

The Welfare and Distributional Effects of Fiscal Uncertainty: a Quantitative Evaluation

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Abstract

This study explores the welfare and distributional effects of fiscal uncertainty using a neoclassical stochastic growth model with incomplete markets. In our model, households face uninsurable idiosyncratic risks in their labor income and discount factor processes, and we allow aggregate uncertainty to arise from both productivity and government purchases shocks. We calibrate our model to key features of the U.S. economy, before eliminating government purchases shocks. We then evaluate the distributional consequences of the elimination of fiscal uncertainty and find that, in our baseline case, welfare gains decline with private wealth holdings.

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1 Introduction

One consequence of the financial crisis has been the perception of high policy uncertainty in both the U.S. and in Europe. In one study, Baker et al. (2015) analyze Internet news and find a (causal) relationship between high policy uncertainty and subdued aggregate economic activity. In another study, Fernández-Villaverde et al. (2011) find large contractionary effects of fiscal uncertainty on economic activity accompanied by inflationary pressure, especially when the nominal interest rate is at the zero lower bound. These findings are reinforced by a 2012 Economic Policy Survey among business economists (Economic Policy Survey (2012)): “the vast majority of a panel of 236 business economists ‘feels that uncertainty about fiscal policy is holding back the pace of economic recovery’.” Finally, Azzimonti (2014) shows that, in recent decades, political polarization in the U.S. has increased, which may lead to heightened fiscal uncertainty.¹

Most of the existing research on fiscal uncertainty has focused on the aggregate effects of short-run fluctuations of uncertainty on various macroeconomic variables (see below for a more detailed discussion of the literature). However, this literature has not explored the welfare and distributional consequences of fiscal uncertainty.² In this paper, we provide such an analysis and address the following question: how large are the welfare costs of fluctuations in government purchases for different wealth households?

To do so, we follow the approach of Krusell and Smith (1998) and use an incomplete market model where heterogeneous households face uninsurable idiosyncratic risks in their labor income and discount factor processes. We then calibrate this model with U.S. data, in particular data on U.S. wealth inequality. Our model has aggregate uncertainty arising from both productivity and government purchases shocks. We thus specify government purchases shocks as the only fundamental source of fiscal uncertainty. In line with the data, we further assume that government purchases shocks are independent of aggregate productivity and employment conditions. Government purchases enter the utility function of the households as separable goods.³

Because the government partially funds its expenditures through taxation, purchases fluctuations generate uncertain household tax rates. To capture the distributional effects of fiscal shocks through taxation, we model key features of the progressive U.S. income tax system. In a progressive tax system,

¹McCarty et al. (2006) makes a similar point in the political science literature.

²We follow the widespread use in the recent literature and treat “fiscal uncertainty” and “fiscal volatility” as synonymous in this paper.

³We consider other utility specifications with complementary and substitutable private-public good relationships, respectively, in extensions to the baseline calibration.

aggregate government purchases fluctuations may lead to changes in the distribution of household-specific tax rates and thus to idiosyncratic after-tax income uncertainty. We also employ an empirical tax revenue response rule, which includes government debt and is estimated from U.S. data.

To eliminate fiscal uncertainty, we follow Krusell and Smith (1999) and Krusell et al. (2009), start from a stochastic steady state of the economy with both productivity and government purchases shocks, and remove the fiscal shocks at a given point in time by replacing them with their conditional expectations, while retaining the aggregate productivity process. We then compute the transition path towards the new stochastic steady state in full general equilibrium. Based on the quantitative solution for this transition path, we then compare the welfare of various household groups in the transition-path equilibrium to their welfare level with both aggregate shocks in place.

Our results show that the magnitude of aggregate welfare costs from fiscal shocks is comparable to that of the welfare costs of business cycle fluctuations reported in Lucas (1987, 2003), even though in our model aggregate (spending) fluctuations lead to idiosyncratic after-tax income uncertainty. The welfare gains of eliminating fiscal uncertainty are decreasing in household wealth. However, alternative implementations of the progressive U.S. federal income tax system and the empirical tax revenue response rule may reverse this pattern. We also find that in two counterfactual tax systems, a linear income tax system and a lump sum tax system, the welfare gains of eliminating fiscal uncertainty increase with household wealth. An investigation of the reasons behind these distributional results uncovers the mechanisms through which fiscal uncertainty influences economic welfare. We conclude that the details of the (progressive) tax system determine which wealth group experiences the tax uncertainty burden caused by government purchases shocks, and thus benefits the most from the elimination of these shocks.

Since uncertain tax rates pre-multiply income levels, they generate – loosely speaking – multiplicative after-tax income risk.⁴ Just as with the additive endowment risk in early incomplete market models, this after-tax income risk leads households to self-insure through precautionary saving. Wealth-rich households can thus achieve a higher degree of self-insurance relative to wealth-poor households. Indeed, we find that wealth-rich households show less consumption volatility both before and after the elimination of fiscal uncertainty. They also experience a smaller reduction in consumption fluctuations due to the policy change. Consequently, from a precautionary saving perspective, wealth-poor households should gain more when fiscal uncertainty is eliminated.

⁴This is cleanest in a linear tax system. However, even in a progressive tax system, the fluctuating average tax rates work like multiplicative after-tax income risk.

However, due to the multiplicative nature of the after-tax income risk, the tax-rate uncertainty induced by government purchases fluctuations also creates a rate-of-return risk to savings, which in turn impacts the quality of capital and bonds as saving vehicles.⁵ From this perspective, eliminating fiscal uncertainty facilitates the intertemporal transfer of resources, and thus benefits the wealth-rich, who have more resources to save. Which of the two effects dominates, depends – as we will show – on the details of the implementation of the progressive tax system in the model. If the tax rate uncertainty induced by government purchases fluctuations is concentrated on households in the middle of the wealth distribution, the precautionary saving effect dominates, and the welfare gains are decreasing in household wealth. Under an implementation where the wealth-rich households are significantly exposed to the tax-rate volatility caused by fiscal uncertainty, the quality-of-saving-vehicle channel starts to matter more, and the distributional welfare gain pattern might be reversed.

Finally, the distributional effects of eliminating fiscal uncertainty can depend on general equilibrium price changes.⁶ The precautionary saving and quality-of-saving-vehicle channels lead to endogenous responses of the aggregate capital stock along the transition path, changing both the pre-tax capital rate-of-return and real wages. In our baseline specification, the aggregate capital stock declines after the elimination of fiscal uncertainty. This favors the wealth-rich through higher pre-tax rates of return, but disadvantages the wealth-poor through lower pre-tax real wages.

In addition to our baseline, we consider three counterfactual fiscal regimes: a balanced budget with a progressive tax system, a linear tax system, and a lump-sum tax system, with the latter two allowing for government debt. In another variation, we show that when private and public consumption are complements, the overall welfare gains from eliminating government purchases fluctuations are higher, because a higher government purchases level leads to a higher marginal utility of private consumption when taxes are high (because government purchases are high). That is, taxes are high when households would receive substantial utility from additional private consumption, making government purchases fluctuations all the more unpleasant. Finally, motivated by recent policy discussions of the possible permanence of heightened fiscal uncertainty, we examine the welfare consequences if we double the historical government purchases volatility level. Our results suggest that the welfare effects of fiscal uncertainty are symmetric between zero and twice the pre-crisis volatility of government purchases.

In addition to its substantive contributions, our study makes a technical contribution to the lit-

⁵Angeletos and Calvet (2006), in a seminal contribution on risk in incomplete markets, discuss this tension between labor endowment risk and rate-of-return uncertainty.

⁶There is also a direct utility effect because households are risk averse with respect to government purchases fluctuations.

erature. Specifically, we merge the algorithm for computing the *deterministic* transition path in heterogeneous-agent economies from Huggett (1997) and Krusell and Smith (1999), and the algorithm for computing a *stochastic* recursive equilibrium in Krusell and Smith (1998), to show that an approximation of the wealth distribution and its law of motion by a finite number of moments can also be applied to a stochastic transition path analysis. Recall that after fiscal uncertainty is eliminated, our economy is still subject to aggregate productivity shocks. This solution method may prove useful for other quantitative studies of stochastic transition-path equilibria.

Related Literature

Besides the general link to the literature on incomplete markets and wealth inequality (see Heathcote et al. (2009) for an overview), our study is most closely related to three strands of literature.

First, our paper contributes to research on the welfare costs of aggregate fluctuations (see Lucas (2003) for a comprehensive discussion). As in Krusell and Smith (1999), Mukoyama and Sahin (2006) and Krusell et al. (2009), we quantify the welfare and distributional consequences of eliminating macroeconomic fluctuations. However, while these studies focus on TFP fluctuations, we examine the welfare consequences of eliminating fluctuations in government purchases. Our study complements theirs by examining fluctuations due to fiscal policy, arguably a more plausible candidate shock to be (fully) eliminated by a policy maker – they are, after all, the result of a policy decision.

Second, our paper relates to the recent literature about the effects of economic uncertainty on aggregate economic activity. Most of the research in this stream of literature has focused on the amplification and propagation mechanisms for persistent, but temporary uncertainty shocks, which are typically modeled and measured as changes to the conditional variance of traditional economic shocks. These uncertainty shocks include second-moment shocks to aggregate productivity, and policy and financial variables, which are often propagated through physical production factor adjustment costs, sticky prices, or financial frictions (see e.g., Arellano et al. (2012), Bachmann and Bayer (2013, 2014), Baker et al. (2015), Basu and Bundick (2012), Bloom (2009), Bloom et al. (2012), Born and Pfeifer (2014), Croce et al. (2012), Fernández-Villaverde et al. (2011), Gilchrist et al. (2014), Kelly et al. (2014), Mumtaz and Surico (2015), Mumtaz and Zanetti (2013), Nodari (2014), Pastor and Veronesi (2012, 2013), and Stokey (2015)). Other studies investigate the effects of uncertainty in the time-varying parameters of monetary or fiscal feedback rules (Bi et al. (2013), Davig and Leeper (2011), and Richter and Throckmorton (2015)). Our study complements this literature by focusing on the welfare and distributional effects of a permanent elimination of fiscal fluctuations.

Finally, our work contributes to the growing body of literature on macroeconomic policy in heterogeneous-agent environments (Auclert (2015), Bachmann and Bai (2013), Bhandari et al. (2013), Boehm (2015), Brinca et al. (2015), Ferriere and Navarro (2014), Gornemann et al. (2012), Gomes et al. (2013), Heathcote (2005), Kaplan and Violante (2014), Li (2013) and McKay and Reis (2013)). There is also a budding empirical literature on the distributional consequences of policy actions: see Coibion et al. (2012) for the case of monetary policy, and Giorgi and Gambetti (2012) for the case of fiscal policy.

The remainder of the paper is structured as follows. Section 2 presents the model. Section 3 discusses its calibration. Section 4 describes our solution method. Section 5 presents the baseline findings on the welfare and distributional effects of eliminating government purchases fluctuations, while Section 6 investigates these welfare and distributional effects in alternative model specifications. We close in Section 7 with final comments and relegate the details of the quantitative procedure to various appendices.

2 Model

Following Aiyagari (1994) and Huggett (1993), we model an incomplete market setting where a continuum of infinitely-lived heterogeneous households face uninsurable idiosyncratic risks in their labor efficiency processes. We also include aggregate productivity shocks as well as shocks to a household's discount factor, as in Krusell and Smith (1998). We then add aggregate uncertainty from government purchases shocks. In our model exposition, we focus our discussion on the fiscal elements.

2.1 The private sector

Our households are *ex-ante* identical, with preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta_t u(c_t, G_t), \quad (2.1)$$

where β_t denotes the cumulative discount factor between period 0 and period t . In particular, $\beta_t = \tilde{\beta} \beta_{t-1}$, where $\tilde{\beta}$ is an idiosyncratic shock following a three-state, first-order Markov process. Furthermore, c_t denotes private consumption, and G_t the public good provided by the government (government purchases).

The strictly concave flow utility function has constant relative risk aversion (CRRA) with respect to a constant-elasticity-of-substitution (CES) aggregate of c and G ,

$$u(c_t, G_t) = \frac{\left(\theta c_t^{1-\rho} + (1-\theta) G_t^{1-\rho}\right)^{\frac{1-\gamma}{1-\rho}} - 1}{1-\gamma}, \quad (2.2)$$

where γ is the risk aversion parameter and $1/\rho$ is the elasticity of substitution between c and G . We discuss the details of the G_t -process in the next subsection.

Our households also face idiosyncratic employment shocks. We denote the employment process by ε , which follows a first-order Markov process with two states $\{0, 1\}$. $\varepsilon = 1$ denotes that the household is employed, providing a fixed amount of labor \tilde{l} to the market, and is paid the market wage, w . $\varepsilon = 0$ represents the unemployed state of a household who receives an unemployment insurance payment that equals a fraction ω of the current wage income of an employed household.

We represent the aggregate production technology as a Cobb-Douglas function:

$$Y_t = z_t F(K_t, L_t) = z_t K_t^\alpha L_t^{1-\alpha}, \quad (2.3)$$

where K_t is aggregate capital, L_t is aggregate labor efficiency input, and z_t is the aggregate productivity level. z_t follows a two-state (z_g, z_b) first-order Markov process, where z_g and z_b denote aggregate productivity in good and bad times, respectively. Note that, because of the law of large numbers, L_t equals $(1 - u_t)\tilde{l}$, where u_t is the unemployment rate. We also allow the unemployment rate to take one of two values: u_g in good times and u_b in bad times. In this way, u_t and z_t move perfectly together.

We now specify the standard aggregate resource constraint:

$$C_t + K_{t+1} + G_t = Y_t + (1 - \delta)K_t, \quad (2.4)$$

where C_t represents aggregate consumption, and δ the depreciation rate.

The markets in our model are perfectly competitive. Labor and capital services are traded on spot markets each period, at factor prices $r(K_t, L_t, z_t) = \alpha z_t K_t^{\alpha-1} L_t^{1-\alpha} - \delta$ and $w(K_t, L_t, z_t) = (1 - \alpha) z_t K_t^\alpha L_t^{-\alpha}$. In addition, we assume that the households can trade one-period government bonds on the asset market in each period t . For computational tractability, we follow Heathcote (2005) and assume that government bonds pay the same rate-of-return as physical capital in all future states in $t + 1$. Because of the assumed perfect substitutability between capital and bonds, each household has

access to effectively only one asset in self-insuring against stochastic shocks. We use a to denote a household's total asset holdings, i.e., the sum of physical capital and government bonds.

2.2 Fiscal uncertainty and the government budget

Our model has three government spending components: government purchases, G_t , aggregate unemployment insurance payments, Tr_t , and aggregate debt repayments, $(1+r_t)B_t$. Government purchases are the only fundamental source of fiscal uncertainty. They follow an AR(1) process in logarithms:

$$\log(G_{t+1}) = (1 - \rho_g) \overline{\log(G)} + \rho_g \log(G_t) + (1 - \rho_g^2)^{\frac{1}{2}} \sigma_g \epsilon_{g,t+1}, \quad (2.5)$$

where ρ_g is a persistence parameter, $\overline{\log(G)}$ is the unconditional mean of $\log(G_t)$, $\epsilon_{g,t+1}$ is an innovation term which is normally distributed with mean zero and variance one, and σ_g is the unconditional standard deviation of $\log(G_t)$. Note that the government purchases process is independent of the process for aggregate productivity. As is well known and as we show below, government purchases are roughly acyclical in U.S. quarterly data.

The aggregate unemployment insurance payment, $Tr_t = u_t \omega w_t \tilde{l}$, depends on both the unemployment rate, u_t , and the size of the unemployment insurance payment for each household, $\omega w_t \tilde{l}$.

We assume that government spending at time t is financed through a combination of aggregate tax revenue, T_t , and new government debt, B_{t+1} . As in Bohn (1998) and Davig and Leeper (2011), we model the aggregate tax revenue net of transfers (as a fraction of GDP) as an (increasing) function of the debt-to-GDP ratio, making the debt-to-GDP ratio stationary. We can thus specify the following fiscal rule for determining tax revenue:

$$\frac{T_t - Tr_t}{Y_t} = \rho_{T,0} + \rho_{T,Y} \log\left(\frac{Y_t}{\bar{Y}}\right) + \rho_{T,B} \frac{B_t}{Y_t} + \rho_{T,G} \frac{G_t}{Y_t}, \quad (2.6)$$

where $(\rho_{T,0}, \rho_{T,Y}, \rho_{T,B}, \rho_{T,G})$ is a vector of positive coefficients and \bar{Y} is a constant number equal to the unconditional mean of GDP in the ergodic distribution.⁷ Furthermore, $\rho_{T,Y}$ captures the automatic stabilizer role of the U.S. tax system when $\rho_{T,Y} > 0$, and $\rho_{T,B}$ and $\rho_{T,G}$ reflect the capability of the endogenous revenue adjustment system in maintaining long-run fiscal sustainability. Note that our fiscal rule implies that the government purchases level (relative to GDP) and the GDP gap are the main non-debt determinants of the primary surplus.

⁷ \bar{Y} serves as a normalization to make the coefficients of the fiscal rule scale-free. Also, while $\rho_{T,B} > 0$ is necessary for the debt-to-GDP ratio to be stationary, this condition is not imposed. Instead, all coefficients in equation (2.6) are estimated from the data, and this estimated $\rho_{T,B}$ just turns out to be positive.

Given the total tax revenue in (2.6), we can use the government budget constraint to determine the dynamics of aggregate government debt B_{t+1} :

$$B_{t+1} = (1 + r_t)B_t + (G_t + Tr_t - T_t). \quad (2.7)$$

2.3 The progressive tax system

Because the distribution of the tax burden across households is important for quantifying the distributional effects of fiscal policies, we model the tax system to approximate the current U.S. tax regime as realistically as possible while maintaining a certain tractability. Specifically, the government uses a flat-rate consumption tax and a progressive income tax to raise the aggregate tax revenue T_t . The consumption tax is given by:

$$\tau^c(c_t) = \tau_c c_t. \quad (2.8)$$

This specification allows the model to capture sources of tax revenue other than income taxes, which in turn provides a total income tax burden that is in line with the data.

