & \text{if } p_t \leq \hat{p} \ (\text{resp. if } p_t < \hat{p}).
\end{cases}
\]

This means that the agent censors bad news for sure when the public belief is above the cutoff \( \hat{p} \) (but below 1) and she stops censoring when the belief is below \( \hat{p} \). The difference between a pure and a mixed strategy is that a mixed strategy allows mixing at the cutoff belief. We call a PBE a cutoff equilibrium if the evaluator uses a cutoff strategy in the equilibrium.

4 Conclusive bad news

We first study the baseline model where bad news cannot arise for the type \( G \) agent (i.e., \( \beta^G = 0 \)). Thus, the type \( G \) agent is a passive player who does not have any action. A piece of publicly revealed bad news is conclusive evidence that the agent’s type is \( B \). In this case, we will show that censorship not only always hurts the evaluator and the type \( G \) agent, but may also hurt the type \( B \) agent.

Now a revealed signal (good or bad news) resolves all uncertainties about the type of the agent. Clearly, the evaluator will never dismiss the agent after seeing a piece of good news, and he will dismiss the agent immediately after seeing a piece of revealed bad news. It follows that the type \( B \) agent will stop censoring after a piece of bad news has already been publicly revealed, even if the evaluator has deviated and has not yet dismissed her. Therefore, what remains to be determined is simply (1) the evaluator’s strategy under the public history, denoted by \( \varnothing_t \), at time \( t \) where no signal has been revealed to the evaluator, and (2) the type \( B \) agent’s strategy under her private history, denoted by \( \bar{\varnothing}_t^{B} \),\(^{16}\) at time \( t \) where no signal has been revealed to the evaluator and a piece of bad news has just arrived.

4.1 No censorship when the censoring cost is high

A prohibitively high cost will prevent censorship. This section determines the cost threshold above which censorship does not exist in equilibrium.

\(^{16}\)More precisely, \( \bar{\varnothing}_t^{n} \) denote the set of such histories. With an abuse of notation, we sometimes also use \( \bar{\varnothing}_t^{B} \) to denote an element in that set.
Note that it is strictly dominant for the evaluator to dismiss the agent when he observes bad news. Given this restriction, the best possible outcome for the type B agent is that she is dismissed only when bad news is publicly revealed. Let \( r_\infty \) denote the evaluator’s strategy under which he dismisses the agent only after a piece of bad news is publicly revealed. Thus, the dismissal time is the first time when the type B agent stops censoring a piece of bad news when it arrives. Given this strategy, the type B agent faces a stationary problem in the history \( \bar{\omega}_t^B \). The agent chooses between censoring or not, and her value function satisfies the following Bellman equation,

\[
V_{r_\infty}^B(\bar{\omega}_t^B) = \max \left\{ \mathbb{E} \left[ -\rho_0 c + \int_0^{\tau_B} \rho_0 e^{-\rho_0 \nu} w \, d\nu + e^{-\rho_0 \tau_B} V_{r_\infty}^B(\bar{\omega}_{\tau_B}^B) \right], 0 \right\},
\]

where \( \tau_B \) is the arrival time of the next piece of bad news from the type B agent. She will be dismissed and obtain zero payoff if she does not censor the arrived bad news. Otherwise, by censoring it, she incurs a cost \( \rho_0 c \), but can stay in the job until the arrival of the next piece of bad news at \( \tau_B \) and obtain her continuation value after \( \tau_B \). Clearly, \( V_{r_\infty}^B(\bar{\omega}_t^B) \) does not depend on \( t \). Thus we use \( V_{r_\infty}^B(\bar{\omega}_t^B) := V_{r_\infty}^B(\bar{\omega}^B) \).

It is easy to check that

\[
V_{r_\infty}^B(\bar{\omega}^B) = \max \left\{ w - (\rho_0 + \beta^B) c , 0 \right\}.
\]  

Thus, the type B agent has a strict incentive not to censor any bad news if \( c > c := \frac{w}{\rho_0 + \beta^B} \). Since any equilibrium strategy of the evaluator gives a weakly lower continuation payoff to the type B agent than the strategy \( r_\infty \) does, she will never censor any bad news if \( c > c \). Consequently, the evaluator faces a decision problem in the absence of censorship. His optimal policy is a cutoff strategy in which the cutoff belief is determined by the trade-off between exploitation and exploration, as demonstrated in the bandit problem literature. The following proposition summarizes the result.\(^{17}\)

**Proposition 1.** Assume \( \beta^G = 0 \) and \( c > c \). There is a unique PBE as follows:

- **Assume that** \( \gamma > \beta^B \). The type B agent never censors any bad news, and the evaluator dismisses the agent whenever the public belief is below \( p_{\text{fast}} := \frac{\rho_1 m}{\rho_1 h + \gamma(h-m)} \).

- **Assume that** \( \gamma < \beta^B \). The type B agent never censors any bad news, and the evaluator dismisses the agent whenever the public belief is below \( p_{\text{slow}} := \frac{\rho_1 m}{\rho_1 h + \beta^B(h-m)} \).

We make a distinction on what news arrives faster. We say **good news arrives faster** if it arrives faster than the bad news from the type B agent (i.e., \( \gamma > \beta^B \)). Otherwise, we say **good news arrives slower** (i.e., \( \gamma < \beta^B \)). A similar distinction is often made in Poisson bandit models where news arrives according to exogenous Poisson processes, which determines the direction in which the belief evolves in the absence of news. Given

\(^{17}\)We disregard the “almost surly” qualification for measure zero events in all statements to simplify the exposition.
that the agent never censors bad news, we have exogenous Poisson news processes. If good news arrives faster, then the public belief drifts down in the absence of news and thus the cutoff belief in the evaluator’s optimal strategy is $p_{fast}$, which is similar to the result in Keller, Rady, and Cripps (2005). But if good news arrives slower, then the public belief drifts up in the absence of news and thus the cutoff belief in the evaluator’s optimal strategy is $p_{slow}$, which is similar to the result in Keller and Rady (2015). We will see later that this distinction is also useful in our analysis where the bad news process is endogenously determined by the equilibrium censorship strategy.

Proposition 1 also shows what would happen if censorship were impossible – the no censorship benchmark (NCB). To study the welfare effect of censorship, we often compare players’ expected payoffs in equilibrium with their expected payoffs in the NCB.

For the rest of the paper, we assume that the censoring cost is not prohibitively high so that censorship may happen in equilibrium. Moreover, we distinguish two information structures according to what news arrives faster.

### 4.2 Faster good news

#### 4.2.1 Equilibrium

We first assume that good news arrives faster (i.e., $\gamma > \beta^B$). In this case, (ODE) indicates that the public belief drifts down in the absence of news, regardless of the agent’s censorship strategy. The greater the intensity of censorship, the faster the public belief declines in the absence of news. However, the evaluator’s optimal strategy (as a function of the public belief) does not depend on how much information is censored, as illustrated in Figure 1.

Figure 1 shows the evaluator’s value functions under different censorship policies. The horizontal axis is the prior belief. Consider two extreme censorship policies – no censorship (the green solid curve) where the agent never censers bad news, and full censorship (the red dashed curve) where the agent always censors bad news. The no censorship policy gives the evaluator the highest payoff for any prior belief, while the full censorship policy gives him the lowest payoff. The evaluator’s best response as a function of the public belief, however, is the same given the two extreme censorship policies, i.e., he dismisses the agent if and only if the public belief is below $p_{fast}$. This is because what matters to the evaluator at the margin\(^1\) is the arrival of good news – he will dismiss the agent if no news is revealed or if a piece of bad news is revealed, so only good news changes his decision. Hence, the optimal cutoff belief $p_{fast}$ does not depend on the arrival of bad news and the censorship policy, and therefore is the same in the two extreme cases. From the evaluator’s perspective, the Blackwell informativeness of any censorship policy lies in between that of the two extreme policies at each instant. Thus, for any censorship strategy of the agent, the evaluator’s value function must also

\(^1\)That is, when the public belief is at the boundary between the continuation and stopping regions.
lie in between his value functions when he faces those two extreme censorship policies. Thus he will use the same cutoff strategy in equilibrium. But we should keep in mind that although the evaluator uses the same cutoff strategy, the more the agent censors, the faster the same cutoff belief is reached in the absence of news.

Given the equilibrium strategy of the evaluator, we can pin down the equilibrium strategy of the type $B$ agent and the unique PBE. The blue dotted curve in Figure 1 illustrates the evaluator’s equilibrium value function given the type $B$ agent’s equilibrium strategy – a pure cutoff strategy with a cutoff belief $p_B$.

**Proposition 2.** Assume $\beta^G = 0$, $c < c$ and $\gamma > \beta^B$. There exists a unique PBE. The equilibrium features a pair of pure cutoff strategies.

1. The evaluator dismisses the agent whenever the public belief is below $p_{\text{fast}}$;

2. The type $B$ agent censors bad news whenever the public belief is above $p^B$, where $p^B > p_{\text{fast}}$.

Figure 2 shows the public belief evolution in the absence of news in equilibrium (the blue dotted curve). The evaluator will dismiss the agent whenever the public belief is below $p_{\text{fast}}$, which will happen eventually without the arrival of good news. Hence, the type $B$ agent knows that she will be dismissed at some point. As the public belief approaches the threshold $p_{\text{fast}}$, she knows that she has less time left before she is fired. Her benefit of censoring – the extra time she is able to stay in the job – becomes relatively small, compared with the cost of censoring. Thus, the type $B$ agent will

---

19 $p^B$ is defined by Equation (3) and (4) in the Appendix.
give up censorship before the public belief hits $p_{fast}$. In addition, before giving up censorship, the type B agent will censor all bad news with probability one. This is because the type B agent’s continuation value is increasing in the public belief, which drifts down in the absence of news. Whenever it is weakly optimal to censor bad news at time $t$ before the public belief drifts down to $p_{fast}$, it is strictly optimal to censor all bad news before time $t$. Hence, the censorship policy has two phases. At the beginning when the public belief is high (i.e., higher than $p^B$), the agent censors all bad news. We call it the Full-Censorship period, the length of which is denoted by $s_1$ in Figure 2. After the Full-Censorship period when the public belief is low (i.e., lower than $p^B$), the agent stops censoring. We call it the No-Censorship period, the length of which is stochastic. In the event that no bad news arrives, the maximal length, denoted by $s_2$ in Figure 2, of this period is reached. But if bad news arrives before the maximal length is reached, the No-Censorship period also ends. If the agent could commit to not censoring at all, then her maximal duration in the job would be $\bar{s}$ in Figure 2.

The indifference condition below determines the threshold at which the agent stops censoring:

$$
\mathbb{E} \left[ -\rho_0 c + \int_0^{s_2 \wedge \tau^B} \rho_0 e^{-\rho w} \, dw \right] = 0,
$$

where $s_2$ is the maximal length of the No-Censorship period and $\tau^B$ is the arrival time of the next piece of bad news from the type B agent. Thus, the expected continuation value in the No-Censorship period must be equal to the censoring cost. The larger the censoring cost, the sooner it becomes not profitable to censor bad news, and the longer the No-Censorship period is. Thus, $s_2$ determines the cutoff belief $p^B$, at which the agent switches from the Full-Censorship policy to the No-Censorship policy.
4.2.2 Welfare: Equilibrium versus NCB

We now compare the welfare consequences of censorship in two scenarios – the equilibrium expected payoff and the expected payoff in the absence of censorship, i.e., the NCB.\textsuperscript{20} Since the type $B$ agent’s equilibrium strategy is to stop censoring when $p_t \leq p^B$, nothing would be different between these two scenarios if the prior belief is below $p^B$. Thus, we assume that we start with a high prior belief $p_0 > p^B$.

The evaluator is worse off in equilibrium compared with the NCB. Figure 1 illustrates the evaluator’s welfare loss due to censorship. This is because censorship garbles information. No news means either nothing really has happened, or bad news has arrived but has been censored. The public belief drifts down faster in equilibrium compared with the NCB. It can be explained by the martingale property of the public belief. Since bad news would be censored, the fact that the public belief cannot jump to zero means that the drifting down process must be accelerated. It implies that the type $G$ agent is also worse off in equilibrium, since, in order to stay in her job, she now has to succeed within a shorter period of time before the public belief drifts below the evaluator’s dismissal cutoff belief.

It is noteworthy that the type $B$ agent may also be worse off in equilibrium. Censorship does provide her a higher expected flow payoff in the Full-Censorship period since she survives bad news when she censors. This is the positive effect for the type $B$ agent from censorship. The longer the Full-Censorship period is, the larger the benefit she enjoys. However, the negative effect is that censorship drives the public belief down faster in the absence of news compared with the NCB. In the event that no bad news arrives, the type $B$ agent indeed suffers from the evaluator’s skepticism. We can show that whenever the Full-Censorship period is short, the positive effect from censorship is dominated by the negative effect, which is summarized below.

\textbf{Proposition 3.} Assume $\beta^G = 0$, $c < \zeta$, $\gamma > \beta^B$ and $p_0 > p^B$. There exists a $s^*_1 > 0$ such that the type $B$ agent is worse off in equilibrium than she is in the NCB if and only if $s_1 < s^*_1$.

The Full-Censorship period is an equilibrium object, but we can reformulate this result in terms of the primitive variables. The most straightforward comparative statics is for the prior belief. When the prior belief is high, the Full-Censorship period is long, thus the type $B$ agent is better off in equilibrium. When the prior belief is low but higher than $p^B$, it means that the Full-Censorship period is short, thus the type $B$ agent is worse off in equilibrium.\textsuperscript{21}

A more interesting comparative statics is for the censoring cost. The censoring cost

\textsuperscript{20} We always compare the equilibrium payoff with the payoff in the NCB, unless stated otherwise.

