Search Frictions and Efficiency in Decentralized Transport Markets

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Abstract

In this paper we explore efficiency and optimal policy in decentralized transportation markets that suffer from search frictions, such as taxicabs, trucks and bulk shipping. We illustrate the impact of two externalities: the well-known thin/thick market externalities and what we call pooling externalities. We characterize analytically the conditions for efficiency, show how they translate into efficient pricing rules, as well as derive the optimal taxes for the case where planner is not able to set prices. We use our theoretical results to explore welfare loss and optimal policy in dry bulk shipping. We find that the constrained efficient allocation achieves 6% welfare gains, while the first-best allocation corresponding to the frictionless world, achieves 14% welfare gains. This suggests that policy can achieve substantial gains even if it does not alleviate search frictions, e.g. through a centralizing platform. Finally, simple policies designed to mimic the optimal taxes perform well.

1 Introduction

The transportation sector is indispensible for economic growth and social development. With both people and goods covering larger distances than ever before, the sector has witnessed a newfound and growing
interest by policymakers. In many transport markets interactions between carriers and customers occur in a decentralized manner. This is for instance the case in the markets for taxis, trucks and bulk shipping among others. In these markets, search frictions may result in unrealized trade, thus posing the question of whether the sector operates efficiently, and if not, what policies can be employed.

In this paper, we study efficiency and optimal policy in decentralized transport markets that suffer from search frictions. In particular, taking the search frictions as given, we dissect the sources of inefficiency that distort the market equilibrium allocation and characterize analytically the conditions for (constrained) efficiency. We derive both a set of efficient pricing rules, as well as the optimal taxes, that the planner can employ to achieve the efficient allocation. Then, we use our theoretical results to explore welfare loss and optimal policy in dry bulk shipping.

Our starting point is a dynamic spatial search model for decentralized transport markets, in the spirit of Lagos (2000) and Brancaccio et al. (2020) (henceforth BKP). There is a network of locations at different distances to each other. In each location, carriers and customers participate in a random matching process. When matched with a customer, carriers transport them to their destination for a price, and restart searching there. Carriers that do not get matched decide where to search next period: they can either wait at their current location or travel empty elsewhere and search there. Matched customers pay the carrier and obtain a value for the service they are receiving, while customers that do not get matched wait at their location. Finally, every period, a large number of potential customers decide whether to start searching for a carrier, as well as their destination, thus replenishing the customer pool seeking transportation. In our setup, we do not impose restrictions on the price setting mechanism, nor the structure of the demand system, in order to nest different modes of transportation (in taxis prices are regulated, while in shipping they are negotiated).

Studying efficiency in this setup is not straightforward due to the dynamic nature of decision-making and the spatial network; yet, we are able to obtain analytical results. In particular, we provide a characterization of the market equilibrium allocation that allows us to directly compare it to the constrained efficient allocation, i.e., the allocation where the planner cannot directly overcome the constraints imposed by search frictions. This comparison allows us to identify the different types of market externalities that can result in this setting and derive conditions for each one to be internalized.
In this setup, we show that search frictions create two types of externalities. First, as is well-known, search frictions generate thin/thick market externalities: when choosing whether to enter, agents affect the matching probabilities faced by other agents both in the same and in the opposite side of the market. If agents’ entry decisions do not internalize this effect, the overall number of agents searching is distorted away from the efficient one. In addition, search frictions generate what we call “pooling externalities”: when choosing their destination, customers may fail to internalize the effect that this choice has on the distribution of carriers over space: a carrier, after taking the customer to his destination, restarts its search there. In that case the composition of customers searching, and thus the composition of trips realized, is distorted away from the efficient one.

Thin/thick market externalities are internalized in equilibrium if and only if the private return from searching are equal to its social return. This amounts to the so-called “Hosios conditions” on surplus sharing (Hosios, 1990): these conditions, which are well-known to characterize efficiency in search models of labor markets with homogeneous workers, require the share of the surplus which is appropriated by agents on each side of the market to be equal to the elasticity of the matching function with respect to the same side.

Pooling externalities are internalized in equilibrium if and only if carriers receive the same surplus regardless of the customer they match with. This condition for efficiency replicates the no-arbitrage condition obtained in a frictionless world, where competition among carriers ensures that prices coincide with the opportunity cost of each trip, until in equilibrium carriers are indifferent among serving different types of customers. In our frictional setup, separate markets for each customer type are missing: if carriers could search for a specific customer type, so that heterogeneous customers were not pooled together, in equilibrium carriers would be indifferent across customer types.

The two efficiency conditions combined characterize analytically the efficient pricing rules. These prices are useful if a central authority is able to set prices, as in the case of taxicabs. In many markets, however, the planner is not able to directly control prices, but he may be able to impose taxes or subsidies. We show that, when prices are set via Nash bargaining, the planner can achieve efficiency using these instruments and we derive their optimal values. We consider three possible instruments: a tax on searching carriers, a tax on searching customers and a tax on trips. The search tax (on either side) is set to equate
the private value of an additional agent searching to its social value and forces agents to internalize the thin/thick market externalities. Taxes on loaded trips can be used to target the pooling externalities. The optimal trip tax depends on the deviation of the trip’s social surplus from the (weighted) average of social surpluses across destinations. In other words, a customer that enters a route with social surplus higher than the average social surplus originating from that region, is subsidized; in contrast, a customer that enters a route with social surplus lower than the average is taxed. The planner can restore efficiency by taxing trips and one of the two searching sides. In practice, some of these instruments may not be feasible or they may be too complex computationally; we return to this in our empirical analysis.

We apply these results to study empirically the dry bulk shipping industry. A number of features of dry bulk shipping, such as information frictions and port infrastructure, can hinder the matching of ships and exporters. We begin by leveraging a rich dataset of vessel movements and bulk shipping prices to document the presence of search frictions. In particular, we propose a novel test to argue that these frictions lead to unrealized potential trade. The test is based on weather shocks at sea that exogenously shift ship arrivals at port: in a frictionless world, in regions with more ships than exporters, the change in the number of ships should not affect matching, since the short side of the market is always matched. Here, instead, matches are indeed affected by weather shocks. In addition, the law of one price does not hold: shipping prices exhibit substantial dispersion within a time-origin-destination triplet, also consistent with frictions. Finally, at any given time, in most countries there are simultaneous arrivals of empty ships that load and departures of empty ships, even though ships are homogeneous. This also suggests wastefulness.

We proceed to estimate the model using the dry bulk shipping data. We use the estimates obtained in BKP for the ship parameters; and we introduce external trade data to estimate the exporter parameters, including the bargaining coefficients.

We first test whether the observed equilibrium is efficient by checking whether the conditions for efficiency hold in the data. Perhaps not surprisingly, we find that neither condition is satisfied, suggesting that the market does not operate efficiently.

Next, we turn to the welfare analysis. We compare, (i) the market equilibrium; (ii) the constrained efficient outcome, i.e., the market allocation under the efficient prices; (iii) the first best, i.e., the efficient outcome in a world without search frictions. We find that total welfare in the market equilibrium
allocation is 6.3% lower than the constrained efficient allocation and 14% lower than the efficient allocation. Moreover, trade volume and net trade value are substantially higher under constrained efficiency (by 13.5% and 11.7% respectively), as well as under efficiency (by 36.5% and 42.7% respectively). This suggests that the externalities have a substantial impact on world trade.

These results relay an important message: under the optimal policy, the market is able to achieve about 44% of the first-best welfare gains. If the first-best allocation is interpreted as a platform (like Uber) that eradicates search frictions, our results imply that policymakers can improve efficiency substantially through simple policies such as taxes without resorting to some form of centralization. This is important because centralization may be infeasible in practice, or, may come with substantial market power if provided by private firms.

We next delve into the different role of the two externalities. We find that both externalities contribute to distorting the equilibrium market allocation. However, they have a qualitatively different impact on the economy. Thin/thick market externalities have a large impact on trade volume, as they essentially distort the numbers of searching agents and therefore the total number of matches formed. In contrast, pooling externalities have an impact on trade value, as they distort the composition of exports and favor destinations with low social value.

In particular, the two externalities operate in different ways. In our empirical setup, restoring thin/thick market externalities requires granting exporters more bargaining power. In other words, the entry of an additional exporter has a substantial positive externality on matching rates, but prices remain too high to appropriately encourage the socially efficient entry of exporters. This imbalance is corrected in the efficient allocation by lowering prices and increasing exporter’s entry. On the other hand, restoring the pooling externalities requires subsidizing routes with high social surplus. Indeed, the planner subsidizes trips with high exporter revenue, short trip duration and/or trips to destinations where there is high option value for the ship (i.e., many customers, high value matches, low travel costs to other locations, etc).

Finally, although the efficient prices and optimal taxes that restore (constrained) efficiency have known expressions, they may not be feasible to implement in practice, either because the planner does not have access to all instruments; or because the expressions may be too complex or computationally challenging.
We thus consider simple policies that are designed to mimic the optimal taxes but may be more easily implementable. We find that a destination-specific tax (customs tax) performs well as it can achieve 44% of welfare gains. In contrast, a tax that is a function of distance achieves no welfare gains, suggesting that pricing schemes used in taxis fail to increase welfare.

Related Literature

This paper broadly relates to four strands of literature: search and matching; transportation; international trade; and industry dynamics.

First, our work naturally relates to the search and matching literature; see Diamond (1982), Mortensen (1982), Pissarides (1985) for the canonical DMP labor market model, as well as Rogerson et al. (2005) for a survey. More specifically, our paper relates to the literature on efficiency of search models. Hosios (1990) considers efficiency in markets with random search and Nash bargaining. He shows that these markets are generically inefficient and derives the well-known “Hosios condition” that restores efficiency. Acemoglu and Shimer (1999) show that the Hosios condition does not guarantee efficiency when firms are heterogeneous, and that efficiency is achieved in models of directed search and posted wages (see also Moen, 1997). In a follow-up paper, Acemoglu (2001) shows that with random search and heterogeneous firms, labor market policies, such as unemployment benefits or minimum wages, can potentially improve welfare.

Our paper extends the existing literature by further investigating the externalities that distort the equilibrium allocation and deriving explicit conditions for efficiency with random search and heterogeneous agents on one side of the market. In particular, our main theorem shows that efficiency is restored if two conditions hold: first, the standard Hosios (1990) condition that ensures that the number of matches in every market is optimal; second, a no arbitrage/indifference condition that ensures that the composition of matches in every market is optimal. This latter condition is novel. In addition, we derive theoretically the set of policy instruments (both efficient pricing rules, and taxes/subsidies) that can restore efficiency.

Second, our paper contributes to a large and rapidly growing literature on transportation. Our model

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1In addition to labor markets, the search and matching framework has been used in other decentralized markets, see Weill (2020) for the case of over-the-counter financial markets, Burdett and Coles (1997) and Shimer and Smith (2000) for the case of marriage markets and Lagos and Wright (2005) for the case of monetary exchange.

2More recently, Bilal (2020) extends Acemoglu (2001)’s results in the spatial context and shows that there are too many low productivity jobs in high productivity locations.
builds on Lagos (2000) (and Lagos, 2003) who provides a micro-foundation of search frictions in the case of taxicabs. More recently, Buchholz (2018) and Frechette et al. (2019) study search frictions and regulation frictions in NYC taxicabs. In particular, Buchholz (2018) relies on a similar framework, and implements numerically tariff pricing changes in order to explore whether welfare improvements can be achieved. Frechette et al. (2019) investigate the welfare impact of changes in the number of active medallions, as well as the introduction of an “Uber-like” platform that is modeled by shutting down search frictions.

We also relate to a growing literature studying different aspects of efficiency in urban transportation; for instance, Shapiro (2018) and Liu and Yang (2019) explore the welfare improvements brought by different centralizing formats; Ghili and Kumar (2020) investigate demand and supply imbalances in ride-sharing platforms; Ostrovsky and Schwarz (2018) focus on carpooling and self-driving cars; Kreindler (2018) studies optimal congestion pricing in India; Cao et al. (2018) explore competition in bike-sharing platforms; while several papers study (platform) pricing (e.g. Ma et al. (2020), Bian (2020), Castillo (2019)).

Third, since our empirical application involves oceanic transportation, we relate to a literature studying transportation in the context of international trade; e.g. Koopmans (1949), Hummels and Skiba (2004), Fajgelbaum and Schaal, 2017, Asturias (2018), Brooks Leah and Rua (2018), Cosar and Demir (2018), Holmes and Singer (2018), Wong (2018), Allen and Arkolakis (2019), Ducruet Cesar and Steinwender (2019), Lee and Xu (2020) and BKP. We also relate to a literature in international trade studying the role of frictions, such as Eaton et al. (2016), and Krolikowski and McCallum (2018) who consider search frictions between importers and exporters and Allen (2014) who investigates information frictions. In our prior work, BKP, we explore the role of the transportation sector in world trade and spell out the impact of endogenous trade costs. Although we rely on the model setup and empirical strategy employed there, our focus here is entirely different. Indeed, this paper considers search frictions and efficiency.

Finally, we relate to the literature on industry dynamics (Hopenhayn, 1992, Ericson and Pakes, 1995), while our empirical methodology borrows from the literature on the estimation of dynamic setups (e.g. Rust, 1987, Bajari et al., 2007, Pakes et al., 2007; applications include Ryan, 2012, Collard-Wexler, 2013, Kalouptsidi, 2014 and Kalouptsidi, 2018). Related to this paper, a small literature lying in the intersection of search and industry dynamics, has explored trading frictions in decentralized markets (e.g. Brancaccio
The rest of the paper is structured as follows: Section 2 describes the model setup. Section 3 provides the efficiency and optimal policy results. Section 4 provides a description of the dry bulk shipping industry and the data used, presents evidence for search frictions and outlines the estimation of the model. Section 5 presents our welfare analysis. Section 6 concludes. The Appendix contains all proofs and additional theoretical results, additional tables and figures, evidence on random search in shipping, as well as some data and computational details.

2 Model

We introduce a model of decentralized transport markets that centers on the interaction between carriers (e.g. ships, taxis, trucks) and customers (e.g. exporters, passengers). The timing is as follows: In every period, in each location, customers and carriers participate in a random matching process. Carriers that get matched transport their customer to their desired destination for a price, and restart there. Carriers that do not get matched decide whether to wait at their current location or travel empty elsewhere to search. Customers that get matched enjoy some utility from arriving at their destination, while customers that do not, wait another period. Finally, a large number of potential customers decide whether to start searching and where to go, thus replenishing the customer pool seeking transportation the following period.

2.1 Environment

Time is discrete and the horizon is infinite. There are $I$ locations, $i \in \{1, 2, \ldots, I\}$. There are two types of agents: customers and carriers. Both are risk neutral and have discount factor $\beta$.

There is a measure $S$ of homogeneous carriers in the economy.\(^3\) At the beginning of every period, a carrier is either in some region $i$, or traveling full or empty, from some location $i$ to some location $j$. Carriers at $i$ can either search or remain inactive. The per-period payoff of staying inactive is set equal

\(^3\)We treat fleet size as an exogenous object. This is consistent with most applications of interest, and can be due to either regulatory constraints or time to build.
to 0 at each location, while searching carriers incur a per-period search cost $c_i^w$. Carriers traveling from $i$ to $j$ incur a per period traveling cost $c_{ij}^s$. The duration of a trip between location $i$ and location $j$ is stochastic: a traveling carrier arrives at $j$ in the current period with probability $d_{ij}$, so that the average duration of the trip is $1/d_{ij}$.\footnote{It is straightforward to have deterministic trip durations instead. Our specification, however, preserves tractability and allows for some variability e.g. due to weather/traffic shocks, without affecting the steady state properties of the model.}

Customers can only be delivered to their destination by carriers and each carrier can carry at most one customer. Customers searching at $i$ meet carriers searching in the same location. Following the search and matching literature, we model the number of matches that take place every period, $m_i$, using a matching function, whereby the number of matchings in region $i$ is

$$m_i = m_i(s_i, e_i) \leq \min\{s_i, e_i\}$$

where $s_i$ is the measure of unmatched carriers in region $i$ and $e_i$ is the number of unmatched customers in region $i$. $m_i(s_i, e_i)$ is increasing and concave in both arguments. We allow for the possibility that $m_i(s_i, e_i) < \min\{s_i, e_i\}$ creating the potential for unrealized trade: two agents searching in the same location might fail to meet, due to impediments such as information frictions or physical constraints. As Petrongolo and Pissarides (2001) note, “[...] the matching function [...] enables the modeling of frictions [...] with a minimum of added complexity. Frictions derive from information imperfections about potential trading partners, heterogeneities, the absence of perfect insurance markets, slow mobility, congestion from large numbers, and other similar factors.”

Since search is random, the probability according to which customers searching at $i$ meet a carrier is $\lambda_i^c = m_i(s_i, e_i) / e_i$, which is the same for all customers. Similarly the probability according to which carriers searching at $i$ meet a customer is $\lambda_i^s = m_i(s_i, e_i) / s_i$.

When a carrier and a customer meet, if they both accept to match, the customer pays a price $\tau_{ij}$ upfront and the carrier begins its trip immediately to $j$. We are agnostic for now as to what the price mechanism is in the market. This allows us to nest several different practices in different markets; for instance prices are fixed by regulation in taxicabs, while prices are bilaterally negotiated in bulk shipping. We return to this later.
Let $q_{ij}$ denote the measure of matches resulting every period on route $ij$. Carriers that remain unmatched decide whether to stay in their current region or travel empty to a different region where they wait for a match. Customers that remain unmatched wait in their current region. Inactive carriers restart the following period in the same region.

Finally, we specify the demand side of the market. Every period, at each location $i$, a large pool of potential customers decide whether to enter the market and start searching for a carrier, in order to be transported to a destination $j$ with $j \neq i$, subject to an entry cost $k_{ij}$. In every route $ij$, we denote by $E_{ij}$ the endogenous measure of customers in $i$ who search for transportation and wish to go to destination $j$. The total measure of customers searching at location $i$ is denoted by $e_i = \sum_{j \neq i} E_{ij}$, while $G_{ij}$ is the share of demand routed from $i$ to $j$, i.e.,

$$\forall ij : G_{ij} \equiv E_{ij}/e_i.$$ 

Once they have entered, customers pay a per period search cost $c^e_i$ every period.

Upon matching with a carrier, customers obtain a valuation from being transported from origin $i$ to destination $j$. We model customer valuations via the function, $w : \mathbb{R}^{I \times I} \rightarrow \mathbb{R}^{I \times I}$, that describes the relationship between all quantities transported and the valuation $w_{ij}(q)$ of the marginal customer on each route $ij$. This can be thought of as a gross inverse demand curve for transportation services, before customer entry and search costs. To give a few examples, in the empirical application of Section 4 we assume that $w_{ij}(q) = w_{ij}$, so that all customers going from $i$ to $j$ obtain the same valuation. In the more general case, customers have heterogeneous valuations for transportation (e.g. passengers looking for cabs who have different value of time); when $q_{ij}$ matches are formed on route $ij$, $w_{ij}(q)$ describes the valuation of the $q_{ij}$-th (i.e. the marginal) consumer entering route $ij$.

Consistent with this interpretation, we assume that $w$ is the gradient of a concave and differentiable function $W : q \mapsto \mathbb{R}_+$, which is interpreted as the total gross customer value from transportation, as a function of the total number of matches $q$.\footnote{In addition, valuations might depend on quantities through general equilibrium effects; for instance in a trade model, traded goods’ prices depend on traded quantities and in equilibrium, the difference between the domestic and foreign price equals the trade costs, so that there is no arbitrage between countries (Samuelson, 1952).}

\footnote{$W$ is identified up to a constant from the marginal valuations. Formally we have, $W(q) = -\int_0^1 w(\alpha q) \cdot q d\alpha + \text{constant.}$}
2.2 Behavior and equilibrium

In this paper, we consider the steady state of our industry model. In a steady state equilibrium, customers and carriers respond optimally to their expectations of the endogenous market variables, which are consistent with agents’ behavior (rational expectations) and constant over time. Market clearing can be achieved either by price adjustment, or rationing (captured by the adjustment of the meeting probabilities faced by the agents at each location), or a mix of these two mechanisms.\(^7\)

We begin by describing the optimal behavior of carriers and customers facing a given tuple \(\tau, \lambda^s, \lambda^e, G\). We then restrict these variables to achieve market clearing.

**Carrier optimality** Let \(V_{ij}\) denote the value of a carrier that begins the period traveling from \(i\) to \(j\) (empty or loaded), \(V_i\) the value of a carrier that begins the period in location \(i\), and \(U_i\) the value of a carrier that remained unmatched at \(i\) at the end of the period. In everything that follows, we suppress the dependence of the value functions on the state of the industry, given our focus on a steady state; in Appendix E we do consider out of steady state dynamics. Given prices \(\tau\) and meeting rates we have,

\[
V_{ij} = -c^s_{ij} + \beta [d_{ij} V_j + (1 - d_{ij}) V_{ij}]
\]

(1)

In words, a carrier that is traveling from \(i\) to \(j\): pays the per period cost of traveling \(c^s_{ij}\); with probability \(d_{ij}\) it arrives at destination \(j\) where it begins unmatched with value \(V_j\); with the remaining probability \(1 - d_{ij}\), the carrier does not arrive and keeps traveling.

A carrier that starts the period in region \(i\) obtains:

\[
V_i = \max \left\{ -c^w_i + \lambda^s_i \sum_{j \neq i} G_{ij} \max \{\tau_{ij} + V_{ij}, U_i\} + (1 - \lambda^s_i) U_i, \beta V_i \right\}.
\]

In words, if the carrier decides to search, it pays the per period search cost \(c^w_i\); it then meets a customer heading to destination \(j\) with probability \(\lambda^s_i G_{ij}\), in which case it accepts to match if and only if the continuation value, inclusive of the price, is higher than its outside option, \(U_i\), otherwise it receives

\(^7\)Consider for instance the case of taxicabs, where prices are regulated. In this case, wait times determine the entry decisions of customers and equilibrate demand and supply. In the case of bulk shipping, both prices and wait times clear the market.
the outside option $U_i$. With the remaining probability the carrier does not meet a customer and receives the value of being unmatched. If the carrier remains inactive, it obtains a flow payoff of zero and restarts the following period at the same region.

Defining the carrier meeting surplus as,

$$\Delta^s_{ij} = \max\{\tau_{ij} + V_{ij} - U_i, 0\}$$

(2)

the value $V_i$ can be written as follows,

$$V_i = \max \left\{ -c_i^w + \lambda_i^s \sum_{j \neq i} G_{ij} \Delta^s_{ij} + U_i, \beta V_i \right\}.$$  

(3)

Next, if the carrier remains unmatched, it chooses where to search: it can either keep waiting at $i$ or travel empty to another region. The unmatched carrier value function is equal to:

$$U_i = \max_j V_{ij}$$

(4)

where we set $V_{ii} \equiv \beta V_i$. In words, if the carrier stays in region $i$, at the beginning of the next period it will be waiting at $i$; otherwise if the carrier chooses another region $j \neq i$ it begins its trip towards $j$.

Having defined all value functions relevant to the carriers, we now characterize their optimal behavior in terms of the three decisions they make (whether to search in the beginning of the period, whether to accept a match and where to search if unmatched at the end of the period). First, carriers search only when it is profitable to do so, so that from equation (3),

$$s_i > 0 \rightarrow V_i = -c_i^w + \lambda_i^s \sum_{j \neq i} G_{ij} \Delta^s_{ij} + U_i.$$  

(5)

Second, they do not reject any match yielding a strictly positive surplus, and they accept only matches yielding a positive surplus:

$$q_{ij} < s_i \lambda_i^s G_{ij} \rightarrow \Delta^s_{ij} = 0$$

(6)

$$q_{ij} > 0 \rightarrow \Delta^s_{ij} = \tau_{ij} + V_{ij} - U_i$$

(7)
Third, we turn to the decision of where to search. Denote by $b_{ij}$ the measure of carriers who decide to relocate empty from $i$ to $j$ (so that $b_{ii}$ is the measure that decides to remain in $i$); optimality requires that this measure is positive only if $j$ achieves the maximum continuation value across all possible choices:

$$b_{ij} > 0 \rightarrow U_i = V_{ij}. \quad (8)$$

Finally, it must be the case that, whenever the measure of inactive carriers is greater than zero, there is some location where some carriers would rather not search at all. Since $q_{ij} + b_{ij}$ carriers depart from $i$ towards $j$ every period, traveling for $1/d_{ij}$ periods on average, the total measure of active carriers in steady state is given by $\sum_{ij} (q_{ij} + b_{ij})/d_{ij}$. Hence this condition can be written as,

$$\sum_{ij} \frac{q_{ij} + b_{ij}}{d_{ij}} < S \rightarrow \exists i : V_i = 0. \quad (9)$$

**Customer Optimality** We now turn to the value functions of customers; we begin with existing customers and then consider customer entry. If a customer meets a carrier it can either agree to form a match, in which case it pays price $\tau_{ij}$ and receives its valuation, or the customer can revert to its outside option and stay unmatched. Hence the meeting surplus of the marginal customer with valuation $w_{ij}(q)$ is given by,

$$\Delta_{ij}^e = \max \left\{ w_{ij}(q) - \tau_{ij} - \beta U_{ij}^e, 0 \right\}, \quad (10)$$

where $U_{ij}^e$ is its value of searching for a carrier in $i$ with destination $j$:

$$U_{ij}^e = -e_i^e + \lambda_i^e \left( \Delta_{ij}^e + \beta U_{ij}^e \right) + (1 - \lambda_i^e) \beta U_{ij}^e$$

$$= -e_i^e + \lambda_i^e \Delta_{ij}^e + \beta U_{ij}^e. \quad (11)$$

In words, the customer pays the cost $e_i^e$ while searching; then with probability $\lambda_i^e$ it meets a carrier and receives the meeting surplus on top of its outside option, while with the remaining probability it remains unmatched and receives its outside option.