Following Castañeda et al. (2003), we specify the progressive income tax function as:

$$\tau^y(y_t) = \begin{cases} \tau_1 \left[y_t - (y_t^{-\tau_2} + s)^{-\frac{1}{\tau_2}} \right] + \tau_0 y_t & \text{if } y_t > 0 \\ 0 & \text{if } y_t \leq 0, \end{cases} \quad (2.9)$$

where $(\tau_0, \tau_1, \tau_2, s)$ is a vector of tax coefficients and y_t is taxable household income; or $y_t = r_t a_t + w_t \varepsilon_t \tilde{l}$.⁸ The first term in the above equation is based on Gouveia and Strauss' (1994) characterization of the effective federal income tax burden of U.S. households. The federal income tax accounts for about 40% of federal government revenue and is the main driver of progressivity in the U.S. tax system (Piketty and Saez, 2007). The linear term, $\tau_0 y_t$, is used to capture any remaining tax revenue, including state income taxes, property taxes and excise taxes.

With these tax specifications, a household's budget constraint can be written as:

$$(1 + \tau_c)c_t + a_{t+1} = a_t + y_t - \tau^y(y_t) + (1 - \varepsilon_t)\omega w(K_t, L_t, z_t)\tilde{l}. \quad (2.10)$$

Note that equation (2.6) specifies a fiscal rule to calculate the aggregate government tax revenue.

⁸Unlike in Castañeda et al. (2003), where households cannot borrow and thus cannot have negative income, y_t can be negative in our model in rare cases, so that we have to specify the tax function also for the case of $y_t < 0$.

Equations (2.8) and (2.9), on the other hand, model the concrete tax instruments with which the government collects the aggregate tax revenue. These two sets of equations are compatible only if we treat one of the parameters in equation (2.9) as an endogenous tax instrument, to be determined in equilibrium, rather than a fixed tax parameter. Following Conesa and Krüger (2006) and Conesa et al. (2009), we choose, in the baseline specification, s for this endogenous parameter, s_t , and denote the resulting tax function by $\tau^y(y_t; s_t)$. Adjusting s means that the overall progressivity of the tax system becomes the main tax instrument to raise the tax revenue required by the fiscal rule, while the top marginal (average) tax rates are approximately constant at $\tau_0 + \tau_1$.⁹ This adjustment allows us to satisfy the empirical fiscal rule that describes aggregate U.S. tax adjustments well, and, more importantly, ensures the stationarity of the debt-to-GDP ratio. Consequently, we take the empirical fiscal rule as given and endogenously adjust one aspect of the tax system to make the two sets of equations compatible as in Davig and Leeper (2011) and Fernández-Villaverde et al. (2011).

Given our tax function specification, we can now specify total tax revenue as follows:

$$T_t = \tau_c C_t + \int_0^1 \left[\tau_0 y_{i,t} + \tau_1 \left(y_{i,t} - (y_{i,t}^{-\tau_2} + s_t)^{-\frac{1}{\tau_2}} \right) \right] * \mathbb{1}(y_{i,t} > 0) di. \quad (2.11)$$

Equation (2.11) defines an implicit function of s_t . Recall that T_t is governed by G_t , Y_t , B_t , and Tr_t through the fiscal rule specified in equation (2.6). This means that, for a given inherited level of bond holdings, B_t , s_t fluctuates in response to changes in both G_t and the income distribution. As a result, in our baseline model the *aggregate* uncertainty in G_t translates into *idiosyncratic* tax rate uncertainty.

2.4 The household's decision problem and the competitive equilibrium

In this subsection, we discuss the household's dynamic decision problem, which is determined by both the idiosyncratic state vector $(a, \varepsilon, \tilde{\beta})$ and the aggregate state vector (Γ, B, z, G) , where Γ denotes the measure of households over $(a, \varepsilon, \tilde{\beta})$. We begin by letting H_Γ denote the equilibrium transition function for Γ :¹⁰

$$\Gamma' = H_\Gamma(\Gamma, B, z, G, z'). \quad (2.12)$$

⁹Both the derivative of equation (2.9) and equation (2.9) divided by y_t converge to $\tau_0 + \tau_1$ for large y_t . In Section 6, we examine two alternative specifications, where we let τ_0 and τ_1 , respectively, be the tax instruments that adjust endogenously.

¹⁰Note that z' , but not G' , is an argument of H_Γ . This is because, in our setting, which reflects the setting in Krusell and Smith (1998), the future z affects the employment transition process, while the G -process is independent of other processes. Note that we also leave time subscripts and switch into recursive notation now.

We next let H_B denote the (exogenous) transition function for B , as described in equation (2.7):

$$B' = H_B(\Gamma, B, z, G). \quad (2.13)$$

Finally, we let S denote the equilibrium function for the endogenous tax parameter s , which is implicitly determined in equation (2.11):

$$s = S(\Gamma, B, z, G). \quad (2.14)$$

The dynamic programming problem faced by a household can now be written as follows:

$$V(a, \varepsilon, \tilde{\beta}, \Gamma, B, z, G; H_\Gamma, S) = \max_{c, a'} \{u(c, G) + \tilde{\beta}E[V(a', \varepsilon', \tilde{\beta}', \Gamma', B', z', G'; H_\Gamma, S) | \varepsilon, \tilde{\beta}, z, G]\}$$

subject to:

$$\begin{aligned} (1 + \tau_c)c + a' &= a + y - \tau^y(y; s) + (1 - \varepsilon)\omega w(K, L, z)\tilde{l} \\ y &= r(K, L, z)a + w(K, L, z)\varepsilon\tilde{l}, \\ a' &\geq \underline{a}, \\ \Gamma' &= H_\Gamma(\Gamma, B, z, G, z'), \\ B' &= H_B(\Gamma, B, z, G), \\ s &= S(\Gamma, B, z, G), \end{aligned}$$

where ε and $\tilde{\beta}$ follow the processes specified in Section 2.1, G follows the process specified in equation (2.5), and \underline{a} is an exogenously set borrowing constraint. Finally, we can summarize the optimal saving decision for households in the following policy function:

$$a' = h(a, \varepsilon, \tilde{\beta}, \Gamma, B, z, G; H_\Gamma, S). \quad (2.15)$$

Our recursive competitive equilibrium is then defined as: the law of motion H_Γ ,¹¹ individual value and policy functions $\{V, h\}$, pricing functions $\{r, w\}$, and the S -function for the endogenous parameter s , such that:

¹¹Note that since H_B is exogenously determined by equation (2.7), it is not an equilibrium object.

1. $\{V, h\}$ solve the household's problem.
2. $\{r, w\}$ are competitively determined.
3. S satisfies equation (2.11) with the fiscal rule (2.6) replacing T_t .
4. H_Γ is generated by h .

The economy without a fluctuating G_t is identical, except for the deterministic G_t -process.

3 Calibration

In this section, we discuss our model calibration beginning with basic parameters. The frequency of our model economy is quarterly. We parameterize the model to match important aggregate and cross-sectional statistics of the U.S. economy (Table 1).

Table 1: Summary of parameters

Parameter	Value	Description	Source / Target
Taken from the literature			
$1/\rho$	1.00	Elasticity of substitution between c and G	Standard value
γ	1.00	Relative risk aversion	Standard value
α	0.36	Capital share	Standard value
δ	0.025	Depreciation rate	Standard value
\bar{l}	0.3271	Hours of labor supply of employed	Normalization
(z_l, z_h)	(0.99, 1.01)	Support of aggr. productivity process	Krusell and Smith (1998)
$\Pi_{z,z'}$	See text	Transition matrix of aggr. productivity process	Krusell and Smith (1998)
(u_g, u_b)	(4%, 10%)	Possible unemployment rates	Krusell and Smith (1998)
$\Pi_{\varepsilon\varepsilon' zz'}$	See text	Transition matrix of employment process	Krusell and Smith (1998)
ω	0.10	Replacement rate	Krusell and Smith (1998)
(τ_1, τ_2)	(0.258, 0.768)	Parameters in the progressive tax function	Gouveia and Strauss (1994)
Estimated from the data			
τ_0	5.25%	Income tax parameter	
τ_c	8.14%	Consumption tax rate	
$\rho_{T,B}$	0.0173	Debt coefficient of fiscal rule	
$\rho_{T,Y}$	0.2820	Output coefficient of fiscal rule	
$\rho_{T,G}$	0.4835	Government purchases coefficient of fiscal rule	
$(G_l/G_m, G_h/G_m)$	(0.951, 1.049)	Size of the G -shock	
$\Pi_{G,G'}$	See text	Transition matrix of the G -process	
Calibrated in the model			
θ	0.7221	Weight on private consumption in utility function	Lindahl-Samuelson condition
G_m	0.2319	Value of the middle grid of the G -process	Mean G/Y (20.86%)
\underline{a}	-4.15	Borrowing constraint	Fraction with negative wealth (11%)
$\rho_{T,0}$	0.1007	Intercept of tax revenue rule	Mean B/Y (30%)
$\bar{\beta}_m$	0.9919	Medium value of discount factor	Mean K/Y (2.5)
$\tilde{\beta}_h - \tilde{\beta}_m, \tilde{\beta}_m - \tilde{\beta}_l$	0.0046	Size of discount factor variation	Gini coeff. (0.79)
$\Pi_{\tilde{\beta},\tilde{\beta}'}$	See text	Transition matrix of discount factor	Share of top 1% in the wealth distribution (30%)

3.1 Basic parameters

We set the relative risk aversion parameter $\gamma = 1$, and the elasticity of substitution between private consumption and the public good $1/\rho = 1$. To calibrate the weight of private consumption in the utility function, θ , we assume that the Lindahl-Samuelson condition holds for our economy in the long-run. This means that there is efficient provision of public goods, i.e., there are equalized marginal

utilities from private and public goods. Mathematically, this is represented as $\int_0^1 \frac{(1-\theta)/G_t}{\theta/c_{it}} di = 1$, on average over many time periods. With this procedure, θ is calibrated to 0.722.

We take other parameter values directly from Krusell and Smith (1998): the depreciation rate is $\delta = 0.025$, the capital elasticity of output in the production function is $\alpha = 0.36$, and labor supply is normalized to $\tilde{l} = 0.3271$. We allow our aggregate productivity process, z_t , to take on two values, $z_g = 1.01$ and $z_b = 0.99$, with unemployment rates of $u_g = 0.04$ and $u_b = 0.1$, respectively. The transition matrix for z_t is as follows:

$$\begin{bmatrix} 0.875 & 0.125 \\ 0.125 & 0.875 \end{bmatrix},$$

where rows represent the current state and columns represent the next period's state. The first row and column correspond to z_g . The transition matrix for the employment status, ε , is a function of both the current aggregate state (z) and the future aggregate state (z'). There are thus four possible cases, (z_g, z_g) , (z_g, z_b) , (z_b, z_g) , and (z_b, z_b) , corresponding to the following employment status transition matrices:¹²

$$\begin{bmatrix} 0.33 & 0.67 \\ 0.03 & 0.97 \end{bmatrix}, \begin{bmatrix} 0.75 & 0.25 \\ 0.07 & 0.93 \end{bmatrix}, \begin{bmatrix} 0.25 & 0.75 \\ 0.02 & 0.98 \end{bmatrix}, \begin{bmatrix} 0.60 & 0.40 \\ 0.04 & 0.96 \end{bmatrix},$$

where the first row and column correspond to $\varepsilon = 0$ (unemployed).

We calibrate the borrowing constraint and the idiosyncratic time preference process to match key features of the overall wealth distribution in the U.S. The borrowing constraint is set to $\underline{a} = -4.15$ to match the fraction of U.S. households with negative wealth holdings, 11%.¹³

$\tilde{\beta}$ takes on values from a symmetric grid, ($\tilde{\beta}_l = 0.9873$, $\tilde{\beta}_m = 0.9919$, $\tilde{\beta}_h = 0.9965$). In the invariant distribution, 96.5% of the population is in the middle state, and 1.75% is distributed across each of the extreme points. The expected duration of the extreme discount factors is set at 50 years, to capture a dynastic element in the evolution of time preferences (Krusell and Smith (1998)). In addition, transitions occur only across adjacent values, where the transition probability from either extreme value to the middle grid is $1/200$, and the transition probability from the middle grid to either extreme value is $7/77200$. This Markov chain for $\tilde{\beta}$ allows our model to generate a long-run U.S. capital-output ratio of 2.5, and a Gini coefficient for the U.S. wealth distribution of 0.79, as reported by Krusell and Smith (1998). It also allows our model to match the wealth share of the top

¹²The numbers are rounded to the second decimal point.

¹³We check that the total resources available to a household, taking into account unemployment insurance benefits and the borrowing limit, are never negative under this calibration.

1% (Krusell and Smith (1998)). An accurate calibration of this moment is important because, as we will show, the welfare effects of fiscal uncertainty for top wealth holders, characterized by high levels of buffer-stock savings and high capital income, can be quantitatively rather different from those for other households.

3.2 Fiscal parameters

Regarding the fiscal parameters, we set the unemployment insurance replacement rate, ω , to 10% of the current market wage income, in line with the data. From Stone and Chen (2014) we know that the overall replacement rate from unemployment insurance is about 46% of a worker’s wage, and its average pre-2008 benefits duration is 15 weeks. This translates to about 53% of a worker’s quarterly wage. In our case, since we spread the unemployment benefits through the agent’s whole unemployment period and the average duration of unemployment in the model is about 2 quarters, this translates to about 27% of the quarterly wage level. Moreover, from Auray et al. (2014) we know about 60% out of all the unemployed workers were eligible for unemployment benefits from 1989 to 2012, and that about 75% of those eligible for benefits actually collected them. Thus, we set our unemployment insurance payment to be 10% of the market wage.¹⁴

To estimate the parameters related to fiscal policy and individual tax rates, we use U.S. quarterly data from 1960I to 2007IV. We restrict the data window up to 2007IV because, arguably, fiscal policy was special during and after the Great Recession and for calibration purposes we want to focus on “normal” times. We provide the details of our fiscal parameter estimation in Appendix A. Here we briefly outline the general procedure.

For the government purchases process, we use the Rouwenhorst method (Rouwenhorst (1995)) to construct a three-state first-order Markov chain approximation to the AR(1) process of the linearly detrended $\log(G)$ series.¹⁵ The middle grid point of the G -process, G_m , is calibrated using the average G/Y -ratio in the data; see Appendix A.1 for the details.

To determine the parameters of our fiscal tax revenue rule, we first estimate the federal revenue

¹⁴Our calibration also matches the aggregate data on unemployment insurance well: 0.0049 for the average unemployment insurance to output ratio (0.0041 in the data), and 0.0021 for its standard deviation, after removing a linear trend (0.0019 in the data). In both the model and the data, the unemployment insurance-to-output ratio is countercyclical. Also note that in Krusell and Smith (1998), the unemployment insurance is treated as a fixed amount, ψ , calibrated to be about 10% of the long-run quarterly wage.

¹⁵Kopecky and Suen (2010) show that the Rouwenhorst method has an exact fit in terms of five important statistical properties: unconditional mean, unconditional variance, correlation, conditional mean and conditional variance. The last two properties are important for our elimination of fiscal uncertainty, where both the conditional mean and variance matter for the transition-path equilibrium.

rule as in Bohn (1998) and Davig and Leeper (2011), and the state and local rule without debt. We then take the weighted average of the federal rule and the state and local rule to get the general government tax revenue function, the empirical counterpart of our model. We describe the details of this procedure in Appendix A.2.

For the consumption tax parameters and the linear part of the income tax function, we follow standard procedures and calculate the time series of the corresponding tax rates from the quarterly NIPA data (see, e.g., Fernández-Villaverde et al. (2011) and Mendoza et al. (1994)). We then take the time-series average values to obtain the following tax rates: $\tau_c = 8.14\%$ and $\tau_0 = 5.25\%$; see Appendix A.3 for the details.

To model the progressivity of the U.S. tax system, we set the parameters of the progressive part of the income tax function to the values estimated by Gouveia and Strauss (1994) for U.S. data from 1989 (see Castañeda et al. (2003) and Conesa and Krüger (2006)), the last year in their sample. Recall that the progressive part of the income tax has the following form:

$$\tau_1 \left[y - (y^{-\tau_2} + s)^{-\frac{1}{\tau_2}} \right]. \quad (3.1)$$

Note that equation (3.1) is linearly homogeneous in y , if s is readjusted appropriately. Consequently, doubling one's income would lead to a doubling of the tax revenue collected from this income, if s adjusts to the new scale. Since the units of s are an innocuous choice, we can use the estimated numbers from Gouveia and Strauss (1994), $(\tau_1, \tau_2) = (0.258, 0.768)$.¹⁶ As mentioned earlier, s is an equilibrium object determined by the fiscal rule.

3.3 The wealth distribution and business cycle moments

In this section, we examine the wealth distribution and the business cycle moments, focusing on the fiscal variables, generated by our calibrated model. For our model to be a suitable laboratory for the experiment of eliminating fiscal uncertainty, and for producing reliable quantitative answers to our welfare and distributional questions, it should broadly match these aspects of the data.

Table 2 compares the long-run wealth distribution generated by our model with both the data and the model results in Krusell and Smith (1998). From Table 2, we see that our wealth distribution is a good match for the U.S. wealth distribution, especially for those in the top 1 percent.

¹⁶Note that the estimation in Gouveia and Strauss (1994) is carried out on annual federal income tax data, whereas our model frequency is quarterly. Given the nonlinear nature of the tax function (equation 3.1), this may raise a time aggregation issue. We therefore estimate the implied tax function from simulated annual income and annual tax payment data (aggregated from simulated quarterly observations). The results from this estimation are very close to those from the annual data.

Table 2: Wealth distribution

	% of wealth held by top					Fraction with wealth < 0	Gini coefficient
	1%	5%	10%	20%	30%		
Model	31%	59%	72%	81%	87%	11%	0.79
K&S	24%	54%	72%	87%	91%	11%	0.81
Data	30%	51%	64%	79%	88%	11%	0.79

Notes: The wealth distribution in the data is taken from Krusell and Smith (1998). Household wealth in our model is the sum of physical capital and government bonds.

Table 3 provides the results of a comparison between the key business cycle moments generated by the model and those from the data. This comparison includes output, tax revenue, and government purchases volatility and persistence. We calculate the same moments for the output ratios of tax revenue, government purchases and federal government debt. Finally, we examine the co-movements of these series with output and government purchases.