\textsuperscript{21} Similarly, if the value of the outside option $m$ is low, then the type $B$ agent is better off in equilibrium. If $m$ is high but not too high such that the Full-Censorship period still exists, then the type $B$ agent is worse off in equilibrium.
stands for how hard it is to hide evidence from the evaluator. It also represents the strength of anti-censorship institutions. The cost directly determines when the agent is willing to give up censorship (i.e., the threshold belief $p^B$) and the length of the No-Censorship period. Thus, it indirectly determines the length of the Full-Censorship period. For a given prior belief, if the censoring cost is very high (but still below $c$), it means that the No-Censorship period is very long such that the Full-Censorship period disappears. Then, obviously, the equilibrium payoff is the same as the payoff in the NCB. If the censoring cost is very low, it means that the No-Censorship period is very short, which in turn implies that the Full-Censorship period is long. If it is long enough, then the type $B$ agent is better off in equilibrium. However, if the censoring cost is moderate, then the Full-Censorship period exists but is short, and the type $B$ agent is worse off in equilibrium. The result is summarized below.

**Corollary 1.** Assume $\beta^G = 0$, $c < \underline{c}$, $\gamma > \beta^B$ and $p_0 > p_{fast}$. There exists $c_1, c_3$, where $0 \leq c_1 < c_3 < \underline{c}$, such that the type $B$ agent is better off (resp. worse off) in the equilibrium than she is in the NCB if $c \in (0, c_1)$ (resp. if $c \in (c_1, c_3)$), and she obtains the same payoff in the equilibrium and in the NCB if $c \geq c_3$. In addition, $c_1 > 0$ if and only if $\bar{s} > s^*$ for some $s^* > 0$.

We can also examine the dependence of the equilibrium payoff with respect to the censoring cost, which is shown in the following result and summarized in Figure 3.

**Corollary 2.** Assume $\beta^G = 0$, $c < \underline{c}$, $\gamma > \beta^B$ and $p_0 > p_{fast}$. There exists a $c_2 \in [c_1, c_3)$ such that the equilibrium payoff of the type $B$ agent is decreasing (resp. increasing) in $c$ if $c \in [0, c_2]$ (resp. if $c \in [c_2, c_3]$). In addition, there exists a $s^{**} \in (0, s^*)$ such that $c_2 > c_1 \geq 0$ if $\bar{s} > s^{**}$, and $c_2 = c_1 = 0$ if $\bar{s} \leq s^{**}$.

<table>
<thead>
<tr>
<th>Eqm. v.s. NCB</th>
<th>Better-off</th>
<th>Worse-off</th>
<th>Same</th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_1$</td>
<td>Decreasing in $c$</td>
<td>$c_2$</td>
<td>Increasing in $c$</td>
</tr>
</tbody>
</table>

**Figure 3:** Comparative statics with respect to the censoring cost

The censoring cost serves as a “commitment” device for the agent. An increase in the censoring cost decreases the type $B$ agent’s expected flow payoff in the Full-Censorship period since censoring is more costly, but it increases her payoff in the No-Censorship period. Moreover, an increase in the cost also increases the length of the No-Censorship period, but decreases the length of the Full-Censorship period.

As illustrated in Corollary 1, when the censoring cost is low (i.e., $c \in (0, c_1)$), the Full-Censorship period is long and the type $B$ agent is better off in equilibrium. In
In this case, the negative effect of an increased censoring cost will dominate the positive effect, since the expected flow payoff in the Full-Censorship period is more important. Thus, the type $B$ agent’s equilibrium payoff is decreasing in the censoring cost. This means that the type $B$ agent who is better off with censorship must prefer a weaker institution (i.e., a smaller censoring cost).

The inverse statement is not true. When the censoring cost is high such that the Full-Censorship period exists but is short (i.e., $c \in (c_1, c_3)$), the type $B$ agent is worse off in equilibrium. However, it does not necessarily mean that she prefers a stronger institution, since her equilibrium payoff is not monotonic in the cost. Only when the cost is high enough (i.e., $c \in (c_2, c_3)$), the positive commitment effect of an increased censoring cost dominates the negative effect. In this case, her equilibrium payoff is increasing in the censoring cost, and a stronger institution would be preferred.

### 4.3 Slower good news

#### 4.3.1 Equilibrium

Now we turn to the case where good news arrives slower (i.e., $\gamma < \beta^B$). In the NCB, the public belief drifts up in the absence of news since the evaluator expects to see bad news more often. Thus, “no news is good news” and the evaluator dismisses the agent only when a piece of bad news arrives, if the game starts with an optimistic prior belief (i.e., $p_0 > p_{\text{slow}}$).

However, the possibility of censorship changes everything. Foremost, the evaluator’s optimal dismissal policy does depend on the agent’s censorship policy, which is different from the previous case where good news arrives faster. In the NCB, the evaluator’s optimal policy is to dismiss the agent whenever the public belief is below $p_{\text{slow}}$. However, if the agent censors all bad news, then the evaluator’s optimal policy is to dismiss the agent when the public belief is below $p_{\text{fast}}$, as shown in the last section. Hence, censorship discourages the evaluator’s incentive for exploring the agent’s competence. In addition, from the above two extreme cases, we know that the evaluator never dismisses the agent when the public belief is above $p_{\text{fast}}$, and he dismisses the agent immediately when the public belief is below $p_{\text{slow}}$, since again the evaluator’s value function for any censorship policy lies in between his value functions given by the two extreme censorship policies. This is illustrated in Figure 4.

The fact that the type $B$ agent can keep her job for sure when the public belief is above $p_{\text{fast}}$ also, perhaps ironically, implies that she censors all bad news in equilibrium under such beliefs. First, when no news is publicly revealed, the public belief must eventually drift down below $p_{\text{fast}}$. Otherwise the type $B$ agent can stay in her job forever by censoring all bad news, which in turn drives the belief down – this is a contradiction. Second, the only way that the belief drifts down below $p_{\text{fast}}$ is that the agent censors bad news before the belief is below $p_{\text{fast}}$. Moreover, if it is the agent’s interest to censor some bad news at a public belief above $p_{\text{fast}}$, she must also find it optimal to censor
bad news when the belief is even higher, since a longer stay in her job gives her a higher incentive for censoring. Those together imply that the type $B$ agent will censor all bad news when the public belief is above $p_{fast}$ and the only direction in which the belief drifts is downward. Thus, the evaluator faces essentially the worst censorship policy, and uses a cutoff strategy with a cutoff belief $p_{fast}$ to dismiss the agent.

However, a pure cutoff strategy that dismisses the agent when the belief hits $p_{fast}$ cannot be supported in equilibrium. If the evaluator were to use such a pure strategy, he would need to dismiss the agent deterministically in the absence of news after some time, since the public belief drifts down deterministically (in the absence of news) when it is above $p_{fast}$. However, this means that the type $B$ agent would give up censorship even when the public belief is slightly above $p_{fast}$, since the evaluator would dismiss her shortly. Such a contradiction implies that we need a mixed strategy for the evaluator. It implies that the type $B$ agent must also mix at the cutoff belief $p_{fast}$, since censoring too much would induce the evaluator to use the pure strategy, while censoring too little would induce the public belief to drift up above $p_{fast}$. The former contradicts with the fact that the evaluator must use a mixed strategy, and the latter contradicts with the fact that belief cannot drift up above $p_{fast}$. We summarize this result below.

**Proposition 4.** Assume $\beta^G = 0$, $c < \xi$, $\gamma < \beta^B$ and $p_0 \neq p_{fast}$. There exists a unique cutoff equilibrium. In the cutoff equilibrium, both players use mixed cutoff strategies with a common cutoff belief $p_{fast}$.

1. The evaluator dismisses (resp. does not dismiss) the agent when the public belief is below (resp. above) $p_{fast}$. He uses a constant hazard rate $z^* := \frac{w}{c} - \rho_0 - \beta^B$ to dismiss the agent at the cutoff belief $p_{fast}$.

2. The type $B$ agent censors (resp. does not censor) bad news when the public belief
is above (resp. below) $p_{\text{fast}}$. She censors bad news with a constant probability $x_{B^*} := 1 - \frac{\gamma}{\beta_B}$ at the cutoff belief $p_{\text{fast}}$.

Figure 5: The belief drifting process (conclusive bad news & slower good news)

Figure 5 shows the public belief evolution in the absence of news in equilibrium (the red dashed curve). The censorship policy still has two phases. At the beginning, we have the Full-Censorship period – the type $B$ agent censors all bad news when the belief is above $p_{\text{fast}}$. When the belief reaches $p_{\text{fast}}$, she censors bad news with a constant probability $x_{B^*}$ such that the arrival rate of good news $\gamma$ equals to the arrival rate of the non-censored bad news $(1 - x_{B^*})\beta_B$. Thus, the public belief stays constant at $p_{\text{fast}}$ in the absence of news and the problem is stationary. Moreover, the evaluator dismisses the agent at a constant hazard rate that is chosen to make the type $B$ agent indifferent between censoring and not. We call this period the Partial-Censorship period.

This is the unique cutoff equilibrium. If the evaluator were to use a cutoff strategy with a different cutoff belief $\hat{p}$, then it must be below $p_{\text{fast}}$, because the evaluator never dismisses the agent when the public belief is above $p_{\text{fast}}$. However, the exact argument that shows the type $B$ agent censors all bad news when the belief is above $p_{\text{fast}}$ in equilibrium applies now so that she must censor all bad news when the public belief is above $\hat{p}$. This means that the evaluator receives even less information. Thus he would not use the cutoff strategy with a lower cutoff belief due to his discouraged incentive for learning. Therefore, we obtain the unique cutoff equilibrium.

4.3.2 Welfare: Equilibrium versus NCB

We now compare payoffs in the cutoff equilibrium with payoffs in the NCB. First, the evaluator is worse off in equilibrium, as illustrated in Figure 4. Actually, he obtains the worst possible payoff. It is the same payoff that the evaluator would have if he faced
a full censorship policy and relied only on good news. That is why his incentive for learning is discouraged – he increases the cutoff belief from $p_{\text{slow}}$ to $p_{\text{fast}}$ in his cutoff strategy. This new effect does not appear when good news arrives faster; we call it the *discouragement effect*. Due to this effect, when the prior belief is in between $p_{\text{slow}}$ and $p_{\text{fast}}$, neither type of agent would have a chance to start her job in equilibrium, but she could at least stay in her job for some time in the NCB. Thus, when $p_0 \in (p_{\text{slow}}, p_{\text{fast}})$, both types of agent are worse off in equilibrium, compared with the NCB.

When the prior belief is above $p_{\text{fast}}$, the type $G$ agent will stay in her job forever in the NCB, but she will be dismissed with a strictly positive probability in the equilibrium. Obviously, she is worse off in equilibrium compared with the NCB. For the type $B$ agent, she has a higher payoff in the Full-Censorship period since she survives bad news, but a lower payoff in the Partial-Censorship period. Her total payoff is the average payoff in those two periods, so she is worse off in equilibrium whenever the Full-Censorship period is short. We summarize the result below.

**Proposition 5.** Assume $\beta^G = 0$, $c < c^B$ and $\gamma < \beta^B$.

1. When $p_0 \in (p_{\text{slow}}, p_{\text{fast}})$, the type $B$ agent’s is worse off in equilibrium than she is in the NCB;

2. When $p_0 > p_{\text{fast}}$, there exists a $\hat{s}^* > 0$ such that the type $B$ agent’s is worse off in equilibrium than she is in the NCB if and only if $\hat{s} < \hat{s}^*$.

The length of the Full-Censorship period is determined in equilibrium. We can easily relate the above result to the primitive variables as in Section 4.2.2. For example, a higher prior belief translates into a longer Full-Censorship period, thus the type $B$ agent is better off in equilibrium; a lower prior belief (but still higher than $p_{\text{fast}}$) translates into a shorter Full-Censorship period, thus she is worse off in equilibrium.

However, the length of the Full-Censorship period now does not depend on the censoring cost, which is different from the faster good news case. This is because the belief $p_{\text{fast}}$ at which the type $B$ agent switches from Full-Censorship to Partial-Censorship is independent of the cost. Thus, whether the type $B$ agent is better off in equilibrium does not depend on the cost. However, an increase in the censoring cost decreases the type $B$ agent’s payoff in the Full-Censorship period since censoring is more costly, but it increases her payoff in the Partial-Censorship period. Her total equilibrium payoff is the average payoff in those two periods. Therefore, if she is better off in equilibrium, it means that the Full-Censorship period must be long and important. Hence, an increase in the cost decreases her equilibrium payoff. The inverse statement is also true. If the type $B$ agent is worse off in equilibrium, it means that the Full-Censorship period must be short and not important. Thus, an increase in the cost increases her equilibrium payoff. Therefore, the type $B$ agent’s payoff in equilibrium is monotonic in the cost, so she prefers either a very strong institution or a very weak institution, depending on how long the Full-Censorship period is. The result is summarized below.
Corollary 3. Assume $\beta^G = 0$, $c < \zeta$ and $\gamma < \beta^B$.

- Whether the type $B$ agent is better off in the equilibrium than she is in the NCB does not depend on the censoring cost;
- When $p_0 > p_{\text{fast}}$, the type $B$ agent’s expected payoff in equilibrium is monotonic in the censoring cost $c$. Moreover, it is decreasing (resp. increasing) in the cost, if and only if, she is better off (resp. worse off) in equilibrium than in the NCB.