Similarly to carriers, optimality requires that the customer does not reject any match yielding a strictly
positive surplus, and that it accepts only matches yielding positive surplus:

\[ q_{ij} < \lambda^e_i E_{ij} \rightarrow \Delta^e_{ij} = 0 \]  
\[ q_{ij} > 0 \rightarrow \Delta^e_{ij} = w_{ij}(q) - \tau_{ij} - \beta U^e_{ij} \]  

Finally, the measure of customers searching on each route \( ij \) is pinned down by a free entry condition for the marginal customer:

\[ U^e_{ij} - \kappa_{ij} \leq 0 \]

with equality if \( E_{ij} > 0 \).  

We adopt the convention that customers in \( i \) choosing \( i \) do not enter the market, and normalize the payoff in that case to zero.–

Feasible allocations  
An allocation for the transportation economy consists of a tuple \((s, E, q, b)\) where \( s = [s_1, \ldots, s_I] \) denotes the measure of carriers waiting in each region, \( E \in \mathbb{R}^{I \times I}_+ \) denotes the measure of customers waiting for transport on each route \( i, j \), \( q \in \mathbb{R}^{I \times I}_+ \) denotes the measure of matches formed on each route, and \( b \in \mathbb{R}^{I \times I}_+ \) denotes the measure of carriers departing empty on each route. Equivalently, we will sometimes denote an allocation by \((s, e, G, q, b)\), where \( e = [e_1, \ldots, e_I] = [\sum_j E_{1j}, \ldots, \sum_j E_{Ij}] \) denotes the measure of customers waiting in each region. This will be useful when we want to emphasize the implications of search behavior on the share of waiting customers in location \( i \) headed towards \( j \), captured by the matrix \( G \). The first triplet \((s, e, G)\) captures search activities; hence we will often refer to it as a search allocation, in contrast with the last pair \((q, b)\), capturing transportation and relocation activities.

An allocation is feasible if it satisfies: (i) the set of recursive constraints defining a steady state, equations (15) and (16) below; (ii) the total fleet capacity constraint, equation (17) below; and (iii) the constraints on the transported quantities imposed by the meeting technology, equation (18) below. Thus,
the following feasibility constraints must hold:

\[ \forall i : \sum_{j} (q_{ij} + b_{ij}) = \sum_{j} (q_{ji} + b_{ji}) \quad (15) \]

\[ \forall i : s_i = \sum_{j} (q_{ij} + b_{ij}) \quad (16) \]

\[ \forall i, j : \sum_{ij} \frac{q_{ij} + b_{ij}}{d_{ij}} \leq S \quad (17) \]

\[ \forall i, j : q_{ij} \leq m_i (s_i, e_i) G_{ij} \quad (18) \]

The first set of constraints requires that the measure of carriers departing (or waiting) at any given location equals the measure of arrivals. The sums on the left and the right hand sides are the measures of all carriers departing every period from and towards \( i \), respectively, so that flows into a region are equal to the flows out of the region. Equation (16) requires that the measure of carriers searching at each location \( s_i \) must equal those that will be matched \( (\sum_j q_{ij}) \), those that will be unmatched and choose to remain \( (b_{ii}) \) and those that will be unmatched and decide to travel elsewhere empty \( (\sum_{j \neq i} b_{ij}) \). Constraint (17) imposes the fleet capacity constraint. Finally, constraints (18) require that the number of matches does not exceed the endogenous number of meetings between carriers and customers.

**Equilibrium** We now define the equilibrium of the transportation economy, which is a tuple \( (s, E, q, b, \tau) \) consisting of an allocation \( (s, E, q, b) \) and prices, \( \tau \in \mathbb{R}^{I \times I} \).

**Definition 1.** An outcome \( (s, E, q, b, \tau) \) is a steady state equilibrium under prices \( \tau \) if:

1. \( (s, E, q, b) \) satisfies the feasibility constraints (15)-(17).
2. \( (s, q, b) \) satisfies carrier optimality conditions (4)-(9) given \( \tau, \lambda^s \) and \( G \).
3. \( E, q \) satisfies customer optimality and free entry conditions (10)-(14) given \( \tau, \lambda^c \).
4. The perceived meeting probabilities are consistent with the true ones, i.e., for all \( i, j, \lambda^s_i s_i = m_i (s_i, e_i), \lambda^c_i e_i = m_i (s_i, e_i) \) and \( G_{ij} e_i = E_{ij} \).

\( (s, E, q, b) \) is an equilibrium allocation if there exists a price matrix \( \tau \) such that \( (s, E, q, b, \tau) \) is an equilibrium outcome.
3 Efficiency

In this section we present our main efficiency results. In Section 3.1 we present the social planner’s optimization problem and we provide a theorem that allows us to compare the solution of that problem to the market equilibrium allocation. In Section 3.2 we discuss the presence of two externalities and provide our main theorem that states the conditions for efficiency. We also discuss the efficient pricing rules. Finally, in Section 3.3 we derive taxes and subsidies that restore efficiency, when prices are set via Nash bargaining.

3.1 Comparing the market equilibrium to the efficient allocation

In this section, we characterize the set of equilibrium allocations as agents become patient, so that $\beta \to 1$, and compare them to the social planner’s solution. In the case of patient (in the limit) agents, welfare only depends on the long-run values of carriers and customers and not on the current state and its ensuing future allocations. In Appendix E we demonstrate that our efficiency results also hold with discounting, as well as out of steady state.

We begin by defining the limits of equilibrium allocations as $\beta \to 1$:

**Definition 2.** $(s, E, q, b, \tau)$ is a limit equilibrium outcome if there exists a sequence $(s^n, E^n, q^n, b^n, \tau^n)_{n \geq 0}$ such that: (i) for each $n$, $(s^n, E^n, q^n, b^n, \tau^n)$ is an equilibrium outcome for the economy populated by agents with discount factor $\beta^n$; and (ii) as $\beta^n \to 1$, $(s^n, E^n, q^n, b^n, \tau^n) \to (s, E, q, b, \tau)$. $(s, E, q, b)$ is a limit equilibrium allocation if there exists a price matrix $\tau$ such that $(s, E, q, b, \tau)$ is a limit equilibrium outcome.

The main task now is to compare the limit equilibrium allocation to the social planner’s solution, who wishes to maximize total welfare. When agents do not discount future payoffs, the (constrained) efficient steady state allocation is a solution to the following problem,

$$\max_{s, E, q, b \geq 0} W(q) - \sum_{ij} q_{ij} k_{ij} - \sum_{ij} (q_{ij} + b_{ij}) \frac{c_{ij}^v}{d_{ij}} - \sum_i s_i c_i^w - \sum_i e_i c_i^e$$

s.t. feasibility constraints (15)-(18)
In words, the social planner maximizes the per-period welfare corresponding to \((s, E, q, b)\): every period, \(q_{ij}\) customers are transported on each route \(ij\), generating gross customer value equal to \(W(q)\); matched customers are replaced by a pool of new entrants of equal measure who pay the entry cost \(k_{ij}\); \(q_{ij} + b_{ij}\) carriers begin traveling on route \(ij\) for \(1/d_{ij}\) periods on average, paying a per-period traveling cost \(c_{ij}^s\), while at every location \(i\), \(s_i\) unmatched carriers pay the search cost \(c_{ij}^w\), and similarly \(e_i\) active customers search for carriers incurring the search cost \(c_i^e\). The planner is subject to the set of steady state feasible allocations (15)-(17). Note that we since focus on constrained efficiency, the planner is subject to the same frictions as the market.

Comparing the socially optimal allocation to the market one is not straightforward, since neither one has a closed-form expression. Indeed, the market equilibrium allocation solves a nonlinear system of equalities and inequalities, as described in Definition 1 (agent optimality conditions and value functions, feasibility constraints and rational expectations constraints), while the efficient allocation solves the planner’s constrained optimization Problem (19) above. Nonetheless, the following theorem establishes that the market allocation can be found by solving an optimization problem that is remarkably similar, in form, to the planner’s problem.

**Theorem 1.** If \((s, E, q, b)\) is a limit equilibrium allocation then it solves

\[
\begin{align*}
\max_{s, E, q, b \geq 0} & \quad W(q) - \sum_{ij} q_{ij} k_{ij} - \sum_{ij} (q_{ij} + b_{ij}) \frac{c_{ij}^s}{d_{ij}} - \sum_i s_i c_{ij}^w - \sum_i e_i c_i^e \\
\text{s.t.} & \quad \text{feasibility constraints (15)-(17)} \\
& \quad \forall i, j : q_{ij} \leq s_i \lambda_i^s G_{ij} \\
& \quad \forall i, j : q_{ij} \leq \lambda_i^e E_{ij}
\end{align*}
\]

(20)

given the perceived probabilities \(\lambda^s, \lambda^e\) and \(G\), and the latter are consistent with the true ones (i.e. they satisfy condition 4 in Definition 1).

Theorem 1 characterizes market equilibrium allocations as solutions to Problem (20), the “market problem”. As in the planner’s Problem (19), the objective function of the market’s Problem (20) is the total market welfare. Moreover, both the market and the planner face the steady state constraints (15)-
(16), and the total fleet constraint (17). However, when it comes to the matching constraints, Problems (19) and (20) differ. Indeed, the social planner faces constraint (18), which treats the meeting rates $\lambda^s, \lambda^e$ and the destination shares $G$ as endogenous objects that are functions of $s, e$; in contrast, constraints (21) and (22) in the market’s Problem (20) treat these objects as exogenous constants.

The proof of Theorem 1, provided in Appendix A, rests heavily on duality. In particular, the dual variables of the market’s Problem (20) are linked to the carrier and customer value functions. This, in turn allows us to show that the carrier optimality conditions, equations (4)-(9), and the customer optimality conditions, (10)-(14), are equivalent in the limit to the first order conditions of the market’s Problem (20).\footnote{Caution is needed however when limits are taken as the discount factor goes to one, because the value functions per se may diverge. The desired correction is obtained by subtracting a reference value function from the remaining ones. Detailed arguments are found in the Appendix A.}

Importantly, when comparing the market’s Problem (20), to the planner’s Problem (19), the only difference is that the latter internalizes the effect of search behavior on the endogenous meeting probabilities and destination shares. The results in the following section formalize the intuition that the market’s failure to optimize with respect to these variables is the unique potential source of inefficiency in the economy.

### 3.2 Externalities and Efficient Prices

In contrast to the frictionless world, in an economy with search frictions prices may fail to balance demand and supply efficiently. As suggested by the comparison of the market’s Problem (20), to the planner’s optimization Problem (19), the key source of inefficiency arises from the effect of agents’ decisions on the endogenous meeting probabilities and destination shares. Although the planner internalizes this effect, the market does not. This hints at the presence of two externalities, one related to the matching rates $\lambda^s, \lambda^e$ and one to the destination shares $G$.

First, when choosing whether to join the search pool, agents may not internalize the effect that their entry has on the matching opportunities faced by other agents. Indeed, an extra carrier (customer) makes it easier for customers (carriers) to find a match and harder for other carriers (customers) to find a match. These are known as “thin/thick market externalities” in the search and matching literature.
Second, when choosing their destination, customers do not internalize the effect that this choice has on the distribution of carriers over space: a carrier will have to take the customer to his destination, and restart its search there. Put differently, the customer only cares about its private surplus of the trip, whereas the planner also cares about the carrier’s surplus, since different destinations have different continuation values. The random matching process creates what we call “pooling externalities”: while customers are heterogeneous in their desired destination, carriers cannot direct their search towards a specific type of customer. As heterogeneous agents on one side of the market are pooled together, prices may fail to fully capture the social value of a match between a carrier and a customer. In turn, this distorts the customers’ destination decisions and the resulting equilibrium destination shares $G$.

We now formalize this intuition. Define the social value of a search allocation $s,e,G$ by

$$V^p (s,e,G) \equiv \max_{q,b \geq 0} W(q) - \sum_{ij} (q_{ij} + b_{ij}) \frac{c^s_{ij}}{d_{ij}} - \sum_{ij} q_{ij} k_{ij} - \sum_i s_i c^w_i - \sum_i e_i c^e_i$$ (23)

s.t. feasibility constraints (15)-(18)

This problem essentially solves for the carriers’ optimal ballast decisions ($b$) and the decision of whether to accept a match or not ($q$), while taking as given the entry decisions of carriers ($s$) and customers ($e$), as well as customers’ destination decisions ($G$).

The social planner Problem (19) is equivalent to,$^9$

$$\max_{s, e, G \geq 0} V^p (s, e, G)$$ (24)

s.t. $\sum_j G_{ij} = 1 \forall i$ and $\sum_i s_i \leq S$.

Intuitively, since the only source of inefficiency results from the agents search behavior, it is useful to “optimize out” the other variables (i.e. $q, b$) in order to focus on the impact of the main variables of interest, $s, e, G$.

**Definition 3.** At a search allocation $(s, e, G)$:

---

$^9$Note that for every feasible solution $s, e, G$ of this problem there exists a pair $q, b \geq 0$ such that the resulting allocation is steady state feasible: simply set $q = 0$, $b_{ij} = 0$ for $i \neq j$ and $b_{ii} = s_i \forall i$.  

---
- Carriers internalize thin/thick market externalities if

\[ s \in \arg \max_{s' \geq 0} V^p(s', e, G) \text{ s.t. } \sum_i s_i \leq S. \]  

(25)

- Customers internalize thin/thick market externalities if

\[ e \in \arg \max_{e' \geq 0} V^p(s, e', G). \]  

(26)

- Customers internalize pooling externalities if

\[ G \in \arg \max_{G' \geq 0} V^p(s, e, G') \text{ s.t. } \sum_j G_{ij} = 1 \forall i. \]  

(27)

Our next theorem states three conditions that determine how the meeting surpluses must be shared between carriers and customers in order for the externalities to be internalized in equilibrium. Since we are dealing with no-discounting limit outcomes, we first need to define the corresponding limits of these surpluses.\(^\text{10}\)

**Definition 4.** Let \((s, e, G, q, b, \tau)\) be a limit equilibrium outcome. Then, \(\bar{\Delta}^s, \bar{\Delta}^e\) is a consistent pair of limit surpluses if there exists a sequence \((s^n, e^n, G^n, q^n, b^n, \tau^n, \Delta^{s,n}, \Delta^{e,n}, \beta^n)_{n \geq 0}\) such that: (i) For each \(n\), \((s^n, E^n, q^n, b^n, \tau^n)\) is an equilibrium outcome for the economy populated by agents with discount factor \(\beta^n\), and \(\Delta^{s,n}, \Delta^{e,n}\) are the corresponding vectors of surpluses, as defined in (2) and (10); (ii) As \(\beta^n \to 1\), \((s^n, E^n, q^n, b^n, \tau^n) \to (s, E, q, b, \tau)\) and \((\Delta^{s,n}, \Delta^{e,n}) \to (\bar{\Delta}^s, \bar{\Delta}^e)\). If \(\bar{\Delta}^s, \bar{\Delta}^e\) is such a pair, the corresponding joint surplus \(\bar{\Delta}\) is defined by

\[ \forall i, j : \bar{\Delta}_{ij} = \bar{\Delta}^s_{ij} + \bar{\Delta}^e_{ij}. \]

For every \(i \in I\), we denote by \(\eta^s_i = d \log m_i(s_i, e_i) / d \log s_i\) and \(\eta^e_i = d \log m_i(s_i, e_i) / d \log e_i\), the elasticities of the matching function with respect to the measure of carriers and customers searching at \(i\), respectively. To avoid delving into corner solutions arising in trivial cases, we assume that the equilibrium

\(^{10}\)Although this definition might suggest that there are multiple consistent pairs of limit surpluses associated with a single limit equilibrium outcome, it turns out that such pairs are essentially unique: see Appendix A.1 for details.
is such that there is a positive measure of customers and carriers searching at each location \((s_i, e_i > 0 \forall i)\) and that \(\sum_i s_i < S\).\(^{11}\)

**Theorem 2.** At a limit equilibrium outcome \((s, e, G, q, b, \tau)\), let \(\Delta^s, \Delta^e\) be a consistent pair of limit surpluses, and \(\Delta\) be the corresponding joint surplus. Suppose that Problem (23) admits a unique optimal solution.\(^{12}\) Then:

(i) Carriers internalize thin/thick market externalities if and only if
\[
\forall i \in I: \frac{\sum_j G_{ij} \Delta^s_{ij}}{\sum_j G_{ij} \Delta_{ij}} = \eta^s_i. \quad (28)
\]

(ii) Customers internalize thin/thick market externalities if and only if
\[
\forall i \in I: \frac{\sum_j G_{ij} \Delta^e_{ij}}{\sum_j G_{ij} \Delta_{ij}} = \eta^e_i. \quad (29)
\]

(iii) Customers internalize pooling externalities if and only if
\[
\Delta^s_{ij} = \max_{k \neq i} \Delta^s_{ik} \quad (30)
\]

for every \(i, j\) such that \(G_{ij} > 0\).

(iv) \((s, e, G, q, b)\) is efficient only if conditions (i)-(iii) hold jointly.

The proof, provided in Appendix A.3, first establishes that the function \(V^p(s, e, G)\) is concave. Therefore the supergradients with respect to each of the arguments \((s, e, G)\) are well-defined at every search allocation and in fact \(V^p\) is differentiable almost everywhere in its domain. Then we demonstrate, through the use of the dual variables associated with a limit equilibrium allocation, that the resulting first order conditions coincide with the conditions internalizing the three externalities.

Conditions (i) and (ii) of Theorem 2 require that the share of the surplus appropriated by agents on

\(^{11}\)If \(s_i = 0\) or \(e_i = 0\) for some \(i\), then the efficiency conditions must hold only at locations with positive number of carriers and customers. The second is a non-triviality assumption which precludes having no ships traveling at all; indeed since \(\sum_i s_i = S\) implies that \((q_{ij}, b_{ij}) = (0, 0)\) for every \(i, j\) such that \(d_{ij} < 1\).

\(^{12}\)This is a technical condition which is generally satisfied, for example, when the function \(W\) is strictly concave.

\(^{13}\)Formally, this condition is necessary only when \(V^p(s, e, G)\) is differentiable in \(s\), which is the case almost everywhere. A similar disclaimer applies to statement (ii), (iii) and (iv), where necessity relies on differentiability with respect to \(e, G\) and \((s, e, G)\), respectively.
each side of the market to be equal to the elasticity of the matching function with respect to the total measure of agents on that same side of the market (Hosios, 1990). As discussed above, when agents (either customers or carriers) choose whether and where to search for a match, they do not take into account the effect that this decision has in improving meeting opportunities for agents on the other side of the market, and in worsening the meeting opportunities for agents on the same side. These externalities drive a wedge between the level of carriers and customers searching in the efficient allocation and in the market equilibrium, generating the well-known thin/thick market externalities. Conditions (28) and (14) have a similar flavor as the standard Coasian conditions in the presence of externalities, where the private value of an action must be equal to its social value. Indeed, we can rewrite equation (28) as

\[
\lambda^s_i \sum_j G_{ij} \Delta s_{ij} = \frac{dm_i(s_i, e_i)}{ds_i} \sum_j G_{ij} \Delta_{ij}.
\]

The left hand side captures the per-period expected private return of a ship entering market \(i\), which equals the expected ship’s surplus from matching \(\sum_j G_{ij} \Delta s_{ij}\) multiplied by its matching probability \(\lambda^s_i\). The right hand side captures the per-period expected social return from an additional ship entering market \(i\) which equals the total expected surplus from an additional match, \(\sum_j G_{ij} \Delta_{ij}\), multiplied by the marginal increase in number of matches \(\frac{dm_i(s_i, e_i)}{ds_i}\).

Condition (iii) of Theorem 2 deals with the pooling externalities. The inefficiency here arises because customer type-specific (in this case destination-specific) markets are missing. When heterogeneous customers are pooled together, carriers cannot direct their search towards a specific type of customer. As a result, carriers cannot compete among themselves to serve a given type of customer; this grants market power to the carriers, creating a wedge between prices and the carriers’ opportunity cost of completing a trip. It is useful to compare our setup with a frictionless environment. In this case, competition among carriers ensures that prices coincide with the carriers’ opportunity cost of each trip. In equilibrium, therefore, carriers are indifferent among serving different types of customers and customers internalize the social cost of their trip. Similarly, in a world with search frictions where carriers can direct their search to a specific customer type, in equilibrium a no-arbitrage condition would make them indifferent across destinations. In our setup, the planner essentially restores this indifference condition in Condition (iii) of Theorem 2.
Moreover, Conditions (i) and (ii) imply that a necessary condition for efficiency is that the matching function exhibits constant returns to scale; indeed, if we add equations (28) and (14), the left-hand-side is equal to one, and thus the elasticities must add to one as well. Corollary A.6 in Appendix A.6 demonstrates that under non-constant returns to scale in matching, efficiency can still be restored, but a tax or subsidy creates a wedge between the price paid by the customer and the one received by the carrier.

Efficient Prices  Condition (iv) of Theorem 2 provides a characterization of the efficient pricing rule:

**Corollary 1.** Let a limit equilibrium outcome \((s, e, G, q, b, \tau)\) be efficient. Then the equilibrium prices are such that for every \(i, j\) such that \(G_{ij} > 0\):

\[
\tau_{ij} = w_{ij}(q) - k_{ij} - \bar{\Delta}_{ij} + \eta_i \sum_j G_{ij} \bar{\Delta}_{ij}.
\]  \hspace{1cm} (31)

Corollary 1 provides the formula for the efficient pricing rule. To gain some intuition for this rule, suppose that prices are set by Nash bargaining in the market. In other words, upon matching, a carrier and a customer negotiate and the price solves the surplus sharing condition \((1 - \gamma_i) \bar{\Delta}_{ij}^s = \gamma_i \bar{\Delta}_{ij}^e\), where \(\gamma_i\) is the bargaining coefficient of the carrier. In that case, the efficient price that the planner would like to set satisfies the following relationship:

\[
\forall i, j: (1 - \eta^s_i) \bar{\Delta}_{ij}^s = \eta^e_i \left[ \bar{\Delta}_{ij}^e - (\bar{\Delta}_{ij} - \sum_j G_{ij} \bar{\Delta}_{ij}) \right].
\]  \hspace{1cm} (32)

Therefore, compared to the market’s Nash bargaining, the planner’s prices are those that would obtain if we (i) replaced the bargaining coefficients with the respective matching function elasticities; and (ii) adjusted the outside option of the customer by the deviation of route \(i, j\)’s social surplus from the (weighted) average of the social surplus across destinations. By adjusting the outside option of the customer, we ensure that customers fully internalize the social value of their destination decision. If the customer has chosen a destination whose social surplus is higher than the mean from origin \(i\), he should enjoy a higher outside option (and thus a lower price), and vice versa.

Corollary 1 suggests that if a central authority could post prices, they should choose them according
to (31). For instance, in the case of taxicabs, prices are regulated by central agencies. In practice, they are roughly set equal to a tariff plus a fee proportional to distance. Corollary 1 indicates that this pricing rule is unlikely to be efficient, since the efficient prices should be origin-destination specific. Naturally, implementing the efficient prices may not be straightforward in practice as there are not many examples of markets where prices can be fully regulated. Thus, in the next section we consider optimal policy under a common pricing mechanism in decentralized markets: Nash bargaining.

### 3.3 Optimal Taxes under Nash Bargaining

In this section we consider the problem of a planner who cannot directly control prices, but can use taxes/subsidies to restore efficiency in the market. We show that the planner can indeed achieve efficiency using such instruments and we derive their optimal values.

Suppose that the planner can impose a tax/subsidy $h^q$ on loaded trips, $h^s$ on searching carriers, and $h^e$ on searching customers. In other words, searching carriers in region $i$ pay $h^s_i$ in addition to their waiting cost $c^w_i$ every period they search; customers searching in $i$ pay $h^e_i$ in addition to their cost $c^e_{ij}$ every period they search; finally, every new match is taxed by the amount $h^q_{ij}$ (as illustrated below which side pays the tax does not matter).

We focus on a specific price mechanism, that of Nash bargaining, which is a commonly employed model used to capture bilateral negotiations. In this setup, prices are determined by the usual surplus sharing condition,

$$
(1 - \gamma_i) \Delta^s_{ij} = \gamma_i \Delta^e_{ij}. 
$$

(33)

where $\gamma_i$ is the carrier bargaining coefficient at $i$.

We extend the definition of equilibrium to accommodate Nash bargaining and taxes in a straightforward manner: $(s, e, G, q, b, \tau)$ is an equilibrium outcome under taxes $h$ and Nash bargaining, if carriers and customers behave optimally given $h$, $\tau$, $\lambda^s$, $\lambda^e$ and $G$; the feasibility constraints are satisfied; $\lambda^s$, $\lambda^e$ and $G$ are consistent with the allocation; and finally, prices are determined by (33) in every $i$.