Table 3: Business cycle moments

A: Data (1960 I - 2007 IV)						
	Y	T-Tr	G	(T-Tr/Y)	(G/Y)	(B/Y)
Standard deviation	0.0149	0.0543	0.0134	0.0123	0.0083	0.0772
Autocorrelation	0.8616	0.8134	0.7823	0.9045	0.9573	0.9945
Corr(Y,X)	1	0.7242	0.0992	0.4791	-0.3826	-0.0472
Corr(G,X)	0.0992	0.0352	1	0.0345	0.4806	-0.0281
B: Model simulation						
	Y	T-Tr	G	(T-Tr/Y)	(G/Y)	(B/Y)
Standard deviation	0.0235 (0.0018)	0.0415 (0.0041)	0.0123 (0.0025)	0.0063 (0.0009)	0.0086 (0.0012)	0.0403 (0.0151)
Autocorrelation	0.5841 (0.0561)	0.5870 (0.0558)	0.6978 (0.0582)	0.8185 (0.0572)	0.8254 (0.0546)	0.9754 (0.0305)
Corr(Y,X)	1 (0)	0.9892 (0.0043)	-0.0013 (0.1294)	0.6939 (0.1044)	-0.6433 (0.0940)	-0.1791 (0.1537)
Corr(G,X)	-0.0013 (0.1294)	0.1316 (0.1296)	1 (0)	0.2488 (0.1116)	0.3804 (0.1079)	-0.0086 (0.0571)

Notes: In Panel A, Y , $T - Tr$ and G are HP-filtered (with a smoothing parameter of 1600) real log series of output, tax revenue net of transfers and government purchases, respectively. $(T - Tr)/Y$, G/Y and B/Y are linearly detrended output ratios of tax revenue net of transfers, government purchases and federal government debt, respectively. The data sources are documented in Appendix A.2. In Panel B, all variables are defined and filtered the same way as those in Panel A. The reported numbers are the average values from 1,000 independent simulations of the same length as the data (192 quarters). We show the standard deviations across these simulations in parentheses.

From Table 3, we see that our baseline model is successful in matching most of the business cycle moments, with the exception of output volatility (which is about 70% larger in the model). Even without fiscal uncertainty, as in Krusell and Smith (1998), the model produces higher output fluctuations than found in the data, while the introduction of fiscal uncertainty does not contribute substantially to the volatility of output. To check whether our welfare results are affected by this

feature of the model, we conduct a robustness check where we recalibrate the aggregate productivity process so that the model matches the output volatility in the data, leaving our baseline results unchanged.

4 Computation

4.1 Stochastic steady state

To compute the model's equilibrium with two aggregate shocks, we use the approximate aggregation technique proposed by Krusell and Smith (1998).¹⁷ This technique assumes that households act as if only a limited set of moments of the wealth distribution matters for predicting the future of the economy, and that the aggregate result of their actions is consistent with their perceptions of how the economy evolves. However, in contrast to Krusell and Smith (1998), we find that higher moments of the wealth distribution are necessary in our model with progressive taxation. That is, the accurate description of our economy's evolution requires a combination of average physical capital and the Gini coefficient of the wealth distribution.

Furthermore, the optimization problem in our model requires households to know the endogenous tax parameter, s . We therefore approximate the function S , as defined in equation (2.14), with a parameterized function of the same moments that represent the wealth distribution.¹⁸ We can now state the following functional forms for H_T and S :¹⁹

$$\begin{aligned} \log(K') = & a_0(z, G) + a_1(z, G)\log(K) + a_2(z, G)B + a_3(z, G)(\log(K))^2 + a_4(z, G)B^2 \\ & + a_5(z, G)B^3 + a_6(z, G)\log(K)B + a_7(z, G)Gini(a), \end{aligned} \quad (4.1)$$

$$\begin{aligned} Gini(a') = & \tilde{a}_0(z, G) + \tilde{a}_1(z, G)\log(K) + \tilde{a}_2(z, G)B + \tilde{a}_3(z, G)(\log(K))^2 + \tilde{a}_4(z, G)B^2 \\ & + \tilde{a}_5(z, G)B^3 + \tilde{a}_6(z, G)\log(K)B + \tilde{a}_7(z, G)Gini(a), \end{aligned} \quad (4.2)$$

¹⁷The solution method for the stochastic steady state of the model with only aggregate productivity shocks is the same, except that $G_t = G_m, \forall t$.

¹⁸This is in the same spirit as the bond price treatment in Krusell and Smith (1997).

¹⁹These specific functional forms perform best among a large set of (relatively parsimonious) functional forms tested.

$$\begin{aligned} \log(s) = & b_0(z, G) + b_1(z, G)\log(K) + b_2(z, G)B + b_3(z, G)(\log(K))^2 + b_4(z, G)B^2 \\ & + b_5(z, G)B^3 + b_6(z, G)\log(K)B + b_7(z, G)Gini(a), \end{aligned} \quad (4.3)$$

where K denotes the average physical capital, and $Gini(a)$ denotes the Gini coefficient of the wealth distribution. We compute the equilibrium using a fixed-point iteration procedure from the parameters in equations (4.1)-(4.3) onto themselves; see Appendix B.1 for the details of the computational algorithm and Appendix B.2 for the estimated equilibrium laws of motions.

A check of the one-step-ahead forecast accuracy yields $R2s$ above 0.999997 for H_Γ (equations (4.1) and (4.2)), and above 0.9992 for S (equation (4.3)). However, as den Haan (2010) points out, high $R2$ -statistics are not necessarily indicative of multi-step-ahead forecast accuracy. Hence, we also examine the 10-year ahead forecast errors of our model. This check shows that our forecast errors are small and unbiased; see Appendix B.2 for the details.

4.2 Transition-path equilibrium

To study the welfare effects of eliminating fiscal uncertainty, we start with the ergodic distribution of the two-shock equilibrium. From time $t = 1$, we let G_t follow its deterministic conditional mean along the transition path until it converges to G_m . While we do not take a stance on how this stabilization is brought about (Lucas (1987) and Krusell et al. (2009)), we do note that, in contrast to stabilizing aggregate productivity shocks, the G_t -process is arguably under more direct government control.

As stated, during the transition periods G_t follows a time-dependent deterministic conditional-mean process until it converges to G_m , i.e.,

$$G_t = [\mathbb{1}(G_1 = G_l), \mathbb{1}(G_1 = G_m), \mathbb{1}(G_1 = G_h)] \Pi_{GG'}^{t-1} \begin{bmatrix} G_l \\ G_m \\ G_h \end{bmatrix}, \quad (4.4)$$

where $\Pi_{GG'}$ is the transition probability matrix of the G -process in the two-shock economy discussed in Appendix A. Note that, depending on G_1 , the G_t -paths will have different dynamics. For example, if $G_1 = G_m$, G_t will stay at G_m for all $t \geq 1$, and the economy will immediately transition to its long-run G level. However, if the economy starts the transition away from G_m , G_t converges to G_m over time through the deterministic process described in (4.4). In this case, the counterfactual economy will go through transitional dynamics to eventually reach the productivity-shock-only stochastic steady state.

Recall the assumption that the government purchases process is independent from other stochastic processes, which implies that none of the other exogenous stochastic processes changes during or after the elimination of the fiscal shocks. Therefore, our counterfactual economy features aggregate productivity shocks both during and after the transition. This creates a new technical challenge in addition to those present in previous transition path analyses of heterogeneous-agent economies (e.g., Huggett (1997) and Krusell and Smith (1999)). While these studies model a deterministic aggregate economy along the transition path, our stochastic setting with aggregate uncertainty produces an exponentially higher number of possible aggregate paths as the length of the transition period increases. This feature precludes computation of the equilibrium for all possible realizations of aggregate productivity shocks.

To address this challenge, we extend the approximate aggregation technique for our two-shock equilibrium to the transition-path setting: that is, we postulate that time-dependent prediction functions govern the evolution of the economy on the transition path, through the following set of laws of motions:

$$\Gamma_{t+1} = H_{\Gamma,t}^{trans}(\Gamma_t, B_t, z_t), \quad (4.5)$$

$$s_t = S_t^{trans}(\Gamma_t, B_t, z_t), \quad (4.6)$$

where t denotes an arbitrary period along the transition path. At the end of the transition path, the laws of motions converge to those in our one-shock equilibrium. Consequently, solving for the transition-path equilibrium is equivalent to finding the appropriate approximations for (4.5) and (4.6), such that the realized evolution of the economy is consistent with the postulated evolution; see Appendix B.3 for the details of the algorithm. We find that the same functional forms we use for the stochastic steady state economy yield accurate predictions also for the transition-path equilibrium. That is, for every period on the transition path, we achieve a similar forecast accuracy as in the stochastic steady state two-shock economy; see Appendix B.4 for the details.

5 Results

Following Lucas (1987), we measure the welfare costs of fiscal uncertainty as the proportional change in a household’s life-time consumption (Consumption Equivalent Variation or λ), such that:

$$E_1\left[\sum_{t=1}^{\infty} \beta_t u((1 + \lambda)c_t, G_t)\right] = E_1\left[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, \tilde{G}_t)\right], \quad (5.1)$$

where c_t is consumption in the baseline economy with G_t -fluctuations, while \tilde{c}_t is consumption in the counterfactual economy with a deterministic \tilde{G}_t -process.

5.1 Baseline results

To obtain our baseline results, we first calculate welfare gains conditional on wealth, employment status and time preference for every sample economy in the transition-path computation,²⁰ using the value functions from our two-shock and transition-path equilibria.²¹ We then average these across the sample economies, including all possible values of G_1 , the government purchases level when fiscal uncertainty is eliminated. The results, presented in Table 4, can thus be interpreted as the *ex-ante* expected welfare gains from eliminating fiscal uncertainty.

Table 4: Expected welfare gains λ (%)

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.0298	0.0303	0.0307	0.0307	0.0304	0.0301	0.0290	0.0240	0.0242
$\varepsilon = 1$	0.0298	0.0302	0.0306	0.0307	0.0304	0.0301	0.0290	0.0240	0.0242
$\varepsilon = 0$	0.0299	0.0304	0.0309	0.0308	0.0305	0.0302	0.0290	0.0240	0.0242
$\tilde{\beta} = \tilde{\beta}_l$	0.0290	0.0293	0.0293	0.0291	0.0289	0.0287	0.0275	0.0240	0.0226
$\tilde{\beta} = \tilde{\beta}_m$	0.0299	0.0312	0.0310	0.0307	0.0304	0.0301	0.0291	0.0241	0.0236
$\tilde{\beta} = \tilde{\beta}_h$	0.0262	0.0316	0.0313	0.0309	0.0304	0.0300	0.0272	0.0232	0.0266

Notes: The wealth groups are presented in ascending order from left to right. The welfare number for a particular combination of ε (or $\tilde{\beta}$) and a wealth group is calculated as follows: we first draw a large set (16,000) of independent joint distributions over $(a, \varepsilon, \tilde{\beta})$ from the simulation of the two-shock equilibrium. These distributions are used to start the computation of the transition-path equilibria. For each sample economy, we then find all the individuals that fall into a particular wealth \times employment status or wealth \times preference category, and calculate their welfare gain according to equation (5.1). We then take the average over the individuals in a particular category to find the welfare numbers for a given sample economy. To arrive at the numbers in this table, we finally take the average across all the 16,000 samples.

²⁰To start the transition-path simulation, we draw a large set (16,000) of independent joint distributions over $(a, \varepsilon, \tilde{\beta})$ from the simulation of the two-shock equilibrium; see Appendix B.3 for the details.

²¹The right side of (5.1) is the value function from the transition-path equilibrium. Given the log-log utility assumption in the baseline calibration, the left side of (5.1) can be expressed using the value function from the two-shock equilibrium and λ ; see Appendix B.5 for the details of the derivation.

The results in Table 4 show that the aggregate welfare gain, i.e., the average welfare change across the whole population, is about 0.03%, comparable in size to the results in Lucas (1987) and Krusell et al. (2009). We further find that the welfare gains decrease with wealth, employment status does not affect the welfare changes, and time preferences do not present a clear welfare gain pattern. In the next sub-section, we examine the mechanisms affecting the welfare gains along the wealth dimension.

5.2 The mechanisms

Our analyses show that the decreasing-with-wealth welfare gain pattern is the result of three interacting channels: a direct utility change channel, a saving channel, and a general equilibrium price channel. In the direct utility change channel, the utility gains resulting from household risk aversion with respect to government purchases fluctuations are isolated. In the saving channel, two types of fiscal risk arising from tax rate fluctuations interact: an after-tax-wage risk and an after-tax-rate-of-return risk. These risks have a different effect on the precautionary saving behavior of households and the quality of both capital and bonds as saving vehicles. Finally, in the general equilibrium price channel, factor price changes along the transition path are reflected.

In the following sub-sections, we discuss each channel in turn. Given the separability of private and public goods in our baseline calibration, we can exactly separate the direct utility channel from the other two. By contrast, we cannot exactly separate the saving channel from the general equilibrium price channel.

5.2.1 The direct utility change channel

Since a household's utility over G is strictly concave, eliminating fluctuations in G leads to a direct increase in expected lifetime utility. To isolate this direct utility gain, we compute a λ_c such that:

$$E_1\left[\sum_{t=1}^{\infty} \beta_t u((1 + \lambda_c)c_t, G_t)\right] = E_1\left[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, G_t)\right], \quad (5.2)$$

where c_t , \tilde{c}_t , and G_t are defined in the same way as before. That is, we calculate the amount of private consumption compensation required to achieve the same lifetime utility under the uncertain G -process as from consumption in the transition-path equilibrium. Since the stochastic G -process enters both sides of equation (5.2), λ_c isolates the joint effect of the saving and general equilibrium price channels. Thus, the difference between λ and λ_c measures the direct utility channel. Furthermore, with a

separable flow utility function, λ_c can be computed using the following simpler equation:

$$E_1\left[\sum_{t=1}^{\infty} \beta_t \log((1 + \lambda_c)c_t)\right] = E_1\left[\sum_{t=1}^{\infty} \beta_t \log(\tilde{c}_t)\right]. \quad (5.3)$$

The results, presented in Table 5, show positive, albeit smaller consumption-related welfare when fiscal uncertainty is eliminated. Thus, we conclude that the direct utility channel is quantitatively important for the overall level of welfare changes, but, distributionally, there is a more pronounced decline in welfare gains as private wealth increases.

Table 5: Expected welfare gains from private consumption, λ_c (%)

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.0087	0.0095	0.0097	0.0096	0.0093	0.0090	0.0078	0.0028	0.0030
$\varepsilon = 1$	0.0087	0.0095	0.0097	0.0096	0.0093	0.0090	0.0078	0.0028	0.0030
$\varepsilon = 0$	0.0088	0.0096	0.0098	0.0097	0.0094	0.0091	0.0078	0.0028	0.0030
$\tilde{\beta} = \tilde{\beta}_l$	0.0086	0.0089	0.0088	0.0086	0.0084	0.0082	0.0070	0.0036	0.0021
$\tilde{\beta} = \tilde{\beta}_m$	0.0088	0.0101	0.0099	0.0096	0.0093	0.0090	0.0080	0.0030	0.0025
$\tilde{\beta} = \tilde{\beta}_h$	0.0044	0.0098	0.0095	0.0091	0.0086	0.0082	0.0054	0.0014	0.0048

Notes: The welfare numbers in this table are calculated as those in Table 4, using (5.3) instead of (5.1).

5.2.2 The saving channel

Fluctuations in government purchases lead to more volatile individual tax rates, resulting in greater after-tax labor and capital income risk. Consequently, the effect of eliminating this uncertainty depends on a household's (heterogeneous) degree of self-insurance against income risks. As in other Bewley-type incomplete market economies, our households engage in precautionary saving. Wealthier households can better insure themselves against after-tax income risk. As a result, wealth-poor households should be more likely to benefit from the elimination of uncertainty.

However, the tax-rate uncertainty induced by the G -shocks also creates a rate-of-return risk on after-tax capital income, affecting the quality of both capital and bonds as saving vehicles. This rate-of-return risk makes households' intertemporal transfer of resources riskier. Since wealth-rich households have more exposure to this rate-of-return risk, they should be more likely to benefit from the elimination of fiscal uncertainty.

In short, the saving channel effects can favor the wealth-poor or the wealth-rich, depending on whether the precautionary saving or the quality-of-assets effect dominates. The strength of the quality-

Table 6: Consumption volatility ($\times 100$)

Percentile	1%	5%	25%	50%	75%	95%	99%
A: The two-shock economy							
All	0.120	0.110	0.102	0.095	0.087	0.040	0.014
$\varepsilon = 0$	0.222	0.125	0.109	0.100	0.091	0.041	0.014
$\varepsilon = 1$	0.113	0.109	0.102	0.095	0.087	0.040	0.014
$\tilde{\beta} = \tilde{\beta}_l$	0.112	0.096	0.088	0.084	0.078	0.037	0.012
$\tilde{\beta} = \tilde{\beta}_m$	0.121	0.110	0.103	0.096	0.087	0.040	0.014
$\tilde{\beta} = \tilde{\beta}_h$	0.123	0.115	0.105	0.099	0.092	0.045	0.016
B: Difference between the two-shock and the one-shock economy							
All	0.029	0.029	0.027	0.025	0.023	0.011	0.004
$\varepsilon = 0$	0.022	0.026	0.027	0.025	0.023	0.011	0.004
$\varepsilon = 1$	0.030	0.029	0.027	0.025	0.023	0.011	0.004
$\tilde{\beta} = \tilde{\beta}_l$	0.027	0.026	0.024	0.023	0.021	0.010	0.004
$\tilde{\beta} = \tilde{\beta}_m$	0.029	0.029	0.027	0.025	0.023	0.011	0.004
$\tilde{\beta} = \tilde{\beta}_h$	0.031	0.030	0.028	0.026	0.024	0.012	0.005

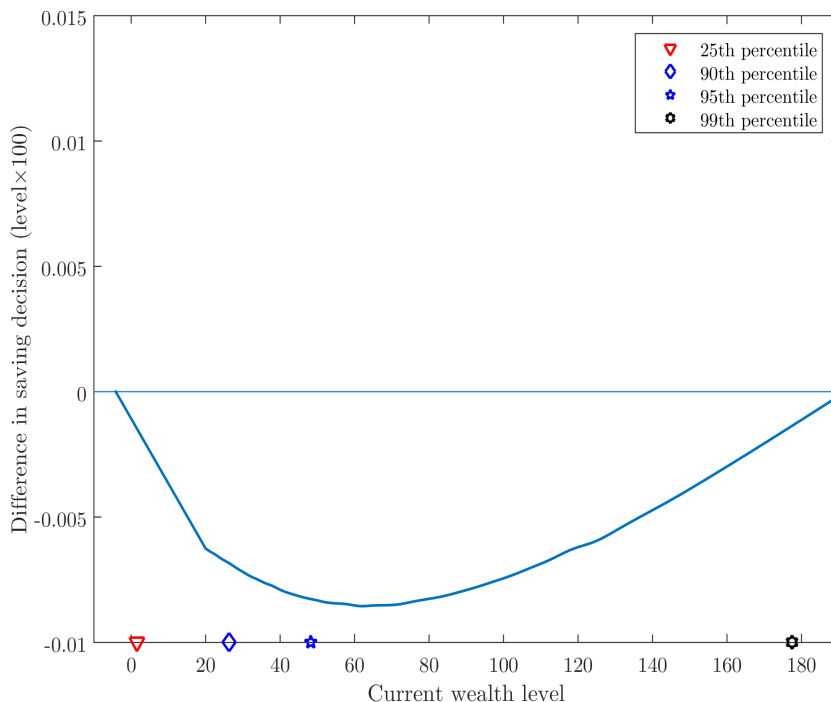
Notes: Consumption volatility is measured as the variance of log consumption and calculated as follows: for each combination of ε , $\tilde{\beta}$, and wealth (see footnote 22 for how the wealth percentiles are calculated), we collect the consumption data of a household with that particular characteristic. We do so for every period of the simulation used in the stochastic steady state equilibrium computation and then construct a set of ε -, $\tilde{\beta}$ -, and wealth-specific consumption time series. Next, we calculate the volatility of these disaggregate consumption time series and take the weighted (by the unconditional probability of each state) average over ε ($\tilde{\beta}$) to get the numbers conditional on $\tilde{\beta}$ (ε). To compute the consumption volatility for the one-shock economy, we use the stochastic steady state equilibrium simulation with only aggregate productivity shocks.

of-assets effect depends on how much individual tax-rate uncertainty fluctuations in government purchases generate for the wealth-rich households.