5 Inconclusive bad news

In this section, we study the more general model where bad news also arises for the type $G$ agent, but it, of course, arises more frequently for the type $B$ agent (i.e., $0 < \beta^G < \beta^B$). Thus, bad news is an informative but imperfect signal in the NCB. The posterior public belief after bad news arrives jumps down, but it is above zero. Hence, after bad news arrives, the evaluator knows that the agent’s type is more likely to be $B$, but he is not certain. The optimal policy for the evaluator’s decision problem in the NCB is still a cutoff strategy, in which the cutoff belief depends on the information structure. However, since bad news is a noisy signal, the evaluator may make both type I and type II errors when he makes a decision based on the bad news.

Inconclusive bad news enriches and complicates the model. We will only focus on new insights derived from inconclusive bad news, but will not fully characterize all equilibria, nor conduct case-by-case discussion. In particular, we will extend our intuition obtained from the conclusive bad news case, focus on simple equilibria in which all players use a cutoff strategy, and show that it is possible that censorship may benefit the evaluator, as well as both types of agent.

5.1 Faster good news

5.1.1 Equilibrium

We assume first that good news arrives faster than the type $B$ agent’s bad news (i.e., $\gamma > \beta^B$). In the NCB, the public belief drifts down in the absence of news.\(^{23}\)

Now both types of agent need to choose a censorship policy. The basic trade-off remains the same – the agent censors bad news if and only if her equilibrium continuation value is higher than the censoring cost. While the censoring cost is the same for both

\(^{22}\)When bad news is inconclusive, we define the NCB in the same way as before. In the NCB, censorship is not possible, for example, due to a prohibitively high censoring cost.

\(^{23}\)In the NCB, the direction in which the public belief drifts is determined by the sign of \(\gamma + \beta^G - \beta^B\). However, as we will show later, the relevant equilibrium term is still determined by the sign of \(\gamma - \beta^B\), since the type $G$ agent censors bad news more aggressively than the type $B$ agent does.
types of agent, the continuation value is different. The type $G$ agent has a higher continuation value than the type $B$ agent does due to the following reasons. First, the type $G$ agent has a chance to succeed in her project, so she stays longer in her job in expectation than the type $B$ agent does. Second, although the cost of censoring one piece of bad news is the same for both types, the fact that bad news arises more often for the type $B$ agent makes it more costly for the $B$ type agent to carry on the same censorship policy. Therefore, the type $G$ agent has a higher continuation value in the job and a higher incentive to censor bad news. This observation significantly changes the welfare effect of censorship, which will be discussed after we introduce the following equilibrium.

**Proposition 6.** Assume $c < c$ and $\gamma > \beta^B$. There exists a $\bar{\beta} \in (0, \beta^B)$ such that for any $\beta^G \in (0, \bar{\beta}]$, there exists a PBE in which every player uses a pure cutoff strategy.

1. The evaluator dismisses the agent whenever the public belief is below $p_{\text{fast}}$;
2. The type $G$ agent censors bad news whenever the public belief is above $p^G$ (but not equal to 1), where $p^G > p_{\text{fast}}$;
3. The type $B$ agent censors bad news whenever the public belief is above $p^{B1}$, where $p^{B1} > p^G$.

Here we keep the assumption that the censoring cost is low. In addition, we assume that the type $G$ agent’s bad news does not happen too often (i.e., $\beta^G \in (0, \bar{\beta}]$). Figure 6 depicts the public belief evolution in the absence of news both in equilibrium (the blue dotted curve) and in the NCB (the green solid curve). Our first observation is that the evaluator’s strategy is the same cutoff strategy with the cutoff belief $p_{\text{fast}}$ in both the equilibrium and the NCB as the same logic in Section 4.2.1 applies so that the cutoff belief does not depend on the arrival rate of bad news and does not depend on the censorship policy. Second, there are three phases in equilibrium. The belief $p^G$ (resp. $p^{B1}$) is the cutoff belief at which the type $G$ (resp. $B$) agent stops censoring. When the public belief is high (i.e., $p_t > p^{B1}$), both types of agent censor bad news for sure – they are pooling on full censorship. Thus, no bad news is publicly revealed in this phase, which also implies that the belief drifts down faster than it does in the NCB. However, if someone deviated and a piece of bad news were publicly revealed in this phase, we would set the off-path public belief to zero under such an off-path event. When the public belief is intermediate (i.e., $p_t \in (p^G, p^{B1})$), the type $G$ agent censors bad news for sure, but the type $B$ agent stops censoring. We call it the Separation period. As explained, the type $G$ agent has a higher incentive and censors bad news more aggressively than the type $B$ agent does. Bad news may arise in this phase. But the noisy inconclusive bad news now becomes to a perfect conclusive signal because of the separation of censorship policies between the two types. Thus, the belief jumps down to zero when bad news is publicly revealed. It implies that the belief must drift down slower than it does in the NCB. This is driven by the martingale property of
the public belief – the jumping process is amplified, thus the drifting process must be mitigated. When the public belief is low (i.e., \( p_t < p^G \)), both types of agent stop censoring – they are pooling on no censorship. In this last phase, a piece of bad news drives the posterior belief down. With the assumption that the type \( G \) agent’s bad news does not happen too often, the posterior belief after bad news arrives jumps down below the evaluator’s cutoff belief \( p_{\text{fast}} \) and the agent is therefore dismissed.

5.1.2 Welfare: Equilibrium versus NCB

Having the above equilibrium in mind, we now compare the payoffs from that equilibrium with the payoffs in the NCB. We find that, under some conditions, every player has a strictly higher payoff from the above equilibrium than he or she does in the NCB.

**Proposition 7.** Assume \( c < \zeta \) and \( \gamma > \beta^B \). There exists a \( \beta \in (0, \beta] \) such that when \( \beta^G \in (0, \beta] \) and \( p_0 \in (p^G, p^{B\dagger}] \), the evaluator and both types of agent have strictly higher payoffs in the equilibrium in Proposition 6, compared with their payoffs in the NCB.

The conditions for the above result are (1) the game starts from the Separation period (i.e., \( p_0 \in (p^G, p^{B\dagger}] \)), and (2) the bad news does not arise too often for the type \( G \) agent (i.e., \( \beta^G \in (0, \beta] \)).\(^{24}\) In the Separation period, a piece of publicly revealed bad

\(^{24}\)Those are sufficient but not necessary conditions to obtain the above result. By continuation, the above result still holds when the prior belief is slightly higher than \( p^{B\dagger} \), but we restrict to those conditions to make a sharp exposition on the underlying economic forces.
news concludes that the agent is of type $B$. This improves the evaluator’s information quality. In this period, he never dismisses a type $G$ agent, and the agent he dismisses must be a type $B$ agent. The endogenously conclusive signal helps the evaluator avoid making type I and type II errors. This is why his payoff is strictly higher than that in the NCB when the game starts from the Separation period. Figure 7 shows the evaluator’s value functions in equilibrium (the blue dotted curve) and in the NCB (the green solid curve). The two value functions may cross when the prior belief is high enough, but this does not happen given the primitive variables in Figure 7. When the prior belief is high ($p_0 > p^{B\dagger}$), the evaluator expects to have low quality information first (i.e., no bad news may arise), and then high quality information (i.e., conclusive bad news may arise). Whether he is better off in equilibrium compared with the NCB depends on whether the benefit from the Separation period dominates the loss from the period in which both types of agent pool on full censorship.

\[
U(p_0) = \begin{cases} 
\text{No censorship} & \quad p_0 > p^{G} \\
\text{Equilibrium} & \quad p^{fast} < p_0 < p^{G} < p^{B\dagger} 
\end{cases}
\]

Figure 7: The evaluator’s value functions (inconclusive bad news & faster good news)

In the Separation period, another implication is that the event that no news is revealed hurts the agent’s reputation less than it would be in the NCB. This is the main reason why both types of agent are better off in equilibrium. Although the equilibrium public belief still drifts down in the absence of news, it drifts down slower than it does in the NCB. Since the dismissal threshold belief is the same in both situations, in equilibrium the agent stays longer in her job in the absence of news than she does in the NCB. For the type $B$ agent, when the game starts from the Separation period, she does not censor bad news in both the equilibrium and the NCB. The assumption that bad news does not arise too often for the type $G$ agent ensures that the posterior belief is driven below the evaluator’s dismissal threshold after a piece of revealed bad news even in the NCB. Thus, a piece of revealed bad news leads to the dismissal of the agent in both
situations. Thus, her payoff is the same in both situations conditional on the event that a piece of bad news arrives. However, in the lucky event that bad news does not arise, she stays longer in her job in the equilibrium compared with the NCB. Thus, the type B agent is strictly better off in equilibrium. The type G agent is also strictly better off in equilibrium. First, censoring bad news in the Separation period gives her a higher expected flow payoff in equilibrium. We have seen the same effect when bad news is conclusive. In addition, she also stays longer in her job in equilibrium. Such an effect is opposite when bad news is conclusive, since there the belief drifts down faster, not slower, in the absence of news. Therefore, both effects imply that the type G agent has a strictly higher payoff in the equilibrium than she does in the NCB.

If the censoring cost increases, the type B agent first gives up censorship as she has a lower value in her job than the type G agent does. Thus, if the cost is neither too low nor too high, and if bad news does not arise too often for the type G agent – a similar condition we assumed in Proposition 6, then there exists an equilibrium in which the evaluator and the type G agent use a similar strategy as in Proposition 6, but the type B agent never censors bad news. Thus, the first phase in Figure 6 where both types are pooling on full censorship disappears – we only have the Separation period and the last period where both types are pooling on no censorship. In this case, the welfare effect is also similar to the one in Proposition 7.

5.2 Slower good news

5.2.1 Equilibrium

Finally, we assume that good news arrives slower than the type B agent’s bad news (i.e., $\gamma < \beta^B$) and maintain the assumption that bad news is inconclusive. Such an information structure has the following implications in the NCB. “Slower good news” does not necessarily mean that “no news is good news.” The type G agent now has two possible signals – good news with intensity $\gamma$ and bad news with intensity $\beta^G$, while the type B agent only has bad news with intensity $\beta^B$. If the total intensity of signals from the type G agent is smaller than that from the type B agent, then indeed “no news is good news” – the public belief will drift up in the absence of news. This happens if good news arrives very slow (i.e., $\gamma < \beta^B - \beta^G$). However, if good news arrives just mildly slow (i.e., $\beta^B - \beta^G < \gamma < \beta^B$), then “no news is bad news” – the public belief will drift down in the absence of news. In both cases, the evaluator’s optimal policy in the NCB is a cutoff strategy in which he dismisses the agent whenever the public belief is below some cutoff $p^* \in (p_{\text{slow}}, p_{\text{fast}}]$. Such a cutoff belief $p^*$ is strictly below $p_{\text{fast}}$ if good news arrives very slow, otherwise it is equal to $p_{\text{fast}}$.

Although the information structure is different from the previous section, our main insight that the type G agent has a higher incentive to censor bad news than the type B agent does still holds. If the censoring cost remains low (i.e., $c < \bar{c}$), we can show that the cutoff equilibrium in Proposition 4 when bad news is conclusive still exists,
despite the fact that bad news is now inconclusive. That is, there exists an equilibrium in which the evaluator and the type B agent use the same strategies as in Proposition 4, and the type G agent censors bad news if and only if the public belief is above or equal to \( p_{\text{fast}} \). We show this result by examining every player’s incentive. First, it is easy to see that, given others’ strategies, the evaluator and the type B agent face the same problem as if bad news were conclusive, because the type G agent censors all bad news. Thus, they choose the same strategies as in the conclusive bad news case. Moreover, because the type G agent has a higher incentive to censor bad news than the type B agent does, the former must censor bad news with probability one if the latter does so with a positive probability. Therefore, such an equilibrium exists.

The type B agent is more reluctant to censor bad news. Therefore, if the censoring cost increases to an intermediate level (i.e., \( c \in (\widehat{c}, \bar{c}) \), where \( \bar{c} := \frac{\gamma + \rho_0 - w}{\beta + \gamma + \rho_0} \)), there exists an equilibrium in which the type B agent never censors but the type G agent censors bad news whenever the public belief is in between \( p_{\text{slow}} \) and 1, while the evaluator dismisses the agent whenever the belief is below \( p_{\text{slow}} \). Note that \( c \) is the cost threshold that determines whether the type B agent is willing to censor all bad news in exchange for staying in her job forever, while \( \bar{c} \) is the cost threshold that determines whether the type G agent is willing to censor all bad news before she succeeds in exchange for staying in her job forever. We again have the separation result – the censoring cost is in a level such that only the type G agent finds it worthwhile to censor bad news. Separation between the two types makes bad news become conclusive in equilibrium. Thus, the evaluator faces a problem as if bad news were conclusive. This is why his equilibrium strategy features the cutoff belief \( p_{\text{slow}} \) – the same cutoff belief in his optimal policy in the NCB when bad news is conclusive. In addition, since good news arrives slower than the type B agent’s bad news in equilibrium, “no news is good news,” i.e., the public belief drifts up in the absence of news. The following proposition summarizes the result.

**Proposition 8.** Assume \( \gamma < \beta^B \) and \( \beta^G \in (0, \beta^B) \).

1. (Low cost) Assume \( c < c \), there exists an equilibrium as follows:
   - The evaluator and the type B agent use the same strategies as in Proposition 4;
   - The type G agent censors bad news whenever the public belief is above and equal to \( p_{\text{fast}} \) (but below 1).

2. (Intermediate cost) Assume \( c \in (\widehat{c}, \bar{c}) \), there exists an equilibrium as follows:
   - The evaluator dismisses the agent whenever the public belief is below or equal to \( p_{\text{slow}} \);
   - The type G agent censors bad news whenever the public belief is above \( p_{\text{slow}} \) (but below 1), and the type B agent never censors.
5.2.2 Welfare: Equilibrium versus NCB

We continue to compare equilibrium payoffs with the payoffs in the NCB. However, the comparison would be tedious if we tried to cover every possible scenario, especially now the NCB has two very different cases, depending on whether good news arrives very slow or mildly slow. Our main interest and focus here will be on how the censoring cost changes the welfare comparison from the evaluator’s point of view.