Corollary 2 derives the tax scheme $h$ that resolves the two externalities:

**Corollary 2.** Let $(s, e, G, q, b, \tau)$ be a limit equilibrium outcome under taxes $h$ and Nash bargaining. Then:
(i) Thin/thick market externalities are internalized if and only if for every $i$

$$\gamma_i \sum_j G_{ij} \Delta_{ij} - \left( \frac{h_i^q}{\lambda_i^q} + \gamma_i \sum_j G_{ij} h_{ij}^q \right) = \eta_i^q \sum_j G_{ij} \Delta_{ij}$$

and similarly,

$$(1 - \gamma_i) \sum_j G_{ij} \Delta_{ij} - \left( \frac{h_i^c}{\lambda_i^c} + (1 - \gamma_i) \sum_j G_{ij} h_{ij}^q \right) = \eta_i^c \sum_j G_{ij} \Delta_{ij}.$$  

(ii) Pooling externalities are internalized if and only if for every $i,j$ we have

$$(1 - \gamma_i)(\Delta_{ij} - h_{ij}^q) = L_i + (\Delta_{ij} - \sum_j G_{ij} \Delta_{ij})$$  

if $G_{ij} > 0$ and where $L_i$ is an arbitrary constant.: 

$$(1 - \gamma_i)(\Delta_{ij} - h_{ij}^q) = L_i + (\Delta_{ij} - \sum_j G_{ij} \Delta_{ij})$$

with equality if $G_{ij} > 0$  

Conditions (i) and (ii) imply that all externalities are internalized if and only if 35 and 36 hold with

$$L_i = -\eta_i^q \sum_j G_{ij} \Delta_{ij} - \frac{h_i^c}{\lambda_i^c}.$$  

Before discussing the result, note that if the planner does not impose any taxes, so that $h = 0$, the conditions required to internalize the thin/thick market externalities (34) and (35) become the well-known Hosios (1990) conditions. Indeed, in that case (34) and (35) imply that

$$\gamma_i = \frac{d \ln m_i(s_i, e_i)}{d \ln s_i} \equiv \eta_i^q$$

$$1 - \gamma_i = \frac{d \ln m_i(s_i, e_i)}{d \ln e_i} \equiv \eta_i^c$$

Notice that, regardless of who pays the matching tax $h_{ij}^q$, we can think of $\gamma_i h_{ij}^q$ as the incidence of this tax on carriers and $(1 - \gamma_i) h_{ij}^q$ as the incidence on customers, since Nash bargaining implies that
the agents split the gross surplus from the match according to $\gamma_i$. In addition, a searching carrier in expectation pays $h_s^i/\lambda_s^i$ while searching and a searching customer pays $h_e^i/\lambda_e^i$ in expectation.

Therefore, Condition (34) states that the private value of an additional carrier searching in $i$ must equal its social value. Indeed, the left-hand-side of (34) consists of the share of the total surplus accruing to the carrier (which is equal to his bargaining coefficient times the total surplus) minus his tax incidence (consisting of the search tax and his share of the trip tax). The right-hand-side is the surplus the planner wants the carrier to receive, which equals the contribution of the extra carrier to total surplus. In other words, the tax incidence of the carrier must be set so that the original condition for thin/thick market externalities, equation (28) of Theorem 2, is satisfied. A similar intuition holds for equation (35).

Equation (36) determines the tax on loaded trips that resolves pooling externalities. The left-hand-side is equal to the after-taxes-surplus accruing to the customer from traveling on route $i, j$ (which is equal to his share of total surplus, minus his tax incidence). The right-hand-side indicates that the customer’s after-tax-surplus should equal the deviation of the route’s social surplus from the (weighted) average of social surpluses across destinations, up to some origin-specific amount $L_i$. Therefore, a customer that enters a route $i, j$ with social surplus higher than the average social surplus originating from region $i$, should be subsidized, and the amount is such that he fully internalizes the social value of the decision to go to destination $j$; similarly a customer that enters a route $i, j$ with social surplus lower than the average should be taxed. To correct the pooling externalities, the amount $L_i$ can be arbitrary; this is intuitive, since pooling externalities are linked to the differences in destinations from a given origin. However, to correct all externalities, the amount $L_i$ must be given by Condition (iii) of Corollary 2.

Corollary 2 does not pin down the efficient taxes uniquely. In particular, we can see that if the planner can only use the search taxes $h^s, h^e$, he can correct the thin/thick market externalities. If he can tax only matches but not search of any side of the market, then he can only correct the pooling externalities (using equation (36)). Finally, the planner can correct all externalities by taxing only searching carriers.

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To see this note that Nash bargaining implies that $\Delta_{ij}^s = \gamma_i \Delta_{ij} - \gamma_i h_{ij}^s$ and $\Delta_{ij}^e = (1-\gamma_i) \Delta_{ij} - (1-\gamma_i) h_{ij}^e$, so that agents split the gross surplus $\Delta_{ij}$ according to $\gamma_i$, and similarly pay their share of the matching tax also according to $\gamma_i$. Indeed, the social surplus is now defined by $\Delta_{ij} = \Delta_{ij}^s + \Delta_{ij}^e + h_{ij}^e$ (as either $\Delta_{ij}^s$ or $\Delta_{ij}^e$ include $-h_{ij}^e$, $\Delta_{ij}$ does not depend on $h_{ij}^e$) and therefore $\Delta_{ij}^s = \gamma_i (\Delta_{ij}^s + \Delta_{ij}^e) = \gamma_i (\Delta_{ij} - h_{ij}^e)$. Note also that the definition of efficiency remains the same, except that we include the planner’s revenue in the welfare.

He can do so by setting by setting $h_{ij}^s/\lambda_s^i = (1-\gamma_i) \sum_j G_{ij} \Delta_{ij} - \eta_i \sum_j G_{ij} \Delta_{ij}$ and $h_{ij}^e/\lambda_e^i + h_{ij}^s/\lambda_s^i = (1-\eta_i - \eta_e) \sum_j G_{ij} \Delta_{ij}$.

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14To see this note that Nash bargaining implies that $\Delta_{ij}^s = \gamma_i \Delta_{ij} - \gamma_i h_{ij}^s$ and $\Delta_{ij}^e = (1-\gamma_i) \Delta_{ij} - (1-\gamma_i) h_{ij}^e$, so that agents split the gross surplus $\Delta_{ij}$ according to $\gamma_i$, and similarly pay their share of the matching tax also according to $\gamma_i$. Indeed, the social surplus is now defined by $\Delta_{ij} = \Delta_{ij}^s + \Delta_{ij}^e + h_{ij}^e$ (as either $\Delta_{ij}^s$ or $\Delta_{ij}^e$ include $-h_{ij}^e$, $\Delta_{ij}$ does not depend on $h_{ij}^e$) and therefore $\Delta_{ij}^s = \gamma_i (\Delta_{ij}^s + \Delta_{ij}^e) = \gamma_i (\Delta_{ij} - h_{ij}^e)$. Note also that the definition of efficiency remains the same, except that we include the planner’s revenue in the welfare.

15He can do so by setting by setting $h_{ij}^s/\lambda_s^i = (1-\gamma_i) \sum_j G_{ij} \Delta_{ij} - \eta_i \sum_j G_{ij} \Delta_{ij}$ and $h_{ij}^e/\lambda_e^i + h_{ij}^s/\lambda_s^i = (1-\eta_i - \eta_e) \sum_j G_{ij} \Delta_{ij}$.
and matches (but not searching customers);\textsuperscript{16} and he can also correct all externalities by taxing only searching customers and matches, (but not searching carriers).

4 Empirical Application: Dry Bulk Shipping

In this section we describe our empirical application using data from the dry bulk shipping industry. We begin in Section 4.1 with a description of the industry and the available data. In Section 4.2 we discuss search frictions in this market. In Section 4.3 we estimate the model and present the parameter estimates. With the exception of Subsection 4.2, this section follows closely BKP. Throughout the following sections, unless otherwise noted, we split ports into 15 geographical regions, depicted in Figure 6 in Appendix B.\textsuperscript{17}

4.1 Industry Description and Data

Dry bulk shipping involves vessels designed to carry a homogeneous unpacked dry cargo, for individual shippers on non-scheduled routes. Bulk carriers operate much like taxi cabs: a specific cargo is transported individually by a specific ship, for a trip between a single origin and a single destination. Dry bulk shipping involves mostly commodities, such as iron ore, steel, coal, bauxite, phosphates, but also grain, sugar, chemicals, lumber and wood chips; it accounts for about half of total seaborne trade in tons (UNCTAD) and 45\% of the total world fleet, which includes also containerships and oil tankers.\textsuperscript{18}

There are four size categories of dry bulk carriers: Handysize (10,000–40,000 DWT), Handymax (40,000–60,000 DWT), Panamax (60,000–100,000 DWT) and Capesize (larger than 100,000 DWT). The industry is unconcentrated, consisting of a large number of small shipowning firms (see Kalouptsidi, 2014). Moreover, shipping services are largely perceived as homogeneous. In his lifetime, a shipowner will contract with hundreds of different exporters, carry a multitude of different products and visit numerous countries.

\textsuperscript{16}He can do so by setting \((1 - \gamma_i) h_{ij}^h = (1 - \gamma_i) \Delta_{ij} + \sum_j G_{ij} \Delta_{ij} - \eta_i^e \sum_j G_{ij} \Delta_{ij},\) if \(G_{ij} > 0\) and \(h_{i}^h/\lambda_i + \sum_j G_{ij} h_{ij}^h = (1 - \eta_i^e - \eta_i^s) \sum_j G_{ij} \Delta_{ij}.\)

\textsuperscript{17}To determine the regions, we employ a clustering algorithm that minimizes the within-region distance between ports. The regions are: West Coast of North America, East Coast of North America, Central America, West Coast of South America, East Coast of South America, West Africa, Mediterranean, North Europe, South Africa, Middle East, India, Southeast Asia, China, Australia, Japan-Korea. We ignore intra-regional trips and entirely drop these observations.

\textsuperscript{18}Bulk ships are different from containerships, which carry cargo (mostly manufactured goods) from many different cargo owners in container boxes, along fixed itineraries according to a timetable. It is not technologically possible to substitute bulk with container shipping.
Trips are realized through individual contracts that are intermediated by a disperse network of brokers. Ships carry the cargo of a single exporter at a time, who fills up the entire ship (much like a taxi, rather than a bus). In this paper, we focus on spot contracts and in particular the so-called “trip-charters”, in which the shipowner is paid in a per day rate.\(^{19}\)

We combine four main data sets. The first is a data set of shipping contracts, from 2010 to 2016, collected by Clarksons Research. An observation is a transaction between a shipowner and a charterer for a specific trip. We observe the vessel, the charterer, the contract signing date, the loading and unloading dates, the price in dollars per day, as well as some information on the origin and destination.

Second, we use satellite AIS (Automatic Identification System) data from exactEarth Ltd for the ships in the Clarksons dataset between July 2010 and March 2016. AIS transceivers on the ships automatically broadcast information, such as their position (longitude and latitude), speed, and level of draft (the vertical distance between the waterline and the bottom of the ship’s hull), at regular intervals of at most six minutes. The draft is a crucial variable, as it allows us to determine whether a ship is loaded or not at any point in time.

Third, we augment the ship data sets above, with international trade data from Comtrade on the export value and volume by country pair for bulk commodities.

Fourth, we use the ERA-Interim archive, from the European Centre for Medium-Range Weather Forecasts (CMWF), to collect global data on daily sea weather. This allows us to construct weekly data on the wind speed (in each direction) on a 6° grid across all oceans.

We provide a brief overview of the data and empirical regularities and we refer the interested reader to BKP for further details. Our final dataset stretches from 2012 to 2016 and involves 5,398 ships (about half the world fleet) and 12,007 shipping contracts with a known price, origin and destination.\(^{20}\) The average trip price is 14,000 dollars per day (or 290,000 dollars for the entire trip), with substantial variation. Trips last on average 2.9 weeks. Contracts are signed close to the loading date, on average six days before. We have 393,058 ship-week observations at which the ship decides to either travel empty someplace (termed “ballast”) or stay at its current location. Ships that do not sign a contract, remain in their current location

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\(^{19}\)Trip-charters are the most common type of contract. Long-term contracts (“time-charters”), however, do exist: on average, about 10% of contracts signed are long-term. Interestingly, though, ships in long-term contracts, are often “relet” in a series of spot contracts, suggesting that arbitrage is possible.

\(^{20}\)We drop the first two years (until May 2012) of vessel movement data, as satellites are still launched at that time and the geographic coverage is more limited.
with probability 77%, while the remaining ships ballast elsewhere. Clarksons reports the product carried in about 20% of the sample and the main commodity categories are grain (29%), ores (21%), coal (25%), steel (8%) and chemicals/fertilizers (6%).

Finally, an important feature of this market revealed by the satellite data, is that most countries are either large net importers or large net exporters. For instance, Australia, Brazil and Northwest America, the world’s biggest exporters, are rich in minerals, grain, coal, etc. At the same time, China and India, the world’s biggest importers, require raw materials to grow further. As a result, commodities flow out of the former, towards the latter. The trade imbalances have implications for both ship ballast behavior and shipping prices. Indeed, at any point in time, 42% of ships are traveling without cargo. At the same time, prices are largely asymmetric and depend on the destination’s trade imbalance: all else equal, the prospect of having to ballast after offloading leads to higher prices. For summary statistics see Table 1 in BKP while for details on the construction of the final dataset see the Supplemental Material of BKP.

4.2 Search Frictions in Dry Bulk Shipping

A number of features of dry bulk shipping, such as information frictions and port infrastructure, can hinder the matching of ships and exporters. In this section we argue that these frictions indeed lead to unrealized potential trade. Consider a geographical region, such as a country or a set of neighboring countries, where there are \( s \) ships available to pick up cargo and \( e \) exporters searching for a ship to transport their cargo. We define search frictions by the inequality:

\[
m < \min \{ s, e \}
\]

where \( m \) is the number of matched ships and exporters. In other words, under frictions there is potential trade that remains unrealized; in contrast, in a frictionless world, the entire short side of the market gets matched, so that \( m = \min \{ s, e \} \). When (39) holds, matches are often modeled via a matching function, \( m = m(s, e) \), as is done in Section 2 above, and also extensively in the labor literature.

In this section, we present three facts consistent with frictions, as defined by (39). In particular, we (i)

---

21The material in this section was included in a previous version of our paper “Geography, Transportation and Endogenous Trade Costs”; please see NBER Working Paper 23581.
provide a direct test for inequality (39); (ii) we document wastefulness in ship loadings; (iii) we document substantial price dispersion. Then, we estimate the matching function $m = m(s, e)$ and gauge the degree of frictions.

**Evidence of search frictions** We begin with a simple test for search frictions. If we observed all variables $s, e, m$, it would be straightforward to test (39); this is essentially what is done in the labor literature, where the co-existence of unemployed workers and vacant firms is interpreted as evidence of frictions. However in our setup, we observe $m$ (i.e. ships leaving loaded) and $s$, but not $e$; we thus need to adopt a different approach.

Assume there are more ships than exporters, i.e. $\min(s, e) = e$. We begin with this assumption, because our sample period is one of low shipping demand and severe ship oversupply due to high ship investment between 2005 and 2008 (see Kalouptsidi, 2014). If there are no search frictions, so that $m = \min(s, e) = e$, small exogenous changes in the number of ships should not affect the number of matches. In contrast, if there are search frictions, an exogenous change in the number of ships changes the number of matches, through the matching function $m = m(s, e)$. We approximate an exogenous change in the number of ships, with ocean weather conditions as an instrument. The intuition for the instrument is that wind affects the speed at which ships travel and therefore exogenously shifts the supply of ships at port. Using this instrument, we can explore whether exogenously changing the number of ships in regions with a lot more ships than exporters affects the realized number of matches.\footnote{We partition the globe into cells of $9^\circ \times 9^\circ$, and for each cell we collect data on the wind speed in different directions, as well as wave period and height. To exclude seasonality, for each cell we residualize the weather measurements on a quarter fixed effect. For each region $i$ and period $t$ we consider as potential regressors all the weather measurements for cells in the sea, for one and two weeks before period $t$. Finally, we follow Belloni et al. (2012) to select the relevant instruments in each region $i$. We use the selected instruments both to perform the test for search frictions and to estimate the matching function.} Since we do not observe exporters directly, to select periods in which there are more ships than exporters, for each region we consider weeks when there are at least twice as many ships waiting in port as matches. Table 1 presents the results. We find that indeed matches are affected by weather conditions in all but one region, consistent with the presence of search frictions.

Second, we document *simultaneous arrivals* of empty ships that then load and *departures of empty ships*. Indeed, the first two panels of Figure 1 display the weekly number of ships that arrive empty and load, as well as the number of ships that leave empty, in two net exporting countries: Norway
Table 1: Test for search frictions. Regressions of the number of matches in each region on the unpredictable component of weather conditions in the surrounding seas. For each region we use weeks in which there are at least twice as many ships as matches. The first column reports the number of observations; the second column joint significance; and the third column the average ratio between matches and ships in each region during these weeks. To proxy for the unpredictable component of weather, we partition the globe into cells of $9^\circ \times 9^\circ$, and for each cell we collect data on the speed of the horizontal (E/W) and vertical (N/S) component of wind, as well as wave period and height. To exclude seasonality, for each cell we residualize the weather measurements on a quarter fixed effect. For each region $i$ and period $t$ we consider as potential regressors one and two weeks lagged values of all the weather measurements for cells in the sea. Finally, we follow Belloni et al. (2012) to select the relevant instruments in each region. We use the selected instruments both to perform the test for search frictions and to estimate the matching function.

<table>
<thead>
<tr>
<th>Region</th>
<th>N</th>
<th>Joint Significance</th>
<th>$\frac{a}{m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>North America West Coast</td>
<td>193</td>
<td>0</td>
<td>2.706</td>
</tr>
<tr>
<td>North America East Coast</td>
<td>200</td>
<td>0</td>
<td>3.172</td>
</tr>
<tr>
<td>Central America</td>
<td>199</td>
<td>0.001</td>
<td>3.451</td>
</tr>
<tr>
<td>South America West Coast</td>
<td>198</td>
<td>0</td>
<td>2.913</td>
</tr>
<tr>
<td>South America East Coast</td>
<td>200</td>
<td>0</td>
<td>4.083</td>
</tr>
<tr>
<td>West Africa</td>
<td>200</td>
<td>0.001</td>
<td>5.862</td>
</tr>
<tr>
<td>Mediterranean</td>
<td>200</td>
<td>0</td>
<td>4.244</td>
</tr>
<tr>
<td>North Europe</td>
<td>200</td>
<td>0</td>
<td>3.577</td>
</tr>
<tr>
<td>South Africa</td>
<td>200</td>
<td>0</td>
<td>2.862</td>
</tr>
<tr>
<td>Middle East</td>
<td>200</td>
<td>0</td>
<td>3.86</td>
</tr>
<tr>
<td>India</td>
<td>200</td>
<td>0.34</td>
<td>8.58</td>
</tr>
<tr>
<td>South East Asia</td>
<td>200</td>
<td>0</td>
<td>3.334</td>
</tr>
<tr>
<td>China</td>
<td>200</td>
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<td>6.194</td>
</tr>
<tr>
<td>Australia</td>
<td>187</td>
<td>0</td>
<td>2.457</td>
</tr>
<tr>
<td>Japan-Korea</td>
<td>200</td>
<td>0</td>
<td>5.311</td>
</tr>
</tbody>
</table>

and Chile. In Norway, several ships arrive empty and load, but almost no ship departs empty. In Chile, however, the picture is quite different: it frequently happens that an empty ship arrives and picks up cargo, while at the same time another ship departs empty. This is suggestive of wastefulness in Chile: why does the ship that depart empty, not pick up the cargo, instead of having another ship arrive from elsewhere to pick it up?

This pattern is observed in many countries. Indeed, the third panel of Figure 1 depicts the histogram of the bi-weekly ratios of outgoing empty ships over incoming empty and loading ships for net exporting countries. In the absence of frictions, one would expect this ratio to be close to zero. However, we see that the average ratio is well above zero. Moreover, this pattern is quite robust in a number of dimensions.\(^{23}\)

\(^{23}\)This figure is robust to alternative market definitions, time periods and ship types. Capesize vessels exhibit somewhat
Figure 1: Simultaneous arrivals and departures of empty ships: The first two panels depict the flow of ships arriving empty and loading, and ships leaving empty in two-week intervals in Norway and Chile. The last panel shows the histogram of the ratio of outgoing empty over incoming empty and loading ships across all net exporting countries.

Third, again inspired by the labor literature, we investigate dispersion in prices. In markets with no frictions, the law of one price holds, so that there is a single price for the same service. This does not hold in labor markets, where there is large wage dispersion among workers who are observationally identical. This observation has generated a substantial and influential literature on frictional wage inequality, i.e. wage inequality that is driven by search frictions.\footnote{See for instance Burdett and Mortensen (1998), Postel-Vinay and Robin (2002), Mortensen (2003) and references therein.} Table 5 in Appendix B shows that there is substantial price dispersion in shipping contracts. More specifically, at best we can account for about 70\% of price variation, controlling for ship size, as well as quarter, origin and destination fixed effects. Moreover, the coefficient of variation of prices within a given quarter, origin and destination triplet is about 30\% (23\%) on average (median). In the most popular trip, from Australia to China, the weekly coefficient of variation is on average 34\% and ranges from 15\% to 65\% across weeks. In addition, it is worth noting that the type of product carried affects the price paid and overall more valuable goods lead to higher contracted prices, as shown in Table 5 in Appendix B. In the absence of frictions, if there are more ships than exporters, as is the case during our sample period, we would expect prices to be bid down to the ships’ opportunity
cost. In contrast, in markets with frictions and bilateral bargaining, since ships now have market power, the price will also depend on the exporter’s valuation and exporters with higher valuations pay more.

As in labor markets, a multitude of factors can lead to frictions (i.e. unrealized matches) in shipping. First, the decentralized and unconcentrated nature of the market and the mere existence of brokers, suggest that information frictions are present. The meeting process involves a disperse network of brokers; oftentimes more than two brokers intervene to close a deal, suggesting that the ship’s and the exporter’s brokers do not always find each other, and that an “intermediate broker” was necessary to bring the two together (Panayides, 2016). In interviews, brokers claimed to receive 5,000-7,000 emails per day; sorting through these emails is reminiscent of an unemployed worker sorting through hundreds of vacancy postings. Port infrastructure, congestion or capacity constraints may also hinder matching.

Finally, we discuss two important assumptions in our setup, no ship heterogeneity and random search. While in labor markets, as some researchers have argued, observed or unobserved heterogeneity may partly explain the documented facts, in shipping heterogeneity is much more limited. Indeed, the data suggests that ship heterogeneity alone is not a prominent explanation for search frictions. Ships do not specialize neither geographically, nor in terms of products: during the period of our data ships deliver cargo to 13 out of 15 regions on average and carry at least 2 of the 3 main products (coal, ore and grain). Moreover, neither shipowner characteristics, nor shipowner fixed effects have any explanatory power in price regressions, as shown in Table 4 in Appendix B, while ballast decisions of ships in the same region are concentrated around the same options. Nonetheless, despite the anecdotal and descriptive evidence presented, it is not possible to reject that heterogeneity can also play a role in the market. Random search is also a reasonable assumption in shipping as meetings occur through an unconcentrated network of brokers. Nonetheless, we examine this assumption more rigorously in Appendix C, where we investigate a standard implication of directed search, whether matching rates differ across destinations from a given origin.

---

25In a frictionless market with more ships than freights and homogeneous ships, in equilibrium the price from an origin to a destination would be such that ships are indifferent between transporting the cargo and staying unmatched.  
26If heterogeneity were an important driver of ships’ ballasting decisions, we would expect ships to choose diverse destinations from a given origin. Yet we find that ballast choices are similar across ships (the CR3 measure for the chosen destinations is higher than 70% in most regions). Moreover, home-ports are not an important consideration for shipowners, as the crew flies to their home country every 6-8 months.
Matching function estimation We close this section by discussing the estimated search frictions that result from the estimated matching function. Here, we follow the same approach to estimate the matching function as BKP; we thus provide a brief overview of the procedure and then present the implications for search frictions.

A sizable literature has estimated matching functions in several different contexts (e.g. labor markets, marriage markets, taxicabs). For instance, in labor markets, one can use data on unemployed workers, job vacancies and matches to recover the underlying matching function. In our data, we observe ships and matches, but not searching exporters; in BKP we simultaneously recovered both exporters, as well as a nonparametric matching function.\(^{27}\) This approach extends the literature in two dimensions. First, we do not take a stance on the presence of search frictions. When one side of the market (in this case exporters) is unobserved or mismeasured, it is difficult to discern whether search frictions are present.\(^{28}\) Second, we avoid parametric restrictions on the matching function; this is important, since as shown in Theorem 2, in frictional markets, the conditions for constrained efficiency depend crucially on the elasticity of the matching function with respect to the search input.