We illustrate the distributional effects from precautionary saving through changes in the volatility of consumption for different wealth levels. Although the quality of assets as saving vehicles also influences consumption volatility, we nevertheless believe this volatility appropriately captures the overall ability of households to self-insure through precautionary saving. Table 6, Panel A, presents the volatilities of $\tilde{\beta}(\varepsilon)$ - and wealth-specific consumption time series for our two-shock economy (measured as the variance of log consumption).²² These results show that household consumption volatility is decreasing along the wealth dimension, which is consistent with the notion that wealthier households

²²The wealth percentiles in Table 6 are constructed differently than those in Table 4 and 5. Recall that, in Table 4 and 5, we calculate wealth percentiles for each sample economy. By contrast, for Table 6, we calculate wealth percentiles from the long-run ergodic distribution of the stochastic two-shock economy. This is more appropriate for analyzing consumption changes for a certain wealth percentile over time, as the associated wealth level remains constant. To compute the type-specific consumption volatilities, we proceed in several steps: First, for each combination of ε , $\tilde{\beta}$ and wealth, we collect the consumption data of a household with that particular characteristic. We do so for every period of the simulation used in the stochastic steady state equilibrium computation. We then construct a set of ε -, $\tilde{\beta}$ - and wealth-specific consumption time series. Next, we calculate the volatility of these disaggregate consumption time series and take the weighted (by the unconditional probability of each state) average over ε ($\tilde{\beta}$) to obtain the volatilities conditional on $\tilde{\beta}$ (ε).

Figure 1: Policy function comparison - saving



Notes: This figure shows the difference between the first-period policy function for saving from the transition equilibrium (with $G_1 = G_m$) and that from the two-shock equilibrium (with $G_1 = G_m$), evaluated at the long-run averages of (K, B) , $z = z_g$, $\varepsilon = 1$, and $\tilde{\beta} = \tilde{\beta}_m$.

have less to gain from the elimination of fiscal uncertainty, as they are already better insured in the two-shock economy. Furthermore, the results in Panel B show that the difference in consumption volatilities between the two-shock and one-shock economies is substantially lower for our wealth-rich households.²³

We next analyze the quality-of-assets effect by examining the change in saving behavior, across wealth groups, after the elimination of fiscal uncertainty. Reduced needs for saving (self-insurance) would indicate that the precautionary saving effect dominates, increased demand for saving would indicate a stronger quality-of-assets effect. The results in Figure 1 show a reduction in saving in the first period of the transition-path equilibrium compared to the two-shock equilibrium across all wealth classes.²⁴ Figure 1 also shows that the quality-of-assets effect is eventually increasing in wealth, but

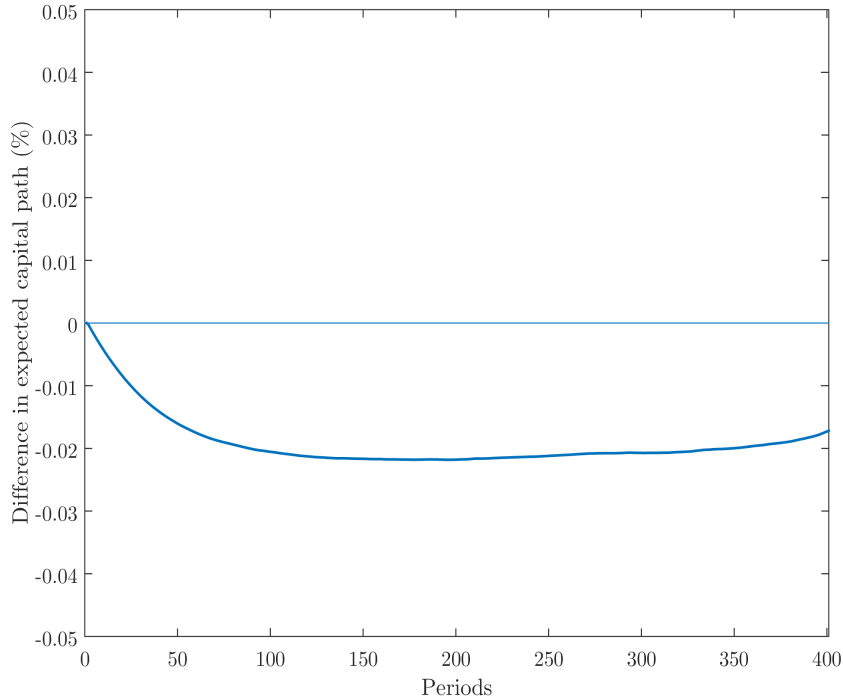
²³For the unemployed households, the relationship is not monotone in wealth. This is because wealth-poor unemployed households have negligible income, so they are not exposed to much tax-rate uncertainty until they are re-employed. They thus experience a relatively smaller volatility reduction between the two and one-shock economies.

²⁴The policy function difference for saving is evaluated at $G_1 = G_m$, the long-run averages of $(K; B)$, $z = z_g$, $\varepsilon = 1$ and $\tilde{\beta} = \tilde{\beta}_m$. However, similar patterns hold for other combinations of state variables.

never dominates the precautionary saving effect. We will show in Section 6.1 that this dominance is specific to the baseline tax system as it depends on which tax instrument is used to raise the tax revenue required by the fiscal rule. Recall, that we use the parameter s in the progressive tax function for the cyclical tax revenue adjustment, and that τ_0 and τ_1 are constants. Since the marginal tax rate faced by the wealth-rich is close to $\tau_0 + \tau_1$, it is largely invariant to fluctuations in aggregate productivity or government purchases. Consequently, the wealth-rich face little tax-rate uncertainty in the baseline two-shock economy.

5.2.3 General equilibrium price channel

Figure 2: Expected aggregate capital path comparison



Notes: This figure shows the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium. We use the same 16,000 sample economies and the same sequences of z -shocks (for both the transition and the two-shock aggregate capital paths) as in the transition-path computation and then take the average. The G -shock sequences in the two-shock simulations are constructed in such a way that the cross-sectional joint distribution of (z, G) -shocks in each period is close to the invariant joint distribution.

We next examine the general equilibrium channel. In our model with a representative neoclassical firm, factor price changes follow aggregate capital stock changes. If the elimination of fiscal uncertainty lowers the aggregate capital stock, then pre-tax capital returns, all else being equal, will increase

relative to wages. Because wealth-rich (wealth-poor) households have higher (lower) capital income shares, the wealth-rich (wealth-poor) households will benefit (lose) from this relative factor price change. As a result, changes in the aggregate capital stock will have distributional effects.

To examine the direction of the general equilibrium price channel for our baseline scenario, we compute the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium. The results in Figure 2 show that the expected aggregate capital path in the transition-path equilibrium is slightly lower than it is in the two-shock equilibrium. We conclude that the general equilibrium factor price changes favor wealth-rich households. Recall that the overall distributional effect from the elimination of fiscal uncertainty favors the wealth-poor households. Overall, we see then that the saving channel dominates the general equilibrium price channel.

5.3 Distributional analysis conditional on G_1

Finally we examine the welfare gains from eliminating fiscal uncertainty *conditional* on G_1 , the level of government purchases at the time the policy change is instituted. The results in Table 7 for λ_c reveal similar overall decreasing-with-wealth welfare gain patterns. However, when $G_1 = G_h$, the welfare gains are essentially flat implying that the general equilibrium price channel must be stronger in this case. This interpretation is consistent with the results in Figure 3, regarding the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium, conditional on G_1 . For $G_1 = G_h$, we find the steepest decline in aggregate capital after the elimination of fiscal uncertainty, but an ultimately flat welfare gain pattern. By contrast, for $G_1 = G_l$, the aggregate capital path is flatter in the transition to a new stochastic steady state, but the welfare gains show the steepest decline.

6 Alternative specifications and additional experiments

In this section, we examine the welfare and distributional consequences of eliminating fiscal uncertainty under the following alternative model specifications: different flow utility functions, different adjustments to the progressive tax function, counterfactual fiscal regimes, and a model without TFP uncertainty. In addition, we examine our results when we double fiscal uncertainty, as well as when the elimination of fiscal uncertainty is accompanied by a sudden change in the level of government

Table 7: Expected welfare gains from private consumption, λ_c (%), conditional on G_1

$G_1 = G_l$									
	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.0140	0.0155	0.0156	0.0153	0.0149	0.0145	0.0128	0.0059	0.0031
$\varepsilon = 1$	0.0140	0.0154	0.0156	0.0153	0.0149	0.0144	0.0128	0.0059	0.0031
$\varepsilon = 0$	0.0142	0.0156	0.0158	0.0155	0.0151	0.0146	0.0129	0.0059	0.0031
$\tilde{\beta} = \tilde{\beta}_l$	0.0144	0.0149	0.0148	0.0145	0.0141	0.0138	0.0118	0.0064	0.0025
$\tilde{\beta} = \tilde{\beta}_m$	0.0141	0.0160	0.0158	0.0153	0.0149	0.0145	0.0129	0.0059	0.0025
$\tilde{\beta} = \tilde{\beta}_h$	0.0091	0.0167	0.0165	0.0160	0.0154	0.0149	0.0112	0.0051	0.0051
$G_1 = G_m$									
	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.0087	0.0095	0.0097	0.0096	0.0094	0.0091	0.0078	0.0027	0.0028
$\varepsilon = 1$	0.0087	0.0095	0.0097	0.0096	0.0093	0.0091	0.0078	0.0027	0.0028
$\varepsilon = 0$	0.0088	0.0097	0.0099	0.0097	0.0094	0.0091	0.0079	0.0027	0.0028
$\tilde{\beta} = \tilde{\beta}_l$	0.0086	0.0089	0.0089	0.0087	0.0085	0.0082	0.0071	0.0036	0.0021
$\tilde{\beta} = \tilde{\beta}_m$	0.0088	0.0101	0.0099	0.0097	0.0094	0.0091	0.0080	0.0029	0.0024
$\tilde{\beta} = \tilde{\beta}_h$	0.0043	0.0097	0.0094	0.0090	0.0086	0.0081	0.0053	0.0013	0.0046
$G_1 = G_h$									
	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.0034	0.0034	0.0037	0.0038	0.0037	0.0036	0.0028	-0.0001	0.0031
$\varepsilon = 1$	0.0033	0.0034	0.0037	0.0038	0.0037	0.0036	0.0028	-0.0001	0.0031
$\varepsilon = 0$	0.0034	0.0035	0.0039	0.0039	0.0037	0.0036	0.0028	-0.0001	0.0031
$\tilde{\beta} = \tilde{\beta}_l$	0.0027	0.0028	0.0027	0.0027	0.0026	0.0025	0.0022	0.0007	0.0019
$\tilde{\beta} = \tilde{\beta}_m$	0.0034	0.0040	0.0040	0.0038	0.0037	0.0036	0.0030	0.0001	0.0026
$\tilde{\beta} = \tilde{\beta}_h$	0.0001	0.0029	0.0026	0.0023	0.0020	0.0017	-0.0003	-0.0018	0.0050

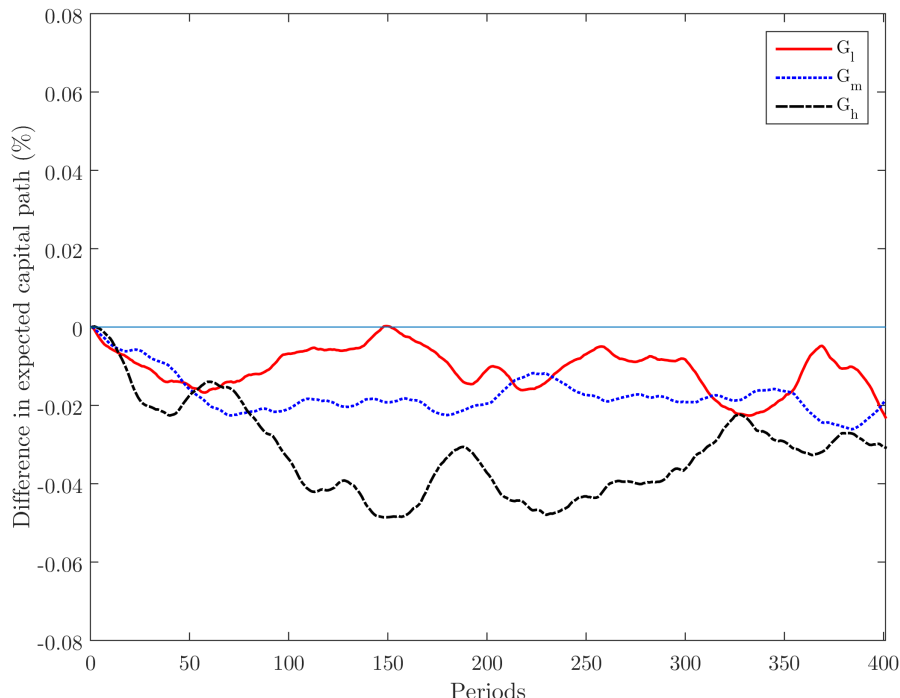
Notes: The welfare numbers in this table are calculated as in Table 5, but separately for $G_1 = G_l, G_m, G_h$, using 8,000 simulations for $G_1 = G_m$ and 4,000 simulations each for $G_1 = G_l, G_h$.

purchases. In each case, we re-calibrate parameter values when necessary to preserve target moment-data consistency. We summarize the welfare change results in terms of λ_c in Table 8. Table 19 in Appendix C reports the corresponding λ -measures.

6.1 Alternative specifications

Non-separable utility. Recall that our baseline specification assumes a separable flow utility function in private and public consumption ($\rho = 1$), which implies that government purchases uncertainty affects the household decisions only indirectly, through equilibrium tax rate changes. By contrast, if public and private consumption are non-separable, then this uncertainty has a direct effect on the

Figure 3: Expected aggregate capital path comparison, conditional on G_1



Notes: This figure shows the percentage difference between the expected aggregate capital path in the transition-path equilibrium and the two-shock equilibrium conditional on G_1 . We use the same 16,000 sample economies and the same sequences of z -shocks (for both the transition and the two-shock aggregate capital paths) as in the transition-path computation and then average by G_1 : 8,000 simulations for $G_1 = G_m$, and 4,000 simulations each for $G_1 = G_l, G_h$. Note that, due to our conditioning on G_1 and the subsequent smaller sample sizes, the expected aggregate capital paths in Figure 3 are more volatile compared to those in Figure 2.

consumption-saving decision, since the government purchases level affects the marginal utility of private consumption.²⁵ We thus examine two alternative specifications where public consumption is an Edgeworth substitute (complement), $\rho = 0.5$ ($\rho = 1.5$), to private consumption.²⁶

The results in rows 2 and 3 of Table 8 show that when G and c are complements (substitutes), the welfare gains from the elimination of fiscal uncertainty are larger (smaller) than in the baseline scenario. This is because the positive conditional comovement between G and taxes in the estimated fiscal rule makes uncertainty in G more costly when c and G are complements.²⁷ Since households face higher tax rates (lower disposable income) when G and the marginal utility of private consumption

²⁵See Fiorito and Kollintzas (2004) for an overview of utility specifications for public consumption.

²⁶To calculate λ_c with a non-separable utility function, we calculate the left side of (5.2) as a discounted sum of flow utilities under various values of λ_c , using the equilibrium policy function. We then find a value of λ_c that satisfies the equation numerically, using a bisection search.

²⁷In the estimated fiscal rule, the tax-output ratio responds to the government-purchases-output ratio with a coefficient of $\rho_{T,G} = 0.484$; see Appendix A.2. for the details.

Table 8: Expected welfare gains from private consumption, λ_c (%), under different model specifications

	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
Baseline	0.0087	0.0095	0.0097	0.0096	0.0093	0.0090	0.0078	0.0028	0.0030
Non-separable Utility Function									
Substitute ($\rho = 0.5$)	-0.0002	-0.0017	0.0008	0.0018	-0.0013	0.0027	-0.0030	-0.0078	-0.0011
Complement ($\rho = 1.5$)	0.0385	0.0267	0.0292	0.0346	0.0293	0.0459	0.0473	0.0372	0.0355
Different Tax Function Adjustment									
Adjusting τ_0	0.0084	0.0083	0.0086	0.0087	0.0084	0.0082	0.0077	0.0091	0.0135
Adjusting τ_1	0.0082	0.0081	0.0085	0.0085	0.0082	0.0079	0.0076	0.0101	0.0159
Counterfactual Fiscal Regimes									
Balanced Budget	0.0076	0.0097	0.0093	0.0087	0.0083	0.0079	0.0068	-0.0002	-0.0015
Linear Tax	0.0072	0.0067	0.0068	0.0070	0.0070	0.0070	0.0071	0.0103	0.0163
Lump-sum Tax	0.0073	0.0070	0.0070	0.0070	0.0069	0.0068	0.0071	0.0129	0.0204
Additional Experiments									
Constant TFP	0.0084	0.0091	0.0093	0.0092	0.0090	0.0087	0.0077	0.0034	0.0040
Double Volatility of G	-0.0071	-0.0081	-0.0083	-0.0081	-0.0077	-0.0073	-0.0061	-0.0011	-0.0012
Sudden change in G	0.0114	0.0148	0.0143	0.0135	0.0127	0.0119	0.0094	0.0001	-0.0015

are high, the utility gain from fiscal uncertainty elimination in the case of complements is larger than in the separable case. An analogous argument applies when G and c are substitutes.