**Proposition 9.** Assume $\gamma < \beta^B$ and $\beta^G \in (0, \beta^B)$.

1. (Low cost) Assume $c < \underline{c}$. When $p_0 \in (p^*, 1)$, the evaluator has a strictly lower payoff in the equilibrium from Proposition 8 than he does in the NCB.

2. (Intermediate cost) Assume $c \in (\underline{c}, \bar{c})$. When $p_0 \in (p_{\text{slow}}, 1)$, the evaluator has a strictly higher payoff in the equilibrium from Proposition 8 than he does in the NCB.

Figure 8 depicts the evaluator’s value functions in the NCB and in the equilibrium from Proposition 8. If the censoring cost is low, then both types of agent censor bad news very aggressively and the evaluator cannot rely on bad news in learning. It is the worst information that the evaluator may have, thus he is worse off in the equilibrium. In addition, the evaluator’s incentive for learning is discouraged when good news arrives very slow. That is, when $\gamma < \beta^B - \beta^G$, the cutoff belief $p^*$ in the evaluator’s optimal policy in the NCB is increased to the cutoff belief $p_{\text{fast}}$ in his equilibrium strategy. We have seen this discouragement effect in Section 4.3.2 when bad news is conclusive. If, instead, the censoring cost is intermediate, then the separation of censorship policies dramatically improves the quality of public information – a piece of bad news is endogenously conclusive. Hence, the evaluator is better off in the equilibrium. Moreover, the improvement in the quality of public information encourages the evaluator to explore the agent’s type even when the belief is in between $p_{\text{slow}}$ and $p^*$. We call it the *encouragement effect* – the cutoff belief $p^*$ in the evaluator’s optimal policy in the NCB is decreased to the cutoff belief $p_{\text{slow}}$ in his equilibrium strategy.

The agent is also affected directly by the encouragement and the discouragement effects. When the censoring cost is intermediate, the encouragement effect implies that both types of agent are better off in the equilibrium if the prior belief is in between $p_{\text{slow}}$ and $p^*$. In the equilibrium, if the game starts at such a prior belief, then the type $G$ agent stays in her job forever and the type $B$ agent stays in her job for some positive time. While in the NCB, neither type has a chance to start her job. When the censoring cost is low and good news arrives very slow (i.e., $\gamma < \beta^B - \beta^G$), the effect is opposite – both types of agent are worse off in the equilibrium if the prior belief is in between $p^*$ and $p_{\text{fast}}$ due to the discouragement effect.
6 Conclusion

This paper studies costly censorship in a dynamic environment. An evaluator learns an agent’s competence level from both good news and bad news. However, the agent who knows her competence level can conceal bad news at some cost.

When bad news is conclusive, it only arises for the incompetent agent. She censors bad news if and only if this secures her a significant increase in tenure. The evaluator suffers from censorship since otherwise valuable information is suppressed. Censorship also makes her suspicious in the event that no news arrives, which hurts the competent agent. It may also hurt the incompetent agent, even though censorship helps her survive bad news. The incompetent agent is worse off with censorship when the period in which she finds it optimal to censor bad news is short. If good news arrives slower than the bad news from the incompetent agent, a discouragement effect occurs – the evaluator increases the public belief threshold at which he dismisses the agent.

When bad news is inconclusive, it arises for both the competent and the incompetent agents. However, the competent agent has a higher incentive to censor it since she has a higher value in the job. When the two types of agent separate in their censorship strategies (i.e., only the competent agent censors bad news), the quality of information for the evaluator is improved – the inconclusive bad news becomes conclusive and perfect. Not only the evaluator benefits from that, but also both types of agent. The event that no news arrives now does not hurt the agent’s reputation as much as it would be when censorship were not possible. Thus, the agent may stay longer in her job. If good news arrives slower than the bad news from the incompetent agent and if
the censoring cost is at an intermediate level, an encouragement effect may occur – the evaluator decreases the public belief threshold at which he dismisses the agent.

Appendices

Proof of Proposition 1. (1) First, we show that when \( c > c_\zeta \), the type B agent never censors bad news under any history \( \hat{\sigma}_t^B \) in any PBE.

In any PBE, a piece of bad news results in an immediate dismissal of the agent. Fix an equilibrium and the equilibrium strategies \( \{ \hat{r}, \hat{x}^B \} \). Suppose, by contradiction, that \( \hat{x}_t^B > 0 \) in history \( \hat{\sigma}_t^B \). Let \( V^\hat{r}_t(\hat{\sigma}_t^B) \geq 0 \) be the type B agent’s equilibrium payoff in history \( \hat{\sigma}_t^B \). Consider an alternative strategy \( r_\infty \) of the evaluator in which he dismisses the agent if and only if a piece of bad news is revealed. If the agent censors that piece of bad news in history \( \hat{\sigma}_t^B \) and resumes her best response to \( r_\infty \), which, according to Equation (1) and \( c > c_\zeta \), is to never censor bad news, she can obtain

\[
-\rho_0 c + \mathbb{E} \left[ \int_0^{\tau^B} \rho_0 e^{-\rho_0 \nu} w \ d\nu + e^{-\rho_0 \tau^B} V^\tau B_0(\hat{\sigma}^B_{\tau^B}) \right] = \frac{\rho_0}{\rho_0 + \beta^B} \left[ w - (\rho_0 + \beta^B) c \right] < 0,
\]

where \( \tau^B \) is the arrival time of the next piece of bad news from the type B agent. Thus, \( v^r B_\infty^\tau(\hat{\sigma}_t^B) \leq \frac{\rho_0}{\rho_0 + \beta^B} \left[ w - (\rho_0 + \beta^B) c \right] < 0 \), where the first inequality comes from the sub-optimality of strategy \( \hat{x}^B \) to the evaluator’s strategy \( r_\infty \).

In addition, \( V^\hat{r}_t(\hat{\sigma}_t^B) = v^\hat{r}_B(\hat{\sigma}_t^B) \leq v^r B_\infty(\hat{\sigma}_t^B) < 0 \) since \( r_\infty \) subscribes the best situation that the agent may face. This contradicts \( V^\hat{r}_t(\hat{\sigma}_t^B) \geq 0 \).

(2) Second, the evaluator faces a standard exponential bandit problem, since the type B agent never censors bad news. The literature on bandit problems characterizes the optimal strategy of the evaluator. For completeness, we sketch the proof here.

Let \( 0 \) denote the strategy of the type B agent in which she never censors bad news. In the history \( \sigma_t \), when no signal is publicly revealed, the evaluator chooses \( r_t \) to maximize

\[
U^{r,0}(\sigma_t) = \mathbb{E} \left[ \int_t^T \rho_1 e^{-\rho_1 (\nu - t)} \ 1_{\theta = G} \ dN^G + e^{-\rho_1 (T - t)} \ m \right | \sigma_t]
= \mathbb{E} \left[ \int_t^T \rho_1 e^{-\rho_1 (\nu - t)} p_\nu \ d\nu + e^{-\rho_1 (T - t)} \ m \right | \sigma_t],
\]

where the second equation comes from the Law of Iterated Expectations.

Since the payoff of the evaluator depends on history only through the public belief \( p_t \), we rewrite the above as \( u^{r,0}(p_t) = u^{r,0}(\sigma_t) \), and \( U^{0}(p_t) = \sup_{r_t} u^{r,0}(p_t) \). When \( \gamma > \beta^B \),
the following Bellman equation solves $U^0(p)$ when it is optimal to experiment,\(^{25}\)

$$
\rho_1 U^0(p) = p \gamma \left[ k (\gamma + \rho_1) - U^0(p) \right] + (1 - p) \beta^B \left[ m - U^0(p) \right] - p(1 - p) (\gamma - \beta^B) U^0'(p).
$$

When the evaluator is indifferent between experimenting and dismissal at a public belief $\hat{p}$, we have $U^0(\hat{p}) = m$ (value matching) and $U^0'(\hat{p}) = 0$ (smooth pasting). Thus,

$$
\rho_1 m = \hat{p} \left[ \rho_1 h + \gamma (h - m) \right],
$$

where $h = \gamma k$ and $\hat{p} = p_{\text{fast}} := \frac{\rho_1 m}{\rho_1 h + \gamma (h - m)}$. The Bellman equation is an ODE with two boundary conditions. For $p < p_{\text{fast}}$ we have $U^0(p) = m$, and for $p \geq p_{\text{fast}}$ it is $U^0(p) = \Phi(p; \bar{s}, \beta^B)$, where $\bar{s}$ is the time length during which the public belief drifts from the prior $p$ to $p_{\text{fast}}$ in the absence of news, i.e., $\bar{s} = \frac{1}{\gamma - \beta^B} \ln \left[ \frac{p_{\text{fast}} - p}{1 - p} \right]$, and

$$
\Phi(p; s, \beta^B) := m + p (h - m) (1 - e^{-(\rho_1 + \gamma)s}) - (1 - p) m \frac{\rho_1}{\rho_1 + \beta^B} (1 - e^{-(\rho_1 + \beta^B)s}).
$$

When $\gamma < \beta^B$, the same Bellman equation applies. However, the smooth pasting condition does not hold, since the public belief at the boundary between the continuation and stopping regions does not drift down to the stopping region in the absence of news.\(^{26}\) A more direct approach is to compute the optimal experimenting time without a conclusive signal. If the experimenting time is $s$, then the evaluator’s expected payoff is $\Phi(p; s, \beta^B)$. It is easy to see from the first order condition that it is either optimal to experiment forever or to stop immediately. The value matching condition gives $\hat{p} = p_{\text{slow}} := \frac{\rho_1 m}{\rho_1 h + \gamma (h - m)}$ that solves

$$
m + \hat{p} (h - m) - (1 - \hat{p}) m \frac{\rho_1}{\rho_1 + \beta^B} = m.
$$

Thus, for $p < p_{\text{slow}}$ we have $U^0(p) = m$, and for $p \geq p_{\text{slow}}$ it is $U^0(p) = \lim_{s \to \infty} \Phi(p; s, \beta^B)$. It is noteworthy that $U^0(p)$ is linear and strictly increasing for $p \geq p_{\text{slow}}$. \(\Box\)

Proof of Proposition 2. (1) First, for any admissible strategy of the type $B$ agent, the evaluator’s best response, if it exists, is a pure cutoff strategy with the cutoff $p_{\text{fast}}$.


\(^{26}\)We say the evaluator experiments when he does not dismiss the agent in the public history $\emptyset_1$. The smooth pasting condition holds in the case where good news arrives faster. This is because when the public belief is at the boundary between the continuation and stopping regions, it drifts down into the stopping region with probability one. For an optimal stopping problem, it means that the boundary is regular. However, the boundary is irregular in the case where good news arrives slower, since the public belief at the boundary drifts up into the continuation region with probability one. Thus, the value function with slower good news has a kink at the boundary between the continuation and stopping regions. See Keller and Rady (2015) and Peskir and Shiryaev (2006) for details.
Let \(0\) and \(1\) denote the type \(B\) agent’s strategy in which she never censors \((x^B_t \equiv 0)\) and always censors bad news \((x^B_t \equiv 1)\), respectively. The standard analysis in bandit problems shows that in both cases the evaluator’s best response is to use a cutoff strategy with the cutoff belief \(p_{fast}\). We have verified that it is the best response to \(0\). To see the best response to full censorship \(1\), let \(U^1(p_0)\) denote the evaluator’s value function. When it is optimal to experiment, it solves the following Bellman equation,

\[
\rho_1 U^1(p) = p \gamma \left[ k (\gamma + \rho_1) - U^1(p) \right] - p(1 - p)\gamma U^1(p).
\]

The value matching and smooth pasting conditions give the cutoff belief \(\hat{p} = p_{fast}\) that solves

\[
\rho_1 m = \hat{p} \left[ \rho_1 h + \gamma (h - m) \right].
\]

Moreover, for \(p \leq p_{fast}\) we have \(U^1(p) = m\), and for \(p > p_{fast}\) we have \(U^1(p) = \Phi(p; \hat{s}, 0)\), where \(\hat{s}\) is the time length during which the public belief drifts from the prior \(p\) to \(p_{fast}\) in the absence of news, i.e.,

\[
\hat{s} = \frac{1}{\gamma} \ln \left[ \frac{p}{1 - p} \frac{1 - p_{fast}}{p_{fast}} \right].
\]

These two cases represent the maximal and the minimal information that the evaluator can use. The minimal information is a garbling of the information induced by any other admissible strategy of the type \(B\) agent, which in turn is a garbling of the maximal information. Consider the case where the type \(B\) agent uses a strategy \(\tilde{\mathbf{x}}^B\) and the public conjecture is \(\tilde{\mathbf{a}}^B\). Let \(r\) be the evaluator’s best response with respect to \(\tilde{\mathbf{a}}^B\), and \(r_0\) and \(r_1\) be his best responses with respect to \(0\) and \(1\), respectively. We have

\[
U^0(p) = u^{\tilde{r}_0,0}(p) \geq u^{r,0}(p) \geq u^{\hat{r},\hat{a}^B}(p) \geq u^{r_1,\hat{a}^B}(p) \geq u^{r_1,1}(p) = U^1(p).
\]

Inequalities \(\geq\) hold because by using the same strategy, the evaluator obtains a higher payoff under the maximal information than under any other information induced by the agent’s strategy, and obtains a lower payoff under the minimal information than under any other information. In addition, when \(p > p_{fast}\), the evaluator’s best response to any publicly conjectured strategy of the agent is to keep experimenting since it is also his best response under the minimal information. Similarly, his best response is to dismiss the agent when \(p < p_{fast}\) as it is his best response under the maximal information.