Briefly, the estimation draws from the literature on nonparametric identification (Matzkin, 2003) and non-separable instrumental variable techniques (e.g. Imbens and Newey, 2009). We require that \(m(s,e)\) is increasing in \(e\), that it exhibits constant returns to scale (although the results are robust to alternative restrictions; see BKP) and that an instrument that shifts the number of ships exists (the weather shocks). The methodology delivers exporters point-wise and the matching function of each location \(i\) nonparametrically. We refer the reader to BKP for details, as well as Brancaccio et al. (forthcoming) for a guide on the implementation of this approach in this and other settings.\(^{29}\)

Figure 2 reports our estimates for search frictions. In particular, to measure the extent of search frictions, we define

\[ m_{it} = \min(s_{it}, e_{it}) \]

which is the highest possible match for the pair \((s_{it}, e_{it})\). This approach extends the literature in two dimensions. First, we do not take a stance on the presence of search frictions. When one side of the market (in this case exporters) is unobserved or mismeasured, it is difficult to discern whether search frictions are present.\(^{28}\) Second, we avoid parametric restrictions on the matching function; this is important, since as shown in Theorem 2, in frictional markets, the conditions for constrained efficiency depend crucially on the elasticity of the matching function with respect to the search input.

To illustrate this assume that \(s\) and \(e\) are independent. We assume that \(m(s,e)\) is continuous and strictly increasing in \(e\), that it exhibits CRS, so that \(m(as,ae) = am(s,e)\) for all \(a > 0\) and that there is a known point \(\{\bar{s}, \bar{e}, \bar{m}\}\), such that \(\bar{m} = m(\bar{s}, \bar{e})\). The intuition behind the identification argument is as follows: the observed correlation between \(s\) and \(m\) informs us on \(\partial m(s,e)/\partial s\); then, due to CRS, this derivative also delivers the derivative \(\partial m(s,e)/\partial e\); once these derivatives are known, the matching function is known and can be inverted to provide the number of exporters.

In particular, let \(F_{m|s}\) denote the distribution of matches conditional on ships, and \(F_e\) the distribution of exporters, \(e\). Then
frictions in different regions, we compute the average percentage of weekly “unrealized” matches; i.e.
\[
\left( \min\{s_i, e_i\} - m_i \right) / \min\{s_i, e_i\}.
\]
The results are plotted in the left panel of Figure 2 along with confidence intervals constructed from 200 bootstrap samples, and reveal that search frictions are heterogeneous over space and may be somewhat sizable, with up to 20% of potential matches “unrealized” weekly in regions like South and Central America and Europe. On average, 13.5% of potential matches are “unrealized”. 30

Moreover, as shown in the right panel of Figure 2, the estimated search frictions are positively correlated with the observed within-region price dispersion (0.47), another indicator of search frictions. We also find that frictions are negatively correlated with the Herfindahl Index of charterers (those reported in the Clarksons contract data) in a region (-0.31); this suggests that when the clientele is disperse, frictions are higher. Finally, when we estimate the matching function separately for Capesize (biggest size) and Handysize (smallest size) vessels, we find that for Capesize, where the market is thinner, search frictions are lower.

4.3 Model Estimation and Results

We make four alterations that render the model presented in 2 amenable to empirical analysis. First, we impose a specific pricing mechanism, Nash bargaining, with \( \gamma_i \) the bargaining coefficient of the ships in market \( i \). Second, we add randomness to the discrete choice problem for ships of where to ballast, by adding idiosyncratic shocks in (4), so that it becomes,

\[
U_i^s = \max_j V_{ij} + \sigma \epsilon_{ij}
\]  

(40)

at a given point \( \{s_t, e_t, m_t\} \) we have:

\[
F_{m|s=s_t} (m|s = s_t) = \Pr (m (s, e) \leq m_t | s = s_t) = \Pr (e \leq m^{-1} (s, m_t) | s = s_t) = \Pr (e \leq m^{-1} (s_t, m_t)) = F_e (e_t)
\]

This equation, along with the CRS assumption, allows us to recover the distribution \( F_e (e) \), for all \( e \): using the known point \( \{\bar{s}, \bar{e}, \bar{m}\} \) and letting \( a = e/\bar{e} \), for all \( e \),

\[
F_e (a\bar{e}) = F_{m|s=a\bar{s}} (m (a\bar{s}, a\bar{e}) | s = a\bar{s}) = F_{m|s=a\bar{s}} (a\bar{m}|s = a\bar{s})
\]

We use this and vary \( a \) to trace out \( \hat{F}_e (e) \), relying on a kernel density estimator for the conditional distribution \( \hat{F}_e (e) \), relying on a kernel density estimator for the conditional distribution \( \hat{F}_{m|s=a\bar{s}} (a\bar{m}|s = a\bar{s}) \).

Since it is unlikely that \( s \) and \( e \) are independent we employ an instrument. We use the weather shocks also employed in the search frictions test, Table 1.

30It is worth noting that this does not imply that in the absence of search frictions we would have 13.5% more matches, as we would need to take into account the optimal response of ships and exporters. This is simply a measure of the severity of search frictions in different regions.
where $\epsilon_{ij}$ are drawn i.i.d. from the Type I extreme value distribution with standard deviation $\sigma$. This implies that ships’ behavior is given by logit choice probabilities,

$$P_{ij} = \frac{\exp \left( \frac{V_{ij}}{\sigma} \right)}{\sum_l \exp \left( \frac{V_{il}}{\sigma} \right)} \quad (41)$$

Third, we consider the version of the model with $\beta < 1$. In Appendix E we demonstrate that our efficiency results hold in this empirical model with discounting and idiosyncratic shocks. Fourth, we also add randomness to the exporters’ problem (14), so that they solve the following discrete choice problem of whether and where to export,

$$\max_j \left\{ U_{ij}^e - \kappa_{ij} + \epsilon_{ij} \right\}$$

with $\epsilon_{ij}$ drawn i.i.d. from the Type I extreme value distribution; we normalize $U_{ii}^e - \kappa_{ii} = 0$. This leads to the exporter logit entry probabilities,

$$P_{ij}^e = \frac{\exp \left( U_{ij}^e - \kappa_{ij} \right)}{1 + \sum_{l \neq i} \exp \left( U_{il}^e - \kappa_{il} \right)} \quad (42)$$

for $j \neq i$ and $P_{ii}^e = 1 / \left( 1 + \sum_{l \neq i} \exp \left( U_{il}^e - \kappa_{il} \right) \right)$ is interpreted as the option of not exporting at all. We also assume for simplicity that $w_{ij}(q) = w_{ij}$ for all $i, j$. 

### Table

<table>
<thead>
<tr>
<th></th>
<th>Correlation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price dispersion</td>
<td>0.47</td>
</tr>
<tr>
<td>Charterer HHI</td>
<td>-0.31</td>
</tr>
</tbody>
</table>

**Figure 2:** Search Frictions. Average weekly share of unrealized matches, with confidence intervals from 200 bootstrap samples.
The main parameters of interest are: the matching functions \( m_i(s_i, e_i) \) for all \( i \), the ship travel and wait costs \( c^s_{ij}, c^w_i \) for all \( i,j \), as well as the standard deviation of the logit shocks \( \sigma \); the exporter valuations \( w_{ij} \), the exporter waiting costs \( c^e_i \), and entry costs \( \kappa_{ij} \) for all \( i,j \); and the bargaining coefficients \( \gamma_i \) for all \( i \). The available data consist of the matches \( m_i \) and ships \( s_i \) for all \( i \), the ship ballast choice probabilities \( P_{ij} \), for all \( i,j \), the average prices \( \tau_{ij} \) on all routes \( i,j \), the exporter entry probabilities \( P^e_{ij} \), for all \( i,j \) as well as total trade values by country pair (Comtrade).

We use the ship parameters \( \{c^s_{ij}, c^w_i, \sigma\} \) estimates from BKP, via a Nested Fixed Point Algorithm (Rust, 1987) from the observed choice probabilities (41). The estimation of the matching functions was discussed in Section 4.2 above.

**Exporter Parameters and Bargaining Coefficients** We are left with four sets of parameters: the exporter valuations \( w_{ij} \), the waiting costs \( c^e_i \), the bargaining coefficients \( \gamma_i \) for all \( i \); and the exporter entry costs \( \kappa_{ij} \), for all \( i,j \). Unlike BKP, we allow the bargaining coefficient to vary by region to allow for flexibility, given the importance of that parameter regarding the thin/thick market externalities. Moreover, we bring in additional data to obtain exporter valuations \( w_{ij} \) and as a result we are able to estimate the extra parameters capturing exporter wait costs \( c^e_i \).

The valuations \( w_{ij} \) are the revenues of exporters in \( i \) from selling their commodities to destination \( j \). We compute them using aggregate trade data from Comtrade, which reports product-level export values and quantities by country pair. We focus on bulk commodities and compute the average value of a cargo of commodities exported from each region \( i \) to each \( j \), which forms our direct estimate for \( w_{ij} \); details are provided in Appendix D.

Next, we turn to \( c^e_i \) and \( \gamma_i \), for all \( i \), which we estimate from observed shipping prices. Nash bargaining implies the surplus sharing condition,

\[
(1 - \gamma_i) (\tau_{ij} + V_{ij} - U_i) = \gamma_i \left[ w_{ij} - \tau_{ij} - U^e_{ij} \right]
\]

\(\text{31}\) In particular, at every guess of the parameters \( \{c^s_{ij}, c^w_i, \sigma\} \) for all \( i,j \), we employ a fixed point algorithm to solve for the ship value functions \( V_i, V_{ij}, U_i \), for all \( i,j \) from equations (3)(1), and (40), using the observed average prices for each route \( i,j \) and the observed meeting probability \( \lambda^s_i \) (which is set equal to the average \( m_i/s_i \)). We maximize over the parameters via Maximum Likelihood on the observed choice probabilities (41). See BKP for further details on identification, estimation, and results.
where if we substitute the exporter value $U_{ij}$ from its steady state value, $U_{ij} = (-c_i^e + \beta \lambda_i^e (w_{ij} - \tau_{ij})) / (1 - \beta (1 - \lambda_i^e))$, we obtain,

$$
\tau_{ij} = \frac{\gamma_i}{1 - \beta (1 - \gamma_i \lambda_i^e)} [\beta c_i^e + (1 - \beta) w_{ij}] + \frac{(1 - \gamma_i) (1 - \beta (1 - \lambda_i^e))}{1 - \beta (1 - \gamma_i \lambda_i^e)} (U_i - V_{ij}),
$$

In this equation, the only unknowns are $\gamma_i$ and $c_i^e$, for all $i$; indeed, note that $\lambda_i^e$ is known from the matching function (set equal to $m_i/e_i$); $U_i, V_{ij}$ are known once the ship cost parameters are known; $w_{ij}$ is obtained from Comtrade data as described above; and $\beta$ is calibrated to 0.995. We thus estimate $\gamma_i$ and $c_i^e$ via non-linear least squares. Identification results from variation over the regions $i, j$: intuitively, the identification of the bargaining coefficient $\gamma_i$ relies on the correlation of prices $\tau_{ij}$ and values $w_{ij}$ across destinations $j$, while the inventory cost $c_i^e$ matches the overall level of prices at origin $i$. To gain power, we restrict the inventory costs to be constant within continent.

Finally, the exporter entry costs $\kappa_{ij}$ are estimated from the observed entry probabilities $P_{ij}^e$, given by (42) following BKP:

$$
\ln P_{ij}^e - \ln P_{i0}^e = U_{ij}^e - \kappa_{ij} = \frac{-c_i^e + \beta \delta \lambda_i^e (w_{ij} - \tau_{ij})}{1 - \beta \delta (1 - \lambda_i^e)} - \kappa_{ij}
$$

where $\kappa_{ij}$ is the only unknown.\textsuperscript{32}

The results are presented in Table 7 in Appendix B. The exporter wait cost, $c_i^e$, is equal to about 3% of the exporters’ valuation, but there is substantial heterogeneity over space; the estimated costs are highest in Central and South America, as well as parts of Africa. These parameters capture inventory expenditures, delay costs, risks of damage or theft, etc. Consistent with this interpretation, we find that exporter wait costs are positively correlated with the recovered wait costs for ships (0.34), and are negatively correlated with the World Bank index of quality of port infrastructure (0.50). Finally, the estimates for the bargaining coefficients suggest that the exporters get a larger share of the surplus in almost all regions.

\textsuperscript{32}To recover $P_{ii}^e$, the share of the “outside good”, corresponding to the choice of not exporting, we use the total production of the relevant commodities for each region $i$. 

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5 Efficiency in Dry Bulk Shipping

In this section we present our main welfare results. We begin by checking whether the conditions of Theorem 2 hold in the case of bulk shipping; this is discussed in Section 5.1. In Section 5.2 we present our main welfare analysis and in Section 5.3 we discuss simple policy implementation.

5.1 Is Dry Bulk Shipping Efficient?

Efficiency requires that the following conditions are met: (i) the elasticity of the matching function with respect to each input must equal the corresponding bargaining coefficient (conditions (28) and (29)); (ii) the ship surplus must equalize across destinations. We test each of these conditions in the data.

Figure 3 examines whether the thin/thick market externalities are internalized. For each region, the left panel presents the histogram of the estimated matching function elasticity with respect to exporters, as well as the estimated bargaining coefficient $\gamma_i$ for each region $i$. In many markets, the two are not close on average, suggesting that the “Hosios conditions” for efficiency (28) and (29) are not satisfied. The table in the right panel of Figure 3 reports the t-statistic for the null hypothesis that the average elasticity of the matching function $\eta^s_i$ is equal to the ships’ bargaining coefficient $\gamma_i$ and shows that for several regions the data lead to reject the null. Thus, the market equilibrium is not efficient. Although the “knife-edge” nature of these conditions implies that this finding is not particularly surprising, it is worth noting that the difference between the elasticity and the bargaining coefficient is often large. Moreover, the ships’ bargaining coefficient tends to be higher than the matching function elasticity, suggesting that the planner would like to see a decrease in the share of the surplus accruing to the carrier.

Figure 4 tests whether the pooling externalities are internalized. For each region $i$, it plots the coefficient of variation of the ship’s surplus from traveling to all destinations $j \neq i$. When pooling externalities are internalized, this coefficient of variation should be equal to zero, since the ship is indifferent across destinations. Figure 4 demonstrates that this is not the case in bulk shipping. In all regions the coefficient of variation is significantly different from zero, and larger than 20%, while in several regions it is substantially higher.

We conclude that the market has not internalized neither the pooling externalities, nor the thin/thick market ones. We now turn to our welfare analysis.
5.2 Welfare Loss

We now come to our main welfare analysis. We begin by a comparison of, (i) the market equilibrium; (ii) the constrained efficient outcome we analyzed in Section 3 above; (iii) the first best, i.e., the efficient outcome in a world without search frictions, so that \( m = \min \{s, e\} \). To compute the constraint efficient outcome, we compute the equilibrium under the efficient prices given in (31) of Corollary 1.\(^{33}\) In terms of policy relevance, one can think of (ii) as what can be achieved by policy makers who are not able to affect the meeting process or the search environment. In contrast, (iii) loosely corresponds to a centralized market; one can think of it as a meeting platform, like Uber, which however does not exercise market power.\(^{34}\) This three-way comparison allows us to assess both the overall impact of frictions on welfare, as well as the impact of the two externalities under study.

The results are shown in Table (2). As reported in the first three columns, total welfare in the market equilibrium allocation is 6% lower than in the constrained efficient allocation and 14% lower than in the efficient allocation. Moreover, externalities coming from search frictions have a substantial impact on world trade, both in terms of value and volume. Indeed, trade volume is 13% higher under constrained efficiency and 36% higher under efficiency, while net trade value (i.e. \( w_{ij} - \kappa_{ij} \)) is 12% higher under constrained efficiency and 42% higher under efficiency. Moreover, ships would ballast 9% and 0.33% less under constrained efficiency and the first best respectively; this suggests that although the majority of ballast traveling is attributed to the natural imbalance in commodities rather than frictions, as expected, some wasteful traveling does exist in the market equilibrium. Finally, ships wait less under constrained and full efficiency (9% and 23% respectively).

These results relay an important message: under the optimal policy, the market is able to achieve about 44% of the first-best welfare gains. It is instructive to consider this in relation to the debate concerning the advantages of market centralization. Platforms that reduce search frictions between agents are emerging in a multitude of markets (naturally the most relevant examples to this paper are ride-sharing apps, like Uber or Lyft; financial and housing markets are other examples). These platforms potentially approach the first best solution considered in Table (2) through centralization; in fact this is how these platforms have

\(^{33}\)Alternatively we can impose the optimal tax/subsidy schedule derived in Corollary 2. The resulting allocation is the same.

\(^{34}\)Other work has indeed modeled platforms as the eradication of search frictions; e.g. Frechette et al. (2019); Buchholz (2018).
been modeled in the literature (e.g. Buchholz, 2018; Frechette et al., 2019). Nonetheless, these platforms
exert market power rather than act as social planners. Hence, the 14% welfare gain in the first-best
allocation can be thought of as just a crude upper bound for the benefit of these platforms. Table (2)
suggests that the policy-makers can, at the very least, achieve almost half of overall welfare gains through
taxes or subsidies, without resorting to some form of centralization. Additionally, centralization may be
infeasible in practice.

We now discuss the different role of the two externalities; in addition to the descriptive value of
this exercise, it will prove useful in providing intuition in the design of implementable policies when
the planner is not able to use all possible tax instruments \((h^s, h^e, h^q)\). In the third and fourth columns
of Table (2), we compute the welfare loss when only thin/thick market externalities or only pooling
externalities are internalized. To do so, we impose the relevant tax derived in Section 3.3, i.e., we impose
\(h^s, h^e\) to internalize the thin/thick market externalities and \(h^q\) to internalize the pooling externalities (see
Footnotes 15 and 16) and compute the equilibrium. The welfare gain is 3.3% (5.1%) when thin/thick
(pooling) market externalities are internalized suggesting that both externalities introduce substantial
distortions in the market equilibrium.

Table (2) also reveals that the two externalities have a qualitatively different impact on the economy;
this is illustrated by the change in the total trade volume and value. The thin/thick market externalities
have a large impact on the volume of trade, as they essentially distort the numbers of searching agents and
therefore the total number of matches formed. Indeed, as shown in Table (2), correcting the thin/thick
market externalities has a bigger impact on trade volume (which rises by 19%) than correcting both
externalities (in which case trade volume rises by 13%). In contrast, pooling externalities have a large
impact on trade value, as they distort the composition of exports. As shown in Table (2), correcting the
pooling externalities leads to a large increase in trade value (by 13%), as destinations with high social
value are subsidized (more on this below).

To provide further intuition on how the market fails to resolve the inefficiency, we analyze the impact of
each externality in turn. As described in Section 3.2, thin-thick market externalities relate to the efficient
entry of agents searching. Based on our estimates, the elasticity \(\eta^e\) of the matching function with respect
to exporters is large. Therefore, the entry of an additional exporter has a substantial positive externality
on matching rates. However, for most markets the exporters’ bargaining coefficient is lower than $\eta_i^e$ (see Figure 3). Therefore, transportation prices are too high to appropriately encourage the socially efficient entry of exporters. When thin/thick market externalities are internalized, this imbalance is corrected by lowering prices and increasing exporter’s entry, as shown in Figure 8 in the Appendix.\textsuperscript{35}

In addition to this direct effect, the final allocation is also determined by “general equilibrium” effects. The heterogeneous decline in prices affects ship behavior, as ships may avoid ballasting to regions with large price declines. As ship supply declines in those regions, prices rise and exporters stop entering, thus dampening the direct effect. This effect is most pronounced in the East Coast of North America and Northern Europe, which experience a large change in prices. Being large exporters, these regions heavily rely on ships ballasting there. Therefore, the dampening accruing from the change in ships ballast behavior is large.

Next, we turn to the pooling externalities. The optimal tax on trips that restores pooling externalities, $h_{ij}^q$, depends on the social surplus $\bar{\Delta}_{ij}$ (see equation (36)). A multitude of factors determine whether a route $i, j$ is associated to a high $\bar{\Delta}_{ij}$, such as the exporter valuation and waiting costs, distance, as well as the options for ships at the destination. In the right panel of Figure (5), we regress the optimal trip tax, $h_{ij}^q$, on some of these features. We see that the planner would like to subsidize exporters with higher value $w_{ij}$; this is not surprising, as the total export revenue is a crucial term in total welfare. The planner would like to tax distant destinations, as they are associated with high travel costs. Beyond this “direct”

\textsuperscript{35}To capture the direct effect only, we do not allow ships to reallocate.
benefit \( (w_{ij}) \) and cost \( (c^s_{ij}) \) however, the planner also values the attractiveness of a destination \( j \) for ships, which also contributes to social value. Attractive regions for ships may involve many customers, high value matches, low travel costs to other locations etc. The right panel of Figure (5) confirms this intuition, as the planner subsidizes destinations that are big exporters, implying that the ship can easily reload there. Instead, he taxes destinations that force the ship to ballast afterwards and/or to ballast somewhere far. This is best illustrated in the left panel of Figure (5), which shows the average import tax for each region (i.e., \( \sum_i q_{ij} h^q_{ij} / \sum_i q_{ij} \)). The planner subsidizes most heavily trips towards the West Coast of North America, as well Australia: these regions are high-value importers \( (w_{ij} \) is high when \( j \) corresponds to these regions) but at the same time, they promise high continuation values for ships that arrive there, as they are also big exporters. In other words, both the trip there is valuable, and the future ship value is high. In contrast, the planner taxes West Africa and India most heavily, as these are both low value importers and provide poor reloading options to ships.

5.3 Policy Implementation

Although the prices and optimal taxes that restore (constrained) efficiency have known expressions, they may not be feasible to implement in practice, either because the planner does not have access to all instruments; or because the expressions may be too complex or computationally challenging. As an example, the planner may not be able to set prices. Moreover, he may be able to tax trips, but not searching agents; indeed, it may be difficult to tax hailing passengers and searching exporters, or waiting taxis/ships. Finally, the matrix \( h^q \) may be very large, in which case the planner might prefer a simpler tax scheme.

In this section we consider simple policies that are designed to mimic the optimal taxes but may be more easily implementable. In particular, we consider the following taxes: (i) an origin-specific tax on matches which, can be interpreted as a flat tax on exports; (ii) a destination-specific tax on matches, which can be interpreted as a customs tax; (iii) a linear in distance tax, resembling the taxi price schedule;

Table 3 reports welfare gains under these tax schemes. The destination-specific tax works best, as it achieves a welfare gain of 2.8%. The origin-specific tax delivers only 0.9% welfare gains. This is consistent with our finding that pooling externalities seem to account for a larger portion of overall welfare gains.
Indeed, to restore the pooling externalities taxing differently across destinations is crucial. In addition, taxes that are a function of distance achieve no welfare gains. This suggests that the pricing scheme used in taxis is far from efficient and cannot alleviate either externality. This finding is not surprising given the optimal taxes derived in Section 3.3, that explicitly target the origin and the destination to correct the thin/thick and pooling externalities respectively. Indeed distance is not a good proxy for the social value of a trip.

<table>
<thead>
<tr>
<th>Optimal taxes</th>
<th>Export tax</th>
<th>Custom Tax</th>
<th>Distance</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h_{ij}^q$</td>
<td>$h_j^q$</td>
<td>$h_i^q$</td>
<td>$\alpha d_{ij}$</td>
</tr>
<tr>
<td>6.33%</td>
<td>2.82%</td>
<td>0.9%</td>
<td>0%</td>
</tr>
</tbody>
</table>

**Table 3:** Simple policy instruments. The table reports the impact on total welfare of three simple policy instruments that the social planners can use to approximate the optimal taxes. Namely, the second column reports the impact of a origin-specific tax on matches; the third reports the impact of a destination-specific tax on matches; the fourth reports the impact a linear in distance tax.
Figure 3: Comparison of the bargaining coefficient and the elasticity of the matching function with respect to exporters, estimated nonparametrically. The red vertical line is the average elasticity and the blue line is the estimated bargaining coefficient. The histogram corresponds to the estimated elasticity at different levels of exporters.
Figure 4: For each region $i$, we plot the coefficient of variation (mean over standard deviation) of ship surplus for all destinations $j \neq i$. When pooling externalities are internalized, the coefficient of variation should be zero.

<table>
<thead>
<tr>
<th>$\text{log}(h_{ij}^q)$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Exporters’ revenues on route $ij$ (log)</td>
<td>0.12**</td>
</tr>
<tr>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Trip duration (log)</td>
<td>−0.02</td>
</tr>
<tr>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>Total value of exports from $j$ (log)</td>
<td>0.03**</td>
</tr>
<tr>
<td>(0.01)</td>
<td></td>
</tr>
<tr>
<td>Probability ballast from $j$ (log)</td>
<td>−0.17**</td>
</tr>
<tr>
<td>(0.07)</td>
<td></td>
</tr>
<tr>
<td>Duration of ballast trip from $j$ (log)</td>
<td>−0.22**</td>
</tr>
<tr>
<td>(0.11)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5: Pooling Externalities. This figure shows the tax on matches which internalizes pooling externalities. The left panel plots the tax averaged across origins for a given destination, $\frac{\sum q_{ij} h_{ij}^q}{\sum q_{ij}}$. This can be interpreted as the average tax on imports for different regions. The right panel regresses the tax $h_{ij}^q$ on several features that affect the social surplus of a trip on route $ij$. 
6 Conclusion

This paper studies efficiency in decentralized transport markets, such as taxis, ships and trucks. In this setup, search frictions create two externalities: thin/thick market and pooling externalities. Because of the latter, we show that the well-known Hosios (1990) conditions are not sufficient to restore efficiency when there is one-sided heterogeneity. We derive explicit and intuitive conditions for efficiency, which lead naturally to the efficient pricing rules. Moreover, we derive the optimal taxes that restore efficiency for a social planner that cannot set prices. Then, using data from dry bulk shipping we demonstrate that search frictions are present and lead to a sizeable social loss. However, through optimal taxes/subsidies the market can achieve substantial welfare gains. In fact, these policies achieve 44% of the first best welfare gains, suggesting that they may constitute a good alternative to centralized platforms. Finally, we use intuition obtained from analyzing the nature of the two externalities to design simple policies that mimic the optimal taxes.
Appendix

A  Proofs

A.1 Preliminaries: limit equilibrium outcomes and associated dual variables

In this section we show that every limit equilibrium outcome can be associated with a set of dual variables corresponding to the no-discounting limits of the agents’ value function. These variables will be instrumental in the proofs of Theorems 1 and 2 below.