Tax function adjustments. Recall that, in our baseline specification, the tax function parameter s is determined endogenously to satisfy the government’s fiscal rule (equation (2.6)), while the linear (τ_0) and top marginal (τ_1) tax rates in the progressive tax function are fixed. In this case, a fluctuating s does not generate substantial tax-rate uncertainty for the very rich households as their marginal tax rate is close to the upper bound $\tau_0 + \tau_1 = \text{const}$. By contrast, it is the tax rates for the middle of the income distribution that respond the most to changes in s .

We therefore consider the following two alternative adjustments in the tax function: adjusting τ_0 , the linear part in the income tax function, and adjusting τ_1 , the parameter that governs the taxes for the top income brackets.²⁸ In both cases, s is fixed at its long-run average from the baseline stochastic two-shock economy. The simulated distributions of both τ_0 and τ_1 are symmetrically centered around their values in the baseline scenario (i.e., the ones estimated from the data).

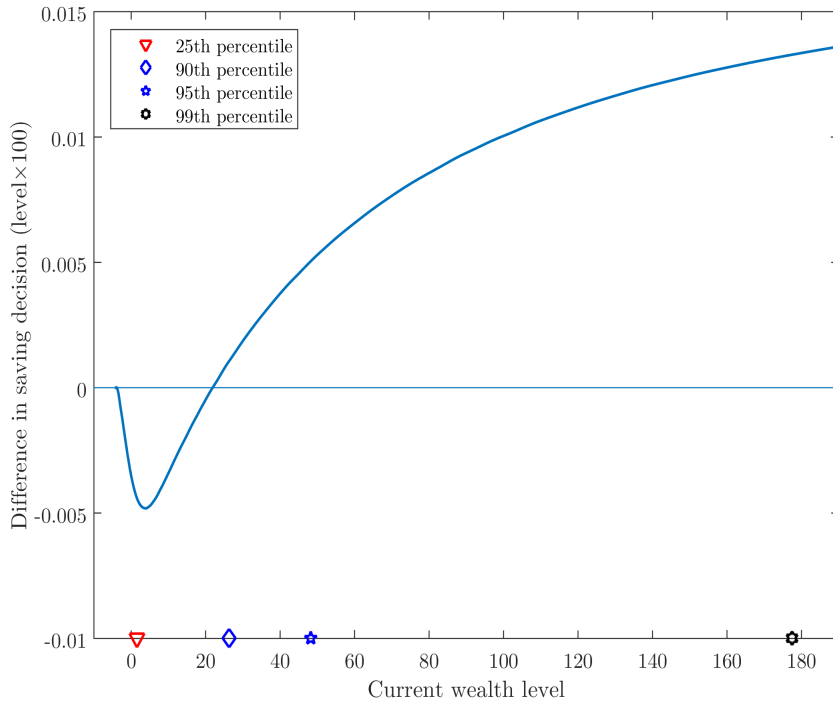
The results in rows 4 and 5 of Table 8 show that other tax function adjustments yield similar overall welfare gains as the baseline case. However, unlike in the baseline scenario, the welfare gain for the top 5% of households is larger than the average welfare gain.

Since our wealth-rich are still better insured against fiscal uncertainty, the precautionary saving aspect does not drive our different distributional results: when we compute the analogue of Table

²⁸In the first case, all the households face the same tax rate changes in terms of absolute magnitude. However, in relative terms, the tax rate change is decreasing with income level. In the second case, all the households face the same percentage change in their tax rates. However, in terms of absolute magnitude, the tax rate change is increasing with income level.

6 for our two alternative cases, the wealth-rich still experience the lowest reduction in consumption volatility. In addition, the general equilibrium channel in our two alternatives favors the wealth-poor households: the aggregate capital stock increases after the elimination of fiscal uncertainty, which leads to higher wages and a lower pre-tax capital rate-of-return. However, when τ_0 or τ_1 are adjusted to satisfy the fiscal rule, the wealth-rich face a higher tax rate uncertainty when government purchases fluctuate. They thus enjoy larger gains from a reduction in rate-of-return risk. We therefore conclude that the quality of capital (bonds) as a saving vehicle increases after the elimination of fiscal uncertainty, meaning that the wealth-rich benefit more.

Figure 4: Policy function comparisons - saving, adjusting τ_1



Notes: This figure shows the difference between the first-period policy function for saving from the transition equilibrium (with $G_1 = G_m$) and that from the two-shock equilibrium (with $G_1 = G_m$), evaluated at the long-run averages of (K, B) , $z = z_g$, $\varepsilon = 1$, and $\tilde{\beta} = \tilde{\beta}_m$.

We illustrate this in Figure 4. From Figure 4, we see that there is a reduction in saving in the first period of the transition-path equilibrium compared to the two-shock equilibrium until approximately the 90th wealth percentile.²⁹ However, this saving reductions for less wealthy households is weaker

²⁹Figure 4 is the analog of Figure 1 for the τ_1 -adjustment case. The τ_0 -adjustment case looks very similar to the τ_1 -adjustment case.

than in the baseline case. In addition, the wealth-rich now increase their saving after the elimination of fiscal uncertainty. Thus, in this case, the quality-of-assets mechanism dominates the precautionary saving mechanism for wealth-rich households.

The different welfare gain patterns in the baseline scenario vis-à-vis the τ_1/τ_0 -adjustments have important policy implications: the distributional effects of eliminating fiscal uncertainty depend on which wealth group experiences the tax uncertainty burden that is caused by government purchases shocks.

Counterfactual fiscal regimes. We next use three counterfactual fiscal regimes and examine their respective welfare results. This analysis will shed additional light on the mechanisms behind the welfare effects of the elimination of fiscal uncertainty. In our first regime, a balanced budget scenario, we dispense with the fiscal rule (equation (2.6)) and assume that government spending is financed exclusively through tax revenue. In our next two regimes, a linear (lump-sum) tax scenario, we keep the fiscal rule but change the progressive tax system to a linear (lump-sum) tax, $\tau_1 = 0$ ($\tau_0 = 0$ and $\tau_2 = -1$). The linear tax rate (the lump-sum tax amount) are then endogenously determined to satisfy the fiscal rule. Note that the lump-sum tax is imposed only on employed households to avoid negative after-tax incomes.³⁰

Table 9: Consumption volatility differences between the two-shock and the one-shock economy

Percentile	1%	5%	25%	50%	75%	95%	99%
A: Baseline							
All	0.029	0.029	0.027	0.025	0.023	0.011	0.004
$\varepsilon = 0$	0.022	0.026	0.027	0.025	0.023	0.011	0.004
$\varepsilon = 1$	0.030	0.029	0.027	0.025	0.023	0.011	0.004
$\tilde{\beta} = \tilde{\beta}_l$	0.027	0.026	0.024	0.023	0.021	0.010	0.004
$\tilde{\beta} = \tilde{\beta}_m$	0.029	0.029	0.027	0.025	0.023	0.011	0.004
$\tilde{\beta} = \tilde{\beta}_h$	0.031	0.030	0.028	0.026	0.024	0.012	0.005
B: Balanced budget							
All	0.016	0.014	0.012	0.011	0.009	0.004	0.001
$\varepsilon = 0$	0.013	0.013	0.011	0.010	0.009	0.004	0.001
$\varepsilon = 1$	0.016	0.014	0.012	0.011	0.009	0.004	0.001
$\tilde{\beta} = \tilde{\beta}_l$	0.017	0.015	0.012	0.011	0.009	0.004	0.001
$\tilde{\beta} = \tilde{\beta}_m$	0.016	0.014	0.012	0.011	0.009	0.004	0.001
$\tilde{\beta} = \tilde{\beta}_h$	0.013	0.012	0.011	0.010	0.009	0.004	0.001

Notes: Panel A is Panel B in Table 6. Panel B is calculated here the same way as Panel A (see notes to Table 6), but from the balanced budget simulations.

³⁰We note that this treatment is slightly different from all the other cases, where tax payments are zero if and only if $y \leq 0$. With lump sum taxes, this could potentially introduce a discontinuity in the budget constraint of the unemployed at $y = 0$ (for the employed, it must hold that $y - T \geq 0$ at the borrowing limit, and thus, *a fortiori*, for all employed households).

We present the results of this set of analyses in rows 6 to 8 of Table 8. Overall, the welfare gains are similar to those obtained in our baseline analysis. Distributionally, the negative slope of the welfare gains along the wealth dimension is steeper in the balanced-budget regime. The results in Table 9 show the changes in consumption volatility between the two and one-shock economy for the balanced budget case (Panel B) compared to those in the baseline case (Panel A). In particular the balanced budget volatility changes for the wealth-rich are quite small, reflecting the negative correlation between the aggregate tax rate $((T - Tr)/Y)$ and the rate-of-return to capital (Table 10). By contrast, our baseline case has a positive correlation between the aggregate tax rate and the rate-of-return to capital, which is higher in the one-shock case.³¹ In short, since in the balanced-budget regime the wealth-rich tend to have higher tax rates when the rate-of-return to capital is low, they enjoy a smaller consumption volatility reduction.

Table 10: Correlation between the aggregate tax rate and the capital rate-of-return

	A: Baseline		B: Balanced Budget	
	two-shock	one-shock	two-shock	one-shock
Corr($r, (T - Tr)/Y$)	0.8307	0.8971	-0.5760	-0.7895
	(0.0666)	(0.0495)	(0.1506)	(0.0942)

Notes: r and $(T - Tr)/Y$ are the rate-of-return to capital and the ratio of tax revenue net of transfers to output from model simulations, both linearly detrended. The correlations are the average values from 1,000 independent simulations of the same length as the data (192 quarters). We show the standard deviations across these simulations in parentheses.

Turning next to the linear tax case, we find that the welfare gains are increasing in wealth, just as in the case of τ_1 - or τ_0 -adjustment, as seen in row 7 of Table 8. Indeed, the mechanism is the same: in a linear tax regime, using the tax rate for cyclical tax adjustment increases the after-tax rate-of-return uncertainty for the wealth-rich. Consequently, the elimination of this uncertainty provides them with a better saving vehicle. Indeed, when we compare the saving policy function between the two-shock and the transition-path equilibrium, we find an almost identical pattern as that in the τ_1 -adjustment case (Figure 4).

Finally, our results for the lump-sum tax case (row 8 of Table 8) show that the welfare gains are again increasing in wealth. We also find again that the policy function comparison between the transition-path and the two-shock equilibrium looks similar to that in Figure 4. In other words, the

³¹The reason for the different sign of these correlations lies in the different cyclicity of the average tax rate (net of transfers) in the two models (the rate-of-return to capital is always procyclical, independent of the fiscal regime). In the baseline model, just as in the data, the average tax rate is procyclical because the estimated fiscal rule captures automatic stabilizers: $\rho_{T,Y} > 0$. In the balanced budget regime, by contrast, the cyclicity of the average tax rate is essentially determined by its denominator, aggregate output, because aggregate taxes are given by the government purchases level, which is assumed to be unrelated to the business cycle. This underscores the empirical importance of having a fiscal rule and including debt in our analysis.

rich save more when fiscal uncertainty is eliminated. However, since for the lump-sum tax case there is no change in the after-tax capital return uncertainty after the elimination of fiscal uncertainty, a different mechanism than for the linear tax (τ_1 -adjustment) case must be at work. Indeed, the aggregate capital stock decreases after the elimination of uncertainty in the lump-sum tax case, that is, the general equilibrium factor price effect favors the wealth-rich. Overall, since there is no distortion from high tax rates, the higher return makes capital more attractive as a saving vehicle after the elimination of uncertainty, leading to benefits for the wealth-rich. The only difference is that the quality-of-asset change is an increase in the average expected return instead of a decrease in the return volatility.

No TFP uncertainty. Recall that our baseline scenario adopts the same TFP and unemployment processes as used in Krusell and Smith (1998). However, these choices produce an output volatility in the model that is 70% larger than that in the data (Section 3.3). To examine whether this difference affects our welfare results, we match the output volatility in the data by keeping TFP constant (at $z = 1$), but allowing unemployment rate fluctuations. The results in row 9 in Table 8 are similar to those from the baseline model, suggesting that the excess output volatility in our model does not influence our welfare results.

6.2 Additional experiments

Transition to a higher level of fiscal uncertainty. As mentioned, one topic that has received vigorous debate is how permanently heightened fiscal policy uncertainty might impact aggregate economic activity and welfare. To address this question, we let the economy transition to a level of fiscal uncertainty which is twice that in our baseline economy. Appendix C provides the details of the computational implementation of this experiment.

The penultimate row in Table 8 shows the welfare changes from this magnified uncertainty experiment. As in the baseline experiment, higher fiscal uncertainty leads to a welfare loss for every wealth group, with wealth-rich households experiencing a smaller loss. Overall, the numbers suggest that, at least for the range between zero and twice the pre-crisis level of fiscal volatility, the welfare effects of fiscal uncertainty are roughly symmetric.

Sudden change in the level of government purchases. In a final experiment, we examine the consequences of a concomitant sudden change in the government purchases level by letting government

purchases move to and stay at their unconditional mean value, G_m , immediately after the elimination of fiscal uncertainty. We view this and the baseline scenario, where government purchases gradually converge to their long-run level, as two extreme ways of how fiscal uncertainty can be eliminated.

From the results in the last row of Table 8, we see that the unconditional welfare gains with a sudden change in the level of government purchases are similar but not identical to those in the baseline case, suggesting that the effect of a sudden change in G is not symmetric between $G_1 = G_l$ and $G_1 = G_h$ (the $G_1 = G_m$ -case is the same as in the baseline scenario).

By contrast, the results in Table 11 show that the welfare changes conditional on $G_1 = G_l$ and $G_1 = G_h$ are one order of magnitude larger than those in the baseline case (Table 7). For the $G_1 = G_l$ -case, the welfare changes are increasing in the wealth level, while the opposite pattern holds for the case of $G_1 = G_h$. However, these patterns are not driven by the elimination of fiscal uncertainty *per se*. The sudden change in the level of government purchases (and hence taxation) leads to a faster aggregate capital stock adjustment and a larger effect on welfare. For instance in the $G_1 = G_l$ -case, the sudden increase in government purchases leads to a faster decrease in aggregate capital, output, and average welfare. However, since lower aggregate capital levels (higher pre-tax rates of return) favor the wealth-rich capital income earners, the welfare change pattern increases with wealth. Following a similar intuition, the distributional effect for the $G_1 = G_h$ -case is reversed.

Table 11: Expected welfare gains from private consumption, λ_c (%), conditional on G_1 , sudden change in G

$G_1 = G_l$	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	-0.3864	-0.5347	-0.5032	-0.4653	-0.4372	-0.4089	-0.3128	0.0591	0.3914
$\varepsilon = 1$	-0.3862	-0.5362	-0.5041	-0.4655	-0.4374	-0.4092	-0.3134	0.0590	0.3914
$\varepsilon = 0$	-0.3877	-0.5263	-0.4954	-0.4628	-0.4345	-0.4060	-0.3056	0.0602	0.3913
$\tilde{\beta} = \tilde{\beta}_l$	-0.5197	-0.5606	-0.5458	-0.5158	-0.4839	-0.4556	-0.3308	-0.0158	0.2921
$\tilde{\beta} = \tilde{\beta}_m$	-0.3908	-0.5110	-0.4927	-0.4648	-0.4372	-0.4091	-0.3194	0.0443	0.3696
$\tilde{\beta} = \tilde{\beta}_h$	-0.0065	-0.4392	-0.4149	-0.3848	-0.3572	-0.3274	-0.1639	0.1501	0.4899
$G_1 = G_h$	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
All	0.4146	0.5748	0.5407	0.5000	0.4692	0.4383	0.3347	-0.0642	-0.4033
$\varepsilon = 1$	0.4144	0.5764	0.5416	0.5002	0.4694	0.4385	0.3352	-0.0641	-0.4033
$\varepsilon = 0$	0.4164	0.5665	0.5333	0.4977	0.4666	0.4356	0.3279	-0.0646	-0.4036
$\tilde{\beta} = \tilde{\beta}_l$	0.5554	0.5995	0.5835	0.5501	0.5154	0.4847	0.3511	0.0154	-0.3067
$\tilde{\beta} = \tilde{\beta}_m$	0.4194	0.5508	0.5304	0.4995	0.4692	0.4385	0.3417	-0.0486	-0.3818
$\tilde{\beta} = \tilde{\beta}_h$	0.0102	0.4786	0.4509	0.4173	0.3868	0.3541	0.1764	-0.1602	-0.4991

7 Conclusion

The recent recession and the economy's slow recovery have sparked a debate over the economic effects of fiscal uncertainty and volatility. Commentators have argued that uncertainty over future taxation and spending policies negatively affects contemporary economic outcomes. In this study, we quantify the welfare costs of fiscal uncertainty and their distribution in a neoclassical stochastic growth environment with incomplete markets. In our model, aggregate uncertainty arises from both productivity and government purchases shocks. Government spending is financed by a progressive tax system, modeled after important features of the U.S. tax system. We calibrate the model to U.S. data and evaluate the welfare and distributional consequences of eliminating government purchases shocks.

Our baseline results show that the welfare gains of eliminating fiscal uncertainty are decreasing in wealth. However, we also find that the effects of eliminating fiscal uncertainty depend on the fiscal regime and which group's after-tax rate-of-return to capital is affected the most by fiscal uncertainty. We also find that the welfare gains of eliminating fiscal uncertainty are higher when public and private consumption are complements.