(2) Given that the evaluator uses a cutoff strategy with the cutoff \(p_{fast}\) in any PBE, the type \(B\) agent will be eventually dismissed. Fix a PBE, let \(s_1 = \sup \{ t \geq 0 : \hat{h}_t^B \in \tilde{\mathbf{a}}^B_t, x_t^B > 0 \}\) be the last time when the agent censors bad news with a positive probability if a piece of bad news has just arrived and no signal has been publicly revealed before. Thus, the agent stops censoring after \(s_1\). Clearly, without any revealed signal until \(s_1\), we must have \(p_{s_1} > p_{fast}\), since otherwise censoring a piece of bad news only incurs a cost, but gives no continuation value.

Moreover, when bad news has not been publicly revealed at time \(t < s_1\), we must have \(x_t^B = 1\). Suppose not, i.e., there exists a \(t < s_1\) and \(x_t^B < 1\). Since \(s_1\) the last time when
the agent censors bad news with a positive probability, there must a \( t' \in (t, s_1] \) such that \( x^B_t > 0 \). The agent’s equilibrium payoff \( V_B^r(\bar{0}^B_t) \) cannot be negative, equivalently,

\[
\mathbb{E} \left[ \int_{t'}^{T} \rho_0 e^{-\rho_0(\nu-t)} (w \, dv - cX^B_{\nu'} \, dN^B_{\nu'}) \bigg| \bar{0}^B_t \right] \geq \rho_0 \, c,
\]

where \( T \) is induced by the equilibrium strategy profile. Consider an alternative strategy \( x^{B'} \) under which the agent censors all bad news for time \( \nu \in [t, t'] \) and then resumes the original strategy. Note that the dismissal time \( T \geq t' \) given \( x^{B'} \). Thus, we have

\[
v^r_{x^{B'}}(\bar{0}^B_t) = -\rho_0 \, c + \int_{t'}^{t} \rho_0 e^{-\rho_0(\nu-t)} (w \, dv - \beta^B \, c \, dv) \\
+ e^{-\rho_0(t'-t)} \mathbb{E} \left[ \int_{t'}^{T} \rho_0 e^{-\rho_0(\nu-t)} (w \, dv - cX^B_{\nu'} \, dN^B_{\nu'}) \bigg| \bar{0}^B_t \right] \\
\geq -\rho_0 \, c + (w - \beta^B \, c)(1 - e^{-\rho_0(t'-t)}) + e^{-\rho_0(t'-t)} \rho_0 \, c \\
= [w - (\rho_0 + \beta^B) \, c] (1 - e^{-\rho_0(t'-t)}) > 0.
\]

Thus, it is strictly optimal to censor that piece of bad news, which contradicts with the optimality of \( x^B_t < 1 \).

At time \( s_1 \), note that

\[
V_B^r(\bar{0}^B_{s_1}) = \max \left\{ \mathbb{E} \left[ -\rho_0 \, c + \int_{0}^{s_2 \wedge \tau^B} \rho_0 e^{-\rho_0 \nu} w \, dv \right], 0 \right\},
\]

where \( s_2 \) is time length during which the public belief drifts from \( p_{s_1} \) to \( p_{fast} \) in the absence of news, i.e., \( s_2 = \frac{1}{\gamma - \beta^B} \ln \left[ \frac{p_{s_1} \, 1 - p_{fast}}{1 - p_{s_1} \, p_{fast}} \right] \), and \( \tau^B \) is the arrival time of the next piece of bad news. It is clear that the term \( \mathbb{E} \left[ -\rho_0 c + \int_{0}^{s_2 \wedge \tau^B} \rho_0 e^{-\rho_0 \nu} w \, dv \right] \) is continuous and increasing in \( s \). Hence, we must have Equation (2) – the indifference condition at \( s_2 \). This pins down

\[
s_2 = \frac{1}{\rho_0 + \beta^B} \ln \left[ \frac{w}{w - (\rho_0 + \beta^B) \, c} \right],
\]

which in turn determines the public belief at which the type \( B \) agent switches her strategy from Full-Censorship to No-Censorship:

\[
p^B := p_{s_1} = \frac{p_{fast}}{p_{fast} + (1 - p_{fast}) e^{-(\gamma - \beta^B) s_2}}.
\]

Thus, if \( p_0 \leq p^B \), the type \( B \) agent does not censor any bad news. But if \( p_0 > p^B \), the type \( B \) agent censors all bad news for \( s_1 \) time length,

\[
s_1 = \frac{1}{\gamma} \ln \left[ \frac{p_0 - 1 - p^B}{p^B} \right] = \frac{\gamma - \beta^B}{\gamma} (s - s_2).
\]

Thus, the type \( B \) agent’s best response is a cutoff strategy with the cutoff belief \( p^B \) in any PBE. Therefore, we have a unique PBE. \( \square \)
Proof of Proposition 3. When $p_0 > p^B$, the equilibrium expected payoff of the type $B$ agent is

$$ (w - \beta^B c)(1 - e^{-\rho_0 s_1}) + e^{-\rho_0 s_1} \rho_0 c. \quad (6) $$

If censorship were not possible, her expected payoff would be

$$ \frac{\rho_0 w}{\rho_0 + \beta^B} (1 - e^{-(\rho_0 + \beta^B)\bar{s}}). $$

Hence, she is better off in the equilibrium compared with the NCB if and only if

$$ (w - \beta^B c)(1 - e^{-\rho_0 s_1}) + e^{-\rho_0 s_1} \rho_0 c > \frac{\rho_0 w}{\rho_0 + \beta^B} (1 - e^{-(\rho_0 + \beta^B)\bar{s}}). $$

Note that we have

$$ w - \beta^B c > \frac{\rho_0 w}{\rho_0 + \beta^B} (1 - e^{-(\rho_0 + \beta^B)\bar{s}}) > \rho_0 c, $$

since $c < \underline{c}$, $s_2 < \bar{s}$ and Equation (3). Together with Equation (5), the type $B$ agent is better off in the equilibrium compared with the NCB if and only if $f(s_1) < 0$, where

$$ f(s) := e^{-\rho_0 s} - \frac{\beta^B + \rho_0 \exp \left[-\frac{\gamma (\rho_0 + \beta^B)}{\gamma - \beta^B} s \right]}{\rho_0 + \beta^B}. \quad (7) $$

Note that $f(s)$ is first increasing and then decreasing in $s$. Moreover, $f(0) = 0$ and $\lim_{\bar{s} \to \infty} f(s) = -\frac{\beta^B}{\rho_0 + \beta^B} < 0$. Hence, there exists a $s_1^* = s_1^*(\rho_0, \gamma, \beta^B) > 0$ such that the type $B$ agent is worse off in the equilibrium if and only if $s_1 < s_1^*$. □

Proof of Corollary 1. Equation (4) that implies $p^B$ is continuous and increasing in $s_2$. Moreover, $s_2$ is continuous and increasing in $c$, $\lim_{c \to 0} p^B = p_{fast}$ and $\lim_{c \to 1} p^B = 1$. There exists a unique $c_3 \in (0, \underline{c})$ such that $p_0 = p^B$ if and only if $c = c_3$. If $c \in [c_3, \underline{c})$, then $p_0 \leq p^B$ and bad news is never censored in equilibrium, so the type $B$ agent obtains the same payoff in equilibrium and in the NCB.

Suppose that $c < c_3$ so we have $p_0 > p^B$. Note that $s_1 < s_1^*$ is equivalent to $\bar{s} < s_2 + s^*$ due to Equation (5), where $s^* := \frac{\gamma}{\gamma - \beta^B} s_1^\ast$. The type $B$ agent is worse off in the equilibrium if and only if $s_2 > \bar{s} - s^*$. If $\bar{s} \leq s^*$, then $s_2 > \bar{s} - s^*$ holds for all $c \in (0, c_3)$, and the type $B$ agent is worse off in the equilibrium. Suppose $\bar{s} > s^*$, where both variables do not depend on the cost $c$. Since $s_2$ is increasing in $c$, $\lim_{c \to 0} s_2 = 0$ and $\lim_{c \to c_3} s_2 = \bar{s}$, there exists a unique $c_1 \in (0, c_3)$ such that $s_2 > \bar{s} - s^*$ if $c \in (c_1, c_3)$, and $s_2 < \bar{s} - s^*$ if $c \in (0, c_1)$. We complete the proof by redefining $c_1$ as $c_1 1_{\bar{s} > s^*}$. □

Proof of Corollary 2. From the proof of Corollary 1, if $c \in [c_3, \underline{c})$, the equilibrium payoff and the NCB payoff are the same for the type $B$ agent, and do not depend on the cost.
Suppose $c < c_3$ so we have $p_0 > p^B$. The type $B$ agent’s equilibrium payoff is characterized by Expression (6). Equation (5) and $\frac{\partial s_2}{\partial c} = \frac{1}{w-(p_0+\beta c)c}$ implies that the partial derivative of Expression (6) with respect to the cost $c$ is

$$-\beta B + \beta B e^{-\rho_0 s_1} \frac{\rho_0 + \gamma}{\gamma}.$$

So the type $B$ agent’s equilibrium payoff is increasing in $c$ if and only if $e^{-\rho_0 s_1} > \frac{\gamma}{\rho_0 + \gamma}$. Let $s_1^{**} = s_1^{**}(\rho_0, \gamma) > 0$ be the unique solution of $e^{-\rho_0 s_1} = \frac{\gamma}{\rho_0 + \gamma}$. Since $e^{-\rho_0 s_1}$ is continuous and decreasing in $s$, the type $B$ agent’s equilibrium payoff is increasing (resp. decreasing) in $c$ if and only if $s_1 < s_1^{**}$ (resp. $s_1 > s_1^{**}$).

Let $s^{**} = \frac{\gamma - \beta s_1}{\rho_0 + \gamma} > 0$. Hence, $s_1 < s_1^{**}$ is equivalent to $s_2 > \bar{s} - s^{**}$.

Note that $s_2$ is increasing in $c$, $\lim_{c \to 0} s_2 = 0$ and $\lim_{c \to c_3} s_2 = \bar{s}$. If $\bar{s} < s^{**}$, then $s_2 > \bar{s} - s^{**}$ holds for all $c \in (0, c_3]$, and the type $B$ agent’s equilibrium payoff is increasing in $c \in (0, c_3]$. Suppose $\bar{s} > s^{**}$, then there exists a unique $c_2 \in (0, c_3]$ such that $s_2 > \bar{s} - s^{**}$ if $c \in (c_2, c_3]$, and $s_2 < \bar{s} - s^{**}$ if $c \in (0, c_2)$. Last, we redefine $c_2$ as $c_2 \equiv \bar{s} > s^{**}$.

Finally, we show that $s^{**} < s_*$, or equivalently $s_1^{**} < s_1^*$.

Let $\nu_1 = s_1^{**}$ be the unique positive solution of $\alpha_1(\nu) = \frac{\gamma}{\rho_0 + \gamma}$, and $\nu_2$ be the unique positive solution of $\alpha_2(\nu) = \frac{\gamma}{\rho_0 + \gamma}$, where $\alpha_1(\nu) = e^{-\rho_0 \nu}$ and $\alpha_2(\nu) = \frac{\beta B + \rho_0 \exp \left( \frac{-\gamma(v_0 + \beta B)}{\gamma - \beta B} \nu \right)}{\rho_0 + \beta B}$. Note that $\nu_1 = \frac{1}{\rho_0} \ln \left( \frac{\rho_0 + \gamma}{\gamma} \right)$ and $\nu_2 = \frac{\gamma - \beta B}{\gamma(v_0 + \beta B)} \ln \left( \frac{\rho_0 + \gamma}{\gamma - \beta B} \right)$. Thus, we have

$$\frac{\partial \nu_2}{\partial \beta B} = \frac{1}{\gamma} \frac{1}{\rho_0 + \beta B} \left( 1 - \ln \left( \frac{\rho_0 + \gamma}{\gamma - \beta B} \right) \frac{\rho_0 + \gamma}{\rho_0 + \beta B} \right).$$

Let $\eta = \frac{\gamma - \beta B}{\rho_0 + \gamma} \in (0, 1)$. Since $1 - \eta + \ln \eta < 0$, we have $\frac{\partial \nu_2}{\partial \beta B} < 0$. Hence, $\nu_2$ is decreasing in $\beta B \in (0, \gamma)$. We also have $\lim_{\beta B \to 0} \nu_2 = \nu_1$. Hence, $\nu_2 < \nu_1$ for $\beta B \in (0, \gamma)$.

Suppose, by contradiction, that $\nu_1 = s_1^{**} \geq s_1^*$. Recall the function $f(\cdot)$ defined in Expression (7), we have $f(\nu) \leq 0$ if and only if $\nu \geq s_1^*$. Thus, $f(\nu_1) \leq 0$. Note that $f(\nu) = \alpha_1(\nu) - \alpha_2(\nu)$ and $\alpha_2(\nu)$ is strictly decreasing in $\nu$, thus $\nu_1 > \nu_2$ implies $f(\nu_1) = \alpha_1(\nu_1) - \alpha_2(\nu_1) > \alpha_1(\nu_1) - \alpha_2(\nu_2) = 0$,

which contradicts with $f(\nu_1) \leq 0$. Thus, $s_1^{**} < s_1^*$ and $s^{**} < s_*$. By the definitions of $c_1$ and $c_2$, $c_1 = c_2 = 0$ when $\bar{s} \leq s^{**}$, $c_1 = 0 < c_2$ when $\bar{s} \in (s^{**}, s_*)$, and $0 < c_1 < c_2$ when $\bar{s} > s_*$. □
Proof of Proposition 4. (1) We first verify that the pair of mixed cutoff strategies constitutes an equilibrium. Given the type $B$ agent’s strategy (call it $x_M$),

$$x_t^B = \begin{cases} 
1, & \text{if } p_t > p_{fast}, \\
1 - \gamma \beta, & \text{if } p_t = p_{fast}, \\
0, & \text{if } p_t < p_{fast},
\end{cases}$$

we solve for the evaluator’s best response conditional on whether the prior belief $p_0$ is equal to, higher or lower than the cutoff belief $p_{fast}$.