Let \((s, E, q, b, \tau)\) be a limit equilibrium outcome and \((s^n, E^n, q^n, b^n, \tau^n, \beta^n)_{n \geq 0}\) be a corresponding sequence of equilibrium outcomes and discount factors. For each \(n\), let \(V^n_i, U^n_i, \Delta^{s,n}_{ij}\) and \(U^{e,n}_i, \Delta^{e,n}_{ij}\) be the corresponding value functions and meeting surpluses for carriers and customers, respectively. Fix an arbitrary reference location \(i^*\).

Lemma 1. The sequences \((V^n_i - V^n_{i^*})_{n \geq 0}\), \((U^n_i - V^n_{i^*})_{n \geq 0}\), \(((1 - \beta^n) V^n_i)_{n \geq 0}\) and \((\Delta^{s,n}_{ij})_{n \geq 0}\) are bounded for every \(i, j\). \(((\Delta^{e,n}_{ij})_{n \geq 0}\) is also bounded provided that \(\lambda^e_i > 0\).

Proof. Taking into account (1) and (3), we can rewrite the carrier’s surplus \(\Delta^{s,n}_{ij}\) defined in (2) as

\[
\Delta^{s,n}_{ij} = \max \{ \tau^n_{ij} + V^n_{ij} - \max_j V^n_{ij}, 0 \}.
\]

Hence \(\Delta^{s,n}_{ij}\) is bounded above by \(\tau^n_{ij}\) and below by zero. Since \(\tau^n\) converges to \(\tau\), it follows that \(\Delta^{s,n}_{ij}\) is bounded. \((1 - \beta^n) V^n_i\) is bounded as an average of bounded prices and the finite set of all possible per-period search and traveling costs. \(V^n_i - V^n_{i^*}\) is bounded below, since we have

\[
V^n_i - V^n_{i^*} \geq -c^w_i + \lambda^{s,n}_i \sum_{j \neq i} G^n_{ij} \Delta^{s,n}_{ij} + \frac{\beta d^*_i V^n_i - c^*_{ii}}{1 - \beta^n (1 - d^*_i)} - V^n_{i^*}
\]

and all sequences on the right-hand-side are bounded. Reversing the roles of \(i\) and \(i^*\) it follows that \(V^n_i - V^n_{i^*}\) is bounded above as well.

Finally, if \(\lambda^e_i > 0\), then \(\lambda^{e,n}_i > 0\) for \(n\) large enough. Hence, based on Equations (11) and (10), for \(n\)

\[\text{In the steady state, equation (1) becomes: } V_{ij} = \left(-c^s_{ij} + \beta d_{ij} V_j\right) / (1 - \beta (1 - d_{ij})).\]
large enough we have,
\[
\frac{c_i^e + \lambda^{e,n}_i (w_{ij} (q^n) - \tau_{ij}^n)}{1 - (1 - \lambda^{e,n}_i) \beta^n} \leq U_{ij}^{e,n} \leq k_{ij}.
\]
Since the left-hand-side converges, \(U_{ij}^{e,n}\) is bounded. Finally, by (10), this implies that \(\Delta_{ij}^{e,n}\) is bounded as well.

Given Lemma 1, there exists a sequence \((n_k)_{k \geq 0} \subseteq \mathbb{N}\) such that we can define the limits
\[
\phi_i = \lim_{k \to \infty} V_i^{n_k} - V_i^{n_k*},
\]
\[
\psi_i = \lim_{k \to \infty} U_i^{n_k} - V_i^{n_k*},
\]
\[
v = \lim_{k \to \infty} (1 - \beta^{n_k}) V_i^{n_k*},
\]
\[
\bar{\Delta}_s^{i} = \lim_{k \to \infty} \Delta_s^{s,n_k}. 
\]
If \(\lambda_i^e > 0\) then we can also define the limit
\[
\bar{\Delta}_e^{i} = \lim_{k \to \infty} \Delta_e^{e,n_k}
\]
only otherwise we simply define
\[
\bar{\Delta}_e^{i} = \max \{w_{ij} (q) - \tau_{ij} - k_{ij}, 0\}.
\]
Note that for every \(i\) and \(i^*\) it holds that
\[
\lim_{k \to \infty} (1 - \beta^{n_k}) V_i^{n_k} = \lim_{k \to \infty} (1 - \beta^{n_k}) (V_i^{n_k} - V_i^{n_k*}) + \lim_{k \to \infty} (1 - \beta^{n_k}) V_i^{n_k*} = v.
\]

**Definition 5.** \((\phi, \psi, v, \Delta_s, \Delta_e)\) is a tuple of equilibrium dual variables associated with the limit equilibrium outcome \((s, E, q, b, \tau)\).

**Lemma 2.** Let \((s, E, q, b, \tau)\) be a limit equilibrium outcome and \((\phi, \psi, v, \Delta_s, \Delta_e)\) be a tuple of dual variables associated with it. Then the following conditions hold for every \(i, j\):
\[ \psi_i \geq \phi_j - \frac{c_{ij}^s}{d_{ij}} - \frac{v}{d_{ij}} \text{ with equality if } b_{ij} > 0 \]  \( \text{(43)} \)

\[ \bar{\Delta}_{ij}^s \geq 0 \text{ with equality if } q_{ij} < s_i \lambda_i^s G_{ij} \]  \( \text{(44)} \)

\[ \phi_i \geq -c_i^w + \lambda_i^s \sum_{j \neq i} G_{ij} \bar{\Delta}_{ij}^s + \psi_i \text{ with equality if } s_i > 0 \]  \( \text{(45)} \)

\[ v \geq 0 \text{ with equality if } \sum_{ij} q_{ij} + \frac{b_{ij}}{d_{ij}} < S \]  \( \text{(46)} \)

\[ \bar{\Delta}_{ij}^e \geq 0 \text{ with equality if } q_{ij} < \lambda_i^e E_{ij} \]  \( \text{(47)} \)

\[ -c_i^e + \lambda_i^e \bar{\Delta}_{ij}^e \leq 0 \text{ with equality if } E_{ij} > 0 \]  \( \text{(48)} \)

\[ \bar{\Delta}_{ij}^s \geq \tau_{ij} + \phi_j - \psi_i - \frac{v}{d_{ij}} \text{ with equality if } q_{ij} > 0 \]  \( \text{(49)} \)

\[ \bar{\Delta}_{ij}^e \geq w_{ij} (q) - k_{ij} - \tau_{ij} \text{ with equality if } q_{ij} > 0. \]  \( \text{(50)} \)

**Proof.** The reader can verify this by taking the no-discounting limits of the equilibrium conditions (2)-(14). For example, the equilibrium conditions (4) and (8) can be written as (taking into account that in steady state

\[ V_{ij} = \left( \beta d_{ij} V_j - c_{ij}^s \right) / (1 - \beta (1 - d_{ij})) \]

\[ U_i^n > \frac{\beta^n d_{ij} V_j^n - c_{ij}^s}{1 - (1 - d_{ij}) \beta^n} \text{ with equality if } b_{ij}^n > 0. \]
Subtracting $V_{i}^{s,n}$ from both sides we obtain,

$$
U_{i}^{n} - V_{i}^{s,n} > \frac{\beta^{n}d_{ij}(V_{j}^{n} - V_{i}^{s,n})}{1 - (1 - d_{ij})\beta^{n}} - \frac{c_{ij}^{s}}{1 - (1 - d_{ij})\beta^{n}} - \frac{1 - \beta^{n}V_{i}^{s,n}}{1 - (1 - d_{ij})\beta^{n}}
$$

with equality if $b_{ij}^{n} > 0$.

Taking limits of both sides as $n \to \infty$ yields Condition (43).

As another example, notice that the equilibrium conditions (2), (6) and (7) are equivalent to,

$$
\Delta_{ij}^{s,n} \geq 0 \text{ with equality if } q_{ij}^{n} < s_{i}^{n}A_{ij}^{s,n}G_{ij}^{n}
$$

and

$$
\Delta_{ij}^{s,n} \geq \tau_{ij}^{n} + V_{ij}^{n} - U_{i}^{n} \text{ with equality if } q_{ij}^{n} > 0.
$$

Taking the limit of the first one gives Condition (44). The second condition can be written as,

$$
\Delta_{ij}^{s,n} \geq \tau_{ij}^{n} + V_{ij}^{n} - V_{i}^{s,n} - (U_{i}^{n} - V_{i}^{s,n}) \text{ with equality if } q_{ij}^{n} > 0.
$$

Taking the limits of both sides gives Condition (49).

Analogous arguments establish the remaining conditions. More precisely, Condition (45) is a consequence of the equilibrium conditions (3) and (5); Condition (46) is a consequence of the equilibrium conditions (3) and (9); Condition (47) results from the equilibrium conditions (10) and (12); Condition (48) is obtained from the equilibrium conditions (11) and (14); and finally, Condition (50) is obtained from the equilibrium conditions (10), (13) and (14). Finally notice that combining conditions (49) and (50) we obtain,

$$
\tilde{\Delta}_{ij}^{s} + \tilde{\Delta}^{e}_{ij} \geq w_{ij}(q) - k_{ij} + \phi_{j} - \psi_{i} - \frac{\nu}{d_{ij}} \text{ with equality if } q_{ij} > 0.
$$

(51)
A.2 Proof of Theorem 1

Consider Problem 20, and let $\phi, \psi, v, \Delta^s$ and $\Delta^e$ be the dual variables associated with constraints (15), (16), (17), (?), and (??), respectively. Since Problem 20 is concave, convex duality implies that $(s, E, q, b, \phi, \psi, v, \Delta^s, \Delta^e)$ is an optimal dual pair of Problem 20 (that is, $(s, E, q, b)$ is an optimal solution of Problem ?? and $(\phi, \psi, v, \Delta^s, \Delta^e)$ are the multipliers associated with the constraints) if and only if it satisfies the Karush-Kuhn-Tucker conditions (see for example Bertsekas, 2009, Prop. 5.2.2 pg. 167 ). The reader can verify that these can be written as (43)-(48)+(51). Hence the result follows by taking $(\phi, \psi, v, \Delta^s, \Delta^e)$ to be a tuple of equilibrium dual variables associated with $s, E, q, b$ and applying Lemma 2.

A.3 Proof of Theorem 2

Lemma 3. For each $(s, e, G) \geq 0$ such that $\sum_i s_i \leq S$ and $\sum_j G_{ij} = 1 \forall i$, consider the set $M(s, e, G)$ of all pairs $(q, b) \geq 0$ satisfying constraints (15)-(17) given $s, e, G$. The multi-valued map $(s, e, G) \rightarrow M(s, e, G)$ satisfies

$$(1 - \lambda) M(u) + \lambda M(u') \subseteq M((1 - \lambda) u + \lambda u')$$

for every $u = (s, e, G)$ and $u' = (s', e', G')$.

Proof. It follows from the concavity of the meeting function. \hfill \Box

Next we show that:

Lemma 4. Let $f(x, u)$ be convex in $(x, u)$ and $M(u)$ satisfies the convexity property of the previous lemma. Then the function:

$$g(u) = \inf_{x \in M(u)} f(x, u)$$

is convex.

Proof. The above is a well-known result when $M(u)$ is convex and does not vary with $u$ (see for instance Boyd et al., 2004). We adapt the proof to this case. Let $u_1, u_2$ and $\lambda \in [0, 1]$ and $\epsilon > 0$. Then there exist
$x_1 \in M(u_1)$ and $x_2 \in M(u_2)$ such that

\[ f(x_1, u_1) \leq g(u_1) + \epsilon \]

\[ f(x_2, u_2) \leq g(u_2) + \epsilon \]

Then,

\[ g(\lambda u_1 + (1 - \lambda) u_2) = \inf_{x \in M(\lambda u_1 + (1 - \lambda) u_2)} f(x, \lambda u_1 + (1 - \lambda) u_2) \]

Since $\lambda x_1 + (1 - \lambda) x_2 \in \lambda M(u_1) + (1 - \lambda) M(u_2) \subseteq M(\lambda u_1 + (1 - \lambda) u_2)$, we have,

\[ \inf_{x \in M(\lambda u_1 + (1 - \lambda) u_2)} f(x, \lambda u_1 + (1 - \lambda) u_2) \leq f(\lambda x_1 + (1 - \lambda) x_2, \lambda u_1 + (1 - \lambda) u_2) \]

Since $f(\cdot)$ is convex in $(x, u)$ we have,

\[ g(\lambda u_1 + (1 - \lambda) u_2) \leq f(\lambda x_1 + (1 - \lambda) x_2, \lambda u_1 + (1 - \lambda) u_2) \]

\[ \leq \lambda f(x_1, u_1) + (1 - \lambda) f(x_2, u_2) \]

\[ \leq \lambda g(u_1) + (1 - \lambda) g(u_2) + \epsilon \]

Since this is true for all $\epsilon$, convexity is established.

Applying this lemma to the function $-V^p(s, e, G)$, defined in (??), we obtain that $V^p(s, e, G)$ is concave. Hence, it is differentiable almost everywhere in its domain. Denote by $\partial V^p(s, e, G)$ the supergradient of $V^p$ at a search allocation $s, e, G$, that is, the set of all vectors

\[ y = (y(s_i)_{i \in I}, y(e_i)_{i \in I}, y(G_{ij}))_{i, j \in I} \in \mathbb{R}^I \times \mathbb{R}^I \times \mathbb{R}^{I \times I} \]

such that for every search allocation $s', e', G'$:

\[ V^p(s', e', G') - V^p(s, e, G) \leq \sum_i y(s_i) (s'_i - s_i) + \sum_i y(e_i) (e'_i - e_i) + \sum_{ij} y(G_{ij}) (G'_{ij} - G_{ij}) \]

Similarly, for every pair $i, j$, we denote by $\partial_{s_i} V^p(s, e, G)$, $\partial_{e_i} V^p(s, e, G)$ and $\partial_{G_{ij}} V^p(s, e, G)$ the super-
gradients of \( V^p \) at \( s, e, G \) with respect to \( s_i, e_i \) and \( G_{ij} \), respectively.

**Lemma 5.** Take a limit equilibrium allocation \((s, e, G, q, b)\), and let \((\phi, \psi, \nu, \bar{\Delta}^s, \bar{\Delta}^e)\) be a tuple of equilibrium dual variables associated with it. For every \( i,j \) define

\[
y(s_i) = -\phi_i - c_i^w + \frac{dm_i(s_i, e_i)}{ds_i} \sum_j G_{ij} (\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e) + \psi_i
\]

\[
y(e_i) = -c_i^e + \frac{dm_i(s_i, e_i)}{de_i} \sum_j G_{ij} (\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e)
\]

\[
y(G_{ij}) = m_i(s_i, e_i) (\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e).
\]

Then \( y \in \partial V^p (s, e, G) \).

**Proof.** Consider Problem 23 defining \( V^p (s, e, G) \). Its Lagrangian can be written as

\[
L (q', b', \psi', \phi', \bar{\Delta}', \nu'| s, e, G) = W(q') + \sum_{ij} (q'_{ij} + b'_{ij}) \left( -\frac{c_{ij}^s}{d_{ij}} + \psi'_{ij} - \frac{\nu}{d_{ij}} \right) - \sum_{ij} q'_{ij} (\bar{\Delta}'_{ij} + k_{ij})
\]

\[
- \sum_i e_i c_i^e - \sum_i s_i (\phi'_i - \psi'_i + c_i^w) + \sum_i m_i(s_i, e_i) \sum_j G_{ij} \bar{\Delta}'_{ij} + S\nu'
\]

and the Karush-Kuhn-Tucker conditions as

\[
\psi_i \geq \phi_j - \frac{c_{ij}^s}{d_{ij}} - \frac{\nu}{d_{ij}} \text{ with equality if } b_{ij} > 0
\]

\[
\bar{\Delta}_{ij} \geq 0 \text{ with equality if } q_{ij} < m_i(s_i, e_i) G_{ij}
\]

\[
\bar{\Delta}_{ij} \geq \phi_j - \frac{c_{ij}^e}{d_{ij}} - \frac{\nu}{d_{ij}} \text{ with equality if } q_{ij} > 0
\]

\[
\nu \geq 0 \text{ with equality if } \sum_{ij} \frac{q_{ij} + b_{ij}}{d_{ij}} < S
\]

which are equivalent to the set of Conditions (43), (44)/(47), (51) and (46), respectively, taking \( \bar{\Delta}_{ij} = \bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e \). Since the problem is concave, the K-K-T conditions are necessary and sufficient for optimality. Hence letting \((\phi, \psi, \nu, \bar{\Delta}^s, \bar{\Delta}^e)\) be a tuple of equilibrium dual variables associated with \((s, e, G, q, b)\), it follows that \((q, b, \psi, \phi, \bar{\Delta}^s + \bar{\Delta}^e, \nu)\) is an optimal dual pair for Problem ??. From the Theorem, it follows
that \((q, b)\) is the unique optimal solution of Problem \ref{prob:opt}. Hence the result follows from Theorem of Marimon and Werner, 2015.

We now proceed with the proof of the main result. By the previous results, Problem \ref{prob:opt} is concave, hence optimality is characterized by the K-K-T conditions. Recall that we are assuming that \(s\) and \(e\) are in the interior of the feasible set (\(e_i, s_i > 0\) for each \(i\) and \(\sum_i s_i < S\)). Hence conditions (25) and (26) are equivalent to the first order conditions,

\[
0 \in \partial s_i V^p(s, e, G) \quad \forall i \quad \text{and} \quad 0 \in \partial e_i V^p(s, e, G) \quad \forall i
\]

respectively. Denoting by \(\nu_{ij}\) and \(\mu_i\) the multipliers associated with the constraints \(G_{ij} \geq 0\) and \(\sum_j G_{ij} = 1\), condition (27) is equivalent to

\[
0 - \nu_{ij} - \mu_i \in \partial G_{ij} V^p(s, e, G)
\]

for some \(\mu, \nu \in \mathbb{R}^I \times \mathbb{R}^I_{+} \times I\) such that \(\nu_{ij} G_{ij} = 0\). It follows from the previous Lemma that conditions

\[
\forall i : y(s_i) = 0 \quad \forall i
\]

\[
\forall i : y(e_i) = 0 \quad \forall i
\]

\[
\forall ij : y(G_{ij}) + \mu_i \leq 0 \quad \text{with equality if} \quad G_{ij} > 0
\]

are sufficient for (25), (26) and (27), respectively, and they are necessary whenever \(V^p(s, e, G)\) is differentiable. Comparing with condition (45), (52) is equivalent to

\[
\forall i : -c_i^u + \frac{dm_i (s_i, e_i)}{ds_i} \sum_j G_{ij} \left( \tilde{\Delta}_{ij}^s + \tilde{\Delta}_{ij}^e \right) + \psi_i = \phi_i = -c_i^w + \lambda_i^s \sum_j G_{ij} \tilde{\Delta}_{ij}^s + \psi_i
\]

\[
\Leftrightarrow \eta_i^s \sum_j G_{ij} \left( \tilde{\Delta}_{ij}^s + \tilde{\Delta}_{ij}^e \right) = \sum_j G_{ij} \tilde{\Delta}_{ij}^s,
\]
Similarly, comparing with condition (48), (53) is equivalent to
\[
\forall i : -c_i^e + \sum_j m_i(s_i, e_i) \frac{d}{de_i} G_{ij} \left( \bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e \right) = 0 = -c_i^e + \lambda_i^e \sum_j G_{ij} \Delta_{ij}^e
\]
\[
\iff \sum_j G_{ij} \bar{\Delta}_{ij}^e = \eta_i^e \sum_j G_{ij} \left( \bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e \right).
\]
Condition (54) requires
\[
\bar{\Delta}_{ij}^s + \bar{\Delta}_{ij}^e \leq -\frac{\mu_i}{m_i(s_i, e_i)} \text{ with equality if } G_{ij} > 0. \tag{55}
\]
To see that this is equivalent to Condition (iii) in the statement, we prove that at a limit equilibrium outcome such that \(e_i > 0\) we have \(\bar{\Delta}_{ij}^e = c_i^e/\lambda_i^e\) for every \(j \neq i\), a slightly stronger version of condition (48) stated in Lemma 2. To see this, let \((s^n, E^n, q^n, b^n, \tau^n, \beta^n)_{n \geq 0}\) be a sequence of equilibrium outcomes and discount factors corresponding to the limit equilibrium outcome \((s, E, q, b, \tau)\). Since \(E^n \to E\) we must have \(e_i^n > 0\) for \(n\) large enough, hence conditions (10), (11) and (14) imply that
\[
\frac{1 - \beta^n}{1 - (1 - \lambda_i^{e,n}) \beta^n} \left[ -c_i^e + \lambda_i^{e,n} \left( w_{ij}(q^n) - \tau^n_{ij} \right) \right] \leq -c_i^e + \lambda_i^{e,n} \Delta_{ij}^{e,n} \leq (1 - \beta^n) k_{ij}
\]
for \(n\) large enough, where \(\lambda_i^{e,n}\) and \(\Delta_{ij}^{e,n}\) are the customers’ meeting probabilities and surpluses associated with the outcome \((s^n, E^n, q^n, b^n, \tau^n)\). Since both the first and the last element of this chain of inequalities converge to zero as \(\beta^n \to 1\), in the limit we have \(-c_i^e + \lambda_i^e \bar{\Delta}_{ij}^e = 0\). Given this, condition (55) is equivalent to
\[
\bar{\Delta}_{ij}^s \leq -\frac{\mu_i}{m_i(s_i, e_i)} - \frac{c_i^e}{\lambda_i^e} \text{ with equality if } G_{ij} > 0
\]
which is equivalent to Condition (iii) in the statement. This completes the proof of Theorem 2.
A.4 Proof of Corollary 1

Suppose that \((s,e,G,q,b,\tau)\) is efficient. Conditions (i) and (ii) of Theorem 2 imply that \(\eta_i^s = 1 - \eta_i^e \equiv \gamma_i\) for all \(i\). For every \(i,j\) such that \(G_{ij} > 0\), Conditions (i) and (iii) of Theorem 2 imply

\[
\bar{\Delta}_{ij}^s = \gamma_i \sum_j G_{ij} \bar{\Delta}_{ij}.
\]

By (50) we have \(\bar{\Delta}_{ij}^e = w_{ij}(q) - k_{ij} - \tau_{ij}\). Substituting \(\bar{\Delta}_{ij} = \bar{\Delta}_{ij} = \bar{\Delta}_{ij} = w_{ij}(q) + k_{ij} + \tau_{ij}\) yields condition (31).

A.5 Proof of Corollary 2

We consider total welfare

\[
\max_{s,E,q,b \geq 0} W(q) - \sum_{ij} q_{ij} \left(k_{ij} + h_{ij}^q\right) - \sum_{ij} (q_{ij} + b_{ij}) \frac{c_{ij}}{d_{ij}} - \sum_i s_i (c_i^w - h_i^q) - \sum_i e_i (c_i^e - h_i^e) + \sum_{ij} q_{ij} h_{ij}^q + \sum_i (e_i h_i^e + s_i h_i^s) \quad (56)
\]

s.t. feasibility constraints (15)-(18)

corresponding the sum of of the agents’ utility and government revenues. In this context, proceeding as in the proof of Theorem 2, one can show that

\[\text{In particular, we proceed by showing that the equilibrium conditions with taxes!!! are a slight modification of the ones without:}\]

\[
w_{ij}(q) - \frac{c_{ij}}{d_{ij}} - k_{ij} + \phi_j - \psi_i - h_{ij}^q - \Delta_{ij}^q - \Delta_{ij}^s - \frac{\nu}{d_{ij}} \leq 0
\]

with equality if \(q_{ij} > 0\)

\[-\frac{c_{ij}}{d_{ij}} + \phi_j - \psi_i - \frac{\nu}{d_{ij}} \leq 0
\]

with equality if \(b_{ij} > 0\)

\[\Delta_{ij}^q, \Delta_{ij}^s \geq 0 \text{ with equality if } q_{ij} \leq m_i(s_i, e_i) G_{ij}\]

\[\nu \geq 0 \text{ with equality if } \sum_{ij} \frac{q_{ij} + b_{ij}}{d_{ij}} < S\]

\[\phi_i = -c_i^w - h_i^q + \psi_i + \lambda_i \sum_j G_{ij} \Delta_{ij}^s\]
• Thin/thick market externalities are internalized if and only if for all $i$,

$$\sum_j G_{ij} \Delta_{ij}^s = \eta_i^s \sum_j G_{ij} \Delta_{ij} + \frac{h_i^s}{\lambda_i^s}$$

and

$$\sum_j G_{ij} \Delta_{ij}^e = \eta_i^e \sum_j G_{ij} \left( h_{ij} + \Delta_{ij}^e + \Delta_{ij}^s \right) + \frac{h_i^e}{\lambda_i^e}$$

• Pooling externalities are internalized if and only if for all $i, j$,

$$\Delta_{ij}^s + h_{ij}^q \leq L_i$$

with equality if $G_{ij} > 0$.