While our study provides insight into the impact of eliminating fiscal uncertainty, it should be viewed as a first step towards a comprehensive analysis of the welfare and distributional implications of fiscal uncertainty. Future research could explore how our results change if nominal frictions that cause relative price distortions are added to the model. It could also examine wait-and-see effects. There is also no role in our model for a direct influence of government purchases on the unemployment process and thus cyclical idiosyncratic risk. Including this feature in a future quantitative analysis would require the development of a statistical model of how government purchases influence idiosyncratic unemployment processes, but such a model is elusive in the literature. Instead, since government purchases appear to be independent of the cycle in U.S. post-war data, we have also used this assumption in the model. Furthermore, we have chosen to place exogenous uncertainty fundamentally within the level of government purchases, while the uncertainty of individual tax rates is derived from our model. Among fiscal data, we view the official aggregate data on government purchases as cleanest and least subject to construction choices, but recognize that the data on tax rates collected in Mertens (2013) could provide an alternative route. Finally, we model government purchases as a symmetric autoregressive process. However, future research could examine fiscal uncertainty in an economy facing the risk of very large government purchases as a very rare and dramatic event.

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A Appendix: Estimation of the fiscal parameters

For the calibration, we use quarterly data from 1960I to 2007IV.

A.1 Government purchases process

We first construct a real government purchases (G) series by deflating the “Government consumption expenditures and gross investment” series (from NIPA table 3.9.5, line 1) with the GDP deflator (from NIPA table 1.1.9, line 1). We then estimate an AR(1) process for the linearly detrended real $\log(G)$ series. We use the Rouwenhorst method (see Rouwenhorst (1995)) to approximate this zero-mean AR(1) process with a three-state Markov Chain. This gives us a transition probability matrix, and a grid in the form $(-m, 0, m)$, where m represents the percentage deviation from the middle grid point. The middle grid point of the G -process, G_m , is then calibrated to match the time series average of nominal G over nominal GDP from U.S. national accounting data (nominal GDP is from NIPA table 1.1.5, line 1), 20.86%. The grid for G is given by (G_l, G_m, G_h) , where $G_h = (1 + m)G_m$ and $G_l = (1 - m)G_m$, and the discretized G -process on $[0.2205, 0.2319, 0.2433]$ has the following transition matrix:

$$\begin{bmatrix} 0.9607 & 0.0389 & 0.0004 \\ 0.0195 & 0.9611 & 0.0195 \\ 0.0004 & 0.0389 & 0.9607 \end{bmatrix}.$$

A.2 Fiscal rule

A.2.1 Methodology and estimation results

We first estimate the fiscal rule separately at two levels of government: the federal government level and the state/local level, allowing for debt only at the federal level.³² We then construct a composite rule, using the share of federal government purchases in total government purchases.

The empirical specification for the federal fiscal rule is based on Bohn (1998) and Davig and Leeper (2011) and takes the following form:

$$\frac{T_t^F - Tr_t^F}{Y_t} = \rho_{T,0}^F + \rho_{T,B}^F \frac{B_t}{Y_t} + \rho_{T,Y}^F \log\left(\frac{Y_t}{\bar{Y}_t}\right) + \rho_{T,G}^F \frac{G_t^F}{Y_t}, \quad (\text{A.1})$$

³²When we estimate one equation, using the sum of federal and the state-local level data, the estimation result implies a non-stationary government debt process.

where:

Y_t : Nominal GDP (Line 1 of NIPA table 1.1.5).

$T_t^F - Tr_t^F$: Federal government current receipts (Line 1 of NIPA table 3.2) minus federal government transfer expenditure (Line 25 of NIPA table 3.2).

B_t : Market value of privately held gross federal debt at the beginning of a quarter: data are from the Federal Reserve Bank of Dallas (<http://www.dallasfed.org/research/econdata/govdebt.cfm>).

\bar{Y}_t : Nominal CBO potential GDP: data are from the CBO website (<http://www.cbo.gov/publication/42912>).

G_t^F : Nominal federal government consumption expenditures and gross investment (Line 23 of NIPA table 1.1.5).

At the state and local level, we drop the debt-to-GDP ratio term, yielding the following equation for the state and local level:

$$\frac{T_t^{SL} - Tr_t^{SL}}{Y_t} = \rho_{T,0}^{SL} + \rho_{T,Y}^{SL} \log\left(\frac{Y_t}{\bar{Y}_t}\right) + \rho_{T,G}^{SL} \frac{G_t^{SL}}{Y_t}, \quad (\text{A.2})$$

where:

$T_t^{SL} - Tr_t^{SL}$: State and local government receipts (Line 1 of NIPA table 3.3) minus state and local government transfer expenditure (Line 24 of NIPA table 3.3).

G_t^{SL} : Nominal state and local government consumption expenditures and gross investment (Line 26 of NIPA table 1.1.5).

We then linearly detrend all ratio variables, except for $\log(Y_t/\bar{Y}_t)$, before estimating equations (A.1) and (A.2).

Table 12 summarizes the estimation results.

Table 12: Estimated coefficients of the fiscal rule

	constant	B_t/Y_t	$\log(Y_t/\bar{Y}_t)$	G_t/Y_t
Federal	-0.009 (0.001)	0.017 (0.004)	0.321 (0.032)	0.146 (0.096)
State and local	0.001 (0.001)	– –	-0.039 (0.015)	0.771 (0.063)

A.2.2 The composite fiscal rule

The composite fiscal rule used in our model is given by:

$$\begin{aligned} \frac{T_t - Tr_t}{Y_t} &= \rho_{T,0} + \rho_{T,B} \frac{B_t}{Y_t} + \rho_{T,Y} \log\left(\frac{Y_t}{\bar{Y}_t}\right) + \rho_{T,G} \frac{G_t}{Y_t} \\ &= \rho_{T,0} + \rho_{T,B}^F \frac{B_t}{Y_t} + (\rho_{T,Y}^F + \rho_{T,Y}^{SL}) \log\left(\frac{Y_t}{\bar{Y}_t}\right) + (\gamma^F \rho_{T,G}^F + (1 - \gamma^F) \rho_{T,G}^{SL}) \frac{G_t}{Y_t}, \end{aligned} \quad (\text{A.3})$$

where γ^F is calibrated as the average share of federal government purchases within total government purchases: 0.46. This yields the following fiscal rule parameters:

$$\rho_{T,B} = 0.017, \quad \rho_{T,Y} = 0.282, \quad \rho_{T,G} = 0.484.$$

We use $\rho_{T,0}$ to match the average debt-to-GDP ratio in the data: 30%.

A.3 Consumption and income tax parameters

For the consumption tax function and the linear part of the income tax function, we use the average tax rate calculated from the data.

To be specific, the average tax rate on consumption is defined as:

$$\tau_c = \frac{TPI - PRT}{PCE - (TPI - PRT)}, \quad (\text{A.4})$$

where the numerator is taxes on production and imports (TPI, NIPA table 3.1, line 4) minus state and local property taxes (PRT, NIPA table 3.3, line 8). The denominator is personal consumption expenditures (PCE, NIPA table 1.1.5, line 2) net of the numerator. We calculate the average $\tau_{c,t}$ over our sample period as our τ_c parameter: 8.14%.

For income taxes, we use the state level tax revenue to approximate the linear part:

$$\tau_0 = \frac{PIT + CT + PRT}{\text{Taxable Income}}, \quad (\text{A.5})$$

where PIT (NIPA table 3.3, line 4) is state income tax, CT (NIPA table 3.3, line 10) is state tax on corporate income, and PRT (NIPA table 3.3, line 8) is state property taxes. Note that we exclude the social insurance contribution in the numerator since we do not have social security expenditures in the model. The denominator is GDP minus consumption of fixed capital (NIPA table 1.7.5, line 6), since our model has a depreciation allowance for capital income. Averaging $\tau_{0,t}$ from 1960I to 2007IV yields $\tau_0 = 5.25\%$.

B Appendix: Computational algorithm

B.1 Computational algorithm for the two-shock stochastic steady state economy

Step 0: We first select a set of summary statistics for the wealth distribution, $\{K, Gini(a)\}$, and fix the functional form of the equilibrium rules in equations (4.1)-(4.3). We then set the interpolation grids for $(a, K, Gini(a), B)$ to be used in the approximation of the household's continuation value function and policy function. We use an initial guess of coefficients $\{a_0^0, \dots, a_7^0\}$, $\{\tilde{a}_0^0, \dots, \tilde{a}_7^0\}$, $\{b_0^0, \dots, b_7^0\}$ to obtain initial conjectures for $\{H_\Gamma^0, S^0\}$, and set up a convergence criterion $\varepsilon = 10^{-4}$.

Step 1: At the n th iteration, imposing $\{H_\Gamma^n, S^n\}$ in the household optimization problem, we use a value function iteration to solve the household's parametric dynamic programming problem as defined in Section 2.4. From this process, we obtain the continuation value function $V^n(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z, G; H_\Gamma^n, S^n)$.

Step 2: We next simulate the economy using N_H households and T periods. In each period t of the simulation, we first calculate the equilibrium $s_t^{eq,n}$ using equation (2.11) and $\{H_\Gamma^n\}$. Then we solve the household's optimization problem for the current $(K_t^n, Gini_t^n(a), B_t^n, z_t^n, G_t^n, s_t^{eq,n})$ using $V^n(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z, G; H_\Gamma^n, S^n)$ as the continuation value function and $\{H_\Gamma^n, H_B\}$. This is a one-shot optimization problem. The aggregate states in the next period follow from our aggregation of the optimal household decisions. From this step, we collect the time series $\{K_t^n, Gini_t^n(a), B_t^n, z_t^n, G_t^n, s_t^{eq,n}\}_{t=1}^T$.

Step 3: With these time series, we obtain separate OLS estimates of $\{\hat{a}_0^n, \dots, \hat{a}_7^n\}$, $\{\hat{a}_0^n, \dots, \hat{a}_7^n\}$, $\{\hat{b}_0^n, \dots, \hat{b}_7^n\}$, for each z and G combination, which, with a slight abuse of notation, we summarize as $(\hat{H}_\Gamma^n, \hat{S}^n)$.

Step 4: If $|H_\Gamma^n - \hat{H}_\Gamma^n| < \epsilon$ and $|S^n - \hat{S}^n| < \epsilon$, we stop. Otherwise, we set

$$\begin{aligned} H_\Gamma^{n+1} &= \alpha_H \times \hat{H}_\Gamma^n + (1 - \alpha_H) \times H_\Gamma^n \\ S^{n+1} &= \alpha_S \times \hat{S}^n + (1 - \alpha_S) \times S^n \end{aligned}$$

with $\alpha_H, \alpha_S \in (0, 1]$, and go to Step 1.

Step 5: Finally, we check whether the $R2s$ (the multiple-step-ahead forecast errors) of the final OLS regressions are sufficiently high (small) for the equilibrium rules to be well approximated. If they are not, we change the functional forms in Step 0 and repeat the algorithm.

In Step 1, we iterate on the value function until it converges at a set of collocation points, chosen to be the grid points of $(a, K, Gini(a), B)$ defined in Step 0. In each step of the value function iteration, we use multi-dimensional cubic splines on this interpolation grid to approximate the continuation value function. For each collocation point of the state variables $(a, K, Gini(a), B)$ as well as the exogenous aggregate state variables (z, G) , we use $\{H_\Gamma^n, S^n, H_B\}$ to infer the values of $(K', Gini'(a), B', s)$. Given the aggregate variables $(K', Gini'(a), B', s)$, we maximize the Bellman equation numerically along the a' -dimension using Brent's method, as described in Press et al. (2007). The same method is used in the numerical optimization part of Step 2.

In Step 2, we use $N_H = 90,000$ households and run 12 parallel simulations of length $T = 18,000$ each.³³ Following Krusell and Smith (1998), we also enforce that at each t these 90,000 households are distributed according to the stationary distribution of the Markov chains governing ε and β . We thus avoid introducing artificial aggregate uncertainty due to small deviations from the law of large numbers. To eliminate sampling error, we use the same series of aggregate shocks for all iterations and all model simulations.

The algorithm is implemented in a mixture of C/C++ and MATLAB, which are then connected through MATLAB's CMEX interface.³⁴

³³Although each simulation has 19,000 periods, we discard the initial 1,000 observations in the estimation.

³⁴On a 12-core 2.67 GHz Intel Xeon X5650 Linux workstation, the typical run time for the value function iteration lies around several hours (it gets shorter as the initial guess gets more accurate), while that for one simulation loop is about 40 minutes. Starting from a guess close to the equilibrium, it takes about 40 iterations to converge.

B.2 Results for the stochastic steady state

B.2.1 Estimated laws of motions for the two-shock equilibrium

H_Γ for aggregate capital in *good* times (state z_g), with low (G_l), medium (G_m), and high (G_h) government purchases levels are, respectively (in the above order):

$$\log(K') = 0.1422 + 0.9125\log(K) - 0.0019B + 0.0124(\log(K))^2 + 0.0000B^2 \quad (G_l)$$

$$+ 0.0000B^3 + 0.0007\log(K)B + 0.0007Gini(a), \quad R2 = 0.999999,$$

$$\log(K') = 0.1374 + 0.9151\log(K) - 0.0017B + 0.0120(\log(K))^2 + 0.0000B^2 \quad (G_m)$$

$$- 0.0000B^3 + 0.0006\log(K)B + 0.0007Gini(a), \quad R2 = 0.999999,$$

$$\log(K') = 0.1223 + 0.9265\log(K) - 0.0014B + 0.0098(\log(K))^2 + 0.0000B^2 \quad (G_h)$$

$$- 0.0000B^3 + 0.0005\log(K)B + 0.0006Gini(a), \quad R2 = 0.999999.$$

H_Γ for aggregate capital in *bad* times (state z_b), with low, medium, and high government purchases levels are, respectively:

$$\log(K') = 0.1116 + 0.9299\log(K) - 0.0016B + 0.0094(\log(K))^2 + 0.0000B^2 \quad (G_l)$$

$$+ 0.0000B^3 + 0.0006\log(K)B + 0.0010Gini(a), \quad R2 = 0.999999,$$

$$\log(K') = 0.1084 + 0.9313\log(K) - 0.0015B + 0.0093(\log(K))^2 + 0.0000B^2 \quad (G_m)$$

$$+ 0.0000B^3 + 0.0005\log(K)B + 0.0009Gini(a), \quad R2 = 0.999999,$$

$$\log(K') = 0.0983 + 0.9385\log(K) - 0.0012B + 0.0079(\log(K))^2 + 0.0000B^2 \quad (G_h)$$

$$- 0.0000B^3 + 0.0004\log(K)B + 0.0008Gini(a), \quad R2 = 0.999999.$$

H_Γ for the Gini coefficient of the wealth distribution in *good* times (state z_g), with low, medium, and high government purchases levels are, respectively:

$$Gini(a') = -0.0300 + 0.0087\log(K) + 0.0024B + 0.0017(\log(K))^2 - 0.0000B^2 \quad (G_l)$$

$$- 0.0000B^3 - 0.0008\log(K)B + 0.9975Gini(a), \quad R2 = 0.999999,$$

$$Gini(a') = -0.0455 + 0.0213\log(K) + 0.0026B - 0.0009(\log(K))^2 - 0.0000B^2 \quad (G_m)$$

$$+ 0.0000B^3 - 0.0009\log(K)B + 0.9975Gini(a), \quad R2 = 0.999999,$$

$$Gini(a') = -0.0431 + 0.0194\log(K) + 0.0025B - 0.0005(\log(K))^2 - 0.0000B^2 \quad (G_h)$$

$$+ 0.0000B^3 - 0.0008\log(K)B + 0.9976Gini(a), \quad R2 = 0.999999.$$

H_Γ for the Gini coefficient of the wealth distribution in *bad* times (state z_b), with low, medium, and high government purchases levels are, respectively:

$$Gini(a') = -0.0747 + 0.0472\log(K) + 0.0025B - 0.0066(\log(K))^2 - 0.0000B^2 \quad (G_l)$$

$$- 0.0000B^3 - 0.0009\log(K)B + 0.9992Gini(a), \quad R2 = 0.999998,$$

$$Gini(a') = -0.0853 + 0.0559\log(K) + 0.0027B - 0.0083(\log(K))^2 - 0.0000B^2 \quad (G_m)$$

$$+ 0.0000B^3 - 0.0010\log(K)B + 0.9992Gini(a), \quad R2 = 0.999998,$$

$$Gini(a') = -0.0804 + 0.0519\log(K) + 0.0027B - 0.0075(\log(K))^2 - 0.0000B^2 \quad (G_h)$$

$$+ 0.0000B^3 - 0.0010\log(K)B + 0.9992Gini(a), \quad R2 = 0.999998.$$

S in *good* times (state z_g), with low, medium, and high government purchases levels are, respectively:

$$\log(s) = 8.3647 - 8.3407\log(K) - 0.5945B + 2.0612(\log(K))^2 + 0.0095B^2 \quad (G_l)$$

$$+ 0.0012B^3 + 0.3437\log(K)B - 0.0907Gini(a), \quad R2 = 0.99992,$$

$$\log(s) = 11.4913 - 10.8071\log(K) - 0.8477B + 2.5649(\log(K))^2 + 0.0109B^2 \quad (G_m)$$

$$+ 0.0021B^3 + 0.4499\log(K)B - 0.0830Gini(a), \quad R2 = 0.99983,$$

$$\log(s) = 16.1515 - 14.4841\log(K) - 1.2486B + 3.3065(\log(K))^2 + 0.0117B^2 \quad (G_h)$$

$$+ 0.0036B^3 + 0.6183\log(K)B - 0.0747Gini(a), \quad R2 = 0.99957.$$

S in *bad* times (state z_b), with low, medium, and high government purchases levels are, respectively:

$$\log(s) = 7.7967 - 7.9011\log(K) - 0.5212B + 1.9637(\log(K))^2 + 0.0076B^2 \quad (G_l)$$

$$+ 0.0018B^3 + 0.3255\log(K)B - 0.1058Gini(a), \quad R2 = 0.99991,$$

$$\log(s) = 11.0908 - 10.4747\log(K) - 0.8318B + 2.4831(\log(K))^2 + 0.0087B^2 \quad (G_m)$$

$$+ 0.0032B^3 + 0.4553\log(K)B - 0.0978Gini(a), \quad R2 = 0.99970,$$

$$\log(s) = 15.8242 - 14.2123\log(K) - 1.2243B + 3.2384(\log(K))^2 + 0.0074B^2 \quad (G_h)$$

$$+ 0.0052B^3 + 0.6220\log(K)B - 0.0935Gini(a), \quad R2 = 0.99924.$$

B.2.2 Multi-step ahead forecast errors

To compute the multi-step ahead forecast errors, we compare the aggregate paths from the equilibrium simulation with those generated by the approximate equilibrium rules. To be specific, starting from the first period, we pick a set of aggregate states 80 periods apart along the final simulation of the stochastic steady state calculation, $\{K_{(i-1)\times 80+1}, B_{(i-1)\times 80+1}, Gini_{(i-1)\times 80+1}(a)\}_{i \in \{1, 2, \dots\}}$. We then simulate each set of aggregate states for 40 periods with the equilibrium laws of motions H_Γ and H_B (with the same aggregate shock sequences as those in the equilibrium simulation), and record the aggregate capital level in the last period of each simulation $\{K_{(i-1)\times 80+1+40}^H\}_{i \in \{1, 2, \dots\}}$. We then calculate the i th 10-year (40-period) ahead forecast error for aggregate capital as $u_i^{40} = K_{(i-1)\times 80+1+40} - K_{(i-1)\times 80+1+40}^H$, where K_t is the aggregate capital level at period t from the equilibrium simulation. Note that the forecast errors generated this way are independent of each other because 40 periods are discarded between each simulation. With the large number of simulations in the stochastic steady state calculation (12 parallel simulations each for 18,000 periods), we generate about 2,700 such forecast errors. The mean of these 10-year-ahead forecast errors is 0.0002%, and the root mean squared error (RMSE) of the 10-year-ahead forecast is about 0.08% of the long-run average capital level, suggesting little bias and high overall forecast accuracy in our laws of motions.