When $p_0 = p_{fast}$, the public belief does not change in the absence of news. The evaluator faces a stationary problem, and his value function solves

$$U^{x_M}(p_{fast}) = \max \left\{ p_{fast} h + \frac{\gamma}{\rho_1} \left[ p_{fast} h + (1 - p_{fast}) m - U^{x_M}(p_{fast}) \right], m \right\}.$$ 

It is easy to check that

$$U^{x_M}(p_{fast}) = \max \left\{ p_{fast} h + (1 - p_{fast}) \frac{\gamma}{\rho_1 + \gamma} m, m \right\}. \quad (8)$$

Note that the first term is exactly $m$ according to the definition of $p_{fast}$, thus the evaluator is indifferent between experimenting and not.

When $p_0 > p_{fast}$, the public belief drifts down until $p_{fast}$ in the absence of news. The strategy of the evaluator is represented by the length of experimentation in the absence of any signal. Let $r_s$ denote this strategy, and $T_s$ denote the corresponding stopping time, and $s \in [0, \bar{s}]$ denote the length of experimentation. Then his payoff would be

$$u^{r_s, x_M}(p_0) = \Phi(p_0; s, 0).$$

That is, $u^{r_s, x_M}(p_0) = \Phi(p_0; s)$. It is easy to show that

$$\frac{\partial u^{r_s, x_M}(p_0)}{\partial s} = \frac{\partial \Phi(p_0; s, 0)}{\partial s} = (1 - p_0) e^{-\rho_1 s} \left[ \frac{p_s}{1 - p_s} (\rho_1 + \gamma)(h - m) - \rho_1 m \right],$$

where $p_s$ is the public belief after time $s$ in the absence of news. Since $p_s \in [p_{fast}, p_0]$, and $\frac{p_{fast}}{1 - p_{fast}} = \frac{\rho_1 m}{(\rho_1 + \gamma)(h - m)}$, we have $\frac{\partial u^{r_s, x_M}(p_0)}{\partial s} > 0$ for $s \in [0, \bar{s})$. Hence, it is optimal for the evaluator to experiment $\bar{s}$ time until the public belief drifts down to $p_{fast}$.

When $p_0 < p_{fast}$, the public belief drifts up until $p_{fast}$ in the absence of news. Let $s \in [0, \bar{s}]$ be the length that the evaluator experiments without a conclusive signal. Then his payoff would be $u^{r_s, x_M}(p_0) = \Phi(p_0; s, \beta^B)$. We can show that

$$\frac{\partial u^{r_s, x_M}(p_0)}{\partial s} = (1 - p_0) e^{-\rho_1 s} \left[ \frac{p_s}{1 - p_s} (\rho_1 + \gamma)(h - m) - \rho_1 m \right],$$

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where \( p_s \) is the public belief after time \( s \) in the absence of news. Since \( p_s \in [p_0, p_{fast}] \), and \( \frac{p_{fast}}{1-p_{fast}} = \left( \frac{\rho_m}{\rho_l} \right) \frac{1}{h-m} \), we have \( \frac{\partial \pi_{s,x_M}(p_s)}{\partial s} < 0 \) for \( s \in [0, \hat{s}] \). Hence, it is optimal for the evaluator to dismiss the agent immediately when \( p_0 < p_{fast} \).

Thus, the evaluator’s best response to the strategy \( x_M \) is a cutoff strategy with a cutoff belief \( p_{fast} \).

We now solve for the type \( B \) agent’s best response \( r_e \) to the evaluator’s mixed cutoff strategy with a constant hazard rate \( z^* = \frac{w}{\ell} - \rho_0 - \beta_B \) at the cutoff belief \( p_{fast} \). Clearly, since the evaluator dismisses the agent when \( p_t < p_{fast} \), thus the agent never censors any bad news when \( p_t < p_{fast} \). When \( p_t = p_{fast} \), the evaluator faces a stationary problem since the evaluator dismisses the agent at a constant rate. Hence,

\[
V_{t}^{r_e}(\bar{\sigma}_t^B) = \sup \{ -\rho_0 c + \mathbb{E} \left[ \int_{0}^{\lambda_B} \rho_0 e^{-\rho_0 \nu} w \ d\nu + e^{-\rho_0 \tau_B} V_x^\lambda(\bar{\sigma}^B_{\tau_B}) \mathbb{1}_{\{\tau_B < \lambda\}} \right] , 0 \},
\]

where \( \lambda \) is the arrival time of dismissal induced by the constant hazard rate \( z^* \).

Since \( \mathbb{E} \left[ \int_{0}^{\lambda_B} \rho_0 e^{-\rho_0 \nu} w \ d\nu \right] = \rho_0 c \), the type \( B \) agent is indifferent between censoring and not. This also means that the continuation value of the type \( B \) agent is \( \rho_0 c \) when no bad news arrives at \( p_t = p_{fast} \).

When \( p_t > p_{fast} \), the evaluator believes that the evaluator censors all bad news until the belief reaches \( p_{fast} \). Hence, the type \( B \) agent can stay in power for some positive time \( s = \frac{1}{\gamma} \ln \left[ \frac{p_t}{1-p_t} \frac{1-p_{fast}}{p_{fast}} \right] > 0 \). Censoring all bad news until then gives her a payoff

\[
v_{t}^{r_e,x_M}(\bar{\sigma}_t^B) = -\rho_0 c + \int_{0}^{\hat{s}} \rho_0 e^{-\rho_0 \nu} (w - \beta_B c) d\nu + e^{-\rho_0 s} \rho_0 c
\]

\[
= [w - (\rho_0 + \beta_B) c] (1 - e^{-\rho_0 s}) > 0.
\]

Hence it is optimal to censor all bad news when \( p_t > p_{fast} \). Thus, the strategy \( x_M \) is the best response to the evaluator’s strategy, and the PBE is verified.

(2) Now we show the above PBE is the unique cutoff equilibrium. First, we show that for any admissible strategy of the agent, the evaluator’s best response, if it exists, is to experiment when \( p_t > p_{fast} \) and to dismiss the agent when \( p_t < p_{slow} \). This helps us restrict our attention to the agent’s cutoff strategy \( \hat{p} \leq p_{fast} \). Second, we show some necessary conditions for the agent’s equilibrium strategy in a cutoff equilibrium. At last, we combine the above results to show no other cutoff equilibrium exists.

(2a) As in the proof of Proposition 2, the maximal information (no censorship) and the minimal information (full censorship) provide two bounds for the value function of the evaluator for any admissible strategy of the agent.

From the Proposition 1 and Proposition 2, we know that in both extreme cases, the evaluator experiments when \( p_t > p_{fast} \), and dismisses the agent when \( p_t < p_{slow} \). Hence, for any admissible strategy of the type \( B \) agent, the evaluator would do the same.
(2b) Fix a cutoff equilibrium where the evaluator uses a cutoff strategy with the cutoff belief \( \hat{p} \). We will show that the type B agent’s equilibrium strategy is to censor all bad news whenever \( p_t > \hat{p} \), and when \( p_t = \hat{p} \), her censoring probability must satisfy \( x_t^B \geq x^{B*} := 1 - \frac{\gamma}{\beta} \).

Suppose, by contradiction, that the type B agent’s equilibrium strategy is \( x_t^B < 1 \) at time \( t \) when \( p_t > \hat{p} \). When the game starts from time \( t \) and \( p_t > \hat{p} \), the belief in the absence of news must eventually drift down to \( \hat{p} \) in finite time \( s = \inf\{\nu > t : p_\nu = \hat{p}\} > 0 \), otherwise the type B agent would benefit from censoring all bad news since the cost is low (i.e., \( c < \zeta \)), which implies that the belief would drift down to \( \hat{p} \) directly in the absence of news. The belief drifts down to \( \hat{p} \) in finite time in the absence of news, thus there must be some \( t' \in (t, s) \) such that \( \hat{p}_{t'} < 0 \) in the absence of news. It implies that we must have \( x_{t'}^B > 0 \) and \( V^*_B(\emptyset_{t'}) \geq 0 \). The exactly same argument in Proposition 2 shows that an alternative strategy \( x^{B'} \) in which censoring all bad news for time \( \nu \in [t, t'] \) and then resuming the original strategy gives \( V^{r,x^{B'}}(\emptyset_{t'}) > 0 \). Thus, it is strictly optimal for the type B agent to censor bad news at time \( t \), i.e., \( x_t^B = 1 \), which contradicts with \( x_t^B < 1 \).

In addition, that the agent censors bad news with probability one when the belief is above \( \hat{p} \) implies that she is not dismissed with probability one when the belief hits \( \hat{p} \). Otherwise, if a piece of bad news arrives at a belief slightly higher than \( \hat{p} \), it is not worth for her to censor it given the same logic in Proposition 2.

Suppose, by contradiction, that the type B agent’s equilibrium strategy is \( x_t^B < x^{B*} \) at time \( t \) when \( p_t = \hat{p} \). In the absence of news, \( \hat{p}_t > 0 \) since \( x_t^B < x^{B*} \). Thus, for a continuation game, \( \hat{p}_\nu > 0 \) for \( \nu \) in some half-neighborhood of \( t \), say \( [t, t'] \), since \( x_t^B \) is continuous from the right. Thus, if no news arrives in \( \nu \in (t, t') \), \( p_\nu > \hat{p} \). Since the agent censors for sure when the belief is above \( p_{fast} \), we have \( x^B_\nu = 1 \) for \( \nu \in (t, t') \), which implies that \( \hat{p}_\nu < 0 \) for \( \nu \in (t, t') \). A contradiction is reached.

(2c) Suppose there is another cutoff equilibrium, in which the evaluator uses a cutoff strategy with a cutoff belief \( \hat{p} \).

According to (2a), \( \hat{p} \leq p_{fast} \). (2b) shows that in the equilibrium, the type B agent censors all bad news when \( p_t > \hat{p} \), her censoring probability when \( p_t = \hat{p} \) cannot be less than \( x^{B*} \), and she is not dismissed with probability one when the belief hits \( \hat{p} \). Moreover, her censoring probability when \( p_t = \hat{p} \) cannot be more than \( x^{B*} \) either. Suppose not, i.e., suppose \( p_t = \hat{p} \) at some time \( t \), and the type B agent’s censoring probability is larger than \( x^{B*} \). Then, the public belief would be drifting down strictly below \( \hat{p} \) after \( t \). Since the evaluator uses a cutoff strategy with a cutoff belief \( \hat{p} \), he must dismiss the agent with probability one at time \( t \). This contradicts with the fact that the agent is not dismissed with probability one when \( p_t = \hat{p} \).

Thus, in this cutoff equilibrium, the censoring probability when \( p_t = \hat{p} \) must be equal to \( x^{B*} \in (0, 1) \), and the public belief would stay at \( \hat{p} \) thereafter. It has two implications.
First, the evaluator’s strategy at this belief must make the type $B$ agent indifferent between censoring bad news and not since $x^{B*} \in (0, 1)$. Second, the evaluator’s strategy is also stationary (i.e., time-independent) since otherwise the type $B$ agent would not be stationary at the belief $\hat{p}$. It implies that the evaluator uses a constant hazard rate $z^*$ to dismiss the agent at this belief. Therefore, all cutoff equilibria have the same mixing structure as in the cutoff equilibrium we have verified in (1). If $\hat{p} = p_{fast}$, this equilibrium is just the cutoff equilibrium we have verified in (1).

If there exists a cutoff equilibrium in which the evaluator uses a cutoff strategy with a cutoff belief $\hat{p} < p_{fast}$, then the type $B$ agent would censor all bad news when the belief is above $\hat{p}$ and censors bad news with probability $x^{B*}$ at the belief $\hat{p}$. This gives the evaluator worse information compared with the case where the agent censors bad news only when the belief is above or equal to $p_{fast}$. In response to the agent’s strategy, the evaluator would strictly prefer to dismiss her at the belief $\hat{p} < p_{fast}$, according to his payoff function Expression (8). But as we have shown, this contradicts with the fact that the agent is not dismissed with probability one when $p_t = \hat{p}$. This contradiction completes the proof.

*Proof of Proposition 5.* When $p_0 \in (p_{slow}, p_{fast})$, the type $B$ agent obtains a zero payoff in the equilibrium and obtains a positive payoff in the NCB. The first result is obvious.

When $p_0 > p_{fast}$, the type $B$ agent’s expected payoff in the equilibrium is

$$(w - \beta^B c)(1 - e^{-p_0 \hat{s}}) + e^{-p_0 \hat{s}} p_0 c. \tag{9}$$

Her expected payoff in the NCB is $\frac{\rho_0 w}{\rho_0 + \beta^B}$. Thus, she is worse off in the equilibrium with censorship if and only if

$$(w - \beta^B c)(1 - e^{-p_0 \hat{s}}) + e^{-p_0 \hat{s}} p_0 c < \frac{\rho_0 w}{\rho_0 + \beta^B}. $$

Thus, she is worse off in the equilibrium with censorship if and only if $e^{-p_0 \hat{s}} > \frac{\beta^B}{\rho_0 + \beta^B}$, since $w - \beta^B c > \frac{\rho_0 w}{\rho_0 + \beta^B} > \rho_0 c$. We complete the proof by defining $\hat{s}^* := \frac{1}{\rho_0} \ln \left[ \frac{p_0 + \beta^B}{\rho_0} \right]$. □

*Proof of Corollary 3.* Obviously, since $\hat{s}$ is not a function of the censoring cost $c$, whether the type $B$ agent is better off in the equilibrium than she is in the NCB does not depend on the cost.