Using the definition $\Delta_{ij} = \Delta_{ij}^e + \Delta_{ij}^s + h_{ij}^q$ (the total surplus includes the planner’s revenue, but notice that $\Delta_{ij}$ does not depend on $h_{ij}^q$ since either the carrier or the customer pays for it and thus either $\Delta_{ij}^s$ or $\Delta_{ij}^e$ includes $-h_{ij}^q$) and the Nash bargaining condition $(1 - \gamma_i) \Delta_{ij}^s = \gamma_i \Delta_{ij}^e$ we obtain the following:

$$\lambda_i^s \sum_j G_{ij} \Delta_{ij}^s = \lambda_i^s \eta_i^s \sum_j G_{ij} \Delta_{ij} + h_i^s$$

$$\iff \lambda_i^s \gamma_i \sum_j G_{ij} \left( \Delta_{ij}^e + \Delta_{ij}^s \right) = \lambda_i^s \eta_i^s \sum_j G_{ij} \Delta_{ij} + h_i^s$$

$$\iff h_i^s + \lambda_i^s \gamma_i \sum_j G_{ij} h_{ij}^q = \lambda_i^s (\gamma_i - \eta_i^s) \sum_j G_{ij} \Delta_{ij}$$

and similarly for the customer. {These conditions are equivalent to (34) and (35).}

On the other hand, the conditions for different externalities to be internalized are unchanged. Comparing the efficiency conditions with the new equilibrium conditions as in the proof of Theorem 2 we get the following.
Moreover, we have that

\[ \Delta^q_{ij} + h^q_{ij} \leq L_i \text{ with equality if } G_{ij} > 0 \]

\[ \iff \gamma_i \left( \Delta_{ij} - h^q_{ij} \right) + h^q_{ij} \leq L_i \text{ with equality if } G_{ij} > 0 \]

\[ \iff (1 - \gamma_i) h^q_{ij} \leq (1 - \gamma_i) \Delta_{ij} - \Delta_{ij} + L_i \text{ with equality if } G_{ij} > 0 \]

\[ (1 - \gamma_i) h^q_{ij} \leq (1 - \gamma_i) \Delta_{ij} + \sum_j G_{ij} \Delta_{ij} - \Delta_{ij} + L_i \text{ with equality if } G_{ij} > 0. \]

Finally, if (34)-(36) hold jointly, then averaging (36) by \( G_i \) and comparing with (34) we obtain

\[ L_i = -\eta^e_i \sum_j G_{ij} \Delta_{ij} - \frac{h^e_i}{\lambda^e_i}. \]

Conversely it is easy to see that if (36) and (35) hold jointly with \( L_i = -\eta^e_i \sum_j G_{ij} \Delta_{ij} - \frac{h^e_i}{\lambda^e_i} \) then (34) holds as well.

### A.6 Efficiency when the planner can charge different prices

In this section, we generalize our main theorem, Theorem 2, to allow for different prices paid by the customer and received by the carrier. This extension is important because it demonstrates that a necessary condition for efficiency is that the matching function is constant returns to scale; nonetheless, efficiency can be restored if the planner is not constrained in setting just one price.

We allow the planner to charge a price to customers, \( \tau^e \), that is different from the prices payed to carriers, \( \tau^s \). We extend the definition of equilibrium to accommodate for different prices in a straightforward manner: \( (s, e, G, q, b, \tau^e, \tau^s) \) is an equilibrium outcome, if carriers behave optimally given \( \tau^s, \lambda^s \) and \( G \); customers behave optimally given \( \tau^e \) and \( \lambda^e \); the feasibility constraints are satisfied; and \( \lambda^s, \lambda^e \) and \( G \) are consistent with the allocation. The definition of efficiency remains the same, except that we include the planner’s revenue in the welfare.\(^{38}\)

**Corollary 3.** Let \( (s, e, G, q, b, \tau^e, \tau^s) \) be a limit equilibrium outcome. Then:

\(^{38}\)This implies that the social surplus is now defined by \( \Delta_{ij} = \Delta^s_{ij} + \Delta^e_{ij} + \tau^e_{ij} - \tau^s_{ij} \). As before, \( \Delta_{ij} \) does not depend on the prices directly (recall that \( \Delta^s_{ij} \) includes the term +\( \tau^s_{ij} \), while \( \Delta^e_{ij} \) includes -\( \tau^e_{ij} \)).
(i) Thin/thick market externalities are internalized if and only if for all $i$,

\[
\sum_{ij} G_{ij} (w_{ij} (q) - k_{ij}) - \sum_{ij} G_{ij} \tau_{ij}^c = \eta_i^e \sum_j G_{ij} \Delta_{ij}
\]  

(57)

and

\[
\sum_{ij} G_{ij} (\tau_{ij}^e - \tau_{ij}^s) = (1 - \eta_i^e - \eta_i^s) \sum_j G_{ij} \Delta_{ij}.
\]  

(58)

(ii) Pooling externalities are internalized if and only if for every $i, j$ we have

\[
w_{ij} (q) - k_{ij} - \tau_{ij}^e \leq L_i + \sum_j G_{ij} \Delta_{ij} - \Delta_{ij}
\]  

(59)

if $G_{ij} > 0$.

(iii) All externalities are internalized if and only if 58) and 59 hold with

\[
L_i = -\eta_i^e \sum_j G_{ij} \Delta_{ij}.
\]

Proof. Proceeding as in the proof of Theorem 2, one can show that

\[\text{In particular, we proceed by showing that the equilibrium conditions with a price wedge are a slight modification of the ones without:}\]

\[
w_{ij} (q) - \frac{c_{ij}}{d_{ij}} - k_{ij} + \phi_j - \psi_i - \frac{\nu}{d_{ij}} - \Delta_{ij}^c - \Delta_{ij}^s - (\tau_{ij}^d - \tau_{ij}^s) \leq 0
\]

with equality if $q_{ij} > 0$

\[
-\frac{c_{ij}^w}{d_{ij}} + \phi_j - \psi_i - \frac{\nu}{d_{ij}} \leq 0
\]

with equality if $b_{ij} > 0$

\[
\phi_i = -c_i^w + \psi_i + \lambda_i \sum_j G_{ij} \Delta_{ij}^s
\]

\[-c_{ij}^w + \lambda_i \Delta_{ij}^s \leq 0 \text{ with equality if } E_{ij} > 0\]

\[
w_{ij} (q) - \tau_{ij}^c - k_{ij} - \Delta_{ij}^c \leq 0
\]

with equality if $q_{ij} > 0$

\[
\Delta_{ij}^c, \Delta_{ij}^s \geq 0 \text{ with equality if } q_{ij} < m_i (s_i, e_i) G_{ij}.
\]
Thin/thick market externalities are internalized if and only if for all $i$,

$$\sum_j G_{ij} \Delta_{ij}^s = \eta_i^s \sum_j G_{ij} \Delta_{ij}$$

and

$$\sum_j G_{ij} \Delta_{ij}^e = \eta_i^e \sum_j G_{ij} \Delta_{ij}$$

Pooling externalities are internalized if and only if for all $i, j$,

$$\Delta_{ij} - \Delta_{ij}^e \leq L_i$$

with equality if $G_{ij} > 0$.

Using the definition $\Delta_{ij} = \Delta_{ij}^e + \Delta_{ij}^s + \tau_{ij}^e - \tau_{ij}^s$ and the equilibrium condition

$$w_{ij}(q) - \tau_{ij}^e - k_{ij} - \Delta_{ij}^e \leq 0$$

with equality if $G_{ij} > 0$

we obtain

$$\sum_j G_{ij} \Delta_{ij}^e = \sum_j G_{ij} \left( \Delta_{ij} - \Delta_{ij}^e + \tau_{ij}^e - \tau_{ij}^s \right) = \sum_j G_{ij} \left( \Delta_{ij} + k_{ij} - w_{ij}(q) + \tau_{ij}^s \right)$$

and hence

$$\sum_j G_{ij} \Delta_{ij}^e = \eta_i^e \sum_j G_{ij} \Delta_{ij} \iff \sum_j G_{ij} \tau_{ij}^e = \sum_j G_{ij} (w_{ij}(q) - k_{ij}) - (1 - \eta_i^s) \sum_j G_{ij} \Delta_{ij}.$$

Moreover,

$$\sum_j G_{ij} \Delta_{ij}^e = \sum_j G_{ij} \left( w_{ij}(q) - \tau_{ij}^e - k_{ij} \right)$$

On the other hand, the conditions for different externalities to be internalized are unchanged. Comparing the efficiency conditions with the new equilibrium conditions as in the proof of Theorem 2 we get the following.
and hence

\[ \sum_j G_{ij} \Delta_{ij}^e = \eta_i \sum_j G_{ij} \Delta_{ij} \iff \sum_j G_{ij} \tau_{ij}^e = \sum_j G_{ij} (w_{ij}(q) - k_{ij}) - \eta_i \sum_j G_{ij} \Delta_{ij}. \]

These conditions together are equivalent to (57) and (58).

Next, we have

\[ \Delta_{ij} - \Delta_{ij}^e = \Delta_{ij} + \tau_{ij}^e + k_{ij} - w_{ij}(q) \]

so that

\[ \Delta_{ij} - \Delta_{ij}^e \leq L_i \iff \tau_{ij}^e \leq w_{ij}(q) - k_{ij} - \Delta_{ij} + L_i \]

with equality if \( G_{ij} > 0 \).

\[ \iff \tau_{ij}^e \leq w_{ij}(q) - k_{ij} + \sum_{ij} G_{ij} \Delta_{ij} - \Delta_{ij} + L_i \]

with equality if \( G_{ij} > 0 \).

Finally, if (57)-(59) hold jointly, then averaging (59) with weights given by \( G_i \cdot \) and comparing with 57 we obtain

\[ L_i = -\eta_i \sum_j G_{ij} \Delta_{ij}. \]

Conversely it is easy to see that if (58) and (59) hold jointly with \( L_i = -\eta_i \sum_j G_{ij} \Delta_{ij} \), then (57) holds as well.

Equation (58) can be seen as a tax for searching at \( i \): the planner wants to tax more if there are decreasing returns to scale in matching. The condition implies that in order to obtain efficiency without taxes we need constant returns to scale, in which case we get the formula for efficient prices of the main text (Corollary 1).\[ \square \]
Figure 7: Average weekly share of unrealized matches due to search frictions, with confidence intervals from 200 bootstrap samples.

B Additional Figures and Tables

Figure 6: Definition of regions. Each color depicts one of the 15 geographical regions.
<table>
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<th>II</th>
<th>III</th>
<th>IV</th>
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<tr>
<td></td>
<td>(0.019)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I {\text{dest. = home country}}$</td>
<td>-0.012</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.015)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>log (Number Employees)</strong></td>
<td>0.008</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>log (Operating Revenues)</strong></td>
<td>0.003</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Qtr×Yr</th>
<th>Qtr×Yr</th>
<th>Qtr×Yr</th>
<th>Qtr×Yr</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Time FE</strong></td>
<td>No</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Shipowner FE</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Ship characteristics</strong></td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td></td>
<td>Orig.</td>
<td>Orig.</td>
<td>Orig.</td>
<td>Orig.</td>
</tr>
<tr>
<td><strong>Region FE</strong></td>
<td>&amp; Dest.</td>
<td>&amp; Dest.</td>
<td>&amp; Dest.</td>
<td>&amp; Dest.</td>
</tr>
</tbody>
</table>

| Observations         | 7,263    | 7,263    | 7,973    | 7,973    |
| Adj. $R^2$           | 0.530    | 0.540    | 0.537    | 0.537    |

* $p<0.1$; ** $p<0.05$; *** $p<0.01$

**Table 4:** Regression of shipping prices on shipowner characteristics and fixed effects. Shipping prices, ships’ characteristics (age and size), and the identity of the shipowner are obtained from Clarksons. Information on shipowner characteristics is obtained from ORBIS. In particular, we match the shipowners in Clarksons to ORBIS; we do so for two reasons: (i) ORBIS allows us to have reliable firm identities, as shipowners may appear under different names in the contract data; (ii) ORBIS reports additional firm characteristics (e.g. number of employees, revenue, headquarters). Here we identify the shipowner with the global ultimate owner (GUO); results are robust to controlling for the identity of the domestic owner (DUO) and the shipowner as reported in Clarksons. Finally, the data used span the period 2010-2016.
<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>log(price per day)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Probability of ballast</td>
<td>0.234**</td>
<td>0.556**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td>(0.081)</td>
<td></td>
</tr>
<tr>
<td>Avg duration of ballast trip (log)</td>
<td>0.166**</td>
<td>0.065**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.014)</td>
<td>(0.032)</td>
<td></td>
</tr>
<tr>
<td>Coal</td>
<td>0.088**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fertilizer</td>
<td>0.245**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Grain</td>
<td>0.131**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.048)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ore</td>
<td>0.124**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.045)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>0.135**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>10.284**</td>
<td>9.127**</td>
<td>8.915**</td>
</tr>
<tr>
<td></td>
<td>(0.103)</td>
<td>(0.099)</td>
<td>(0.408)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>I</th>
<th>II</th>
<th>III</th>
</tr>
</thead>
<tbody>
<tr>
<td>Destination FE</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Origin FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Ship type FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Quarter FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Obs</td>
<td>11,014</td>
<td>11,011</td>
<td>1,662</td>
</tr>
<tr>
<td>R²</td>
<td>0.694</td>
<td>0.674</td>
<td>0.664</td>
</tr>
</tbody>
</table>

** $p < 0.05$, * $p < 0.1$

**Table 5:** Shipping price regressions. The dependent variable is the logged price per day in USD. The independent variables include combinations of: the average frequency of ballast traveling after the contract’s destination (Probability of ballast), the average logged duration (in days) of the ballast trip after the contract’s destination, as well as ship type, origin, destination and quarter FE’s. The product is reported in only 20% of the sample, so the regression in column III has substantially fewer observations. The omitted product category is cement.
<table>
<thead>
<tr>
<th>Region</th>
<th>F-stat</th>
</tr>
</thead>
<tbody>
<tr>
<td>North America West Coast</td>
<td>21.132</td>
</tr>
<tr>
<td>North America East Coast</td>
<td>18.429</td>
</tr>
<tr>
<td>Central America</td>
<td>17.877</td>
</tr>
<tr>
<td>South America West Coast</td>
<td>18.671</td>
</tr>
<tr>
<td>South America East Coast</td>
<td>16.889</td>
</tr>
<tr>
<td>West Africa</td>
<td>16.333</td>
</tr>
<tr>
<td>Mediterranean</td>
<td>46.072</td>
</tr>
<tr>
<td>Baltic States</td>
<td>28.651</td>
</tr>
<tr>
<td>South Africa</td>
<td>13.153</td>
</tr>
<tr>
<td>Middle East</td>
<td>68.037</td>
</tr>
<tr>
<td>India</td>
<td>29.521</td>
</tr>
<tr>
<td>South East Asia</td>
<td>34.909</td>
</tr>
<tr>
<td>China</td>
<td>28.642</td>
</tr>
<tr>
<td>Australia</td>
<td>35.977</td>
</tr>
<tr>
<td>Japan-Korea</td>
<td>32.794</td>
</tr>
</tbody>
</table>

Table 6: First Stage, Matching Function Estimation. Regressions of the number of ships in each region on the unpredictable component of weather conditions in the surrounding seas. The table reports the F-statistic. For the construction of the instrument, see Table 4.

![Figure 8](image.png)

Figure 8: The vertical axis reports the change in prices when only thin/thick market externalities are internalized (i.e. when the exporter bargaining coefficient is set equal to the elasticity of the matching function with respect to exporters). The horizontal axis reports the difference between the estimated exporter bargaining coefficient and the estimated elasticity of the matching function with respect to exporters.
<table>
<thead>
<tr>
<th>Region</th>
<th>Exporter wait costs</th>
<th>Ship bargaining coefficient</th>
<th>Exporter value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c_{inv}$</td>
<td>$\gamma$</td>
<td>$\bar{w}_i$</td>
</tr>
<tr>
<td>North America West Coast</td>
<td>83.49</td>
<td>0.384</td>
<td>13,738</td>
</tr>
<tr>
<td></td>
<td>(10.72)</td>
<td>(0.018)</td>
<td></td>
</tr>
<tr>
<td>North America East Coast</td>
<td>83.49</td>
<td>0.585</td>
<td>12,192</td>
</tr>
<tr>
<td></td>
<td>(10.72)</td>
<td>(0.012)</td>
<td></td>
</tr>
<tr>
<td>Central America</td>
<td>302.3</td>
<td>0.344</td>
<td>14,350</td>
</tr>
<tr>
<td></td>
<td>(69.28)</td>
<td>(0.038)</td>
<td></td>
</tr>
<tr>
<td>South America West Coast</td>
<td>302.3</td>
<td>0.259</td>
<td>20,096</td>
</tr>
<tr>
<td></td>
<td>(69.28)</td>
<td>(0.017)</td>
<td></td>
</tr>
<tr>
<td>South America East Coast</td>
<td>302.3</td>
<td>0.371</td>
<td>6,971</td>
</tr>
<tr>
<td></td>
<td>(69.28)</td>
<td>(0.042)</td>
<td></td>
</tr>
<tr>
<td>West Africa</td>
<td>396.9</td>
<td>0.292</td>
<td>4,547</td>
</tr>
<tr>
<td></td>
<td>(512.74)</td>
<td>(0.078)</td>
<td></td>
</tr>
<tr>
<td>Mediterranean</td>
<td>3.44</td>
<td>0.412</td>
<td>10,508</td>
</tr>
<tr>
<td></td>
<td>(8.60)</td>
<td>(0.014)</td>
<td></td>
</tr>
<tr>
<td>Baltic States</td>
<td>3.44</td>
<td>0.517</td>
<td>14,577</td>
</tr>
<tr>
<td></td>
<td>(8.60)</td>
<td>(0.014)</td>
<td></td>
</tr>
<tr>
<td>South Africa</td>
<td>396.9</td>
<td>0.24</td>
<td>6,224</td>
</tr>
<tr>
<td></td>
<td>(512.74)</td>
<td>(0.075)</td>
<td></td>
</tr>
<tr>
<td>Middle East</td>
<td>20.53</td>
<td>0.615</td>
<td>7,160</td>
</tr>
<tr>
<td></td>
<td>(9.04)</td>
<td>(0.026)</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>20.53</td>
<td>0.568</td>
<td>6,305</td>
</tr>
<tr>
<td></td>
<td>(9.04)</td>
<td>(0.022)</td>
<td></td>
</tr>
<tr>
<td>South East Asia</td>
<td>174.01</td>
<td>0.215</td>
<td>4,918</td>
</tr>
<tr>
<td></td>
<td>(44.00)</td>
<td>(0.036)</td>
<td></td>
</tr>
<tr>
<td>China</td>
<td>282.8</td>
<td>0.194</td>
<td>8,231</td>
</tr>
<tr>
<td></td>
<td>(77.86)</td>
<td>(0.028)</td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>174.01</td>
<td>0.388</td>
<td>12,475</td>
</tr>
<tr>
<td></td>
<td>(44.00)</td>
<td>(0.037)</td>
<td></td>
</tr>
<tr>
<td>Japan-Korea</td>
<td>282.8</td>
<td>0.265</td>
<td>2,977</td>
</tr>
<tr>
<td></td>
<td>(77.86)</td>
<td>(0.038)</td>
<td></td>
</tr>
</tbody>
</table>

**Table 7:** Exporter valuation, wait costs and bargaining coefficients estimates. All the estimates are in 1,000 USD. Standard errors computed from 200 bootstrap samples.
C  Random Search in Bulk Shipping

In this section we investigate whether search in bulk shipping is random (or undirected), as assumed in the model of Section 2. We contrast this with the case of directed search (see e.g. Moen, 1997), where carriers choose to search in a specific “market”, i.e. a market for customers heading to a specific destination. Under directed search, profitable markets attract more carriers, thereby reducing their matching probabilities compared to less profitable markets. We can directly test this implication of directed search by checking whether in a given origin, \(i\), ships’ waiting time is equal across destinations \(j\), on average. We use 15 regions, so for a given region there are (up to) 14 possible destinations; therefore there are \(\binom{14}{2} = 91\) such equalities to test for every origin \(i\). Using a simple F-test we are only able to reject the null of no difference for 16% of the equalities.

In addition, we examine the coefficient of variation of matching probabilities within a given origin. Weighted by trade shares, the average coefficient of variation is just 8%. In contrast, the coefficient of variation of trip prices from a given origin is substantially higher and equal to 46%, suggesting that differences in the attractiveness of different types of customers is reflected in prices, but not in matching probabilities, as would be the case in directed search.

D  Data, Estimation and Computation Details

D.1  Exporter Valuations

We construct exporter valuations \(w_{ij}\) from product-level data on export value and quantity by country-pair, obtained from Comtrade. We select bulk commodities among all possible 4-digit HS product codes. The list includes cereals (except rice and barley); oil seeds (this consists of mostly soybeans); cocoa beans; salt and cement; ores; mineral fuels (except petroleum coke); fertilizers; fuel wood and wood pulp; metals; cermets and articles thereof.\(^{40}\)

\(^{40}\)Specifically, the products included are: Wheat and meslin, Rye, Maize, Grain sorghum, Soya beans, Copra Oil seeds; linseed, Rape or colza seeds; Sunflower seeds; Oil seeds and oleaginous fruits, Cereal straw and husks, unprepared; Cocoa beans; Cocoa; cocoa waste; Salt; Iron pyrites; Sulphur of all kinds; Graphite; natural Sands; Quartz; Kaolin; Clays; Chalk; Natural calcium phosphates; Gypsum; Limestone; cement; Iron ores and concentrates; Manganese ores and concentrates; Copper ores and concentrates; Nickel ores and concentrates; Cobalt ores and concentrates; Aluminium ores and concentrates; Lead ores and concentrates; Zinc ores and concentrates; Tin ores and concentrates; Molybdenum ores and concentrates; Granulated slag (slag sand) ; Coal; Lignite; Peat; (including peat litter); Fertilizers nitrogenous; Fertilizers chemical; phosphatic Fertilizers; potassic Fertilizers; Fuel wood; Wood pulp; Chemical wood pulp; Ferrous products obtained by direct reduction of iron ore;
To compute the average value of a cargo exported from region $i$ to $j$, we first compute the average "price" of a ton exported by dividing total export value by total export quantity from $i$ to $j$. Then, we multiply this price by the average ship tonnage capacity in our sample.\footnote{This is robust to using the average ship tonnage capacity on route $ij$.}

Finally, although most countries belong to one of our regions (depicted in Figure 6), the USA and Canada each belong to two regions (according to the coast). We thus need to split the Comtrade data for the USA and Canada into east and west coast export values. To do so, we employ data on state-level exports from the US Census, as well as on province-level exports from the Canadian International Merchandise Trade Database.\footnote{See https://www.census.gov/data/tables/2012/econ/cfs/state.html and https://ww150.statcan.gc.ca/n1/en/catalogue/65F0013X69}

In particular, we assign every state (province) to either the east or the west coast and compute, for every product, the share of the total value of trade in that commodity that is exported by west and east coast states (provinces). Then, we compute the total value and quantity of trade for the region East Coast of North America (West Coast of North America) by summing over products the share of the value of east coast trade by the total value of the country’s trade for the USA and Canada. Implicitly, this approach assumes that export values from these two regions are only different due to the composition of products, not their prices.

\section{Algorithm to compute the efficient allocation}

Here, we describe the algorithm employed to compute the steady state of our model in order to obtain the welfare maximizing allocation described in Section 5.2. Throughout the paper, in order to simulate both the equilibrium and the efficient allocation we approximate the estimated matching function with a Cobb Douglas functional form. In particular, for each region we impose $m_{it} = A_s^{1-\alpha_i}e^{\alpha_i t}$, and select the parameters $(A, \alpha_i)$ through non-linear least-squares (and the estimated exporters).