B.2.3 Estimated laws of motions for the one-shock equilibrium

H_Γ for aggregate capital in *good* times (state z_g) and *bad* times (state z_b) are, respectively:

$$\begin{aligned} \log(K') &= 0.1329 + 0.9189\log(K) - 0.0016B + 0.0112(\log(K))^2 + 0.0000B^2 & (z_g) \\ &\quad - 0.0000B^3 + 0.0006\log(K)B + 0.0007Gini(a), \quad R2 = 0.999999, \end{aligned}$$

$$\begin{aligned} \log(K') &= 0.1045 + 0.9346\log(K) - 0.0014B + 0.0086(\log(K))^2 + 0.0000B^2 & (z_b) \\ &\quad + 0.0000B^3 + 0.0005\log(K)B + 0.0009Gini(a), \quad R2 = 0.999999. \end{aligned}$$

H_Γ for Gini coefficient of wealth distribution in *good* times (state z_g) and *bad* times (state z_b) are, respectively:

$$\begin{aligned} Gini(a') &= -0.0509 + 0.0258\log(K) + 0.0026B - 0.0018(\log(K))^2 - 0.0000B^2 & (z_g) \\ &\quad + 0.0000B^3 - 0.0009\log(K)B + 0.9976Gini(a), \quad R2 = 0.999999, \end{aligned}$$

$$\begin{aligned} Gini(a') &= -0.0997 + 0.0678\log(K) + 0.0030B - 0.0108(\log(K))^2 - 0.0000B^2 & (z_b) \\ &\quad - 0.0000B^3 - 0.0011\log(K)B + 0.9992Gini(a), \quad R2 = 0.999997. \end{aligned}$$

S in *good* times (state z_g) and *bad* times (state z_b) are, respectively:

$$\begin{aligned} \log(s) &= 10.0396 - 9.6184\log(K) - 0.7878B + 2.3216(\log(K))^2 + 0.0117B^2 & (z_g) \\ &\quad + 0.0017B^3 + 0.4253\log(K)B - 0.0841Gini(a), \quad R2 = 0.99991, \end{aligned}$$

$$\begin{aligned} \log(s) &= 9.3062 - 9.0204\log(K) - 0.7350B + 2.1870(\log(K))^2 + 0.0096B^2 & (z_b) \\ &\quad + 0.0025B^3 + 0.4157\log(K)B - 0.1008Gini(a), \quad R2 = 0.99987. \end{aligned}$$

B.3 Computational algorithm for the transition-path equilibrium

Step 0: Set up:

We choose the starting G level (three possibilities) and guess a length for the transition period

T_{trans} .

We next assume specific functional forms for $\{H_{\Gamma,t}^{trans}, S_t^{trans}\}_{t=1}^{T_{trans}}$.

We then select the interpolation grid for $(a, K, Gini(a), B)$ used in the spline approximation of the household's continuation value function.

This calculation also requires the following inputs:

1. H_{Γ}^{1s} and S^{1s} : laws of motions from the one-shock equilibrium.
2. $V_{1s}(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z; H_{\Gamma}^{1s}, S^{1s})$: the value function for households from the one-shock equilibrium.
3. N_{trans} independent joint distributions over $(a, \varepsilon, \tilde{\beta})$ (each with N_H households) drawn from the two-shock equilibrium simulation to start the transition-path equilibrium simulations. To get a balanced sample for each combination of z and t , exactly half of these distributions are collected during good times ($z = z_g$).
4. N_{trans} different aggregate productivity shock paths $\{\{z_t^i\}_{t=1}^{T_{trans}}\}_{i=1}^{N_{trans}}$, where z_t^i matches with the productivity level in the i th collected joint distribution and the $z_{t>1}^i$ are randomly drawn following its Markov process.

Step 1: We start from an initial coefficient guess $\{\{a_{0,t}^0, \dots, a_{7,t}^0\}, \{\tilde{a}_{0,t}^0, \dots, \tilde{a}_{7,t}^0\}, \{b_{0,t}^0, \dots, b_{7,t}^0\}\}_{t=1}^{T_{trans}}$ to get our initial conjectures $\{H_{\Gamma,t}^{trans,0}, S_t^{trans,0}\}_{t=1}^{T_{trans}}$. Set up a convergence criterion ε .

Step 2: In the n th iteration, we compute the household's value function at each period by backward induction, with imposed laws of motions $\{H_{\Gamma,t}^{trans,n}, S_t^{trans,n}\}_{t=1}^{T_{trans}}$. To be specific, $\forall t \in \{1, \dots, T_{trans}\}$, given the continuation value $V_{t+1}^{trans,n}(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z; H_{\Gamma,t+1}^{trans,n}, S_{t+1}^{trans,n})$, we calculate $V_t^{trans,n}(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z; H_{\Gamma,t}^{trans,n}, S_t^{trans,n})$. Note that $V_{T_{trans}+1}^{trans,n}(\cdot) = V_{1s}(\cdot)$. We store the value functions at each point in time of the transition path.

Step 3: In this step, we simulate the N_{trans} economies with the corresponding productivity shock paths. For the simulation of the i th economy, in each period t , we first calculate the equilibrium $s_t^{eq,n,i}$ using equation (2.11) and $H_{\Gamma,t}^{trans,n}$. Then we solve the household's optimization problem for $(K_t^{n,i}, Gini_t^{n,i}(a), B_t^{n,i}, z_t^{n,i}, s_t^{eq,n,i})$ using $V_{t+1}^{trans,n}(a, \varepsilon, \tilde{\beta}, K, Gini(a), B, z; H_{\Gamma,t+1}^{trans,n}, S_{t+1}^{trans,n})$ and $\{H_{\Gamma,t}^{trans,n}, S_t^{trans,n}, H_B\}$. The aggregate states in the next period follow from aggregating the optimal household decisions. We finally collect the following panel data $\{K_t^{n,i}, Gini_t^{n,i}(a), B_t^{n,i}, z_t^{n,i}, s_t^{eq,n,i}\}_{t=1}^T\}_{i=1}^{N_{trans}}$.

Step 4: $\forall t \in \{1, \dots, T_{trans}\}$, we run OLS for each point in time along the transition path to get estimates of $\{\hat{a}_{0,t}^n, \dots, \hat{a}_{7,t}^n\}, \{\hat{\tilde{a}}_{0,t}^n, \dots, \hat{\tilde{a}}_{7,t}^n\}, \{\hat{b}_{0,t}^n, \dots, \hat{b}_{7,t}^n\}$, which, with a slight abuse of notation, we summarize as $\{\hat{H}_{\Gamma,t}^{trans,n}, \hat{S}_t^{trans,n}\}_{t=1}^{T_{trans}}$.

Step 5: If $\max_t |H_{\Gamma,t}^{trans,n} - \hat{H}_{\Gamma,t}^{trans,n}| < \varepsilon$ and $\max_t |S_t^{trans,n} - \hat{S}_t^{trans,n}| < \varepsilon$, we stop. Otherwise, $\forall t \in \{1, \dots, T\}$, we set:

$$\begin{aligned} H_{\Gamma,t}^{trans,n+1} &= \alpha_H \times \hat{H}_{\Gamma,t}^{trans,n} + (1 - \alpha_H) \times H_{\Gamma,t}^{trans,n} \\ S_t^{trans,n+1} &= \alpha_S \times \hat{S}_t^{trans,n} + (1 - \alpha_S) \times S_t^{trans,n} \end{aligned}$$

with $\alpha_H, \alpha_S \in (0, 1]$, and go to Step 2.

Step 6: We check the convergence of the last period's laws of motion to those from the one-shock equilibrium. To be specific, starting from the aggregate states observed in the last period's simulations, $\{z_T^i, K_T^i, B_T^i, Gini_T^i(a)\}_{i=1}^{N_{trans}}$, we calculate the differences in the predicted values of aggregate capital, the wealth Gini coefficient and s , between when we use the converged last period's laws of motions of the transition-path equilibrium, $\{H_{\Gamma, T_{trans}}^{trans}, S_{T_{trans}}^{trans}\}$, and when we use the laws of motions from the one-shock economy, $\{H_{\Gamma, 1s}, S_{1s}\}$. If the differences are comparable in size to those of the one-step prediction errors of the laws of motions from the one-shock stochastic steady state equilibrium, we go to Step 7. Otherwise, we go back to Step 0 and increase T_{trans} .

Step 7: We check whether the $R2s$ (the multiple-step-ahead forecast errors) of the final OLS regressions are sufficiently high (small) for the equilibrium rules to be well approximated. If they are not, we change the functional forms in Step 0 and repeat the algorithm.³⁵

The numerical methods for interpolation and optimization used to solve the household's maximization problem in Step 2 and Step 3 are the same as those in the computation of the stochastic steady state. The only difference is that the procedure does not involve a value function iteration since we use backward induction to solve for the value function for each period, starting from the value function from the stochastic steady state without fiscal uncertainty.

B.4 Estimated laws of motions for transition-path equilibrium

Here we present the estimated laws of motions for selected periods and every combination of z_t and G_1 .

³⁵We choose $\varepsilon = 10^{-4}$, $N_H = 90,000$, and $T_{trans} = 400$. N_{trans} is set to be 8,000 when we start from G_m and 4,000 for the other two cases, to keep the ratios between the numbers of observations across different cases the same as those implied by the ergodic distribution of the G -process. On a 32-core 2.4 GHz Intel Xeon E5-4640 Linux workstation, the typical run time for the value function calculation takes about an hour, and for one simulation loop, it takes about six hours. Starting from a guess based on the weighted average between the two-shock and the one-shock laws of motion, it takes about 40 to 50 iterations to converge.

Table 13: Transition-path equilibrium: starting from $G_1 = G_l, z_t = z_g$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	R2
1	0.1390	0.9150	-0.0017	0.0118	0.0000	-0.0000	0.0006	0.0007	0.2205	0.999999
2	0.1423	0.9122	-0.0018	0.0124	0.0000	-0.0000	0.0007	0.0007	0.2210	0.999999
3	0.1446	0.9104	-0.0018	0.0128	0.0000	-0.0000	0.0007	0.0007	0.2214	0.999999
...
388	0.1278	0.9231	-0.0015	0.0103	0.0000	-0.0000	0.0005	0.0006	0.2319	0.999999
399	0.1303	0.9210	-0.0016	0.0108	0.0000	-0.0000	0.0006	0.0006	0.2319	0.999999
400	0.1321	0.9196	-0.0016	0.0110	0.0000	-0.0000	0.0006	0.0006	0.2319	0.999999
$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	R2	
1	-0.0394	0.0162	0.0027	0.0002	-0.0000	0.0000	-0.0009	0.9975	0.2205	0.999999
2	-0.0340	0.0117	0.0026	0.0011	-0.0000	-0.0000	-0.0009	0.9975	0.2210	0.999999
3	-0.0451	0.0208	0.0028	-0.0008	-0.0000	0.0000	-0.0009	0.9975	0.2214	0.999999
...
388	-0.0524	0.0271	0.0025	-0.0021	-0.0000	0.0000	-0.0008	0.9975	0.2319	0.999999
399	-0.0475	0.0231	0.0025	-0.0013	-0.0000	0.0000	-0.0008	0.9975	0.2319	0.999999
400	-0.0548	0.0291	0.0026	-0.0025	-0.0000	0.0000	-0.0009	0.9976	0.2319	0.999999
$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	R2	
1	10.1857	-9.8044	-0.6990	2.3558	0.0105	0.0012	0.3858	-0.0880	0.2205	0.99985
2	10.7885	-10.3002	-0.7003	2.4586	0.0099	0.0014	0.3867	-0.0892	0.2210	0.99985
3	10.8166	-10.3177	-0.7052	2.4617	0.0100	0.0014	0.3888	-0.0890	0.2214	0.99986
...
388	9.2488	-8.9958	-0.7253	2.1997	0.0126	0.0013	0.3996	-0.0880	0.2319	0.99993
399	9.4318	-9.1361	-0.7483	2.2264	0.0134	0.0011	0.4088	-0.0873	0.2319	0.99993
400	9.4819	-9.1765	-0.7500	2.2345	0.0134	0.0011	0.4095	-0.0871	0.2319	0.99993

Table 14: Transition-path equilibrium: starting from $G_1 = G_l, z_t = z_b$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	R2
1	0.1138	0.9280	-0.0018	0.0098	0.0000	0.0000	0.0006	0.0010	0.2205	0.999999
2	0.1213	0.9219	-0.0020	0.0110	0.0000	0.0000	0.0008	0.0010	0.2210	0.999999
3	0.1098	0.9312	-0.0016	0.0092	0.0000	0.0000	0.0006	0.0010	0.2214	0.999999
...
388	0.1000	0.9382	-0.0013	0.0079	0.0000	0.0000	0.0004	0.0009	0.2319	0.999999
399	0.0977	0.9402	-0.0013	0.0074	0.0000	-0.0000	0.0005	0.0009	0.2319	0.999999
400	0.0987	0.9393	-0.0013	0.0076	0.0000	-0.0000	0.0005	0.0009	0.2319	0.999999
$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	R2	
1	-0.1048	0.0716	0.0035	-0.0115	-0.0000	-0.0000	-0.0013	0.9991	0.2205	0.999997
2	-0.1013	0.0685	0.0037	-0.0108	-0.0000	-0.0000	-0.0014	0.9991	0.2210	0.999998
3	-0.0858	0.0556	0.0037	-0.0081	-0.0000	-0.0000	-0.0014	0.9991	0.2214	0.999997
...
388	-0.1108	0.0768	0.0032	-0.0126	-0.0000	0.0000	-0.0012	0.9992	0.2319	0.999998
399	-0.1003	0.0681	0.0034	-0.0108	-0.0000	-0.0000	-0.0012	0.9991	0.2319	0.999997
400	-0.0972	0.0655	0.0034	-0.0103	-0.0000	-0.0000	-0.0013	0.9991	0.2319	0.999997
$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	R2	
1.0000	9.6792	-9.4256	-0.6023	2.2721	0.0084	0.0018	0.3583	-0.1046	0.2205	0.99988
2.0000	8.7794	-8.6901	-0.5785	2.1223	0.0087	0.0017	0.3485	-0.1027	0.2210	0.99989
3.0000	10.3272	-9.9497	-0.6354	2.3792	0.0092	0.0018	0.3717	-0.1024	0.2214	0.99986
...
388	8.5682	-8.4447	-0.6632	2.0752	0.0123	0.0016	0.3855	-0.1017	0.2319	0.99993
399	8.7976	-8.6271	-0.6809	2.1114	0.0120	0.0017	0.3930	-0.1025	0.2319	0.99993
400	8.7089	-8.5561	-0.6756	2.0973	0.0120	0.0016	0.3908	-0.1025	0.2319	0.99993

Table 15: Transition-path equilibrium: starting from $G_1 = G_m$, $z_t = z_g$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	R2
1	0.1345	0.9176	-0.0016	0.0115	0.0000	-0.0000	0.0006	0.0007	0.2319	0.999999
2	0.1348	0.9173	-0.0016	0.0115	0.0000	0.0000	0.0006	0.0007	0.2319	0.999999
3	0.1351	0.9170	-0.0016	0.0116	0.0000	0.0000	0.0006	0.0007	0.2319	0.999999
...
388	0.1311	0.9203	-0.0015	0.0109	0.0000	0.0000	0.0006	0.0007	0.2319	0.999999
399	0.1326	0.9190	-0.0015	0.0112	0.0000	0.0000	0.0005	0.0008	0.2319	0.999999
400	0.1327	0.9189	-0.0015	0.0112	0.0000	0.0000	0.0005	0.0008	0.2319	0.999999
$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	R2	
1	-0.0470	0.0224	0.0028	-0.0011	-0.0000	-0.0000	-0.0009	0.9975	0.2319	0.999999
2	-0.0506	0.0253	0.0029	-0.0017	-0.0000	-0.0000	-0.0010	0.9975	0.2319	0.999999
3	-0.0523	0.0267	0.0029	-0.0020	-0.0000	-0.0000	-0.0010	0.9975	0.2319	0.999999
...
388	-0.0520	0.0265	0.0029	-0.0019	-0.0000	-0.0000	-0.0010	0.9975	0.2319	0.999999
399	-0.0497	0.0247	0.0027	-0.0016	-0.0000	-0.0000	-0.0009	0.9975	0.2319	0.999999
400	-0.0537	0.0279	0.0028	-0.0022	-0.0000	-0.0000	-0.0010	0.9975	0.2319	0.999999
$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	R2	
1	11.6816	-10.9556	-0.8705	2.5939	0.0122	0.0018	0.4589	-0.0846	0.2319	0.999984
2	11.6892	-10.9601	-0.8751	2.5946	0.0120	0.0019	0.4607	-0.0855	0.2319	0.999983
3	11.8277	-11.0691	-0.8836	2.6160	0.0123	0.0019	0.4641	-0.0857	0.2319	0.999983
...
388	10.4730	-9.9769	-0.7954	2.3957	0.0125	0.0015	0.4283	-0.0848	0.2319	0.999992
399	10.3018	-9.8403	-0.7797	2.3687	0.0122	0.0015	0.4220	-0.0855	0.2319	0.999993
400	9.9651	-9.5653	-0.7746	2.3124	0.0121	0.0015	0.4199	-0.0853	0.2319	0.999993