When $p_0 > p_{fast}$, the type $B$ agent’s expected payoff in the equilibrium is characterized by Expression (9). Its first derivative with respect to $c$ is $-\beta^B + (\rho_0 + \beta^B) e^{-p_0 \hat{s}}$. Thus, the type $B$ agent’s expected payoff in the equilibrium is decreasing in the censoring cost if and only if $e^{-p_0 \hat{s}} < \frac{\beta^B}{\rho_0 + \beta^B}$, and it is increasing in the cost if and only if $e^{-p_0 \hat{s}} > \frac{\beta^B}{\rho_0 + \beta^B}$. We complete the proof by comparing these conditions with the conditions in the proof of Proposition 5. □
Proof of Proposition 6. (1) First, given the strategies of all players, we characterize the evolution of the public belief. When $p_t > p^{B\dagger}$, no bad news is revealed in equilibrium, and we assume the off-path belief will jump to zero after off-path bad news is revealed.\footnote{For our result, it is enough to assume that a piece of revealed bad news when $p_t > p^{B\dagger}$ leads to an off-path belief that is below $p_{fast}$.} In addition, the drifting process in the absence of news follows

$$dp_t = -p_t (1 - p_t) \gamma dt.$$

When $p_t \in (p^G, p^{B\dagger})$, the public belief will jump to $J(p_t, 1, 0) = 0$ after a piece of bad news is revealed, and the drifting process in the absence of news follows

$$dp_t = -p_t (1 - p_t) (\gamma - \beta) dt. \quad (10)$$

When $p_t < p^G$, the public belief will jump to $j(p_t) > 0$ after a piece of bad news is revealed, and the drifting process in the absence of news follows

$$dp_t = -p_t (1 - p_t) (\gamma + \beta^G - \beta) dt. \quad (11)$$

In addition, we will see later that $j(p^G) \leq p_{fast}$ is equivalent to $\beta^G \leq \bar{\beta}$. Thus, $j(p_t) \leq j(p^G) \leq p_{fast}$ when $p_t < p^G$, due to the monotonicity of $j(\cdot)$. Thus, for any belief $p_t < 1$, it will jump down below $p_{fast}$ after a piece of bad news is revealed.

(2) Suppose that the evaluator uses a cutoff strategy with the cutoff belief $p_{fast}$, therefore a piece of revealed bad news leads to the dismissal of the agent. We now solve for the best response for each type of the agent.

The type $B$ agent faces a similar problem as in the conclusive bad news case, thus her best response is a cutoff strategy with some cutoff belief $p^{B\dagger} > p_{fast}$.\footnote{Note that $p^{B\dagger} > p^B$, by the public belief evolution in the absence of news, and the indifference conditions – Equation (2) and (12).} She censors bad news if and only if $p_t > p^{B\dagger}$, and at the belief $p^{B\dagger}$, she is indifferent between censoring bad news and not, thus an indifference condition similar to Equation (2) applies,

$$E \left[ -\rho_0 c + \int_0^{s^B + \tau^B} \rho_0 e^{-\rho_0 w} d\nu \right] = 0, \quad (12)$$

where $s^B$ is the time during which the belief drifts from $p^{B\dagger}$ to $p_{fast}$ in the absence of news, and $\tau^B$ is the arrival time of a piece of bad news. Thus, we have

$$\frac{\rho_0}{\beta + \rho_0} (1 - e^{-\rho_0 s^B}) w = \rho_0 c, \quad (13)$$

which implicitly defines $s^B$.\footnote{For our result, it is enough to assume that a piece of revealed bad news when $p_t > p^{B\dagger}$ leads to an off-path belief that is below $p_{fast}$.}
Similarly, the type $G$ agent’s best response is also a cutoff strategy with some cutoff belief $p^G > p_{\text{fast}}$. Thus, she censors bad news if and only if $p_t \in (p^G, 1)$, and at the belief $p^G$, she is indifferent between censoring bad news and not. Thus, we have

$$
\mathbb{E} \left[ -\rho_0 c + \int_0^T \rho_0 e^{-\rho_0 \nu} w \, d\nu \right] = 0, 
$$

(14)

where

$$
T = \begin{cases} 
\infty, & \text{if } \chi \leq s^G \wedge \tau^G, \\
s^G \wedge \tau^G, & \text{if } \chi > s^G \wedge \tau^G, 
\end{cases}
$$

is the period length for her to stay in the job, $\tau^G$ and $\chi$ are the arrival times of a piece of bad news and a piece of good news from the type $G$ agent, respectively, and $s^G$ is the time during which the public belief drifts from $p^G$ to $p_{\text{fast}}$ in the absence of news. Thus, we have

$$
\frac{\gamma + \rho_0}{\beta^G + \gamma + \rho_0} (1 - e^{-(\beta^G + \gamma + \rho_0)s^G}) w = \rho_0 c,
$$

(15)

which implicitly defines $s^G$. Since $c < c$, we have

$$
\rho_0 c < \frac{\rho_0}{\beta^B + \rho_0} w < \frac{\gamma + \rho_0}{\beta^G + \gamma + \rho_0} w.
$$

It is easy to see that $\frac{\gamma + \rho_0}{\beta^B + \gamma + \rho_0} (1 - e^{-(\beta^G + \gamma + \rho_0)s^G}) w$ is increasing in $\gamma > 0$, and decreasing in $\beta^G < \beta^B$, thus, according to Equation (13) and Equation (15), we have

$$
\frac{\rho_0}{\beta^B + \rho_0} (1 - e^{-(\beta^B + \rho_0)s^B}) w = \frac{\gamma + \rho_0}{\beta^G + \gamma + \rho_0} (1 - e^{-(\beta^G + \gamma + \rho_0)s^G}) w > \frac{\rho_0}{\beta^B + \rho_0} (1 - e^{-(\beta^B + \rho_0)s^G}) w.
$$

Hence, $s^B > s^G$, which implies $p^{B^*} > p^G$.

Finally, we show that there exists a $\tilde{\beta} \in (0, \beta^B)$ such that $j(p^G) \leq p_{\text{fast}}$ if and only if $\beta^G \leq \tilde{\beta}$. Note that the belief jumping process implies that

$$
\frac{j^{-1}(p_{\text{fast}})}{1 - j^{-1}(p_{\text{fast}})} = \frac{p_{\text{fast}} \beta^B}{1 - p_{\text{fast}} \beta^G},
$$

(16)

and the belief drifting process implies that

$$
\frac{p^G}{1 - p^G} = \frac{p_{\text{fast}} e^{(\gamma + \beta^G - \beta^B)s^G}}{1 - p_{\text{fast}} e^{(\gamma + \beta^G - \beta^B)s^G}}.
$$

Thus, $p^G \leq j^{-1}(p_{\text{fast}})$ is equivalent to $\beta^G e^{(\gamma + \beta^G - \beta^B)s^G} \leq \beta^B$, which, according to Equation (15), is also equivalent to

$$
\phi(\beta^G) := \beta^G \left[ \frac{c(\beta^G)}{c(\beta^G) - c} \right]^{\xi(\beta^G)} \leq \beta^B,
$$
where $\xi(\beta^G) := \frac{\gamma + \beta^G \alpha B}{\gamma + \beta^G \alpha B + \rho_0}$ and $\tilde{c}(\beta^G) := \frac{\gamma + \rho_0}{\beta^G + \gamma + \rho_0} > \xi$.

It is easy to show that $\psi(\beta^G)$ is continuous and increasing in $\beta^G$ for $\beta^G \in [0, \beta^B]$, $\psi(0) = 0$, and $\psi(\beta^B) > \beta^B$. Thus, there exists a $\beta \in (0, \beta^B)$ such that $\beta^G \in (0, \beta]$ if and only if $p^G \leq j^{-1}(p_{\text{fast}})$.

(3) At last, given the strategies of both types of agent, the evaluator faces a standard Poisson bandit problem when $p_t < p^G$, and his best response is a cutoff strategy with the cutoff belief $p_{\text{fast}}$. When $p_t \geq p^G$, by the continuity of the evaluator’s value function, it is easy to verify that he has a strict incentive to not dismiss the agent. This completes the proof. \hfill \Box

**Proof of Proposition 7.** (1) We first establish that there exists a $\underline{\beta} \in (0, \beta^B)$ such that $j(p_{\text{fast}}) \leq p^G$ if and only if $\beta^G \in (0, \underline{\beta}]$.

Note that the belief drifting process implies that
\[
\frac{p^B}{1 - p^B} = \frac{p^G}{1 - p^G} e^{\beta^B (s^B - s^G)} = \frac{p_{\text{fast}}}{1 - p_{\text{fast}}} e^{\beta^B (s^B - s^G)}.
\]

Together with Equation (16), it implies that $p^G \leq j^{-1}(p_{\text{fast}})$ is equivalent to $\psi(\beta^G) := \beta^G e^{\beta^B (s^B - s^G)} \leq \beta^B$. According to Equation (15), we have
\[
\beta^G s^G = \beta^G \frac{\beta^G s^G}{\beta^G + \gamma + \rho_0} \ln \frac{w (\gamma + \rho_0)}{w (\gamma + \rho_0) - \rho_0 c (\beta^G + \gamma + \rho_0)}
\]
which is increasing in $\beta^G$. Thus, $\psi(\beta^G)$ is continuous and increasing in $\beta^G$ for $\beta^G \in [0, \beta^B]$. In addition, $\psi(0) = 0$, and $\psi(\beta^B) > \beta^B$. Thus, there exists a $\underline{\beta} \in (0, \beta^B)$ such that $p^G \leq j^{-1}(p_{\text{fast}})$ if and only if $\beta^G \leq \underline{\beta}$. Moreover, $\beta < \underline{\beta}$ since $p^G > p^B$.

(2) We now show that the evaluator is better off in the equilibrium when $p_0 \in (p^G, p^B]$. When $p \in [p^G, p^B]$, it is optimal for the evaluator to not dismiss the agent. Thus, his value function $U^0$ in the NCB solves the following Bellman equation:
\[
\rho_1 U^0(p) = \frac{p \gamma [k (\gamma + p_1) - U^0(p)]}{\text{Expected benefit from good news}} + \beta(p) \left[ U^0(j(p)) - U^0(p) \right] \text{Expected loss from bad news} - (\gamma + \beta^G - \beta^B) p (1 - p) U^0(p),
\]

where $\beta(p) := p\beta^G + (1 - p)\beta^B$ is the average arrival rate of bad news.

Similarly, his value function $U^\beta$ in the equilibrium solves the Bellman equation:
\[
\rho_1 U^\beta(p) = \frac{p \gamma [k (\gamma + p_1) - U^\beta(p)]}{\text{Expected benefit from good news}} + (1 - p) \beta^B \left[ m - U^\beta(p) \right] \text{Expected loss from bad news} - (\gamma - \beta^B) p (1 - p) U^\beta(p),
\]

where $m = p \beta^G + (1 - p) \beta^B$ is the average arrival rate of bad news.
For \( p \leq p^G \), they coincide, i.e., \( U^\pi(p) = U^0(p) \), since there is no censorship in either case. Thus, comparing Equation (17) and (18) at \( p = p^G \), we have

\[
U^0(p^G) < U^\pi(p^G),
\]

where \( U^\pi(p^G) \) is the right derivative. Since the value functions are continuously differentiable, the above relation holds for some neighborhood \([p^G, \hat{p}]\), thus in that neighborhood \( U^\pi(p) - U^0(p) \) is increasing in \( p \). We have \( U^\pi(p) > U^0(p) \) for \( p \in (p^G, \hat{p}) \).

Suppose, by contradiction, that there is some \( \hat{p} \in (p^G, p^{B^\dagger}] \) - let it be the smallest one - such that \( U^\pi(\hat{p}) \leq U^0(\hat{p}) \). Thus, \( U^\pi(p) > U^0(p) \) for \( p \in (p^G, \hat{p}) \) and \( U^\pi(\hat{p}) = U^0(\hat{p}) \) by continuity. Note that we must have \( U^0(\hat{p}) \geq U^\pi(\hat{p}) \). Otherwise, if \( U^0(\hat{p}) < U^\pi(\hat{p}) \), then it holds in a neighborhood of \( \hat{p} \) by continuity so that \( U^\pi(p) - U^0(p) \) is increasing in that neighborhood of \( \hat{p} \), which, together with \( U^\pi(p) > U^0(p) \) for \( p \in (p^G, \hat{p}) \), implies that \( U^\pi(\hat{p}) > U^0(\hat{p}) \) - a contradiction. Thus, we have \( U^0(\hat{p}) \geq U^\pi(\hat{p}) \). Comparing Equation (17) and (18) at \( p = \hat{p} \) and using \( U^0(\hat{p}) \geq U^\pi(\hat{p}) \), we have

\[
\beta(\hat{p})[U^0(\hat{p}) - U^0(j(\hat{p}))] + \beta^G \hat{p}(1 - \hat{p})U^0(\hat{p}) - (1 - \hat{p})\beta^G[U^0(\hat{p}) - m] \leq 0. \tag{19}
\]

According to standard arguments in Poisson bandit problems, \( U^0(p) \) is strictly increasing and convex in \( p \in [p_{fast}, 1] \). If \( j(\hat{p}) \leq p_{fast} \), we have \( U^0(j(\hat{p})) = m \). The left-hand side of Inequality (19) is

\[
\beta^G \hat{p} \left[ U^0(\hat{p}) - m \right] + \beta^G \hat{p}(1 - \hat{p})U^0(\hat{p}) > 0,
\]

which contradicts with Inequality (19). Thus, it must be \( j(\hat{p}) > p_{fast} \). The convexity of \( U^0(p) \) also implies that

\[
U^0(\hat{p}) \geq \frac{U^0(\hat{p}) - U^0(j(\hat{p}))}{\hat{p} - j(\hat{p})} \geq \frac{U^0(\hat{p}) - U^0(p_{fast})}{\hat{p} - p_{fast}}.
\]

Thus, since \( U^0(p_{fast}) = m \), we have the left-hand side of Inequality (19) equal to

\[
\beta(\hat{p}) \left[ U^0(\hat{p}) - U^0(j(\hat{p})) \right] + \beta^G \hat{p}(1 - \hat{p}) U^0(\hat{p}) - (1 - \hat{p}) \beta^G \left[ U^0(\hat{p}) - U^0(p_{fast}) \right] \geq \frac{U^0(\hat{p}) - U^0(p_{fast})}{\hat{p} - p_{fast}}.
\]

(3) Now we show the type B agent is better off in the equilibrium compared with the NCB when \( \beta^G \in (0, \beta] \) and \( p_0 \in (p^G, p^{B^\dagger}] \).