The algorithm proceeds as follows:

1. Make an initial guess for $\{U^{e,0}, \tau^0, s^0, E^0\}$.

2. At each iteration $k$, inherit $\{U^{e,k-1}, \tau^{k-1}, s^{k-1}, E^{k-1}\}$. Let $G^{k-1}$, $e^{k-1}$, and $q^{k-1}$ denote the asso-

\begin{itemize}
\item Tungsten (wolfram) articles thereof; Molybdenum articles thereof; Tantalum articles thereof; Magnesium articles thereof; Cobalt, mattes; Bismuth; Cadmium, articles thereof; Titanium articles thereof; Zirconium, articles thereof; Manganese; Cermets; articles thereof, including waste and scrap.
\end{itemize}
associated destination shares, searching exporters, and matches respectively.\footnote{That is, } Moreover, let \( \lambda^{k-1} \) and \( \lambda^{k-1} \) denote the associated matching rates. We update our guess according to the following steps:

(a) First, in an inner loop we compute the ship’s optimal policies and value function implied by the matching rates \( \lambda^{k-1} \), prices \( \tau^{k-1} \), and destination shares \( G^{k-1} \). In particular, after initializing \( V^0 \), repeat the following steps until convergence

i. At iteration \( h \), compute the value of traveling \( V^h_{ij} \) from:

\[
V^h_{ij} = \frac{-c^{s}_{ij} + d^{s}_{ij} \beta V^{h-1}_{i}}{1 - \beta (1 - d^{s}_{ij})}
\]

ii. Compute the value \( U^h_i \) from:

\[
U^h_i = \sigma \log \left( \frac{\beta V^{h-1}_{i}}{\sigma} + \sum_{j \neq i} \exp \left( \frac{V^h_{ij}}{\sigma} \right) \right) + \sigma \gamma_{euler}
\]

iii. Update \( V^h \)

\[
V^h_i = -c^{w}_{i} + \left( 1 - \lambda^{s,k-1}_{i} \right) U^h_i + \lambda^{s,k-1}_{i} \sum_{j} G^{k-1}_{ij} \left( V^h_{ij} + \tau^{k-1}_{ij} \right)
\]

iv. Upon convergence, we set \( V^\infty_{ij} = V^\infty_{ij} \), \( V^\infty_i = V^\infty_i \), \( U^\infty_i = U^\infty_i \), and compute the ships’ optimal choice probabilities based on:

\[
P^k_{ij} = \frac{\exp \left( V^k_{ij} \right)}{\sum_{l} \exp \left( \frac{V^k_{il}}{\sigma} \right) + \exp \left( \frac{\beta V^k_{il}}{\sigma} \right)} \quad \text{for } i \neq j
\]

\[
P^k_{ii} = \frac{\exp \left( V^k_{ii} \right)}{\sum_{l} \exp \left( \frac{V^k_{il}}{\sigma} \right) + \exp \left( \frac{\beta V^k_{il}}{\sigma} \right)} \quad \text{for } i \neq j
\]

(b) Next, we update the efficient prices \( \tau^k \) based on Corollary 1. In particular, we compute the
total surplus from matching as,

$$\Delta_{ij}^k = w_{ij} - \delta \beta U_{ij}^{e,k-1} + V_{ij}^k - U_i^k,$$

and compute the efficient prices based on,

$$\tau_{ij}^k = w_{ij} - \delta \beta U_{ij}^{e,k-1} - \Delta_{ij}^k + \alpha_i \sum_j G_{ij}^{k-1} \Delta_{ij}^k,$$

where $\alpha_i$ denotes the elasticity of the matching function with respect to the number of exporters.

(c) We then update the exporters’ value function $U_{e,k}$ based on the efficient prices $\tau^k$ and matching rates $\lambda_{e,k}^{e,k-1}$ setting

$$U_{e,k}^{e,k} = -c_i e_i + \beta \delta \lambda_{i}^{e,k-1} \left[ (w_{ij} - \tau_{ij}^k) \left( 1 - \beta \delta \lambda_{i}^{e,k-1} \right) \right].$$

(d) Finally, we update the number of ships and exporters searching $\{s^k, E^k\}$ according to

$$E_{ij}^k = E_i \frac{\exp \left( U_{ij}^{e,k} - \kappa_{ij} \right)}{1 + \sum_{l \neq i} \exp \left( U_{il}^{e,k} - \kappa_{il} \right)} + \delta \left( q_i^{k-1} - q_{ij}^{k-1} \right) \text{ unmatched}.$$

and

$$s_{i}^k = \sum_j P_{ij}^k \left( s_{j}^{k-1} - q_{ij}^{k-1} \right) + \sum_j q_{ij}^{k-1}.$$

3. If $\|s^k - s^{k-1}\| < \epsilon$, $\|E^k - E^{k-1}\| < \epsilon$, $\|J^{e,k} - J^{e,k-1}\| < \epsilon$, and $\|\tau^k - \tau^{k-1}\| < \epsilon$ stop; otherwise go back to point (a).

E Discounting, preference shocks and out of steady state dynamics

In this section we show that the main results of Section 3.2 are valid in a more general setup. In particular, we extend the model of Section 2 to allow for idiosyncratic preference shocks (relevant in our empirical

---

44Following BKP we assume unmatched exporters survive with probability $\delta$ so that their effective discount factor is $\beta \delta$. This would make no difference in our theoretical analysis.
application), as well as out of steady state dynamics, and we derive an efficiency result analogous to that of Theorem 2 in this framework.

E.1 Model

We begin by laying out the model focusing on the changes made compared to Section 2.

States and transitions In this section we do not consider the steady state equilibrium. Hence, we now state explicitly the dependence of actions and value functions on the relevant state variables and transitions, which were only implicit in the model of Section 2. At the beginning of a given time period, the state of the economy is described by a vector,

\[ z = (x, y) \in \mathbb{R}_+^{I \times I} \times \mathbb{R}_+^{I \times I}. \]

The first element of \( z \), \( x = (x_{ij})_{i,j \in I} \), corresponds to the supply: for every origin \( i \),

- \( x_{ii} \) is the measure of carriers waiting at location \( i \)
- for every destination \( j \neq i \), \( x_{ij} \) is the measure of carriers traveling from \( i \) to \( j \)

The second element of \( z \), \( y = (y_{ij})_{i,j \in I} \), corresponds to demand. For every origin-destination pair \( i,j \), \( y_{ij} \) is the measure of customers who are waiting for transport on route \( ij \). These are customers that entered in some previous period and have not yet been matched with a carrier.

At a given state \( z \), the choice sets that agents face, as well as the search and matching process are the same as in Section 2. At each origin \( i \), a measure \( s_i \leq x_{ii} \) of carriers choose to search for a customer, while the remaining measure \( x_{ii} - s_i \) choose to remain inactive. Similarly, a measure \( E_{ij} \geq y_{ij} \) of customers search for a carrier on route \( ij \), so that \( E_{ij} - y_{ij} \) is the measure of new customers joining (or existing customers leaving, if negative) the existing search pool.

Once a customer and a carrier meet, they can choose whether to match or remain unmatched. The outcome of this process is a vector \((b, q)\) describing the measure of carriers that start traveling empty \( (b_{ij}) \)
or full \((q_{ij})\) on each route \(ij\). The state transitions as a function of the allocation are as follows:

\[
\forall i,j : x_{ii}^{+1} (s, E, q, b) = x_{ii} - s_i + \sum_j d_{ji} (q_{ji} + b_{ji})
\]

\[
x_{ij}^{+1} (s, E, q, b) = (1 - d_{ij}) (x_{ij} + q_{ij} + b_{ij})
\]

\[
y_{ij}^{+1} (s, E, q, b) = E_{ij} - q_{ij}.
\]

(60)

The feasibility constraints on the allocation \((s, E, q, b)\) are:

\[
\forall i,j : x_{ii} \geq s_i
\]

\[
E_{ij} \geq y_{ij}
\]

\[
\sum_j (q_{ij} + b_{ij}) = s_i
\]

\[
m_i(s_i, e_i) G_{ij} \geq q_{ij}
\]

\[
s_i, E_{ij}, q_{ij}, b_{ij} \geq 0.
\]

(61)

**Prices, expectations and allocation rules**  The pricing rule maps each state into the associated vector of transportation prices on each route:

\[
\tau : z \mapsto \tau (z) = (\tau_{ij} (z))_{i,j \in I}.
\]

As in Section 2, we begin by remaining agnostic regarding the structure of the pricing rule, and later we characterize the pricing rules that are consistent with efficient equilibria and compare them with Nash bargaining.

In state \(z\) carriers expect to meet customers at rate \(\lambda^s_i (z)\), and customers expect to meet carriers at rate \(\lambda^c_i (z)\), where

\[
\lambda^s : z \mapsto \lambda^s (z) = (\lambda^s_i (z))_{i \in I}, \lambda^c : z \mapsto \lambda^c (z) = (\lambda^c_i (z))_{i \in I}.
\]

Given their expectations regarding the transition of the state \(z\), agents make optimal choices, generat-
ing an allocation rule \((s, E, q, b) : z \mapsto (s(z), E(z), q(z), b(z))\), mapping states into feasible allocations. That is, for every state \(z\), \((s(z), E(z), q(z), b(z))\) satisfies 61.

Similarly to Section 2, we will sometimes denote an allocation rule by \((s, e, G, q, b)\), where \(e_i(z) = \sum_j E_{ij}(z)\) and \(G_{ij}(z) = E_{ij}(z) / e_i(z)\), and we will often refer to the first triplet \((s, e, G)\) as a search rule.

**Preference shocks and carriers’ optimality**  Carriers’ payoff structure is the same as that in Section 2. To explore the properties of our empirical model of Section 4.3, however, we allow for stochasticity in carriers’ preferences for destinations. The stochastic component at each origin \(i\) is represented by a random vector \(\epsilon_i = (\epsilon_{ij})_{i,j \in I}\) that enters the carriers’ utility of relocating to different destinations additively, is i.i.d. across carriers and satisfies the conditional independence assumption,

\[
\epsilon_i^{+1} | z^{+1} \perp \epsilon_i, z.
\]

To simplify the exposition, we assume that \(\epsilon_i\) is independent of \(z\) and \(i\), so that

\[
\forall i, z : \epsilon_i \sim \mathbb{P} \in \Delta \mathbb{R}^I
\]

although this assumption is not needed for the results. We assume that \(\mathbb{P}\) has full support and that it admits a continuous density.

The value of a carrier unmatched at origin \(i\) at state \(z\) depends on the particular realization of the shock. We denote its expectation by

\[
U_i^s(z) = \mathbb{E}_\mathbb{P} \max_j \left( V_{ij}^s(z) + \epsilon_{ij} \right).
\]  

The values of traveling and waiting are given by:

\[
V_{ij}^s(z) = -c_{ij}^s + \beta \left[ d_{ij} V_j^s(z^{+1}) + (1 - d_{ij}) V_{ij}^s(z^{+1}) \right]
\]

\[
V_i(z) = \max \left\{ -c_i^w + \lambda_i^s(z) \sum_{j \neq i} G_{ij} \Delta_{ij}^s(z) + U_i^s(z), \beta V_i(z^{+1}) \right\}
\]
as before, where
\[ \Delta_{ij}^s (z) = \max \{ \tau_{ij} (z) + V_{ij}^s (z) - U_i^s (z), 0 \} \]
is the carriers’ expected surplus of being matched with respect to being unmatched.

Denote by \( P^b \) the matrix of carriers’ relocation choice probabilities associated with \( b \in \mathbb{R}^{I \times I} \):
\[ \forall i, j : P^b_{ij} = \frac{b_{ij}}{\sum_k b_{ik}}. \]

Optimality in state \( z \) requires that
\[ \forall i, j : P_{ij}^b (z) = P \left[ V_{ij}^s (z) + \epsilon_{ij} = \max_k (V_{ik}^s (z) + \epsilon_{ik}) \right]. \quad (63) \]
The remaining optimality conditions of Section 2 still hold. In particular, carriers search only when it is profitable to do so:
\[ s_i (z) > 0 \rightarrow V_i (z) = -c_i^w + \lambda_i^s (z) \sum_{j \neq i} G_{ij} \Delta_{ij}^s (z) + U_i^s (z) \quad (64) \]
Moreover, they do not reject any match yielding a strictly positive surplus, and they accept only matches yielding a positive surplus:
\[ q_{ij} (z) < s_i (z) \lambda_i^s (z) G_{ij} \rightarrow \Delta_{ij}^s (z) = 0 \]
\[ q_{ij} (z) > 0 \rightarrow \Delta_{ij}^s (z) = \tau_{ij} (z) + V_{ij}^s (z) - U_i^s (z). \quad (65) \]

**Customers’ entry**

Customers value functions are the same as in Section 2, making explicit the dependence on the state of the economy. In state \( z \), the meeting surplus of the marginal customer (with respect to being unmatched) is given by
\[ \Delta_{ij}^c (z) = \max \left\{ w_{ij} (q (z)) - \tau_{ij} (z) - \beta U_{ij}^c (z + 1), 0 \right\}, \]
where \( U_{ij}^c (z) \) is his value of searching for a carrier in location \( i \) with destination \( j \):
\[ U_{ij}^c (z) = -c_i^c + \lambda_i^c (z) \Delta_{ij}^c (z) + \beta U_{ij}^c \left( z^{e+1} \right). \quad (66) \]
Optimality requires that the marginal customer does not reject a match yielding a strictly positive surplus:

\[ q_{ij}(z) < \lambda^e_i(z) E_{ij}(z) \rightarrow \Delta^e_{ij}(z) = 0. \] (67)

The measure of customers searching on each route \( ij \) is pinned down by a free entry condition for the marginal customer:

\[ U^e_{ij}(z) - \kappa_{ij} \leq 0 \]

with equality if \( E_{ij}(z) > y_{ij} \). (68)

**Equilibrium** An outcome is a tuple \((s, E, q, b, \tau)\) consisting of an allocation rule and a price rule.

**Definition 6.** An outcome is a Markovian equilibrium if, for every state \( z \):

1. \((s(z), E(z), q(z), b(z))\) satisfies the feasibility constraints 61.
2. \((s(z), q(z), b(z))\) satisfies carriers optimality conditions (62)-(65) given \( \tau(z), \lambda^e(z), z^{+1} \) and \( G(z) \).
3. \((E(z), q(z))\) satisfies customers optimality and free entry conditions (66)-(68) given \( \tau(z), \lambda^e(z) \) and \( z^{e,+1} \).
4. Expectations are consistent with the realized outcomes:

\[ \forall i : \lambda^e_i(z) s_i(z) = m_i(s_i(z), e_i(z)) \]
\[ X^e_i(z) e_i(z) = m_i(s_i(z), e_i(z)) \]
\[ z^{+1} = z^{+1} (s(z), E(z), q(z), b(z)). \]

\((s, E, q, b)\) is an equilibrium allocation rule if there exists a price rule \( \tau \) such that \((s, E, q, b, \tau)\) is a Markovian equilibrium.

**E.2 Externalities and Efficiency**

The social planner wishes to maximize the discounted sum of future social payoffs given the state of the economy. We formulate the problem as an infinite horizon constrained Markov decision problem.

The first goal is to express the social welfare in terms of the aggregate choice outcomes only, without
any explicit reference to the particular allocation of single agents. To do so, for every vector of choice probabilities \( p = (p_j)_{j \in I} \in \Delta I \), let \( \Pi (p) \) be the set of all probability measures \( \pi \in \Delta (\mathbb{R}_I \times I) \) such that the marginal of \( \pi \) over \( I \) is \( p \) and the marginal of \( \pi \) over \( \mathbb{R}_I \) is \( \mathbb{P} \). Define the function \( f : \Delta I \to \mathbb{R} \) by

\[
    f (p) = \max_{\pi \in \Pi (p)} E_{\pi} (\epsilon_j)
\]

where the expectation on the right hand side is with respect to a joint realization of the vector \( \epsilon = (\epsilon_j)_{j \in I} \) and the destination \( j \in I \). \( f (p) \) should be interpreted as value associated with the best allocation of shocks to destinations at \( i \) conditional on the aggregate choice probabilities being given by \( p \). The maximum instantaneous payoff attainable under \( (s, e, G, q, b) \) in state \( z \) can be written as

\[
    W^p (s, e, G, q, b; z) = W (q) - \sum_{ij} (x_{ij} + q_{ij} + b_{ij}) e^s_{ij} - \sum_i e_i \sum_j G_{ij} c^u_{ij} - \sum_{ij} (e_i G_{ij} - y_{ij}) \kappa_{ij} + \sum_i b_i f (P^b_i).
\]

where \( P^b_i = (P^b_{ij})_{j \in I} \) is the vector of choice probabilities induced by \( b \) at \( i \), and the last term captures the component of welfare due to carriers’ preference shocks.

In what follows, we use the upper bar notation \( \bar{a} = (a^t)_{t=0}^\infty \) for infinite sequences. When dealing with a sequence of allocations \( (\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}) \) and an initial state \( z^0 \), unless stated otherwise, it is understood that \( \bar{z} \) refers to the sequence of states induced by \( (\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}) \) from \( z^0 \):

\[
    \forall t \geq 0 : z^{t+1} = z^{t+1} (s^t, e^t, G^t, q^t, b^t; z^t).
\]

Moreover, when dealing with a feasible allocation rule \( (s, e, G, q, b) \) and an initial state \( z^0 \), it is understood that \( (\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}) \) refers to the sequence of allocations induced by \( (s, e, G, q, b) \) from \( z^0 \):

\[
    \forall t \geq 0 : (s^t, e^t, G^t, q^t, b^t) = (s (z^t), e (z^t), G (z^t), q (z^t), b (z^t)).
\]
The planner’s dynamic problem at state \( z^0 \) is given by

\[
V^p (z^0) = \max_{(s,e,G,q,b)} \sum_{t=0}^{\infty} \beta^t W^p \left( s^t, e^t, G^t, q^t, b^t; z^t \right)
\]

\[
\text{s.t. } \forall t, i, j : x^t_{ii} \geq s^t_i
\]

\[
e^t_i G^t_i \geq y^t_{ij}
\]

\[
\sum_j \left( q^t_{ij} + b^t_{ij} \right) = s^t_i
\]

\[
m_i \left( s^t_i, e^t_i \right) G^t_i \geq q^t_{ij}
\]

\[
\sum_j G^t_{ij} = 1
\]

\[
s^t_i, e^t_i, G^t_i \geq 0.
\]

**Definition 7.** An allocation rule \((s,e,G,q,b)\) is efficient at a state \( z^0 \) if \((\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b})\) solves Problem 69.

Similarly to Section 2, we distinguish three different potential sources of inefficiency. To do so, for each state \( z^0 \), let \( A(z^0) \) be the set of allocation sequences \((\bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b})\) which are feasible from \( z^0 \), so that they satisfy the constraints of Problem 69. Let also

\[
SA(z^0) = \left\{ (\bar{s}, \bar{e}, \bar{G}) : \exists (\bar{q}, \bar{b}) \in A(z^0) \right\}
\]

be the set of feasible sequences of search allocations, and

\[
SA(z^0|\bar{e}, \bar{G}) = \left\{ \bar{s} : (\bar{s}, \bar{e}, \bar{G}) \in SA(z^0) \right\}
\]

\[
SA(z^0|\bar{s}, \bar{G}) = \left\{ \bar{e} : (\bar{s}, \bar{e}, \bar{G}) \in SA(z^0) \right\}
\]

\[
SA(z^0|\bar{s}, \bar{e}) = \left\{ \bar{G} : (\bar{s}, \bar{e}, \bar{G}) \in SA(z^0) \right\}
\]

For every \((\bar{s}, \bar{e}, \bar{G}) \in SA(z^0)\), we define the maximum dynamic welfare attainable by this sequence by:
\[ V^p(s, \tilde{e}, \tilde{G}, z^0) = \max_{q, b} \sum_{t=0}^{\infty} \beta^t W^p(s^t, e^t, G^t, q^t, b^t; z^t) \]

\[ \text{s.t. } (\tilde{s}, \tilde{e}, \tilde{G}, \tilde{q}, \tilde{b}) \in A(z^0) \]

so that we have

\[ V^p(z^0) = \max_{(\tilde{s}, \tilde{e}, \tilde{G}) \in SA(z^0)} V^p(\tilde{s}, \tilde{e}, \tilde{G}, z^0). \]

**Definition 8.** Given an equilibrium allocation rule \((s, e, G, q, b)\) and an initial state \(z^0\) we say that:

(i) Carriers internalize thin/thick market externalities at \(z^0\) if \(\tilde{s}\) solves

\[ \max_{s' \in SA(z^0|\tilde{e}, \tilde{G})} V^p(s', \tilde{e}, \tilde{G}, z^0) \]

(ii) Customers internalize thin/thick market externalities at \(z^0\) if \(\tilde{e}\) solves

\[ \max_{e' \in SA(z^0|\tilde{s}, \tilde{G})} V^p(\tilde{s}, e', \tilde{G}, z^0) \]

(iii) Customers internalize pooling externalities at \(z^0\) if \(\tilde{G}\) solves

\[ \max_{\tilde{G}' \in SA(z^0|\tilde{s}, \tilde{e})} V^p(\tilde{s}, \tilde{e}, \tilde{G}', z^0). \]

Next we state the equivalent of Theorem 2 in the current framework. Given \((s, e, G)\), we denote by \(\eta^s_i(z) = d \log m_i(s_i(z), e_i(z))/d \log s_i\) and \(\eta^e_i(z) = d \log m_i(s_i(z), e_i(z))/d \log e_i\). For simplicity, in order to avoid delving into corner conditions, in the statement below we assume that the equilibrium path originating from \(z^0\) is such that we have \(s^t_i, e^t_i > 0\) for every \(t, i\).

**Theorem 3.** Let \((s, e, G, q, b)\) be an equilibrium allocation rule, fix a state \(z^0\), and suppose that Problem 70 has a unique optimal solution. Then the following holds:\footnote{Formally, the only if parts of statements (i) to (iii) hold for almost every sequence \((\tilde{s}, \tilde{e}, \tilde{G}) \in SA(z^0)\). That is, there exists a dense subset \(D\) of \(SA(z^0)\) such that the only if part of the statements hold whenever \((\tilde{s}, \tilde{e}, \tilde{G}) \in D\). See Section E.3}

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(i) Carriers internalize thin/thick market externalities at \( z^0 \) if and only if, for every \( t \geq 0 \):

\[
\forall i \in I : \sum_j G_{ij}^t \left( z^t \right) \Delta_{ij}^s \left( z^t \right) = \eta_i^s \left( z^t \right) \sum_j G_{ij}^t \left( z^t \right) \left( \Delta_{ij}^s \left( z^t \right) + \Delta_{ij}^e \left( z^t \right) \right).
\]

(ii) Customers internalize thin/thick market externalities at \( z^0 \) if and only if, for every \( t \geq 0 \):

\[
\forall i \in I : \sum_j G_{ij} \left( z^t \right) \Delta_{ij}^e \left( z^t \right) \leq \eta_i^e \left( z^t \right) \sum_j G_{ij} \left( z^t \right) \left( \Delta_{ij}^s \left( z^t \right) + \Delta_{ij}^e \left( z^t \right) \right)
\]

with equality if \( e_i \left( z^t \right) > 0 \).

(iii) Customers internalize pooling externalities at \( z^0 \) if and only if, for every \( t \geq 0 \), for each origin \( i \), \( \Delta_{ij}^s \left( z^t \right) \) is constant across all destinations \( j \neq i \) such that \( G_{ij} \left( z^t \right) > 0 \).

The following section provides the proof.

### E.3 Proof

The proof follows the same reasoning as under the steady state assumption, but has to overcome a number of technical subleties associated with the infinite dimensional form of the planner’s optimization problem.

#### E.3.1 Preliminaries

Let \( X \) be a compact and convex subset of \( \mathbb{R}^N \) for some \( N \in \mathbb{N} \), and \( \beta \in (0, 1) \). For each pair of sequences \( \bar{x}, \bar{y} \in X^{\mathbb{N} \cup \{0\}} \) we define the inner product

\[
\forall \bar{x}, \bar{y} : \langle \bar{x}, \bar{y} \rangle = \sum_{t=0}^{\infty} \beta^t x^t \cdot y^t
\]

where \( \cdot \) denotes the standard inner product on \( \mathbb{R}^N \). Define the norm \( \| \bar{x} \| = \sqrt{\langle \bar{x}, \bar{x} \rangle} \). Then \( \left( X^{\mathbb{N} \cup \{0\}}, \| \cdot \| \right) \) is a Banach space.

Let \( \mathcal{X} \subseteq X^{\mathbb{N} \cup \{0\}} \) be a convex set and \( f : \mathcal{X} \rightarrow \mathbb{R} \) be a continuous and concave function.

**Definition 9.** For every \( \bar{x} \in \mathcal{X} \), the super gradient of \( f \) at \( \bar{x} \), denoted \( \partial f \left( \bar{x} \right) \), is the set of all sequences for details.
\( y \in X^{\mathbb{N} \cup \{0\}} \) such that, for every \( \bar{x}' \in X \):

\[
f(\bar{x}') - f(\bar{x}) \leq \langle \bar{x}' - \bar{x}, \bar{y} \rangle.
\]

\( f \) is differentiable at \( \bar{x} \) if its super gradient at \( \bar{x} \) contains a unique element.

**Lemma 6.** Let \( D \subset X \) be the set of sequences at which \( f \) is differentiable. Then \( D \) is a dense subset of \( X \).

*Proof.* See (Asplund et al., 1968), Theorem 2.

**Lemma 7.** \( \bar{x} \) maximizes \( f \) over \( X \) if and only if there exists \( \bar{y} \in \partial f(\bar{x}) \) such that \( y^t = 0 \) for every \( t \).

*Proof.* Immediate from the definition of \( \partial f(\bar{x}) \).