Table 16: Transition-path equilibrium: starting from $G_1 = G_m$, $z_t = z_b$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	R2
1	0.1072	0.9322	-0.0015	0.0091	0.0000	-0.0000	0.0005	0.0010	0.2319	0.999999
2	0.1071	0.9324	-0.0015	0.0091	0.0000	-0.0000	0.0005	0.0010	0.2319	0.999999
3	0.1077	0.9319	-0.0015	0.0092	0.0000	-0.0000	0.0005	0.0010	0.2319	0.999999
...
388	0.1003	0.9379	-0.0013	0.0079	0.0000	0.0000	0.0004	0.0010	0.2319	0.999999
399	0.1018	0.9367	-0.0014	0.0082	0.0000	0.0000	0.0005	0.0010	0.2319	0.999999
400	0.1001	0.9381	-0.0014	0.0079	0.0000	0.0000	0.0005	0.0010	0.2319	0.999999
$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	R2	
1	-0.0976	0.0660	0.0030	-0.0104	-0.0000	-0.0000	-0.0011	0.9991	0.2319	0.999998
2	-0.0829	0.0538	0.0028	-0.0079	-0.0000	-0.0000	-0.0010	0.9991	0.2319	0.999998
3	-0.0868	0.0570	0.0029	-0.0085	-0.0000	-0.0000	-0.0011	0.9991	0.2319	0.999998
...
388	-0.0944	0.0633	0.0031	-0.0098	-0.0000	0.0000	-0.0011	0.9992	0.2319	0.999998
399	-0.0977	0.0662	0.0028	-0.0105	-0.0000	-0.0000	-0.0010	0.9991	0.2319	0.999998
400	-0.0957	0.0645	0.0028	-0.0101	-0.0000	0.0000	-0.0010	0.9992	0.2319	0.999998
$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	R2	
1	11.1724	-10.5413	-0.8306	2.4971	0.0106	0.0026	0.4544	-0.1022	0.2319	0.999983
2	10.4475	-9.9508	-0.8006	2.3768	0.0101	0.0026	0.4424	-0.1020	0.2319	0.999986
3	10.9516	-10.3594	-0.8225	2.4595	0.0103	0.0026	0.4514	-0.1017	0.2319	0.999985
...
388	9.2879	-9.0151	-0.7167	2.1881	0.0112	0.0021	0.4077	-0.1022	0.2319	0.999993
399	9.7104	-9.3685	-0.7219	2.2617	0.0113	0.0020	0.4100	-0.1005	0.2319	0.999992
400	9.6848	-9.3502	-0.7156	2.2586	0.0113	0.0020	0.4074	-0.1008	0.2319	0.999992

Table 17: Transition-path equilibrium: starting from $G_1 = G_h, z_t = z_g$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	R2
1	0.1279	0.9219	-0.0015	0.0107	0.0000	-0.0000	0.0006	0.0007	0.2433	0.999999
2	0.1296	0.9206	-0.0016	0.0110	0.0000	-0.0000	0.0006	0.0007	0.2429	0.999999
3	0.1272	0.9226	-0.0015	0.0106	0.0000	-0.0000	0.0006	0.0007	0.2424	0.999999
...
388	0.1408	0.9124	-0.0019	0.0125	0.0000	-0.0000	0.0007	0.0008	0.2319	0.999999
399	0.1454	0.9086	-0.0019	0.0133	0.0000	0.0000	0.0007	0.0008	0.2319	0.999999
400	0.1372	0.9153	-0.0017	0.0119	0.0000	0.0000	0.0006	0.0008	0.2319	0.999999
$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	R2	
1	-0.0416	0.0179	0.0026	-0.0002	-0.0000	0.0000	-0.0009	0.9975	0.2433	0.999999
2	-0.0341	0.0119	0.0024	0.0011	-0.0000	-0.0000	-0.0008	0.9975	0.2429	0.999999
3	-0.0472	0.0227	0.0025	-0.0012	-0.0000	0.0000	-0.0008	0.9975	0.2424	0.999999
...
388	-0.0451	0.0209	0.0026	-0.0008	-0.0000	-0.0000	-0.0009	0.9975	0.2319	0.999999
399	-0.0376	0.0148	0.0024	0.0005	-0.0000	-0.0000	-0.0008	0.9974	0.2319	0.999999
400	-0.0393	0.0161	0.0025	0.0002	-0.0000	-0.0000	-0.0008	0.9974	0.2319	0.999999
$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	R2	
1	17.8423	-15.8786	-1.2955	3.5942	0.0142	0.0032	0.6363	-0.0752	0.2433	0.99960
2	17.5167	-15.6076	-1.2901	3.5373	0.0147	0.0030	0.6337	-0.0768	0.2429	0.99962
3	16.0065	-14.3765	-1.2447	3.2855	0.0146	0.0029	0.6151	-0.0760	0.2424	0.99966
...
388	10.0928	-9.6530	-0.8079	2.3267	0.0132	0.0014	0.4329	-0.0827	0.2319	0.99993
399	10.5458	-10.0150	-0.8425	2.3989	0.0144	0.0012	0.4466	-0.0823	0.2319	0.99993
400	10.7527	-10.1863	-0.8392	2.4344	0.0134	0.0014	0.4457	-0.0824	0.2319	0.99993

Table 18: Transition-path equilibrium: starting from $G_1 = G_h, z_t = z_b$

t	$a_{0,t}$	$a_{1,t}$	$a_{2,t}$	$a_{3,t}$	$a_{4,t}$	$a_{5,t}$	$a_{6,t}$	$a_{7,t}$	G values	R2
1	0.1020	0.9355	-0.0014	0.0085	0.0000	-0.0000	0.0005	0.0009	0.2433	0.999999
2	0.1025	0.9351	-0.0014	0.0086	0.0000	-0.0000	0.0005	0.0009	0.2429	0.999999
3	0.1020	0.9355	-0.0014	0.0085	0.0000	-0.0000	0.0005	0.0009	0.2424	0.999999
...
388	0.1131	0.9274	-0.0016	0.0101	0.0000	-0.0000	0.0006	0.0010	0.2319	0.999999
399	0.1134	0.9271	-0.0017	0.0101	0.0000	-0.0000	0.0006	0.0011	0.2319	0.999999
400	0.1193	0.9223	-0.0019	0.0111	0.0000	-0.0000	0.0007	0.0011	0.2319	0.999999
$\tilde{a}_{0,t}$	$\tilde{a}_{1,t}$	$\tilde{a}_{2,t}$	$\tilde{a}_{3,t}$	$\tilde{a}_{4,t}$	$\tilde{a}_{5,t}$	$\tilde{a}_{6,t}$	$\tilde{a}_{7,t}$	G values	R2	
1	-0.0671	0.0408	0.0024	-0.0052	-0.0000	-0.0000	-0.0009	0.9991	0.2433	0.999998
2	-0.0875	0.0576	0.0029	-0.0087	-0.0000	-0.0000	-0.0010	0.9991	0.2429	0.999998
3	-0.0912	0.0605	0.0032	-0.0092	-0.0000	-0.0000	-0.0012	0.9992	0.2424	0.999998
...
388	-0.0876	0.0576	0.0028	-0.0086	-0.0000	-0.0000	-0.0010	0.9992	0.2319	0.999998
399	-0.0987	0.0669	0.0027	-0.0106	-0.0000	-0.0000	-0.0009	0.9991	0.2319	0.999998
400	-0.0983	0.0666	0.0025	-0.0105	-0.0000	-0.0000	-0.0009	0.9991	0.2319	0.999998
$b_{0,t}$	$b_{1,t}$	$b_{2,t}$	$b_{3,t}$	$b_{4,t}$	$b_{5,t}$	$b_{6,t}$	$b_{7,t}$	G values	R2	
1	14.7092	-13.2959	-1.2257	3.0504	0.0144	0.0037	0.6194	-0.0944	0.2433	0.99961
2	16.4578	-14.7219	-1.2851	3.3401	0.0133	0.0040	0.6442	-0.0937	0.2429	0.99952
3	19.4670	-17.1930	-1.3600	3.8468	0.0131	0.0041	0.6750	-0.0948	0.2424	0.99945
...
388	10.2379	-9.7773	-0.7763	2.3407	0.0123	0.0019	0.4317	-0.1007	0.2319	0.99992
399	9.7252	-9.3655	-0.7442	2.2579	0.0108	0.0023	0.4191	-0.0994	0.2319	0.99992
400	9.7874	-9.4211	-0.7399	2.2703	0.0116	0.0020	0.4171	-0.0994	0.2319	0.99993

B.5 Calculating the welfare gain

In this appendix, we show how to use the value functions from the recursive decision problems in the transition-path equilibrium and the two-shock equilibrium to conduct the welfare cost calculation implicitly defined in equation (5.1), which we restate here for convenience:

$$E_1\left[\sum_{t=1}^{\infty} \beta_t u((1+\lambda)c_t, G_t)\right] = E_1\left[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, \tilde{G}_t)\right]. \quad (5.1)$$

We denote the value functions from the transition-path equilibrium and the two-shock equilibrium by \tilde{V} and V , respectively, where

$$\tilde{V} = E_1\left[\sum_{t=1}^{\infty} \beta_t u(\tilde{c}_t, \tilde{G}_t)\right],$$

and

$$V = E_1\left[\sum_{t=1}^{\infty} \beta_t u(c_t, G_t)\right].$$

Note that the right side of (5.1) is exactly \tilde{V} . Under the assumption of a log separable utility function, the left side of (5.1) can be expressed as:

$$E_1\left[\sum_{t=1}^{\infty} \beta_t u((1+\lambda)c_t, G_t)\right] = E_1\left[\sum_{t=1}^{\infty} \beta_t u(c_t, G_t)\right] + E_1\left[\sum_{t=1}^{\infty} \beta_t \theta \log(1+\lambda)\right] = \theta \log(1+\lambda) E_1\left[\sum_{t=1}^{\infty} \beta_t\right] + V.$$

This allows us to rewrite (5.1) as:

$$\theta \log(1+\lambda) E_1\left[\sum_{t=1}^{\infty} \beta_t\right] + V = \tilde{V}.$$

Thus, λ can be calculated as follows:

$$\lambda = \exp\left(\frac{\tilde{V} - V}{\theta E_1\left[\sum_{t=1}^{\infty} \beta_t\right]}\right) - 1.$$

Note that, since β_1 is known at time 1, the value of the denominator in the parentheses is a function of β_1 . The calculation of λ is straightforward using the transition matrix governing the Markov process for $\tilde{\beta}$.

Under a non-separable utility function, we solve for λ numerically. That is, we calculate the left side of (5.1) as a discounted sum of flow utilities under various values of λ and the equilibrium policy function and then find a value of λ that satisfies the equation, using a bisection search.

C Appendix: Alternative specifications and additional experiments

C.1 λ under different model specifications

Table 19 reports the λ -measure of welfare gains under different model specifications.

Table 19: Expected welfare gains, λ (%), under different model specifications

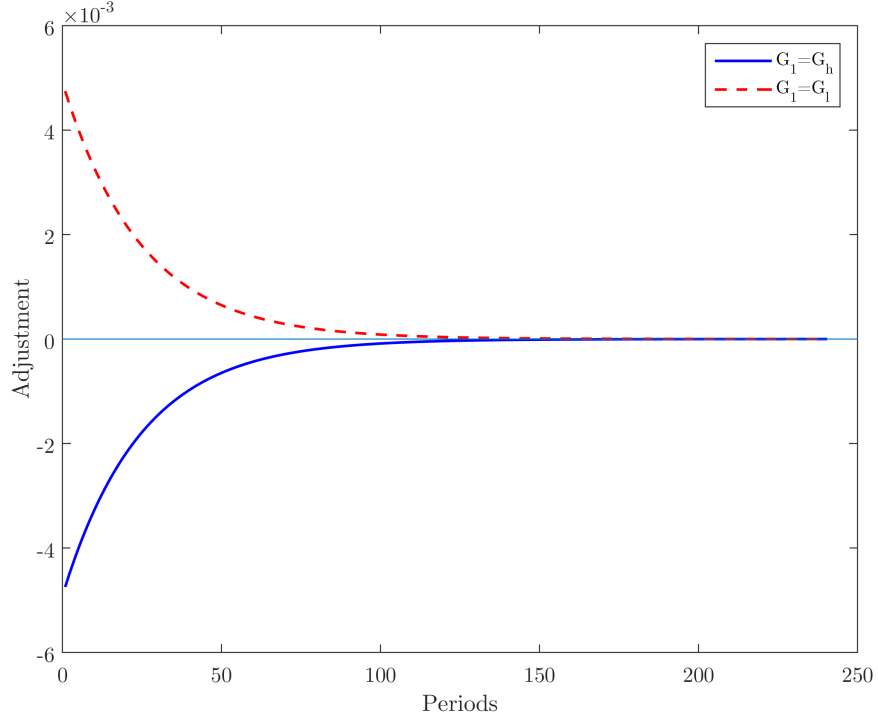
	Wealth Group								
	All	<1%	1-5%	5-25%	25-50%	50-75%	75-95%	95-99%	>99%
Baseline	0.0298	0.0303	0.0307	0.0307	0.0304	0.0301	0.0290	0.0240	0.0242
Non-separable Utility Function									
Substitute ($\rho = 0.5$)	0.0062	0.0046	0.0071	0.0082	0.0052	0.0090	0.0034	-0.0015	0.0053
Complement ($\rho = 1.5$)	0.1233	0.1101	0.1142	0.1196	0.1136	0.1309	0.1324	0.1233	0.1218
Different Tax Function Adjustment									
Adjusting τ_0	0.0295	0.0291	0.0296	0.0298	0.0295	0.0293	0.0288	0.0303	0.0347
Adjusting τ_1	0.0293	0.0289	0.0295	0.0296	0.0293	0.0290	0.0287	0.0313	0.0371
Counterfactual Fiscal Regimes									
Balanced Budget	0.0288	0.0306	0.0303	0.0299	0.0294	0.0291	0.0280	0.0210	0.0198
Linear Tax	0.0284	0.0277	0.0279	0.0282	0.0282	0.0282	0.0283	0.0315	0.0375
Lump-sum Tax	0.0280	0.0276	0.0276	0.0277	0.0276	0.0275	0.0279	0.0336	0.0412
Additional Experiments									
Constant TFP	0.0295	0.0299	0.0303	0.0303	0.0301	0.0298	0.0288	0.0246	0.0252
Double Volatility of G	-0.0282	-0.0290	-0.0293	-0.0293	-0.0288	-0.0284	-0.0272	-0.0223	-0.0225
Sudden change in G	0.0343	0.0371	0.0370	0.0363	0.0355	0.0347	0.0324	0.0237	0.0227

C.2 Doubling fiscal uncertainty

To implement the experiment of doubling fiscal uncertainty, we start by solving a new two-shock equilibrium with a government purchases variance twice that in the baseline model, while keeping the other parameter values the same as in the baseline calibration. We use the Rouwenhorst method to discretize the counterfactual AR(1) process and to obtain a new grid for G ($G_l^{new}, G_m^{new}, G_h^{new}$) $= ((1 - m_{new})G_m, G_m, (1 + m_{new})G_m)$, with the same transition probability matrix as in the baseline model. We then start from the ergodic distribution of the original two-shock equilibrium and let the economy gradually transit to the new two-shock equilibrium as follows: at $t = 1$, after a particular G -state is realized from the old grid, (G_l, G_m, G_h) , the households in the economy learn that, from the next period on ($t \geq 2$), the G -states will evolve according to a different process, with the same transition probabilities, but with new period- t grid values that are conditional on the period-1 state, G_1 , as $(G_l^{new}, G_m^{new}, G_h^{new}) + G_{t,G_1}^{adj}$. G_{t,G_1}^{adj} is an adjustment term that makes the period- t conditional mean as of $t = 1$ equal to those of the original process.³⁶ Mechanically, G_{t,G_1}^{adj} is zero for all t when $G_1 = G_m$, and positive (negative) and decreasing (increasing) to zero when $G_1 = G_l$ ($G_1 = G_h$). We plot G_{t,G_1}^{adj} in Figure 5.

³⁶Note that this adjustment term isolates the effect of a change in fiscal uncertainty without a sudden level adjustment.

Figure 5: Government purchases adjustment (G_{t,G_1}^{adj})



We follow similar steps as in the baseline case to solve the model. Note that, unlike in the baseline case, we now have an uncertain government purchases level along the transition path in addition to the aggregate productivity shocks. However, we do not condition on G in the transition-equilibrium laws of motions. Instead, we incorporate G as a continuous variable in $H_{\Gamma_{t+1}}$ and S_{t+1} , and pool the regressions for the laws of motion for $t = 1, 2, \dots, 30$. Note that we do condition the laws of motion on the period-1 value of government purchases (G_1). We find that the following (relatively parsimonious) functional forms perform well:

$$\begin{aligned}
 \log(K'_{t+1}) &= a_{0,t}(z_t, G_1) + a_{1,t}(z_t, G_1)\log(K_t) + a_{2,t}(z_t, G_1)B_t + a_{3,t}(z_t, G_1)Gini(a_t) \\
 &\quad + a_{4,t}(z_t, G_1)B_t^5 + a_{5,t}(z_t, G_1)G_t^2 + a_{6,t}(z_t, G_1)B_t^5G_t^2, \\
 Gini(a'_{t+1}) &= \tilde{a}_{0,t}(z_t, G_1) + \tilde{a}_{1,t}(z_t, G_1)\log(K_t) + \tilde{a}_{2,t}(z_t, G_1)B_t + \tilde{a}_{3,t}(z_t, G_1)Gini(a_t) \\
 &\quad + \tilde{a}_{4,t}(z_t, G_1)B_t^5 + \tilde{a}_{5,t}(z_t, G_1)G_t^2 + \tilde{a}_{6,t}(z_t, G_1)B_t^5G_t^2, \\
 \log(s_t) &= b_{0,t}(z_t, G_1) + b_{1,t}(z_t, G_1)\log(K_t) + b_{2,t}(z_t, G_1)B_t + b_{3,t}(z_t, G_1)Gini(a_t) \\
 &\quad + b_{4,t}(z_t, G_1)B_t^2 + b_{5,t}(z_t, G_1)G_t^2 + b_{6,t}(z_t, G_1)B_t^5G_t^2.
 \end{aligned}$$