In both the equilibrium and the NCB, the type B agent does not censor bad news when \( p_0 \leq p^{B^\dagger} \). Thus, in both cases, she is dismissed when either a piece of bad news
arrives or when the public belief drift down to \( p_{\text{fast}} \) in the absence of news. Note that in the absence of news the public belief drifts down according to the ODE (10) and (11) when \( p \in (p^G, p^{B*}] \) and \( p \in [p_{\text{fast}}, p^G] \), respectively, in the equilibrium. But in the NCB, the public belief drifts down according to the ODE (11) for \( p \in [p_{\text{fast}}, p^{B*}] \). Since the drifting rate is higher in the ODE (11) than it is in the ODE (10), the public belief drifts down slower in the equilibrium than it does in the NCB, and the type \( B \) agent is better off in the equilibrium compared with the NCB.

(4) Finally, we show the type \( G \) agent is also better off in the equilibrium compared with the NCB when \( \beta^G \in (0, \beta] \) and \( p_0 \in (p^G, p^{B*}] \).

In the equilibrium, the type \( G \) agent’s payoff when \( p_0 \in (p^G, p^{B*}] \) is

\[
V^E_G(p_0) = \mathbb{E} \left[ \int_0^{T^E} \rho_0 e^{-\rho_0 \nu} \left( w - c \sum_{p_\nu \in (p^G, 1)} X^G_\nu \right) dN^G_\nu \right],
\]

where

\[
T^E = \begin{cases} 
\infty, & \text{if } \chi \leq s^1 + s^G \wedge \tau^G, \\
 s^1 + s^G \wedge \tau^G, & \text{if } \chi > s^1 + s^G \wedge \tau^G,
\end{cases}
\]

is the period length for her to stay in the job, \( \tau^G \) and \( \chi \) are the arrival times of a piece of bad news and a piece of good news from the type \( G \) agent, respectively, and \( s^1 \) and \( s^G \) are the times during which in the absence of news the public belief drifts from \( p_0 \) to \( p^G \) (according to rate \( \gamma - \beta^B \)) and from \( p^G \) to \( p_{\text{fast}} \) (according to rate \( \gamma + \beta^G - \beta^B \)), respectively. It can be shown that

\[
V^E_G(p_0) = \frac{(\gamma + \rho_0) w - \rho_0 c \beta^G}{\gamma + \rho_0} (1 - e^{-(\rho_0 + \gamma)s^1}) + e^{-(\rho_0 + \gamma)s^1} \rho_0 c.
\]

In the NCB, when \( p_0 \in (p^G, p^{B*}] \), a piece of bad news terminates the type \( G \) agent’s job because \( j(p_0) \leq j(p^{B*}) \leq p_{\text{fast}} \), according to the assumption \( \beta^G \leq \beta \). Thus, her payoff in the NCB is

\[
V^N_G(p_0) = \mathbb{E} \left[ \int_0^{T^N} \rho_0 e^{-\rho_0 \nu} w \ d\nu \right],
\]

where

\[
T^N = \begin{cases} 
\infty, & \text{if } \chi \leq (s^0 + s^G) \wedge \tau^G, \\
 (s^0 + s^G) \wedge \tau^G, & \text{if } \chi > (s^0 + s^G) \wedge \tau^G,
\end{cases}
\]

is the period length for her to stay in the job, and \( s^0 \) is the times during which in the absence of news the public belief drifts from \( p_0 \) to \( p^G \) (according to rate \( \gamma + \beta^G - \beta^B \)).
It can be shown that

\[ V_N^G(p_0) = \frac{(\gamma + \rho_0) w}{\beta^G + \gamma + \rho_0} \left( 1 - e^{-(\beta^G + \gamma + \rho_0)s^0} \right) + e^{-(\beta^G + \gamma + \rho_0)s^0} \rho_0 c. \]

First, \( c < \xi \) implies that

\[ \frac{(\gamma + \rho_0) w - \rho_0 c \beta^G}{\gamma + \rho_0} > \frac{(\gamma + \rho_0) w}{\beta^G + \gamma + \rho_0} > \rho_0 c. \]

Second, according to the definition of \( s^1 \) and \( s^0 \), we have

\[ (\gamma - \beta^B) s^1 = \ln \left[ \frac{p_0}{1 - p_0 p^G} \right] = (\gamma + \beta^G - \beta^B) s^0, \]

which, due to \( \beta^G > 0 \), implies that

\[ (\rho_0 + \gamma) s^1 > (\beta^G + \gamma + \rho_0) s^0. \]

Thus, the type \( G \) agent has a higher expected payoff in the equilibrium than she does in the NCB, i.e., \( V_E^G(p_0) > V_N^G(p_0) \) for \( p_0 \in (p^G, p^B] \). \( \square \)

Proof of Proposition 8. (1) (Low cost) Given the strategies of both types of agent, there is no bad news in equilibrium when \( p_t > p_{fast} \). We assume that the public belief jumps down to zero after off-path bad news. At \( p_t = p_{fast} \), only the type \( B \) agent may not censor bad news, so the belief also jumps down to zero after a revealed bad news. In addition, the belief drifting process is the same as in Proposition 4 when \( p_t \geq p_{fast} \).

Given the strategy of the type \( G \) agent, the evaluator and the type \( B \) agent face the same problem, hence have the same best response as in Proposition 4 when \( p_t \geq p_{fast} \).

Given the strategy of the evaluator, when \( p_t = p_{fast} \), if the type \( G \) agent censors all bad news before a piece of good news arrives, then her expected payoff is

\[ E \left[ \int_0^T \rho_0 e^{-\rho_0 \nu} w \, d\nu - \int_0^{T_{\lambda}} \rho_0 e^{-\rho_0 \nu} c X_{\nu}^G \, dN_{\nu}^G \right], \]

where

\[ T = \begin{cases} \infty, & \text{if } \chi \leq \lambda, \\ \lambda, & \text{if } \chi > \lambda, \end{cases} \]

is the period length for her to stay in the job, \( \chi \) is the arrival time of a piece of good news, and \( \lambda \) is the arrival time of dismissal induced by the evaluator’s strategy \( z^* = \frac{w}{c} - \rho_0 - \beta^B \). We can derive her expected payoff as follows:

\[ \frac{(\gamma + \rho_0) w - \rho_0 c \beta^G}{z^* + \gamma + \rho_0}. \]
which can be shown strictly larger than $\rho_0 c$. Thus, the type $G$ agent has a strict incentive to censor bad news when $p_t = p_{fast}$, which confirms that she has a higher incentive to censor bad news than the type $B$ agent does – recall that the latter is indifferent between censoring and not when $p_t = p_{fast}$. It is also easy to verify that the type $G$ agent has a strict incentive to censor bad news when $p_t > p_{fast}$.

At last, when $p_t < p_{fast}$, we can show that the evaluator has a strict incentive to dismiss the agent, given the strategies of both types of agent. Thus, neither agent has an incentive to censor bad news when $p_t < p_{fast}$.

(2) (Intermediate cost) Given the strategies of both types of agent, when $p_t > p_{slow}$, the public belief jumps down to zero after a piece of revealed bad news since only the type $G$ agent engages in censorship. In the absence of news, the belief drifts up according to the ODE (10). The type $B$ agent faces the same problem as in Proposition 1 (with $\gamma < \beta^B$) since the cost is not low ($c > \tilde{c}$), so she never censors any bad news. The evaluator also faces the same problem as in Proposition 1 (with $\gamma < \beta^B$), thus he uses a cutoff strategy with the cutoff belief $p_{slow}$.

When $p_t \in (p_{slow}, 1)$, if the type $G$ agent censors all bad news before a piece of good news arrives, she stays in the job forever as the belief only goes up and the evaluator never dismisses her, so she obtains a discounted expected value of $w$. But she has to pay all the censoring costs until the arrival of a piece of good news, which costs her a discounted expected value of $\rho_0 \tilde{c} \beta^G \rho_0 + \gamma$. Given such a strategy, her expected payoff is $w - \frac{\rho_0 \tilde{c} \beta^G \rho_0}{\rho_0 + \gamma}$. Note that $\tilde{c} = \frac{\rho_0 \tilde{c} \beta^G \rho_0}{\rho_0 + \gamma}$ is the cost threshold such that the type $G$ agent is indifferent between censoring and not, i.e., $w - \frac{\rho_0 \tilde{c} \beta^G \rho_0}{\rho_0 + \gamma} = \rho_0 \tilde{c}$. Thus, the type $G$ agent has a strict incentive to censor bad news due to $c < \tilde{c}$.

At last, when $p_t < p_{slow}$, we can show that the evaluator has a strict incentive to dismiss the agent, given the strategies of both types of agent. Thus, neither agent has an incentive to censor bad news when $p_t \leq p_{slow}$.

**Proof of Proposition 9.** (1) First, we summarize the property of $p^*$ – the strategy of the evaluator in the NCB – from the bandit problem literature. The optimal policy of the evaluator is a cutoff strategy with a cutoff belief $p^* \in (0, 1)$. His value function $U^0(p_0)$ is continuously differentiable everywhere, with a possible exception at the cutoff belief $p^*$. In addition, $U^0(p_0)$ is strictly increasing and convex when $p_0 \geq p^*$.

When $\gamma + \beta^G \geq \beta^B$, $p^* = p_{fast}$ and the evaluator’s value function $U^0(p_0)$ is continuously differentiable at $p^*$, because the smooth pasting condition holds.

When $\gamma + \beta^G < \beta^B$, $p^* \in (p_{slow}, p_{fast})$ and the evaluator’s value function $U^0(p_0)$ has a kink at $p^*$ – the smooth pasting condition does not hold since the public belief in the absence of news does not drift down to the stopping region.

(2) (Low cost) Assume $c < \tilde{c}$ and $p_0 \in (p^*, 1)$. In the NCB, the following “pretending no bad news exists” strategy is feasible for the evaluator: if good news arrives before
\( \tilde{s} \), the evaluator never dismisses the agent, otherwise he dismisses the agent after time \( \tilde{s} \) is reached, where \( \tilde{s} \) is the period length during which the public belief drifts from \( p_0 \) to \( p_{fast} \) according to the rate \( \gamma \), while at the same time ignoring all bad news. Such a strategy is strictly sub-optimal for the evaluator, however, it delivers an expected payoff that is equal to his expected payoff in the equilibrium. Thus, the evaluator is strictly worse off in the equilibrium when \( p_0 \in (p^*, 1) \).

(3) (Intermediate cost) Assume \( c \in (\underline{c}, \bar{c}) \). Let \( U^0(p_0) \) and \( U^\bar{p}(p_0) \) be the evaluator’s value functions in the NCB and in the equilibrium, respectively. Note that the cutoff beliefs in the evaluator’s strategy are \( p^* \) and \( p_{slow} \) in the NCB and in the equilibrium, respectively. We also know that \( U^0(p_0) \) is strictly increasing and convex when \( p_0 \geq p^* \), and, from the Proof of Proposition 1, \( U^\bar{p}(p_0) \) is strictly increasing and linear when \( p_0 \geq p_{slow} \). Thus, for \( p_0 \in (p_{slow}, p^*) \), \( U^0(p_0) = m < U^\bar{p}(p_0) \) – the evaluator is better off in equilibrium.

Suppose, by contradiction, that there exists some \( \hat{p} \in (p^*, 1) \) – let it be the smallest one – such that \( U^\bar{p}(\hat{p}) \leq U^0(\hat{p}) \). Thus, \( U^\bar{p}(p) > U^0(p) \) for \( p \in (p^*, \hat{p}) \) and \( U^\bar{p}(\hat{p}) = U^0(\hat{p}) \) by continuity. From the convexity of \( U^0(p) \) and the linearity of \( U^\bar{p}(p) \), we have

\[
\frac{U^0(1) - U^0(\hat{p})}{1 - \hat{p}} \geq \frac{U^0(\hat{p}) - U^0(p^*)}{\hat{p} - p^*} \geq \frac{U^\bar{p}(\hat{p}) - U^\bar{p}(p_{slow})}{\hat{p} - p_{slow}},
\]

This implies that

\[
\frac{U^0(1) - U^0(\hat{p})}{1 - \hat{p}} > \frac{U^\bar{p}(1) - U^\bar{p}(\hat{p})}{1 - \hat{p}},
\]

but \( U^0(1) = U^\bar{p}(1) \) and \( U^0(\hat{p}) = U^\bar{p}(\hat{p}) \), which is a contradiction. \( \square \)

References