The following lemma will be useful in the derivation of each of the three statements internalizing the respective externalities in Theorem 3. As before, \( z \) denotes the state. The variables \( x, \theta \) will change based on each externality considered; for instance, in the case of carrier externalities, \( x \) corresponds to \( s \), while \( \theta \) corresponds to \( e, G \). The function \( f \) summarizes all constraints, \( T \) defines the state dynamics and \( u \) the welfare.

**Lemma 8.** Let \( Z, X, \Theta \), compact and convex subsets of \( \mathbb{R}^L, \mathbb{R}^M \) and \( \mathbb{R}^N \), respectively, \( u : X \times \Theta \times Z \to \mathbb{R} \) be a concave and continuously differentiable function, \( T : X \times \Theta \times Z \to Z \) be a linear function, and for each \( k = 1, ..., K > 0 \), let \( f_k(x, \theta, z) \) be a continuously differentiable and convex function. Let \( z^0 \in Z \), and \( \Theta \subseteq \Theta^{\mathbb{N} \cup \{0\}} \) be such that for every \( \bar{\theta} \in \Theta \) problem

\[
P(\bar{\theta}) : \max_{x \in X^{N \cup \{0\}}} \sum_{t=0}^{\infty} \beta^t u \left( x^t, \theta^t, z^t \right)
\]

\[
\forall t, k : f_k \left( x^t, \theta^t, z^t \right) \geq 0
\]

\[
\forall t : z^{t+1} = T \left( x^t, \theta^t, z^t \right)
\]

is feasible, and let \( V(\bar{\theta}) \) denote its value. Then \( V \) is concave. Moreover, suppose that \( \bar{\theta} \in \Theta \) is such that \( P(\bar{\theta}) \) admits a unique optimal solution, and let \( \bar{x} \in X^{N \cup \{0\}}, \bar{\lambda} \in \left( \mathbb{R}^K \right)^{N \cup \{0\}} \) and \( \bar{\phi} \in \left( \mathbb{R}^L \right)^{N \cup \{0\}} \) be such
that, for every $t, k, l, m$:

$$\lambda_k^t \geq 0 \text{ with equality if } f_k(x^t, \theta^t, z^t) > 0$$

(71)

$$\phi_l^t = \frac{\partial u(x^t, \theta^t, z^t)}{\partial z_l} + \sum_k \lambda_k^t \frac{\partial f_k(x^t, \theta^t, z^t)}{\partial z_l} + \beta \sum_{l'} \frac{\partial T_{l'}(x^t, \theta^t, z^t)}{\partial z_l} \phi_{l'}^{t+1}$$

(72)

$$\frac{\partial u(x^t, \theta^t, z^t)}{\partial x_m} + \sum_k \lambda_k^t \frac{\partial f_k(x^t, \theta^t, z^t)}{\partial x_m} + \beta \sum_{l'} \frac{\partial T_{l'}(x^t, \theta^t, z^t)}{\partial x_m} \phi_{l'}^{t+1} = 0$$

(73)

where the sequence $\bar{z}$ is defined recursively by

$$\forall t : z^{t+1} = T \left( x^t, \theta^t, z^t \right).$$

Then $\bar{\theta}$ maximizes $V$ over $\Theta$ if

$$\forall t, n : \frac{\partial u(x^t, \theta^t, z^t)}{\partial \theta_n} + \sum_k \lambda_k^t \frac{\partial f_k(x^t, \theta^t, z^t)}{\partial \theta_n} + \sum_{l'} \frac{\partial T_{l'}(x^t, \theta^t, z^t)}{\partial \theta_n} \phi_{l'}^{t+1} = 0$$

(74)

and the above condition is also necessary whenever $V$ is differentiable at $\bar{\theta}$.

**Proof.** For a generic sequence $\bar{a} = (a^t)_{t=0}^\infty$ and for every $T > 0$, we use the notation $\tau \bar{a} \equiv (a^t)_{t=0}^\infty$ to denote the truncation of $\bar{a}$ at $T$. When dealing with a sequence $\left( \bar{\theta}, \bar{x} \right)$ and an initial state $z^0$, unless stated otherwise, it is understood that $\bar{z}$ refers to the sequence of states induced by $\left( \bar{\theta}, \bar{x} \right)$ and the map $T$ from $z^0$:

$$\forall t \geq 0 : z^{t+1} = T \left( x^t, \theta^t, z^t \right).$$

Let $\bar{\theta}, \bar{x}, \bar{\lambda}, \bar{\phi}$ be as in the statement. For every $T > 0$ consider the finite horizon problem,

$$P \left( T, \bar{\theta} \right) : V^T \left( T \bar{\theta} \right) = \max_{T \bar{x} \in X^{T+1}} \sum_{t=0}^T \beta^t u \left( x^t, \theta^t, z^t \right) + \beta^{T+1} \sum_{l} z_{l}^{T+1} \phi_{l}^{T+1}$$

s.t. $\forall t = 0, ..., T : \forall k : f_k \left( x^t, \theta^t, z^t \right) \geq 0$.

By standard convex optimization theory, Conditions 71, 72 and 73 imply that $\left( T \bar{x}, T \bar{\lambda} \right)$ is an optimal
dual pair for Problem \( P (T, \tilde{\theta}) \). Hence for every feasible sequence \( \tilde{x}' \) and for every \( T > 0 \) we have
\[
\sum_{t=0}^{T} \beta^t u \left( x^t, \theta^t, z^t \right) + \beta^{T+1} \sum_{l} z_l^{T+1} \phi_l^{T+1} \geq \sum_{t=0}^{T} \beta^t u \left( x'^t, \theta^t, z'^t \right) + \beta^{T+1} \sum_{l} z_l^{T+1} \phi_l^{T+1}
\]
Sine \( Z \) and \( u \) are bounded\(^{46}\), taking limits on both sides implies that \( \tilde{x} \) is optimal for \( P (\tilde{\theta}) \). Hence by our assumptions it must be the unique optimal solution for \( P (\tilde{\theta}) \). Define
\[
\forall t, n : y'_n = \frac{\partial u \left( x^t, \theta^t, z^t \right)}{\partial \theta_n} + \sum_k \lambda_k^t \frac{\partial f_k \left( x^t, \theta^t, z^t \right)}{\partial \theta_n} + \sum_l \frac{\partial T_l \left( x^t, \theta^t, z^t \right)}{\partial \theta_n} \phi_l^{t+1}
\]
We show that \( \tilde{y} \in \partial V (\tilde{\theta}) \). From (Marimon and Werner, 2015) it follows that \( T_{\tilde{y}} \in \partial V (\tilde{\theta}) \) for all \( T > 0 \):
\[
\forall \tilde{\theta}' \in \Theta : V^T (\tilde{\theta}') - V^T (\tilde{\theta}) \leq \sum_{t=0}^{T} \beta^t \sum_n y_n^t \left( \theta_n^t - \theta_n^t \right)
\]
Pick \( \tilde{\theta}' \in \Theta \) and let \( \tilde{x}' \) be an optimal solution for \( P (\tilde{\theta}') \). For each \( T \) we have
\[
\sum_{t=0}^{T} \beta^t u \left( x^n, \theta^n, z^n \right) + \beta^{T+1} \sum_{l} z_l^{T+1} \phi_l^{T+1} \leq V^T (\tilde{\theta})
\]
and
\[
V^T (\tilde{\theta}) = \sum_{t=0}^{T} \beta^t u \left( x^t, \theta^t, z^t \right) + \beta^{T+1} \sum_{l} z_l^{T+1} \phi_l^{T+1}
\]
therefore
\[
\sum_{t=0}^{T} \beta^t u \left( x^n, \theta^n, z^n \right) + \beta^{T+1} \sum_{l} \left( z_l^{T+1} - \phi_l^{T+1} \right) \phi_l^{T+1} \leq \sum_{t=0}^{T} \beta^t \sum_n y_n^t \left( \theta_n^t - \theta_n^t \right)
\]
Taking limits of both sides we get \( V (\tilde{\theta}') - V (\tilde{\theta}) \leq \sum_{t=0}^{\infty} \beta^t \sum_n y_n^t \left( \theta_n^t - \theta_n^t \right) \). Since \( \tilde{\theta}' \) was arbitrary, this implies \( y \in \partial V (\tilde{\theta}) \). Hence, by Lemma 7, \( \tilde{\theta} \) maximizes \( V \) over \( \Theta \) if \( \tilde{y} = 0 \), and this condition is also necessary whenever \( V \) is differentiable at \( \tilde{\theta} \). This completes the proof. \( \square \)

\(^{46}\)u is bounded, being a continuous function on a compact space.
E.3.2 Proof of Theorem 3

This subsection is devoted to the proof of Theorem 3. We first establish two auxiliary lemmas.

**Lemma 9.** Function \( f \) is continuously differentiable. Moreover, given a vector of choice probabilities \( p \in \Delta I \), a vector \( \phi \in \mathbb{R}^I \) and a scalar \( \psi \in \mathbb{R} \), the following are equivalent:

(i) \[
\psi = E_P \max \left( \phi_j + \epsilon_j \right) \text{ and } \forall j: p_j = P \left[ \phi_j + \epsilon_j = \max_k \left( \phi_k + \epsilon_k \right) \right].
\]

(ii) \[
\forall j: f(p) + \frac{\partial f(p)}{\partial p_j} - \sum_k p_k \frac{\partial f(p)}{\partial p_k} + \phi_j - \psi = 0
\]

**Proof.** To see that \( f \) is continuously differentiable, it is well known (Galichon, 2018) that

\[
\forall p \in \Delta I : -f(p) = \min_\phi \left[ \sum_j p_j \phi_j - E_P \max_j \left( \phi_j + \epsilon_j \right) \right].
\]

By our assumptions on \( P \), the objective function of the above problem is continuous and strictly convex, hence:

- The set of its solutions is a singleton. By the envelope theorem, this implies that
  \[
  \partial f(p) = \{-\phi^*(p)\}
\]

where \( \phi^*(p) \) is the unique optimal solution of the above problem. Hence \( f \) is differentiable

- Continuity of \( \partial f \) follows by noting that \( \phi^* \) is continuous by the Maximum Theorem.

For the second part of the statement, it is well known (see Galichon, 2018) that (i) is equivalent to

\[
\phi \in \partial (-f(p)) \text{ and } -f(p) + \psi = \sum_j p_j \phi_j.
\]

When \( f \) is differentiable, the condition above is equivalent to (ii). This completes the proof. \( \square \)

**Lemma 10.** Let \( \bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b}, \bar{\psi}, \bar{\mu}^c, \bar{\mu}^s, \bar{\Delta}, \bar{\phi}^s, \bar{\phi}^e \) be such that \( \left( \bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b} \right) \in \mathcal{A}(z^0) \) and, for every \( t, i, j, \)
\( s^t_i, e^t_i > 0 \) and the following conditions hold:

\[
\mu_{s,t}^{e,t} \geq 0 \text{ with equality if } s^t_i < x^t_{ii}
\]

\[
\mu_{e,t}^{e,t} \geq 0 \text{ with equality if } e^t_i G^t_{ij} > y^t_{ij}
\]

\[
\Delta^t_{ij} \geq 0 \text{ with equality if } q^t_{ij} < m_i \left( s^t_i, e^t_i \right) G^t_{ij}
\]

\[
\Delta^t_{ij} \geq w_{ij} \left( q^t_{ij} \right) + \phi_{ij}^{s,t} - \beta \mu_{ij}^{e,t+1} - \psi^t_i
\]

with equality if \( q^t_{ij} > 0 \)

\[
\psi^t_i = E_{\Phi} \max_j \left( \phi_{ij}^{s,t} + \epsilon_{ij} \right)
\]

\[
P^B_{ij} = P \left[ \phi_{ij}^{s,t} = \max_k \left( \phi_{ik}^{s,t} + \epsilon_{ik} \right) \right]
\]

\[
\phi_{ii}^{s,t} = \mu_{i}^{s,t} + \beta \phi_{ii}^{s,t+1}
\]

\[
\phi_{ij}^{e,t} = \kappa_{ij} - \mu_{ij}^{e,t}
\]

\[
\phi_{ij}^{s,t} = -c_{ij} + \beta \left[ d_{ij} \phi_{jj}^{s,t+1} + (1 - d_{ij}) \phi_{ij}^{s,t+1} \right].
\]

Then:

(i) \( \bar{s} \) maximizes the function \( \bar{s}' \mapsto V \left( \bar{s}', \bar{e}, \bar{G}, z^0 \right) \) over \( \mathcal{SA} \left( z^0|\bar{e}, \bar{G} \right) \) if, for every \( i, t \):

\[
-c^w_i + \frac{\partial m_i \left( s^t_i, e^t_i \right)}{\partial s_i} \sum_j G^t_{ij} \Delta^t_{ij} + \psi^t_i - \beta \phi_{ii}^{s,t+1} - \mu_{i}^{s,t} = 0. \tag{75}
\]

This condition is also necessary whenever the function \( \bar{s}' \mapsto V \left( \bar{s}', \bar{e}, \bar{G}, z^0 \right) \) is differentiable at \( \bar{s} \).
(ii) \( \bar{e} \) maximizes the function \( \bar{e}' \mapsto V \left( \bar{s}, \bar{e}', \bar{G}, z^0 \right) \) over \( \mathcal{S} \mathcal{A} \left( z^0 | \bar{s}, \bar{G} \right) \) if, for every \( i, t \):

\[
\frac{\partial m_i (s^i_t, e^i_t)}{\partial e_i} \sum_j G^t_{ij} \Delta^t_{ij} - \sum_j G^t_{ij} \left( e^e_{ij} + \kappa_{ij} - m_i \bar{e}_i - \phi_{ij} \right) = 0. \tag{76}
\]

This condition is also necessary whenever the function \( \bar{e}' \mapsto V \left( \bar{s}, \bar{e}', \bar{G}, z^0 \right) \) is differentiable at \( \bar{e} \).

(iii) \( \bar{G} \) maximizes the function \( \bar{G}' \mapsto V \left( \bar{s}, \bar{e}, \bar{G}', z^0 \right) \) over \( \mathcal{S} \mathcal{A} \left( z^0 | \bar{s}, \bar{e} \right) \) if there exists a sequence \( \bar{\omega} \) such that, for every \( i, t \):

\[
m_i \left( s^i_t, e^i_t \right) \Delta^t_{ij} - e^i_t \left( e^e_{ij} + \kappa_{ij} - m_i \bar{e}_i - \phi_{ij} \right) \leq \omega^t_i, \tag{77}
\]

with equality if \( G_{ij} > 0 \).

This condition is also necessary whenever the function \( \bar{G}' \mapsto V \left( \bar{s}, \bar{e}, \bar{G}', z^0 \right) \) is differentiable at \( \bar{G} \).

\textbf{Proof.} We apply Lemma 8 to Problem 70

\[
P \left( \bar{s}, \bar{e}, \bar{G} \right) : V^p \left( \bar{s}, \bar{e}, \bar{G}, z^0 \right) = \max_{\bar{q}, \bar{b}} \sum_{t=0}^{\infty} \sum_{i,j} \beta^t W^p \left( s^i_t, e^i_t, G^t_{ij}, q^t_{ij}, b^t_{ij}, z^t \right)
\]

\[
s.t. \left( \bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b} \right) \in \mathcal{A} \left( z^0 \right).
\]

In doing so, notice that the assumptions of Lemma 8 are satisfied, since by Lemma 9 function \( W^p \) is continuously differentiable, and we can take feasible allocations and states to live inside compact set.\footnote{Indeed, let \( M = \sum_{ij} x^0_{ij} \). Then for every \( \left( \bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b} \right) \in \mathcal{A} \left( z^0 \right) \) we must have}

\[
\forall t, i, j : 0 \leq s^i_t, q^t_{ij}, b^t_{ij}, z^t_{ij} \leq M.
\]

Moreover, letting \( e^*, q^*, b^* \) be a solution of

\[
\max_{e, q, b \geq 0} W \left( q \right) - \sum_i e_i \left( \min_j c^e_{ij} + \min_j \kappa_{ij} \right)
\]

\[
s.t. q_{ij} \leq M
\]

\[
\sum_j q_{ij} \leq m_i \left( M, e_i \right)
\]

we have that every sequence \( \bar{e} \) such that \( e^t_i > e^*_i \) for some \( t, i \) is clearly sub optimal, hence without loss of generality we can take

\[
0 \leq e^t_i, b^t_{ij} \leq e^*_i.
\]
We use the following notation for the Lagrangian multipliers:

multiplier constraint

\[
\begin{align*}
\mu_{i}^{s,t} & \geq 0 \\
\mu_{ij}^{e,t} & \geq 0 \\
\psi_{i}^{t} & = \sum_{j} (q_{ij}^{t} + b_{ij}^{t}) \\
\Delta_{ij}^{t} & \leq m_{i} \left( s_{i}^{t}, e_{i} \right) G_{ij}^{t} \\
-w_{i}^{t} & \sum_{j} G_{ij}^{t} = 1
\end{align*}
\]

Moreover, we denote \( \bar{\phi} = (\bar{\phi}^{s}, \bar{\phi}^{e}) \), where \( \bar{\phi}^{s} \) is the component of \( \bar{\phi} \) associated with the supply component of the state and \( \bar{\phi}^{e} \) is associated with the demand component. With this notation in hand, the set of Conditions 71 is given by

\[
\forall t, i, j : \mu_{i}^{s,t} \geq 0 \text{ with equality if } s_{i}^{t} < x_{i}^{t} \\
\mu_{ij}^{e,t} \geq 0 \text{ with equality if } e_{i}^{t} G_{ij}^{t} > y_{ij}^{t} \\
\Delta_{ij}^{t} \geq 0 \text{ with equality if } q_{ij}^{t} < m_{i} \left( s_{i}^{t}, e_{i} \right) G_{ij}^{t}
\]

the set of Conditions 72 is given by

\[
\forall t, i, j : \phi_{ii}^{s,t} = \mu_{i}^{s,t} + \beta \phi_{ii}^{s,t+1} \\
\phi_{ij}^{e,t} = \kappa_{ij} - \mu_{ij}^{e,t} \\
\phi_{ij}^{s,t} = -c_{ij} + \beta \left[ d_{ij} \phi_{jj}^{s,t+1} + (1 - d_{ij}) \phi_{ij}^{s,t+1} \right]
\]

and the set of Conditions 73 is given by

\[
\forall t, i, j : \Delta_{ij}^{t} \geq w_{ij} \left( q_{ij} ^{t} \right) + \phi_{ij}^{s,t} - \beta \mu_{ij}^{e,t+1} - \psi_{i}^{t} \text{ with equality if } q_{ij}^{t} > 0 \\
f \left( P_{i}^{b} \right) + \frac{\partial f \left( P_{i}^{b} \right)}{\partial P_{ij}} - \sum_{k} P_{ik}^{b} \frac{\partial f \left( P_{i}^{b} \right)}{\partial P_{ik}} + \phi_{ij}^{s,t} - \psi_{i}^{t} = 0.
\]
By Lemma 9, the set of conditions in the second line above is equivalent to
\[ \forall t, i, j : P^{h}_{ij} = \mathbb{P} \left[ \phi_{ij}^{s} = \max_k \left( \phi_{ik}^{s} + \epsilon_{ik} \right) \right] \text{ and } \psi_{i}^{t} = \mathbb{E}_{P} \max_j \left( \phi_{ij}^{s} + \epsilon_{ij} \right). \]

To prove Statement (i), we apply Lemma 8 to the function \( s' \mapsto V \left( s', \bar{e}, \bar{G}, z^{0} \right) \). Given our assumption that \( s_{t} > 0 \) for every \( t, i \), Condition 74 is given by
\[ \forall i, t : -c_{i}^{w} + \frac{\partial m_{i}}{\partial s_{i}} \sum_{j} G_{ij}^{t} \Delta_{ij}^{t} + \psi_{i}^{t} - \beta \phi_{ij}^{s} - \mu_{ij}^{s} = 0. \]

To prove Statement (ii), we apply Lemma 8 to the function \( e' \mapsto V \left( \bar{s}, \bar{e}', \bar{G}, z^{0} \right) \). Given our assumption that \( e_{t} > 0 \) for every \( t, i \), Condition 74 is given by
\[ \forall i, t : \frac{\partial m_{i}}{\partial e_{i}} \sum_{j} G_{ij}^{t} \Delta_{ij}^{t} - \sum_{j} G_{ij}^{t} \left( e_{ij}^{e} + \kappa_{ij} - \mu_{ij}^{e} - \beta \phi_{ij}^{e} \right) = 0. \]

To prove Statement (iii), we apply Lemma 8 to the function \( \bar{G}' \mapsto V \left( \bar{s}, \bar{e}, \bar{G}', z^{0} \right) \). Condition 74 is given by
\[ \forall i, j, t : m_{i} \left( s_{i}^{t}, e_{i}^{t} \right) \Delta_{ij}^{t} - e_{i}^{t} \left( e_{ij}^{e} + \kappa_{ij} - \mu_{ij}^{e} - \beta \phi_{ij}^{e} \right) - \omega_{i}^{t} + pos_{ij}^{t} = 0 \]
where \( pos_{ij}^{t} \) is the multiplier associated to the positivity constraint \( G_{ij} \geq 0 \), which must satisfy
\[ pos_{ij}^{t} \geq 0 \text{ with equality if } G_{ij}^{t} > 0. \]

This completes the proof.

**Proof of main result** In order to prove the main result, let everything be as in the statement. Let \( (V^{s,t}, V^{e,t}, U^{e,t}, \Delta^{s,t}, \Delta^{e,t})_{t=0}^{\infty} \) be the sequence of carriers and customers’ value functions and meeting surpluses associated with the sequence \( \bar{s}, \bar{e}, \bar{G}, \bar{q}, \bar{b} \) evaluated at the state trajectory \( z' \), \( t \geq 0 \). For every
$t \geq 0$ define $\phi^{s,t} = V^{s,t}$, $\phi^{e,t} = U^{e,t}$, $\psi^t = E_Z U^{s,t}$ ($e$), $\Delta^t = \Delta^{s,t} + \Delta^{e,t}$ and

$$
\mu^{s,t}_i = \max \left\{ -c^w_i + \lambda^s_i \left( z^t \right) \sum_{j \neq i} G^t_{ij} \Delta^{s,t}_{ij} + U^t_i - \beta V^{t+1}_i, 0 \right\}
$$

$$
\mu^{e,t}_{ij} = \kappa_{ij} - U^{e,t}_{ij}.
$$

Then $s, e, \tilde{G}, \tilde{q}, \tilde{b}, \tilde{\psi}, \tilde{\mu}^s, \tilde{\mu}^e, \tilde{\Delta}, \tilde{\phi}^s, \tilde{\phi}^e$ satisfies the conditions of Lemma 10. Moreover, notice that:

- Condition 75 writes as

$$
\forall i, t : \frac{\partial m_i \left( s^t_i, e^t_i \right)}{\partial s_i} \sum_j G^{t}_{ij} \left( \Delta^{s,t}_{ij} + \Delta^{e,t}_{ij} \right) - \lambda^s_i \left( z^t \right) \sum_{j \neq i} G^t_{ij} \Delta^{s,t}_{ij} = 0.
$$

Using $\lambda^s_i \left( z^t \right) = \frac{m_i \left( s^t_i, e^t_i \right)}{s_i}$ and rearranging, this is equivalent to

$$
\eta^s_i \left( z^t \right) \sum_j G^{t}_{ij} \left( \Delta^{s,t}_{ij} + \Delta^{e,t}_{ij} \right) = \sum_j G^t_{ij} \Delta^{s,t}_{ij}.
$$

- Condition 76 writes as

$$
\forall i, t : \frac{\partial m_i \left( s^t_i, e^t_i \right)}{\partial e_i} \sum_j G^{t}_{ij} \left( \Delta^{s,t}_{ij} + \Delta^{e,t}_{ij} \right) - \sum_j G^t_{ij} \left( c^e_{ij} + U^{e,t}_{ij} - \beta U^{e,t+1}_{ij} \right) = 0.
$$

Using $U^{e,t}_{ij} = -c^e_{ij} + \lambda^e_i \left( z^t \right) \Delta^{e,t}_{ij} + \beta U^{e,t+1}_{ij}$, $\lambda^e_i \left( z^t \right) = \frac{m_i \left( s^t_i, e^t_i \right)}{e_i}$ and rearranging, this is equivalent to

$$
\eta^e_i \left( z^t \right) \sum_j G^{t}_{ij} \left( \Delta^{s,t}_{ij} + \Delta^{e,t}_{ij} \right) = \sum_j G^t_{ij} \Delta^{e,t}_{ij}.
$$

- Condition 77 writes as

$$
\forall i, j, t : m_i \left( s^t_i, e^t_i \right) \Delta^t_{ij} - c^e_i \left( c^e_{ij} + U^{e,t}_{ij} - \beta U^{e,t+1}_{ij} \right) \leq \omega^t_i
$$

with equality if $G^t_{ij} > 0$. 

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Using $U_{ij}^e = -c_{ij}^e + \lambda_i^e(z^t) \Delta_{ij}^e + \beta U_{ij}^e(t+1)$, $\lambda_i^e(z^t) = \frac{m_i(s_i^e, e_i)}{e_i}$ and rearranging, this is equivalent to

$$\forall i, j, t : \Delta_{ij}^e \leq -\frac{\omega_i^t}{\lambda_i^e(z^t)}.$$  

with equality if $G_{ij}^t > 0$.

This completes the proof.

References


